Nonlocal Nucleon-Nucleon Interaction*

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It has been explicitly demonstrated that the proton-proton scattering data require that the singlet even parity nucleon-nucleon interaction be nonlocal. This result comes from an examination of the energy dependence of both the $^1S_0$ and $^1D_2$ phase shifts. A nonlocal singlet even parity potential of the form $V(\mathbf{r},\mathbf{r}')=V(r)^{1/2}V(r')^{1/2}R^{-3}[\exp(-R^{-1}|\mathbf{r}-\mathbf{r}'|)]/(4\pi R^{-1}|\mathbf{r}-\mathbf{r}'|)$ is examined. The potential function $V(r)$ is taken to have the form of a monotonic attraction outside a "hard core." The radius of the "hard core," the depth and range of $V(r)$, and the nonlocal distance R provide four parameters which can be adjusted to fit the experimental values of the singlet scattering length and effective range, and the 310-MeV $^1S_0$ and $^1D_2$ phase shifts. For a long-range attractive potential function, such as the Yukawa, the energy dependence of the $^1S_0$ and $^1D_2$ phase shifts is strikingly similar to that suggested by the phase-shift analysis of the Yale group. Although the radius of the "hard core" is decreased somewhat from the local value, a "hard core" is still required.

THE energy variation of the $^1S_0$ nucleon-nucleon phase shift[1,2] suggests that the nuclear interaction in the singlet even parity state may be characterized by means of a strong short-range repulsion and a longer range attraction.[3] However, it had been noted that a local potential[4] designed to reproduce this energy variation of the $^1S_0$ phase shift produces much too large a $^1D_2$ phase shift at 310 MeV, where the most complete proton-proton scattering data exists.[5] Subsequently, great progress has been made toward mapping the nucleon-nucleon phase shifts[2] for all energies below $\sim$350 MeV. This work puts the tentative conclusions made earlier on a firmer footing. The evidence is now very striking that the $^1D_2$ phase shift is incompatible with the idea of a local potential.

In what follows we shall be quite specific as to the meaning of the term nonlocal potential. The nonlocal potential to which we refer is energy and momentum independent and is of the form $V(\mathbf{r},\mathbf{r}')$, where $\mathbf{r}$ and $\mathbf{r}'$ are relative coordinates. Thus, for proton-proton scattering, in the singlet state, the Schrödinger equation in the center-of-mass system is

$$-\frac{\hbar^2}{M_p}\nabla^2\Psi(\mathbf{r})+V_{\mathrm{Coul}}(r)\Psi(\mathbf{r})+\int d\mathbf{r}'V(\mathbf{r},\mathbf{r}')\Psi(\mathbf{r}')=E\Psi(\mathbf{r}), \quad (1)$$

where all the symbols have the standard meanings.

In the calculations reported below we have chosen $V(\mathbf{r},\mathbf{r}')$ to be of the form

$$V(\mathbf{r},\mathbf{r}')=V(r)^{1/2}V(r')^{1/2}R^{-3}[\exp(-R^{-1}|\mathbf{r}-\mathbf{r}'|)]/\times(4\pi R^{-1}|\mathbf{r}-\mathbf{r}'|). \quad (2)$$

A nonlocal potential of this form has already been suggested,[4] and some qualitative results obtained[6] which indicate that such a nonlocal potential might make possible a fit to the singlet phase shifts. Results of exact numerical calculations are presented here.

A glance at Eq. (2) shows us that in the limit as R goes to zero, the nonlocal potential becomes

$$V(\mathbf{r},\mathbf{r}')=V(r)^{1/2}V(r')^{1/2}\delta(\mathbf{r}-\mathbf{r}'), \quad (3)$$

that is, the potential is local. The length parameter R, we refer to as the nonlocal distance. Note that we need not distinguish local and nonlocal potentials, since the local case occurs when R is zero. The function $V(r)$, which appears in Eq. (2) or Eq. (3) we call the potential function.

We may wish to choose the potential function $V(r)$ to be an attractive square well with a hard core. In that case we have

$$\begin{aligned} V(r)&=+\infty, && r\leqslant r_c\\ &=-V_0, && r_c<r\leqslant b \\ &=0, && r>b, \end{aligned} \quad (4)$$

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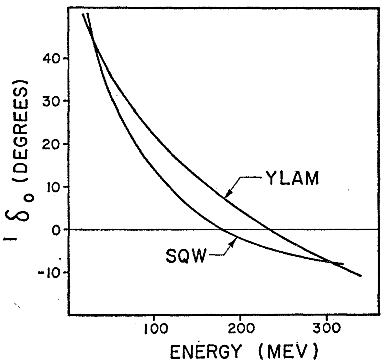

FIG. 1. The singlet S-wave phase shift, $^1\delta_0$, plotted against energy. The curve labeled SQW represents the results of the local and nonlocal square well. On this scale the results of the local and nonlocal Yukawa potential, as well as the results of Rojo and Simmons, are indistinguishable from the results of the Yale group (YLAM).

where r_c is the radius of the hard core, and V_0 and b are, respectively, the depth and range of the attractive portion. For this choice of potential function shape, the potential [Eq. (2)] is characterized by four parameters: r_c, V_0, b, and R. For a given choice of the nonlocal distance R, we may adjust the remaining three parameters to fit the singlet scattering length (-7.68 F), the singlet effective range (2.65 F), and the $^1S_0$ phase shift at 310 MeV (~ -8 deg).

In the local case, the parameters which fit these data are

$$\begin{aligned} r_c &= 0.200 \text{ F}, \\ b &= 2.434 \text{ F}, \\ V_0 &= 18.46 \text{ MeV}, \\ R &= 0. \end{aligned} \tag{5}$$

This choice of parameters leads to a value of 15.5 deg, compared to the experimental value of about 10 deg for the $^1D_2$ phase shift at 310 MeV.

The nonlocal square well which fits the scattering length, effective range, and the $^1S_0$ phase shift at 310 MeV, as well as the $^1D_2$ phase shift at 310 MeV (~ 10 deg) is characterized by the parameters

$$\begin{aligned} r_c &= 0.157 \text{ F}, \\ b &= 2.605 \text{ F}, \\ V_0 &= 19.89 \text{ MeV}, \\ R &= 0.461 \text{ F}. \end{aligned} \tag{6}$$

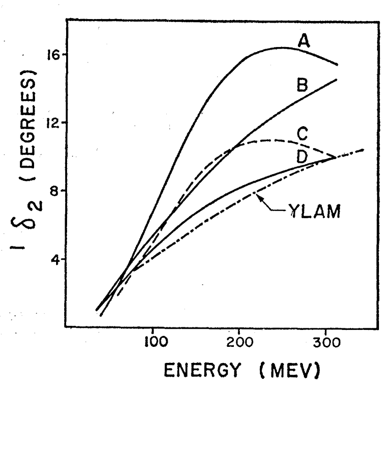

FIG. 2. The singlet D-wave phase shift, $^1\delta_2$, plotted against energy. The results of the local square well and Yukawa potentials are indicated by curves A and B, respectively. The results of the nonlocal square well and nonlocal Yukawa potentials are indicated by C and D, respectively. The results of the Yale group are indicated by (YLAM).

Similarly, we may choose the potential function to have a "Yukawa" shape, in which case we write

$$\begin{aligned} V(r) &= +\infty, && r \leqslant r_c \\ &= -V_0[\exp(-r/b)]/(r/b), && r_c < r < r_d \\ &= 0, && r > r_d. \end{aligned} \tag{7}$$

The Yukawa was cut off for $r > r_d$, where $r_d \sim 4.5$ F. This was done primarily for practical reasons relating to the method of calculation.

The parameters which fit the S-wave data for the the local Yukawa potential are

$$\begin{aligned} r_c &= 0.388 \text{ F}, \\ b &= 0.70 \text{ F}, \\ V_0 &= 396.1 \text{ MeV}, \\ R &= 0, \\ r_d &= 4.63 \text{ F}. \end{aligned} \tag{8}$$

This yields a $^1D_2$ phase shift at 310 MeV of 14.6 deg.

A nonlocal Yukawa potential adjusted to fit the $^1D_2$ phase shift at 310 MeV as well as the S-wave data is characterized by the parameters

$$\begin{aligned} r_c &= 0.329 \text{ F}, \\ b &= 0.74 \text{ F}, \\ V_0 &= 325.3 \text{ MeV}, \\ R &= 0.286 \text{ F}, \\ r_d &= 4.51 \text{ F}. \end{aligned} \tag{9}$$

The variation of the $^1S_0$ nuclear phase shift with energy is shown in Fig. 1. It is seen that a square well (SQW) is not compatible with the energy variation of the $^1S_0$ phase shift according to the phase-shift analysis of Breit and co-workers (YLAM). Further, the local square well leads to too large a $^1D_2$ phase shift as shown by curve A in Fig. 2.

The $^1S_0$ phase shift calculated from the nonlocal square well is virtually indistinguishable from that calculated from the local square well. For this reason, in Fig. 1, a single curve for the "square well" suffices.

The behavior of the $^1D_2$ phase shift is sensitive to the nonlocality. The $^1D_2$ phase shift for the nonlocal square well (curve C of Fig. 2) is considerably reduced by the nonlocality but differs appreciably at intermediate energies from the results of the Yale group.

The energy dependence of the $^1S_0$ phase shift for both the local and nonlocal Yukawa potentials was found to be indistinguishable, on the scale chosen for Fig. 1 from the phase-shift analysis of Breit and co-workers. On the other hand, the $^1D_2$ phase shift calculated from the local Yukawa (curve B of Fig. 2) differs markedly from the Yale result.

The nonlocal Yukawa potential results in an energy map of the $^1D_2$ phase shift (curve D of Fig. 2) which differs appreciably from the local case and is in good agreement with the $^1D_2$ phase shift obtained by the Yale group.

The $^1D_2$ phase shift for the nonlocal Yukawa potential is redrawn as curve A in Fig. 3, which also includes for comparitive purposes, the primary and secondary results of the Yale group (YLAM and YLA). Curves B and C represent the results of Rojo and Simmons[7] using a velocity-dependent potential without a repulsive hard core which fits the energy dependence of the S-wave phase shift.

Thus, it is apparent that a careful study of the data can yield much detailed information. The relative energy dependence of the $^1S_0$ and $^1D_2$ phase shifts shows that the interaction cannot be local. The shape of the potential likewise is seen to be more diffuse than the square well. Further, the speculation that the nonlocality might so reduce the hard core as to render its role unimportant seems to be unwarranted.

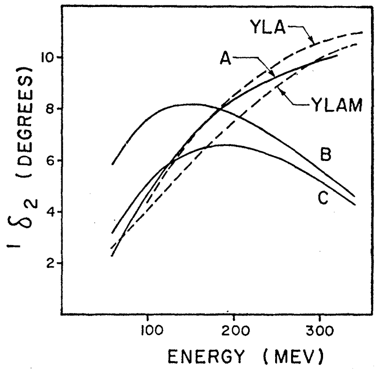

FIG. 3. The singlet D-wave phase shift, $^1\delta_2$, plotted against energy. Curve A is the result of the nonlocal Yukawa calculation. YLAM and YLA are the results of the phase-shift analysis of the Yale group. Curve B and C are the results of Rojo and Simmons.

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Interference of K-Capture, Gamma Transitions through a Virtual State, and Inner Bremsstrahlung Transitions*

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The energy distribution of photons emitted during the transition from the ground state of Ni^{59} to that of Co^{59} has been calculated with allowance being made for interference between the K-capture, γ-ray transition through a virtual intermediate excited state of Co^{59} and the second forbidden K-capture transition, accompanied by inner bremsstrahlung, between ground states. The effect of the transition through the virtual state is to introduce terms in the photon energy distribution whose maxima are at higher energies than those resulting from inner bremsstrahlung transitions. In particular, the energy distribution for the leading interference term is similar to that used by Schmorak, who found that a better fit to his data was obtained by assuming that the virtual capture transition exists. Thus, the present calculation confirms Schmorak's method of analysis and, therefore, gives support for the existence of the virtual capture transition.

THE possibility of observing a combined beta (β)-decay, gamma (γ)-ray transition through a virtual intermediate nuclear state has recently been investigated by Rose, Perrin, and Foldy.[1] By considering the phase-space factors and energy denominators entering into the transition probability, these authors concluded that the K-capture transition in Ni^{59} offers the best opportunity for the experimental verification of such an effect. This combined K-capture, gamma transition proceeds from the ground state of Ni^{59} to the first excited state of Co^{59} by a Gamow-Teller allowed transition and then to the ground state of Co^{59} by emission of $M1$ or $E2$ radiation, as illustrated in Fig. 1. This gamma ray has a continuous spectrum since energy is not conserved in the intermediate state. The detection of a gamma ray is not sufficient, however, to establish that the virtual transition occurred, since the second forbidden K-capture transition between the two ground states is accompanied by inner bremsstrahlung. In fact, the two transitions are coherent, and interference

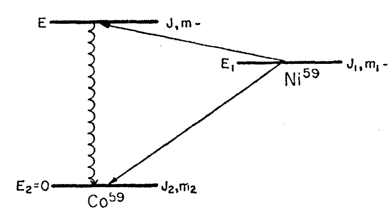

FIG. 1. Decay scheme of Ni^{59}. The transition through the virtual state goes by K capture from the ground state of Ni^{59}, labeled by the total angular momentum $J_1=\frac{3}{2}$ with z component m_1, to the first excited state of Co^{59} identified by the quantum numbers J, m, and then by emission of $M1$ or $E2$ gamma radiation to the ground state of Co^{59}, labeled with J_2, m_2. All states shown have negative parity. The values of the energies indicated by E_1, E, and E_2 are 1.076, 1.098, and 0 MeV, respectively.

* Supported by the U. S. Atomic Energy Commission.
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