

Properties of Excited States of P^{31} . II. Gamma-Ray Angular Distributions and Correlations*

GALE I. HARRIS

Aeronautical Research Laboratories, Wright-Patterson Air Force Base, Ohio

AND

L. W. SEAGONDOLLAR

Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas

(Received 15 March 1963)

Angular distributions and triple correlations of gamma rays emitted from several resonances in the range $E_p=1177$ to 1509 keV in the $Si^{30}(p,\gamma)P^{31}$ reaction have been measured using thin, highly enriched Si^{30} targets. The decay schemes of these resonances and of intermediate levels excited in P^{31} were presented in an earlier paper (I). The angular distributions and correlations lead to the resonance level assignments $1177(\frac{3}{2}^-)$, $1204(\frac{3}{2}^-)$, $1324(\frac{3}{2}^-)$, $1392(\frac{3}{2}^-)$, $1400(\frac{3}{2}^-)$, $1489(\frac{3}{2}^-)$, and $1509(\frac{3}{2}^-)$ where the proton resonance energies are in keV. The results indicate even parity for all of the resonances if it is assumed that all gamma-ray multipolarity mixings are of the form $E(L+1)/M(L)$. The mixing ratios (all of the type $E2/M1$) are presented for most of the observed primary transitions and of several secondary transitions. The results of this study are consistent with the assignments $\frac{3}{2}^+$, $\frac{5}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, $\frac{3}{2}^+$, and $\frac{5}{2}^+$ made elsewhere for the 1.27-, 2.23-, 3.13-, 3.29-, 3.51-, and 4.19-MeV levels of P^{31} . The 3.41-MeV level is identified as $\frac{3}{2}^+$ by means of angular-distribution and correlation measurements at the 1509-keV resonance. A $\frac{3}{2}^+$ assignment for the 3.41-MeV level casts doubt upon the existence of a weak transition, which has been reported by earlier workers and in I, from that level to the $\frac{1}{2}^+$ ground state. Results of several measurements designed to determine the origin of the weak gamma-ray peak of approximately 3.4 MeV in coincidence spectra at the 1509-keV resonance were inconclusive. Angular distributions at the 1204-keV resonance lead to the assignment of $\frac{3}{2}^+$ to the 4.26-MeV level. Limitations are placed on the possible assignments for the 4.43- and 5.01-MeV levels. Both members of the recently discovered doublet near $E_p=1481$ keV are found to decay mainly to the ground state, and each have $J=\frac{3}{2}$ on the basis of angular distributions. The value of $\Gamma_p\Gamma_\gamma/\Gamma_t$ has been determined for each of the resonances. Results of this study are discussed in terms of recent collective model calculations for P^{31} .

INTRODUCTION

THIS paper is the second of a series which presents the results of an investigation of states of P^{31} which are excited by the bombardment of Si^{30} with protons of incident energy between 1177 and 1509 keV. The first paper¹ (referred to herein as I) gave the results of a study of decay properties of $Si^{30}(p,\gamma)P^{31}$ resonances at 1177, 1204, 1324, 1392, 1400, 1481, 1489, and 1509 keV, and of intermediate states of P^{31} through which these resonance levels decay. Brief reference is given in I to attempts which have been made to apply the unified model to the known properties of states of P^{31} . In particular, it now appears²⁻⁴ that the predictions of the Nilsson model are in poor agreement with the observed order and electromagnetic decay properties of levels of P^{31} . However, as mentioned in I, the level order predicted by calculations based upon the weak-coupling vibrational model has appeared to be in fair agreement with observation with the exception that a heretofore unobserved $\frac{3}{2}^+$ level is required below an excitation of

about 4 MeV.⁵ It will be shown in the present paper that a $\frac{3}{2}^+$ level does indeed exist at 3.41 MeV.⁶ This result was unexpected because of the apparent observation in I and in earlier work⁷ that the 3.41-MeV level decays partially to the $\frac{1}{2}^+$ ground state (G.S.).

The above result will be presented in proper context in the following systematic presentation of results of angular-distribution and correlation measurements at the resonances discussed in I. The data and arguments leading to resonance level assignments will first be presented and discussions which pertain to properties of intermediate states will follow.

As mentioned in a footnote added in proof in I, the "1481-keV resonance level" has been found by H. Van Rinsvelt and P. B. Smith to be a closely spaced doublet with a separation of 1.5 keV.⁸ Subsequent to the work reported in I, the members of the doublet have been observed individually with very thin targets and greatly improved beam resolution. The yield curve and angular distributions of ground-state transitions at these resonances will be presented.

*The work performed at the University of Kansas was supported by the U. S. National Science Foundation and the U. S. Office of Naval Research.

¹G. I. Harris and L. W. Seagondollar, Phys. Rev. **128**, 338 (1962).

²E. E. Baart, L. L. Green, and J. C. Willmott, Proc. Phys. Soc. (London) **79**, 237 (1962).

³L. L. Green, G. Kaye, and J. C. Willmott, Nucl. Phys. **25**, 278 (1961).

⁴L. L. Green (private communication).

⁵V. K. Thankappan and S. P. Pandya, Nucl. Phys. **19**, 303 (1960).

⁶G. I. Harris, Bull. Am. Phys. Soc. **7**, 462 (1962).

⁷C. Broude, L. L. Green, and J. C. Willmott, Proc. Phys. Soc. (London) **72**, 1097, (1958).

⁸P. M. Endt (private communication).

PROCEDURE

The angular distribution measurements, with the exception of those at the 1481-keV doublet, were performed at the University of Kansas, with the experimental arrangement essentially the same as described in I. The 5-in.-diam \times 5-in.-long NaI(Tl) crystal was used as the movable detector and the 3-in.-diam \times 3-in.-long crystal was used as a monitor fixed at 90° relative to the incoming proton beam direction. The front face of the movable detector was located 8 in. from the center of the target, thus subtending a solid angle of 0.28 sr. Collimation of the detectors was not used for the angular correlation measurements. The effect on the data by the small reduction in quality of the line shapes thus introduced was more than compensated by the better statistics allowed by the higher count rates obtainable without collimation.

At each resonance a pulse-height spectrum was recorded from the 5-in. \times 5-in. detector at each of the angles 0°, 30°, 45°, 60°, 90°, and 120°, with the 0° run then being repeated as a check. An "off-resonance run" was also made to account for background and contamination effects. The runs were normalized by means of the fixed monitor data and corrections were made for the effects of differences in "live times" of on- and off-resonance runs where necessary.

The same stripping procedure was employed to analyze the spectra at each angle as was used in analyzing the spectra discussed in I. The relative intensities of corresponding gamma rays at the various angles were obtained in this case, however, by measuring only that part of the area under the full-energy, first-escape, and second-escape peaks for gamma rays of energy greater than 3 MeV, and only under the full-energy peak for those of energy lower than 3 MeV. This procedure reduced the error caused by uncertainties in the magnitude of the Compton tails. The intensities used at 0° were the weighted means of the two 0° measurements, and those used at 60° were the weighted means of the measurements at 60° and 120°. This latter average was appropriate because of the symmetry about 90° of the radiation emitted in reactions which proceed through an isolated compound-nucleus state. The observed absence of statistically significant differences in intensity at 60° and 120° may be considered as evidence for the absence of interfering states of opposite parity.

The coefficients A_k in the expression

$$W(\theta) = \sum_k A_k P_k(\cos\theta), \quad (k \text{ even}),$$

for the intensity of each gamma ray were determined from the data by means of a least-squares analysis. The least-squares calculations were performed by means of an IBM-650 computer using a program prepared by Prosser and Graber⁹ and based upon the method of treatment of angular-correlation data suggested by

⁹ F. W. Prosser, Jr., and H. D. Graber (private communication).

Rose.¹⁰ Separate calculations were performed for $k_{\max} = 2, 4, \text{ and } 6$, successively. The coefficients so obtained were corrected for small effects due to slight misalignments of the target and axis of rotation of the movable counter with respect to the proton beam, and for the more significant effect of finite-detector geometry. The curves of Stanford and Rivers,¹¹ which are based upon the formulas of Rose,¹⁰ were used to determine the attenuation factors appropriate to this work. The factors for the A_2 and A_4 coefficients were 0.94 and 0.82, respectively. No significant A_6 terms were observed.

The triple-correlation (p, γ_1, γ_2) measurements and the investigation of the 1481-keV doublet were performed with the Aeronautical Research Laboratories (ARL) 2-MeV Van de Graaff accelerator. The basic experimental arrangement was similar to that at the University of Kansas except that instead of using an electrostatic analyzer for beam-energy control, a control signal was obtained from electrodes located about 8 ft beyond the beam-analyzing magnet. The energy resolution thus provided was sufficient for the triple-correlation measurements.

In order to study the individual members of the 1481-keV doublet, however, a more elaborate beam stabilizing system was necessary. A modified form¹² of the Duke homogenizing system¹³ was used to detect, by means of a small capacitive pickup plate, the alternating voltage variation of the high-voltage terminal, to amplify the signal so obtained to match the terminal variations, and to apply a correction voltage to the insulated target. Hence, the terminal-to-target voltage remained at a fixed value even though the terminal-to-ground voltage may have been fluctuating.

The separated Si³⁰ targets used at the ARL facilities were taken from the same set of targets used at the University of Kansas. All Si³⁰ targets used in the present work and in I were prepared in a single evaporation. Provisions for water cooling were incorporated into the target chamber used at ARL, thus allowing sustained bombardment with about 10- μ A beam currents.

For the triple-correlation measurements, a 5-in.-diam \times 5-in.-long crystal was used as the movable detector, and an 8-in.-diam \times 8-in.-long crystal was used as a fixed detector at an angle relative to the incoming beam determined by the geometrical arrangement desired. The experimental geometries used in the triple-correlation measurements are defined¹⁴ as follows:

¹⁰ M. E. Rose, Phys. Rev. **91**, 610 (1953).

¹¹ A. L. Stanford, Jr., and W. K. Rivers, Jr., Rev. Sci. Instr. **30**, 719 (1959).

¹² R. E. Smith, thesis, Air Force Institute of Technology, Air University, Wright-Patterson Air Force Base, Ohio, 1961 (unpublished).

¹³ P. B. Parks, H. W. Newson, and R. M. Williamson, Rev. Sci. Instr. **29**, 834 (1958).

¹⁴ A. J. Ferguson and A. R. Rutledge, "Coefficients for Triple Angular Correlation Analysis in Nuclear Bombardment Experiments," Atomic Energy of Canada, Ltd., CRP-615, AECL-420, 1957 (unpublished).

Geometry I: θ_1 variable, $\theta_2=90^\circ$, $\phi=180^\circ$,
 Geometry II: $\theta_1=90^\circ$, θ_2 variable, $\phi=180^\circ$,
 Geometry IV: $\theta_1=0^\circ$, θ_2 variable, $\phi=180^\circ$,
 Geometry VI: $\theta_1=90^\circ$, θ_2 variable, $\phi=90^\circ$.

The spectra of pulses from the movable crystal in coincidence with gate pulses from selected regions of the spectrum from the fixed crystal were recorded by a 256-channel Nuclear Data analyzer. In some cases, correlations of two separate cascades were measured simultaneously by using two single-channel analyzers to provide gate pulses, and by accumulating separate spectra in the 0–128 and 129–256 channel groups of the analyzer. A transistorized, multiple-coincidence unit was used for these measurements. Coincidence spectra were recorded for movable-detector angular positions in the sequence 0° , 90° , 30° , 60° , and 45° . The sequence was then repeated several times (each time reversing the order) in order to minimize any effects due to electronic drifts. A similar procedure was used in the case where the fixed detector was located at $\theta=0^\circ$. The data were analyzed in a manner similar to that used for the angular distribution measurements. The detectors were located such that each subtended a solid angle of about 0.33 sr.

For finite-sized detectors the results of a triple-correlation measurement using a particular geometrical arrangement of the detectors is influenced by the, in general, different correlations which correspond to other geometrical arrangements. The “pure geometries” corresponding to point detectors are, in a sense, mixed together when finite detectors are used. The procedure used in correcting for finite geometry effects in the triple correlations, therefore, required a correction of the theoretical correlation functions. Prosser *et al.*¹⁵ have shown that, for finite detectors, when through particular choices of position for the fixed detector we have the expression¹⁴

$$W_{\omega} = \sum_r A_r P_r(\cos \xi),$$

where ξ is the variable angle of the set $(\theta_1, \theta_2, \phi_1 - \phi_2)$, then A_r is of the form

$$A_r = \sum_{KMN} D_{KM} N \alpha^r_{KMN} J_K(1) J_M(2),$$

where J_l is the absorption integral of Rose.¹⁰ The labels 1 and 2 refer to the counters at θ_1, ϕ_1 and θ_2, ϕ_2 , respectively. All theoretical triple-correlation functions referred to in this paper have been corrected accordingly.

INTERPRETATION OF CORRELATION DATA

In the analysis of the correlation data the following assumptions are used: (1) All resonances (Res.) are formed with protons of orbital angular momentum $l \leq 4$. (2) Transitions of lowest order L of the type $E(L)$ are

pure and those of lowest order L of the type $M(L)$ may include the type $E(L+1)$. (3) For “observed” radiations the highest multipole order that need be considered is octupole, and for “unobserved” radiations the highest order that need be considered is quadrupole. Since Si³⁰ has a ground-state spin of zero, the first assumption implies all resonances have $J \leq \frac{9}{2}$. From penetrability considerations one would not expect to observe, at the energies involved, a resonance of $J > \frac{9}{2}$. The second assumption is commonly used and is consistent with the relative transition probabilities computed from the single-particle model. The third assumption is further restricted to the consideration of octupole radiations only in the combination $M2-E3$. No pure $E3$ transitions are considered. This assumption is justified by the occurrence of levels at low excitation of spins $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ in P³¹. The occurrence of this spin sequence virtually guarantees that for any observed level with spin J , there is a lower level to which it can de-excite such that $\Delta J \leq 2$. This circumstance, incidentally, gives further basis for considering only compound states with $J \leq \frac{9}{2}$ since all resonances decay to at least one of these three low-lying levels. The possibility of $M2-E3$ mixtures in the unobserved radiations is ignored because of the incoherent interference. The possibility of $M1-E2$ mixtures in both observed and unobserved radiations is retained, however.

The analysis of angular correlation data is greatly simplified when the target nucleus has zero spin. The channel spin has the unique value $S = \frac{1}{2}$. As a consequence of this and the requirement of conservation of parity, only incoming protons having unique orbital angular momenta contribute to the formation of any state J^π . The formation channels are, therefore, pure. The tables of Sharp *et al.*¹⁶ were used to compute the theoretical coefficients for the (p, γ) and (p, γ_1, γ_2) , γ_1 unobserved, angular distributions, and those of Ferguson and Rutledge¹⁴ to compute those for the (p, γ_1, γ_2) triple correlations. The sign convention for interference terms used in this study is that of Ferguson and Rutledge which includes the Huby corrections. That of Sharp *et al.* has been changed correspondingly.¹⁷

The mixing ratio for primary radiations will be represented by δ_p , and that for secondary radiations by δ_s , where δ is the amplitude ratio $E(L+1)/M(L)$. The multipolarity L is implied by the spin change under consideration, and will not always be stated explicitly. The comparison of the experimental with the (appropriately corrected) theoretical coefficients was greatly facilitated by plotting the theoretical coefficients versus $\tan^{-1} \delta$. Contour plots (as suggested by Gove¹⁸) were also used

¹⁶ W. T. Sharp, J. M. Kennedy, B. J. Sears, and M. G. Hoyle, “Tables of Coefficients for Angular Distribution Analysis,” Atomic Energy of Canada, Ltd., CRT-556, AECL-97, 1957 (unpublished).

¹⁷ This convention results in phases for mixing ratios of primary radiations opposite those in Ref. 7. The phases for secondary radiations are, however, the same.

¹⁸ H. E. Gove, in *Nuclear Reactions*, edited by P. M. Endt and M. Demeur, (North-Holland Publishing Company, Amsterdam,

¹⁵ F. W. Prosser, Jr., R. W. Krone, and J. J. Singh, Phys. Rev. 129, 1716 (1963). See also, M. E. Rose, Oak Ridge National Laboratory Report ORNL-2516, 1958 (unpublished).

TABLE I. Angular-distribution coefficients observed at the 1177-keV resonance.

E_γ (MeV)	Transition	A_2	A_4
6.19	$R \rightarrow 2.23(5/2+)$	-0.28 ± 0.03	-0.02 ± 0.05
5.13	$R \rightarrow 3.29(5/2+)$	-0.26 ± 0.03	0.00 ± 0.04
4.23	$R \rightarrow 4.19(5/2+)$	-0.12 ± 0.04	-0.06 ± 0.06
2.2	a	0.62 ± 0.04	-0.23 ± 0.05
2.0	a	0.69 ± 0.03	0.12 ± 0.04

* Arise from transitions between more than one pair of levels.

in many cases where both δ_p and δ_s appear in the theoretical coefficient. In the following discussion of resonance levels, the assignments G. S. ($\frac{1}{2}+$), $1.27(\frac{3}{2}+)$, $2.23(\frac{5}{2}+)$, $3.13(\frac{3}{2}+)$, $3.29(\frac{5}{2}+)$, $3.51(\frac{3}{2}+)$, and $4.19(\frac{5}{2}+)$, given by earlier workers^{7,19} for lower lying states of P^{31} will be assumed.

RESONANCE LEVEL ASSIGNMENTS

1177-keV Resonance

The coefficients A_k (normalized such that $A_0=1$) of the polynomials $P_k(\cos\theta)$ in angular distributions observed at this resonance are listed in Table I. The gamma-ray energies and transitions to which they correspond are taken from I.

The presence of significant P_4 terms in the 2.2- and 2.0-MeV gamma rays, even though they arise from transitions between more than one pair of levels, rules out $J=\frac{1}{2}$ or $\frac{3}{2}$ for the resonance level. If $J^\pi=\frac{3}{2}+$, the 6.19-, 5.13-, and 4.23-MeV transitions would each have the unique theoretical distribution $W(\theta)=1+0.476P_2-0.286P_4$. If $J^\pi=\frac{3}{2}-$ is assumed, the transitions would consist of interfering $M2$ and $E3$ radiations. The interference, characterized by δ_p , affects sensitively the expected angular distributions. By the proper choice of δ_p it is possible to reduce the expected value of A_2 to a minimum of -0.14 . The value of δ_p at which this minimum occurs requires $A_4=-1.08$ and $A_6=0.34$. Hence, a spin of $\frac{3}{2}$ for the resonance level is not possible. The primary transitions, if $\frac{5}{2}-$ and $\frac{7}{2}-$ are assumed, would have $W(\theta)=1+0.457P_2$ and $W(\theta)=1-0.357P_2$, respectively; neither of which agree with the experimental data.

For $J^\pi=\frac{5}{2}+$ and $\delta_p=+0.71$ or $+13.3$ (δ_p here refers to $M1-E2$ mixing) one calculates $A_2=-0.28$ which agrees with that measured for the 6.19-MeV gamma ray. However, for these values of δ_p , A_4 must be 0.123 and -0.367 , respectively. A similar situation exists for the 5.13-MeV gamma ray. The only remaining possibility is $\frac{7}{2}+$. For $\frac{7}{2}+$ and $\delta_p=-0.03$ it is required that $A_2=-0.28$ and $A_4 \approx 0$ which agree with the observed values for the 6.19-MeV transition. The lack of a P_4

(1959) pp. 310, 311. The contour diagram (Fig 21, p. 311) of this reference was computed before the Huby phase corrections were applied to Ref. 14. [H. E. Gove (private communication)].

¹⁹A. E. Litherland, E. B. Paul, G. A. Bartholomew, and H. E. Gove, *Can. J. Phys.* **37**, 53 (1959).

TABLE II. Angular-distribution coefficients observed at the 1204-keV resonance.

E_γ (MeV)	Transition	A_2	A_4
7.18	$R \rightarrow 1.27(3/2+)$	-0.45 ± 0.06	0.09 ± 0.07
6.22	$R \rightarrow 2.23(5/2+)$	0.61 ± 0.05	0.00 ± 0.07
5.16	$R \rightarrow 3.29(5/2+)$	0.46 ± 0.05	0.03 ± 0.08
4.94	$R \rightarrow 3.51(3/2+)$	-0.18 ± 0.05	0.00 ± 0.08
4.2	a	-0.16 ± 0.03	0.00 ± 0.03
3.51	$3.51 \rightarrow \text{G.S.}(1/2+)$	0.13 ± 0.04	-0.08 ± 0.06
2.99	$4.26 \rightarrow 1.27(3/2+)$	0.8 ± 0.2	0.3 ± 0.2
2.23	$2.23 \rightarrow \text{G.S.}(1/2+)$	0.35 ± 0.05	0.04 ± 0.07
2.03	$3.29 \rightarrow 1.27(3/2+)$	0.33 ± 0.06	-0.02 ± 0.08

* Represents both members of a cascade through the 4.26-MeV level.

term in the distribution precludes a high value of δ_p . For appropriate δ_p , the distributions of the 5.13- and 4.23-MeV transitions are also in agreement with $\frac{7}{2}+$. It is concluded, therefore, that $J^\pi=\frac{7}{2}+$ for the 1177-keV resonance. (The values of δ_p obtained at all resonances are summarized in Table XII.)

1204-keV Resonance

The coefficients A_k observed at this resonance are shown in Table II. The anisotropy of the radiations rules out $J=\frac{1}{2}$ for the resonance level. $J=\frac{3}{2}$ need not be considered because of the presence of primary transitions to spin $\frac{3}{2}$ levels. The most negative value of A_2 for a $\frac{7}{2} \rightarrow \frac{3}{2}$ primary transition is -0.003 , and at the value of δ_p for which this occurs, $A_4=-1.2$. Hence, $J=\frac{7}{2}$ is eliminated. The maximum possible value of A_2 for a $\frac{3}{2} \rightarrow \frac{3}{2}$ transition is 0.38. The observed value of 0.61 ± 0.05 for the 6.22-MeV gamma ray, thus, eliminates $J=\frac{3}{2}$. If $J^\pi=\frac{5}{2}-$ the same gamma ray would have the unique distribution $W(\theta)=1+0.457P_2$ which disagrees with that observed. For appropriate δ_p all observed distributions are consistent with $J^\pi=\frac{5}{2}+$ for the 1204-keV resonance.

1324-keV Resonance

The coefficients A_k observed at this resonance are shown in Table III. The remarks made for $J=\frac{1}{2}$ and $\frac{3}{2}$ assignments for the 1204-keV resonance apply here also.

TABLE III. Angular-distribution coefficients observed at the 1324-keV resonance.

E_γ (MeV)	Transition	A_2	A_4
7.30	$R \rightarrow 1.27(3/2+)$	-0.38 ± 0.02	-0.04 ± 0.03
6.34	$R \rightarrow 2.23(5/2+)$	0.36 ± 0.05	0.08 ± 0.08
5.44	$R \rightarrow 3.13(3/2+)$	-0.48 ± 0.09	-0.01 ± 0.13
5.06	$R \rightarrow 3.51(3/2+)$	-0.03 ± 0.04	0.03 ± 0.05
4.3	a	0.20 ± 0.06	0.03 ± 0.09
3.51	$3.51 \rightarrow \text{G.S.}(1/2+)$	0.50 ± 0.04	0.02 ± 0.05
3.0	b	0.12 ± 0.08	-0.04 ± 0.14
2.2	b	0.34 ± 0.03	-0.03 ± 0.04

* Represents both members of a cascade through the 4.26-MeV level.

b Arise from transitions between more than one pair of levels.

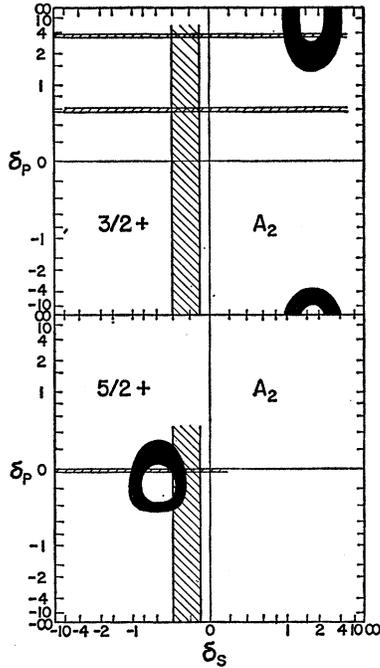


FIG. 1. Contour plots of Geometry II triple-correlation results for the Res. \rightarrow 1.27 \rightarrow G.S. cascade at the 1324-keV resonance. The darkly shaded regions represent the experimental value of the coefficient A_2 , the horizontal bands represent the values of δ_p allowed by the angular distribution results, and the vertical band represents the value of δ_s determined by McCallum. The upper plot assumes $\frac{3}{2}+$ for the 1324-keV resonance, and the lower, $\frac{5}{2}+$. It is clear that the resonance is $\frac{5}{2}+$ and $\delta_s \approx -0.3$. As discussed in the text, similar results were obtained at the 1392-keV resonance and, therefore, $\frac{5}{2}+$ is assigned to that resonance also.

A $J = \frac{7}{2}$ assignment can be eliminated by applying the same argument used against a $\frac{7}{2}$ assignment for the 1204-keV resonance to the 7.30-MeV transition here. If $J^\pi = \frac{3}{2}-$, the 7.30-MeV transition would have the unique distribution $W(\theta) = 1 + 0.400P_2$. If $J^\pi = \frac{5}{2}-$ the 6.34-MeV transition would have the distribution $W(\theta) = 1 + 0.457P_2$ which is outside the range of error for the experimental result. The remaining possibilities are $\frac{3}{2}+$ and $\frac{5}{2}+$; both of which are consistent with the observed primary distributions for appropriate values of δ_p . One notes first, however, that if $\frac{3}{2}+$ is assumed the δ_p are all unusually large. On the other hand, for $\frac{5}{2}+$ the δ_p are reasonably small. In order to make a definite choice between $\frac{3}{2}+$ and $\frac{5}{2}+$, a Geometry II triple correlation measurement was made on the Res. \rightarrow 1.27 ($\frac{3}{2}+$) \rightarrow G.S. ($\frac{1}{2}+$) cascade. The result was $W(\theta) = 1 - (0.77 \pm 0.05)P_2$. This experimental value for A_2 is indicated by the darkly shaded regions on the contour plots of the theoretical values of A_2 versus δ_p and δ_s in Fig. 1. The horizontal bands represent the possible values of δ_p allowed by the result of the angular-distribution measurement of the 7.30-MeV primary. The vertical band represents the value -0.25 ± 0.15 for δ_s determined by McCallum.²⁰ (It will subsequently be

²⁰ G. J. McCallum, Phys. Rev. **123**, 568 (1961).

TABLE IV. Angular-distribution coefficients observed at the 1392-keV resonance.

E_γ (MeV)	Transition	A_2	A_4
7.37	$R \rightarrow 1.27(3/2+)$	-0.49 ± 0.02	-0.02 ± 0.03
6.41	$R \rightarrow 2.23(5/2+)$	0.42 ± 0.04	0.07 ± 0.06
5.35	$R \rightarrow 3.29(5/2+)$	0.31 ± 0.07	-0.13 ± 0.12
5.13	$R \rightarrow 3.51(3/2+)$	-0.22 ± 0.16	0.22 ± 0.23
4.45	$R \rightarrow 4.19(5/2+)$	1.2 ± 0.4	...
4.21	$R \rightarrow 4.43(?)$	0.12 ± 0.05	0.03 ± 0.08
3.51	$3.51 \rightarrow$ G.S. ($1/2+$)	0.61 ± 0.06	0.04 ± 0.08
2.2	a	0.33 ± 0.03	0.10 ± 0.04
2.0	a	0.25 ± 0.08	0.15 ± 0.09
1.27	$1.27 \rightarrow$ G.S. ($1/2+$)	-0.66 ± 0.03	-0.06 ± 0.04

^a Arise from transitions between more than one pair of levels.

shown that it is this value which is consistent with all results of the present work.) By inspection of the diagrams, it can be seen that the data are consistent only with the sequence $\frac{5}{2}+ \rightarrow \frac{3}{2}+ \rightarrow \frac{1}{2}+$ and, thus, it is concluded that $J^\pi = \frac{5}{2}+$ for the 1324-keV resonance.

1392-keV Resonance

The coefficients A_k observed at this resonance are shown in Table IV. The comments concerning $J = \frac{1}{2}, \frac{7}{2}$, and $\frac{9}{2}$ assignments in the discussion of the 1204- and 1324-keV resonances apply here also. If $J^\pi = \frac{3}{2}-$, the 7.37-MeV gamma ray would have $A_2 = 0.400$. Similarly, if $J^\pi = \frac{5}{2}-$, the 5.35-MeV gamma ray would have $A_2 = 0.457$ which is outside the range of error of the observed value. Hence, $J^\pi = \frac{3}{2}+$ or $\frac{5}{2}+$. A Geometry II triple-correlation measurement on the Res. \rightarrow 1.27 ($\frac{3}{2}+$) \rightarrow G.S. ($\frac{1}{2}+$) cascade gave the result $W(\theta) = 1 - (0.72 \pm 0.02)P_2$. By reference to Fig. 1, which displays a similar result for the 1324-keV resonance, it can be seen that the 1392-keV resonance must also have $J^\pi = \frac{5}{2}+$.

1400-keV Resonance

The coefficients A_k observed at this resonance are shown in Table V. $J = \frac{1}{2}$ and $\frac{9}{2}$ assignments are again precluded by the observed anisotropies and by the decay to spin- $\frac{3}{2}$ levels. A $\frac{7}{2}$ assignment is not allowed by the observed distribution of the 7.38-MeV gamma ray. If $J^\pi = \frac{3}{2}-$, the 7.38-MeV gamma ray would have

TABLE V. Angular-distribution coefficients observed at the 1400-keV resonance.

E_γ (MeV)	Transition	A_2	A_4
7.38	$R \rightarrow 1.27(3/2+)$	0.53 ± 0.02	0.02 ± 0.03
6.42	$R \rightarrow 2.23(5/2+)$	0.19 ± 0.04	0.09 ± 0.16
5.52	$R \rightarrow 3.13(3/2+)$	-0.37 ± 0.03	0.01 ± 0.07
5.36	$R \rightarrow 3.29(5/2+)$	-0.22 ± 0.07	0.11 ± 0.12
4.46	$R \rightarrow 4.19(5/2+)$	0.28 ± 0.08	0.16 ± 0.16
3.13	$3.13 \rightarrow$ G.S. ($1/2+$)	0.09 ± 0.07	-0.01 ± 0.12
1.27	$1.27 \rightarrow$ G.S. ($1/2+$)	-0.28 ± 0.08	0.01 ± 0.10

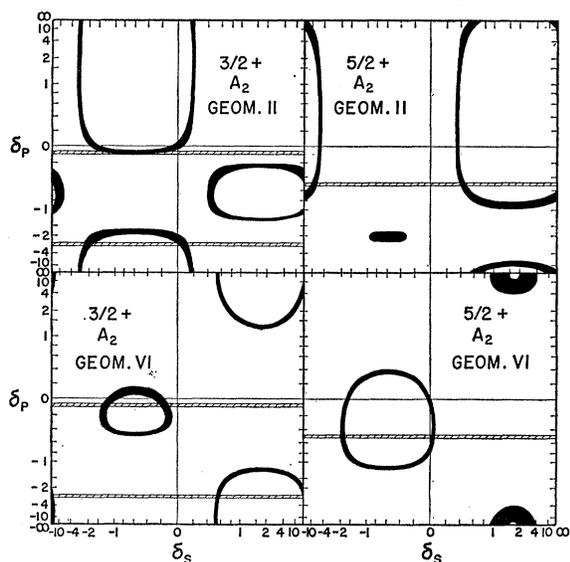


FIG. 2. Contour plots of Geometry II and VI triple-correlation results for the Res. $\rightarrow 1.27 \rightarrow$ G.S. cascade at the 1400-keV resonance. The two diagrams on the left assume $\frac{3}{2}+$ and the two on the right assume $\frac{5}{2}+$ for the 1400-keV resonance. As in Fig. 1, the horizontal bands are derived from angular-distribution data. For $\frac{3}{2}+$ it is seen that the results are self-consistent, whereas those for $\frac{5}{2}+$ are not.

$A_2=0.400$, which disagrees with the observed value. Similarly, for $\frac{5}{2}-$ this gamma ray would have $A_2=-0.400$. The choice between the remaining $\frac{3}{2}+$ and $\frac{5}{2}+$ possibilities can be most easily made by reference to Fig. 2 where the results $W(\theta)=1+(0.14\pm 0.04)P_2$ and $W(\theta)=1-(0.40\pm 0.03)P_2$ for Geometries II and VI triple-correlation measurements, respectively, are indicated for the Res. $\rightarrow 1.27(\frac{3}{2}+) \rightarrow$ G.S. $(\frac{1}{2}+)$ cascade. It is evident that the data are consistent only with $J^\pi=\frac{3}{2}+$ and that the high value for δ_p is excluded.

1481-keV Doublet

The resonance discussed in I as occurring at 1481 keV has recently been discovered by Van Rinsvelt and Smith to be a doublet consisting of two about equally strong components with a separation of about 1.5 keV.⁸ The decay scheme proposed in I is, therefore, a combined system for both members of the doublet since target thicknesses of about 3 keV and about 1-keV beam resolution were used in that work.

By use of the special techniques discussed in the procedures section, the presence of the doublet was verified. By observing the relative magnitudes of the two members of the doublet and that of the 1509-keV resonance which decays strongly to the 1.27-MeV first excited state (and not to the ground state) as the discriminator bias was increased in steps up to a setting at which both members failed to appear in the yield curve, it was found that both members decay strongly to the ground state. It can be estimated from these observations and from the combined decay scheme shown in I that the ground-

state branching ratio for each member must be approximately 70%. In Fig. 3 is shown the yield of 8.73-MeV ground-state radiation in the region of the doublet. Three sets of curves are shown. The upper curve in each set represents the yield measured by a fixed detector at 270° relative to the incoming beam direction, and the lower curves are from a movable detector located at 0° , 45° , and 90° successively. The upper and lower curves in each set were obtained simultaneously. (The slight differences in the location between sets of the resonances relative to the magnet galvanometer scale are probably due to hysteresis effects in the beam analyzing magnet.)

By a least-squares analysis of the areas under the resonance peaks shown in Fig. 3 and those of a similar set not shown, the angular distributions $W(\theta)=1-(0.44\pm 0.02)P_2$ and $W(\theta)=1-(0.47\pm 0.03)P_2$ are obtained for the lower- and upper-energy resonance ground-state radiations, respectively. $J=\frac{1}{2}$ and $J\geq\frac{7}{2}$ for the resonances are excluded by the observed anisotropy of the radiations and the strong decay to the $\frac{1}{2}+$ ground state. $J=\frac{5}{2}+$ would require $A_2=0.57$ and $A_4=-0.57$.

For $\frac{5}{2}-$, the minimum possible value of A_2 is $+0.29$, and at this minimum, $A_4=-1.28$. For $J^\pi=\frac{3}{2}-$, A_2 must be -0.50 which will agree with the result for the upper-energy resonance, but not with that for the lower. Both the observed distributions are consistent with $\frac{3}{2}+$. Hence, both members of the doublet have $J=\frac{3}{2}$, and the lower member must have even parity. The parity of the upper member is undetermined. The values of δ_p for the ground-state transitions are -0.038 ± 0.010 or $+1.91\pm 0.05$ for the lower resonance, and -0.02 ± 0.02 or $+1.83\pm 0.07$ for the upper resonance.

The total widths for each of the two resonances can be estimated from the data to be $\Gamma_t \leq 300$ eV. The target thickness (≈ 500 eV) used in this estimate was obtained by comparing the area under the resonance at 1509 keV with the step in a thick-target curve at that resonance.

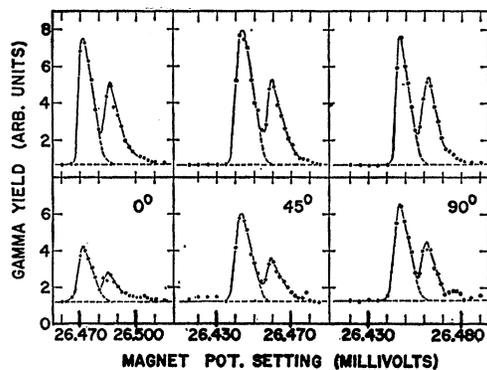


FIG. 3. Yield of ground-state radiation in the region of the doublet near 1481 keV. The upper curves represent the yield observed by a fixed monitor as discussed in the text and the lower curves represent the yield observed at 0° , 45° , and 90° by a movable detector. The separation of the two members as determined from these data is 1.7 ± 0.1 keV.

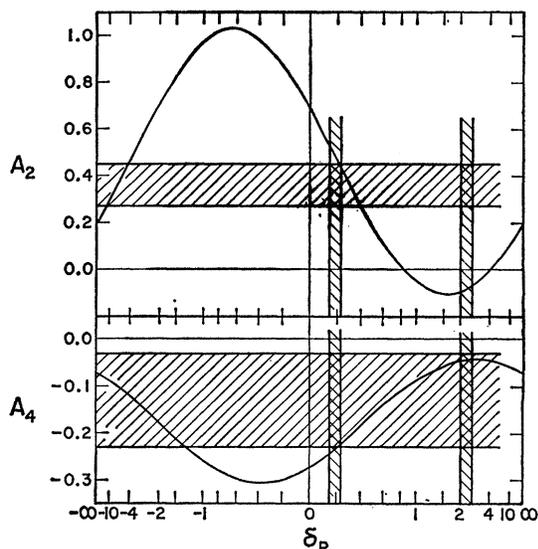


FIG. 4. Diagram of Geometry II triple-correlation results for the Res. \rightarrow 2.23 \rightarrow G.S. cascade at the 1489-keV resonance. The resonance is assumed to be $\frac{3}{2}+$. The horizontal bands represent the experimental values of the A_2 and A_4 coefficients, and the vertical bands represent the possible values of δ_p determined from angular-distribution data. The correlation depends only upon δ_p in this case since it is assumed the 2.23 ($\frac{3}{2}+$) \rightarrow G.S. ($\frac{3}{2}+$) transition is pure E2. The results are seen to be consistent with $\frac{3}{2}+$ for the resonance and the small value of δ_p .

1489-keV Resonance

The coefficients A_k observed at this resonance are shown in Table VI. $J=\frac{1}{2}$ and $\frac{3}{2}$ are not possible for reasons given in discussions of other resonances. If $J^\pi=\frac{3}{2}-$, the 6.51-MeV gamma ray would have $A_2=-0.100$ which disagrees with that observed. Similarly, if $J^\pi=\frac{7}{2}-$ this gamma ray would have $A_2=-0.357$. For $J^\pi=\frac{7}{2}+$ and $\frac{5}{2}-$, the 7.47-MeV gamma ray would have $A_2=0.510$ and -0.400 , respectively. Hence, $J^\pi=\frac{3}{2}+$ or $\frac{5}{2}+$. Of several triple correlations measured at this resonance, the one which makes the clearest distinction between the two remaining resonance assignments is a Geometry II correlation on the Res. \rightarrow 2.23($\frac{5}{2}+$) \rightarrow G.S.($\frac{1}{2}+$) cascade. The result was $W(\theta)=1+(0.36\pm 0.09)P_2-(0.13\pm 0.10)P_4$. For a $\frac{5}{2}$ resonance spin, the Geometry II correlation would have

TABLE VI. Angular-distribution coefficients observed at the 1489-keV resonance.

E_γ (MeV)	Transition	A_2	A_4
7.47	$R \rightarrow 1.27(3/2+)$	0.42 ± 0.04	0.09 ± 0.06
6.51	$R \rightarrow 2.23(5/2+)$	0.11 ± 0.04	-0.03 ± 0.05
5.6	a	0.12 ± 0.02	0.01 ± 0.03
3.96	$R \rightarrow 4.78(?)$	0.02 ± 0.03	-0.06 ± 0.05
3.75	a	-0.27 ± 0.04	-0.05 ± 0.05
3.51	$3.51 \rightarrow \text{G.S.}(1/2+)$	-0.3 ± 0.2	...
3.13	$3.13 \rightarrow \text{G.S.}(1/2+)$	0.14 ± 0.05	0.03 ± 0.04
2.23	$2.23 \rightarrow \text{G.S.}(1/2+)$	0.38 ± 0.02	-0.02 ± 0.02
2.03	$3.29 \rightarrow 1.27(3/2+)$	0.53 ± 0.04	0.05 ± 0.03

^a Arise from transitions between more than one pair of levels.

TABLE VII. Angular-distribution coefficients observed at the 1509-keV resonance.

E_γ (MeV)	Transition	A_2	A_4
7.48	$R \rightarrow 1.27(3/2+)$	0.12 ± 0.02	0.04 ± 0.02
6.52	$R \rightarrow 2.23(5/2+)$	0.55 ± 0.04	0.05 ± 0.06
5.34	$R \rightarrow 3.41(7/2+)^a$	0.10 ± 0.03	0.04 ± 0.04
2.2	b	0.38 ± 0.04	-0.04 ± 0.04
1.27	$1.27 \rightarrow \text{G.S.}(1/2+)$	-0.61 ± 0.04	-0.05 ± 0.04

^a See discussion of 3.41-MeV level.

^b Arises from transitions between more than one pair of levels.

$A_2 \leq 0$ and $A_4 \geq 0$ for any choice of δ_p . The consistency of the measured correlation with a $\frac{3}{2}+$ assignment is evident in Fig. 4. The vertical bands represent the values of δ_p allowed by the angular distribution of the 6.51-MeV primary gamma ray. The remaining correlations measured at this resonance are consistent with a $\frac{3}{2}+$ resonance assignment and will be discussed in a subsequent section.²¹

1509-keV Resonance

The coefficients A_k observed at this resonance are shown in Table VII. Again, $J=\frac{1}{2}$ and $\frac{3}{2}$ are not possible for the resonance level. For $J=\frac{3}{2}$, the maximum possible value of A_2 for the 6.52-MeV gamma ray is 0.38. If $J^\pi=\frac{7}{2}+$, the 7.48-MeV gamma ray would have $A_2=0.510$ and $A_4=-0.367$. For $J^\pi=\frac{7}{2}-$ and $\delta_p=0.44$ or 1.9 it is possible that $A_2=0.12$, but for these values of δ_p the 7.48-MeV gamma ray would have $W(\theta)=1+0.12P_2-1.04P_4+0.12P_6$ and $W(\theta)=1+0.12P_2-0.91P_4+0.60P_6$, respectively. Thus, $J=\frac{7}{2}$ is not possible. For $J^\pi=\frac{5}{2}-$, the same gamma ray would have $A_2=-0.400$. The distributions are all consistent with $\frac{5}{2}+$. A $\frac{5}{2}+$ assignment is confirmed by triple-correlation measurements on the Res. $\rightarrow 1.27(\frac{3}{2}+) \rightarrow \text{G.S.}(\frac{1}{2}+)$ cascade which will be discussed in a subsequent section on the 1.27-MeV level.

RESONANCE YIELDS

From the thin-target yield curve and decay schemes presented in I, the resonance level spins and the angular distributions of emitted radiations presented in the preceding section, and the result $\Gamma_\gamma \Gamma_p / \Gamma_t = 1.61 \pm 0.12$ eV for the 620-keV resonance in the $\text{Si}^{30}(p,\gamma)\text{P}^{31}$ reaction,²² the yields of the resonances can be obtained. The fraction of the total radiation which arises from the resonance level in each case was determined from the proposed decay schemes. The decay schemes were also

²¹ The results, available at the time of writing of I, of preliminary angular-distribution measurements at this resonance led to a report [G. I. Harris and L. W. Seagondollar, Bull. Am. Phys. Soc. 7, 301 (1962)] that $J^\pi=\frac{3}{2}+$ for the 1489-keV resonance and, as a consequence, to a statement in I that the resonance probably does not decay significantly to the ground state. The more recent measurements discussed in this paper show conclusively that $J^\pi=\frac{3}{2}+$, and thus there is no longer reason to exclude the presence of a ($\approx 5\%$) ground-state transition.

²² P. B. Smith and P. M. Endt, Phys. Rev. 110, 1442 (1958).

TABLE VIII. Values of $\Gamma_\gamma\Gamma_p/\Gamma_t$ for resonances in the $^{31}\text{Si}(\beta,\gamma)\text{P}^{31}$ reaction.

E_p (keV)	J	$\Gamma_\gamma\Gamma_p/\Gamma_t$ (eV)
620 ^a	1/2	1.61 ± 0.12
1177	7/2	0.048 ± 0.007
1204	5/2	0.21 ± 0.02
1324	5/2	0.34 ± 0.03
1392	5/2	0.52 ± 0.05
1400	3/2	0.61 ± 0.06
1480.5	3/2	1.00 ± 0.12
1482	3/2	0.63 ± 0.09
1489	3/2	0.49 ± 0.05
1509	5/2	0.56 ± 0.05

^a Comparison resonance. See Ref. 22.

used to determine the fraction of the total radiation which has energy above 2.8 MeV in order to correct for the 2.8-MeV discriminator bias used in obtaining the thin-target yield curve. The results were then corrected for the effect of angular distributions of the more prominent gamma rays with $E \geq 2.8$ MeV, and for the effects of target deterioration where necessary.

From the familiar expression for the resonance yield,

$$\omega\gamma = \frac{2J+1}{(2I+1)(2S+1)} \frac{\Gamma_\gamma\Gamma_p}{\Gamma_t} = \frac{2\epsilon Y}{\lambda^2 \xi},$$

where ϵ is the stopping power of the target material, ξ is the target thickness, λ is the wavelength of the incoming proton, and Y is the area (corrected as discussed above) under the thin-target curve; and the fact that the quantity ϵ/ξ is independent of bombarding energy for a given target, one finds that

$$\left(\frac{\Gamma_\gamma\Gamma_p}{\Gamma_t}\right)_r = \frac{(E_p)_r Y_r (2J_r+1)}{(E_p)_R Y_R (2J_r+1)} \left(\frac{\Gamma_\gamma\Gamma_p}{\Gamma_t}\right)_R,$$

where r refers to the resonance for which the yield is desired, and R refers to a comparison resonance observed in the same target. The values of $\Gamma_\gamma\Gamma_p/\Gamma_t$ obtained in this manner are shown in Table VIII. The quoted errors arise primarily from the uncertainty in the absolute yield of the comparison resonance.

PROPERTIES OF INTERMEDIATE STATES

With the angular momenta and parities of the resonance states established, interpretation of the correlation data which relate to the spin and parity assignments of intermediate states and to multipolarity mixings in their decay can be made. The angular distribution (intermediate radiation unobserved) of secondary radiations and the results of triple-correlation measurements will now be more fully utilized.

1.27- and 2.23-MeV Levels

The 2.23-MeV level has been shown^{7,19} to have spin and parity $\frac{5}{2}+$. All results in the present work are con-

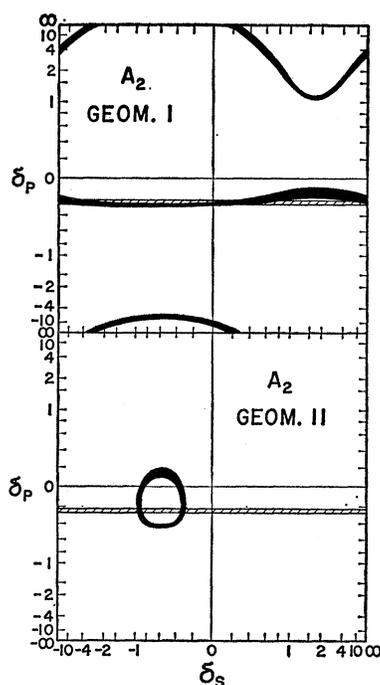


Fig. 5. Contour plots of Geometry I and II triple-correlation results for the Res. $\rightarrow 1.27 \rightarrow$ G.S. cascade at the 1509-keV resonance. The resonance is assumed to be $\frac{5}{2}+$ in these diagrams. Similar diagrams for $\frac{3}{2}+$ are not self-consistent. These data, when combined with the results of McCallum, lead to an accurate and unique determination of the $M1-E2$ mixing in the decay of the 1.27-MeV level of P^{31} as discussed in the text. The Geometry I, A_4 diagram (not shown) is consistent with those shown, but otherwise yields no additional information.

sistent with this assignment and with decay of the level to the ground state via pure $E2$ radiation.

All results in this work are also in agreement with the assignment⁷ $\frac{3}{2}+$ for the 1.27-MeV level. The value of δ_s for the $M1-E2$ mixing in the decay to the ground state has been reported as $\delta_s = -1.18 \pm 0.24$ or -0.20 ± 0.06 by Broude *et al.*⁷ with the former value being the more probable, and as $\delta_s = -0.25 \pm 0.15$ by McCallum.²⁰ A relatively accurate and consistent determination of this value is possible from the triple-correlation measurements of the present work. Geometry I and Geometry II triple correlations of the Res. $\rightarrow 1.27(\frac{3}{2}+) \rightarrow$ G.S. ($\frac{1}{2}+$) cascade were measured at the 1509-keV resonance with the results:

$$\text{Geometry I: } W(\theta) = 1 + (0.12 \pm 0.07)P_2,$$

$$\text{Geometry II: } W(\theta) = 1 - (0.78 \pm 0.02)P_2.$$

These results are indicated by the curved bands in Fig. 5 which assumes the $\frac{5}{2}+$ resonance assignment given in the preceding section. By inspection, it is clear that two values, $\delta_s = -0.91 \pm 0.06$ and -0.31 ± 0.03 , are allowed by these data. The latter value is consistent with the results of McCallum. Both are in reasonable agreement with those of Broude *et al.* Results consistent with those obtained at the 1509-keV resonance were

TABLE IX. E2/M1 amplitude ratio for the decay of the 1.27-MeV level.

Resonance (keV)	δ_s
1324	-0.89 ± 0.16 or -0.35 ± 0.10
1392	-0.99 ± 0.07 or -0.27 ± 0.04
1400	-1.23 ± 0.16 or -0.16 ± 0.07
1509	-0.91 ± 0.06 or -0.31 ± 0.03
Av.	-0.96 ± 0.04 or -0.28 ± 0.02

obtained from triple-correlation measurements of the corresponding cascades at the 1324-, 1392-, and 1400-keV resonances (see Figs. 1 and 2). The values of δ_s so obtained are summarized in Table IX.

The value of the $M1-E2$ mixing ratio of the decay of the 1.27-MeV level which is consistent with these results and with that of McCallum is $\delta_s = -0.28 \pm 0.02$. It is worth noting that if $J^\pi = \frac{3}{2}^+$ is assumed for the 1509-keV resonance, diagrams similar to those in Fig. 5 show the results not to be compatible with that assignment. The $\frac{5}{2}^+$ assignment for the 1509-keV resonance is thereby confirmed.

3.13- and 3.29-MeV Levels

The results of this work are consistent with the assignments $\frac{3}{2}^+$ and $\frac{5}{2}^+$, respectively, by other workers⁷ for the 3.13- and 3.29-MeV levels. There is a relatively strong, well-isolated cascade through the 3.13-MeV level at the 1400-keV resonance. At the 1489-keV resonance there occurs a cascade through this level which, however, is not well resolved experimentally from a stronger cascade through the 3.29-MeV level (see I). Cascades through the 3.29-MeV level also appear at the 1204- and 1392-keV resonances. The angular distributions and correlations of these cascade gamma rays can be utilized to determine the $M1-E2$ mixings in the decay of the 3.13-MeV level to the ground state, and of the 3.29- to the 1.27-MeV level.

At the 1400-keV resonance, the results of triple-correlation measurements on the Res. ($\frac{3}{2}^+$) \rightarrow 3.13 ($\frac{3}{2}^+$) \rightarrow G.S. ($\frac{1}{2}^+$) cascade were as follows:

$$\text{Geometry II: } W(\theta) = 1 + (0.08 \pm 0.03)P_2,$$

$$\text{Geometry VI: } W(\theta) = 1 + (0.07 \pm 0.03)P_2.$$

The distributions of the primary and secondary radiations are given in Table V. The results are shown in Fig. 6 where the horizontal bands represent the values of δ_p derived from the angular distributions. There are two possible solutions; both of which correspond to the high value of δ_p . The two solutions, representing a weighted average of the two independent measurements, are $\delta_s = 0.22 \pm 0.02$ and -3.1 ± 0.2 . Neither of these two possible values can be eliminated by present data. Furthermore, from an investigation of the theoretical correlation coefficients it was found that ambiguities in δ_s can not be resolved for any value of δ_p or any possible arrangement of the detectors by means of angular-

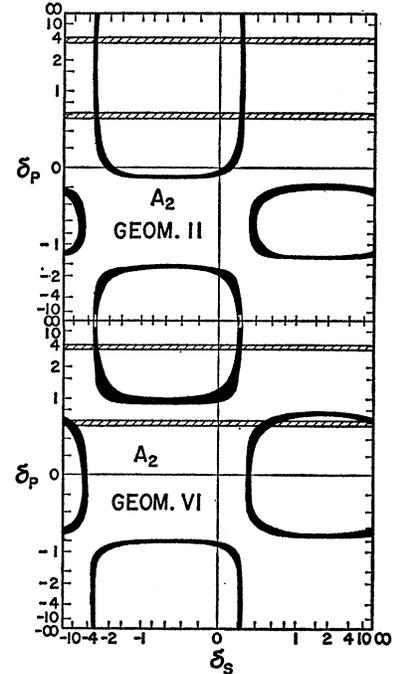


FIG. 6. Contour plots of Geometry II and VI triple-correlation results for the Res. \rightarrow 3.13 \rightarrow G.S. cascade at the 1400-keV resonance. These data show that the unusually high value $\delta_p \approx 3.7$ is the appropriate value for this cascade. The two possible values for δ_s are seen to be approximately 0.2 and -3 .

distribution and triple-correlation measurements on a $\frac{3}{2}^+ \rightarrow \frac{3}{2}^+ \rightarrow \frac{1}{2}^+$ cascade such as the one considered here.

From angular-distribution measurements of the members of the cascade through the 3.13-MeV level at the 500-keV resonance, Broude *et al.*⁷ obtain $\delta_s = +0.16$ or -3.0 (no errors quoted) which agree reasonably well with the present values. They also performed triple-correlation experiments on the same cascade with the results:

$$\text{Geometry I: } W(\theta) = 1 - (0.43 \pm 0.06)P_2,$$

$$\text{Geometry II: } W(\theta) = 1 + (0.11 \pm 0.07)P_2,$$

by means of which they exclude the value $\delta_s = -3.0$. However, by plotting these results, along with the values of δ_p and δ_s allowed by their angular distribution data, on a diagram similar to Fig. 6 with appropriate allowance for the difference in phase convention, it is found that, as stated above, no choice between the two values of δ_s can be made. The origin of the discrepancy appears to lie in the calculated values of the Geometry I triple correlations for the cases $\delta_p = -1$, $\delta_s = -3$ and $\delta_p = -2$, $\delta_s = -3$ in Ref. 7. It must, therefore, be concluded that on the basis of presently available data, the value of the $M1-E2$ mixing ratio for the decay of the 3.13-MeV level is either 0.22 or -3 .

An analysis of the cascades through the 3.13- and 3.29-MeV levels from the 1489-keV resonance is complicated by the fact that the primary gamma rays are not experimentally resolved. The angular distributions of the 5.6-MeV peak (which represents a combination of the two primaries), the 3.13-MeV secondary from the

TABLE X. $E2/M1$ amplitude ratios for the decay of the 3.13-MeV level to the ground state, and for the decay of the 3.29-MeV level to the 1.27-MeV level.

Resonance (keV)	$\delta_s(3.13 \rightarrow \text{G.S.})$	$\delta_s(3.29 \rightarrow 1.27)$
1204	...	0.47 ± 0.08
1392	...	0.46 ± 0.09
1400	0.22 ± 0.02 or -3.1 ± 0.2	...
1489	0.18 ± 0.06 or -2.8 ± 0.4	0.63 ± 0.08
Av.	0.22 ± 0.02 or -3.0 ± 0.2	0.52 ± 0.05

3.13-MeV level, and the 2.03-MeV secondary from the 3.29-MeV level were as follows:

$$5.6 \text{ MeV: } W(\theta) = 1 + (0.12 \pm 0.02)P_2,$$

$$3.13 \text{ MeV: } W(\theta) = 1 + (0.14 \pm 0.05)P_2,$$

$$2.03 \text{ MeV: } W(\theta) = 1 + (0.53 \pm 0.04)P_2.$$

The Geometry II, triple correlations of the

$$\text{Res.}(\frac{3}{2}+) \rightarrow 3.13(\frac{3}{2}+) \rightarrow \text{G.S.}(\frac{1}{2}+)$$

and

$$\text{Res.} \rightarrow 3.29(\frac{5}{2}+) \rightarrow 1.27(\frac{3}{2}+)$$

cascades were measured to be

$$W(\theta) = 1 + (0.08 \pm 0.15)P_2$$

and

$$W(\theta) = 1 + (0.66 \pm 0.11)P_2 - (0.08 \pm 0.12)P_4,$$

respectively. By using the values $\delta_s = 0.18$ or -2.8 for the decay of the 3.13-MeV level, the value of $\delta_s = 0.63$ for the decay of the 3.29-MeV level to the 1.27-MeV level, and the values $\delta_p = -4$ and $\delta_p = +0.08$ for the primary transitions to the 3.13- and 3.29-MeV levels, respectively; and by utilizing the values given in I for the relative intensities of these primaries, one calculates for the theoretical correlations:

$$5.6 \text{ MeV: } W(\theta) = 1 + 0.10P_2,$$

$$3.13 \text{ MeV: } W(\theta) = 1 + 0.06P_2,$$

$$2.03 \text{ MeV: } W(\theta) = 1 + 0.49P_2,$$

$$3.13 \text{ T.C.: } W(\theta) = 1 + 0.07P_2,$$

$$3.29 \text{ T.C.: } W(\theta) = 1 + 0.59P_2 + 0.08P_4,$$

where T.C. signifies the triple correlation of the cascade through the level indicated. By means of a detailed graphical analysis of the theoretical correlation functions, it can be shown that the mixing ratios given are unique, i.e., there exists no other significantly different set which will give values of the coefficients in reasonable agreement with those measured. The δ_p with their estimated errors are shown in Table XII, and the δ_s are shown in Table X. Also, in Table X are shown values of the mixing ratio obtained for the decay of the 3.29-MeV level by the analysis of the angular distributions of the 2.03-MeV gamma rays in the spectra of the 1204- and 1392-keV resonances. (High values of δ_s were also allowed by the data at these resonances but they are

not shown in the Table because they are excluded by the data at the 1489-keV resonance.)

3.41-MeV Level

A brief discussion of the results of correlation measurements involving this level has been given earlier.⁶ These are the first measurements reported from which the spin and parity of the 3.41-MeV level can be deduced. This level is the only known level below 4-MeV excitation in P^{31} for which no experimental determination of spin or parity has been previously reported.

From a study of the cascades through the 3.41-MeV level from the 675-, 760-, 775-, 840-, 980-, and 995- (978.8 on our scale) keV resonances, Broude *et al.*⁷ reported consistent evidence that a transition from the 3.41-MeV level to the 1.27-MeV level accounts for about 85% of the radiation from this level, and that the remaining 15% is due to a transition to the ground state. They also conclude that the possible decay to the 2.23-MeV level cannot account for more than 7 or 8% of the radiation from the level. At the 1509-keV ($\frac{5}{2}+$) resonance studied in the present work, 28% of the radiation from the resonance level is due to a transition to the 3.41-MeV level. The results of measurements at this resonance seemed to indicate that the relative intensities for decay of the 3.41-MeV level are 90% to the 1.27-MeV level and 10% to the ground state.²³ Again no evidence was found for a transition to the 2.23-MeV level.

$J = \frac{1}{2}$ assignment for the 3.41-MeV level can be eliminated immediately by considering the angular distribution of the primary radiation from the 1509-keV resonance. For $J^\pi = \frac{1}{2}+$, the 5.34-MeV primary gamma ray would have $W(\theta) = 1 + 0.571P_2 - 0.571P_4$ which is definitely not in agreement with the observed results (see Table VII). If the level were $\frac{1}{2}-$, the minimum possible value for A_2 would be 0.29, and at this minimum, $A_4 = -1.28$. For $J^\pi = \frac{3}{2}-$, the 5.34-MeV gamma ray would have $A_2 = -0.400$, and for $\frac{5}{2}-$ it would have $A_2 = 0.457$. $J \geq \frac{3}{2}$ is clearly unreasonable in light of the observed decay properties of the 3.41-MeV level. Hence, $J^\pi = \frac{3}{2}+$, $\frac{5}{2}+$, or $\frac{7}{2}\pm$. The observed distribution of the 5.34-MeV gamma ray will not distinguish between these remaining possibilities.

The peak at about 2.2 MeV in the 1509-keV resonance spectrum arises from the 2.15-MeV transition between the 3.41- and 1.27-MeV levels, the 2.23-MeV transition from the 2.23-MeV level, and the 2.20-MeV transition between the 4.43- and 2.23-MeV levels, with relative intensities 54%, 33%, and 13%, respectively. From Table XII, the 6.52-MeV primary to the 2.23-MeV level has $\delta_p = -0.09$. By using this value of δ_p the distribution expected for the 2.23-MeV gamma ray can be calculated. The result is $W(\theta) = 1 + 0.37P_2 + 0.08P_4$. From Table VII, the observed "2.2-MeV gamma ray" has $W(\theta) = 1$

²³ See I, Figs. 10 and 11. (By an unfortunate oversight, the captions for these two figures were interchanged.)

TABLE XI. Theoretical distributions of the 2.15-MeV gamma ray from the decay of the 3.41-MeV level at the 1509-keV resonance.

$J^\pi(3.41)$	δ_p	δ_s	A_2	A_4
3/2+	-0.24	0.09 or 2.9	0.39	0
5/2+	0.31	0.58	0.39	-0.03
5/2+	0.31	4.5	0.39	-0.11
7/2+	0.17	...	0.43	-0.18

+ (0.38±0.04) P_2 - (0.04±0.04) P_4 . If the small contribution of the 2.20-MeV gamma ray is ignored, calculation yields $W(\theta) = 1 + 0.39P_2 - 0.23P_4$ for the 2.15-MeV gamma ray. Theoretically expected $W(\theta)$ for various possible assignments for the 3.41-MeV level, and for appropriately chosen δ_s , are shown in Table XI. The values of δ_p are fixed by the distribution of the 5.34-MeV primary.

The A_2 coefficients in all cases agree well with the value extracted from the experimental data. The A_4 coefficient for the $\frac{7}{2}+$ case is in best agreement with the experimental value. However, the errors inherent in the method used to extract A_4 from the data are large, and one cannot exclude $\frac{3}{2}+$ or $\frac{5}{2}+$ assignments on this basis alone.

In order to obtain additional independent and more accurate information, Geometry I, II, IV, and VI triple-correlation measurements were performed at the 1509-keV resonance on the Res. ($\frac{5}{2}+$) → 3.41(J^π) → 1.27($\frac{3}{2}+$) cascade with the following results:

Geometry I:

$$W(\theta) = 1 + (0.23 \pm 0.05)P_2 - (0.05 \pm 0.05)P_4,$$

Geometry II:

$$W(\theta) = 1 + (0.53 \pm 0.03)P_2 - (0.19 \pm 0.03)P_4,$$

Geometry IV:

$$W(\theta) = 1 + (0.29 \pm 0.07)P_2 - (0.01 \pm 0.08)P_4,$$

Geometry VI:

$$W(\theta) = 1 + (0.34 \pm 0.04)P_2 - (0.15 \pm 0.05)P_4.$$

Theoretically, $A_4 = 0$ for a $\frac{5}{2}+ \rightarrow \frac{3}{2}+ \rightarrow \frac{3}{2}+$ sequence and the Geometry II and VI arrangements. Since, however, the observed values of A_4 were definitely non-zero, $J^\pi = \frac{3}{2}+$ for the 3.41-MeV level is excluded.

The Geometry VI results are indicated by the solid bands in Fig. 7 where it is assumed $J^\pi = \frac{5}{2}+$ for the 3.41-MeV level. The possible values of δ_p and δ_s allowed by the angular distribution measurements are indicated by the horizontal and vertical bands. By inspection of the figure, it is clear that there are no values of δ_p and δ_s which yield solutions consistent with the observed correlation coefficient. The inconsistency is even more marked in similar diagrams (not shown) for the Geometry II case. Also, in that case the maximum theoretical value of A_2 after finite-geometry corrections is

approximately 0.35, which is considerably smaller than the observed value, 0.53 ± 0.03 . Hence, $J^\pi = \frac{5}{2}+$ for the 3.41-MeV level is also excluded.

The observed Geometry II and VI coefficients are indicated in Fig. 8, for $J^\pi = \frac{7}{2}+$, by the horizontal bands. (The coefficients in this case depend only upon δ_p since it is assumed the $\frac{7}{2}+ \rightarrow \frac{3}{2}+$ secondary transition would be pure $E2$.) The vertical band indicates the value of δ_p determined by the angular distribution of the primary radiation. In each case it is seen that the results are in statistically significant agreement with $J = \frac{7}{2}$ for the 3.41-MeV level. The same is true for the Geometry I and IV results which are not illustrated. Since multipolarity mixing is required to explain the results, the parity is considered to be even.

The assignment of $\frac{7}{2}+$ to the 3.41-MeV level and the requirements imposed by gamma-ray selection rules leads to the unusual conclusion that the reported 3.41-MeV ground-state transition, if it actually exists, must be of order $M3$, or possibly even a $M3-E4$ mixture. The unusual character of this transition is rendered even more acute by the fact that it must compete with a possible, but unobserved, $M1$ transition to the 2.23-MeV state. For these reasons it was stated in I (Introduction) that a $\frac{7}{2}+$ assignment suggested by the weak-coupling vibrational model⁵ for the 3.41-MeV level is unlikely.

Because of the possible significance and unusual consequences of a $\frac{7}{2}+$ assignment, further experimental checks were performed. For example, all triple-correlation measurements were repeated with particular emphasis being given to the possibility that the Geometry II, A_4 term may be an experimental effect. The

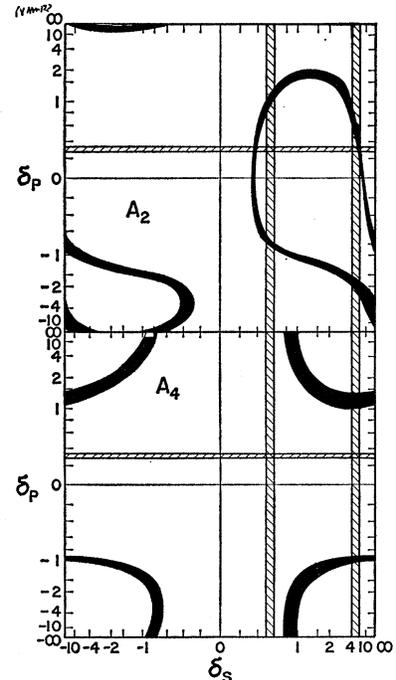


FIG. 7. Contour plots of the Geometry VI triple-correlation results for the Res. → 3.41 → 1.27 cascade at the 1509-keV resonance. The 3.41-MeV level is assumed to be $5/2+$ in these diagrams. It is evident that for this assumption the triple-correlation and angular-distribution results are not self-consistent.

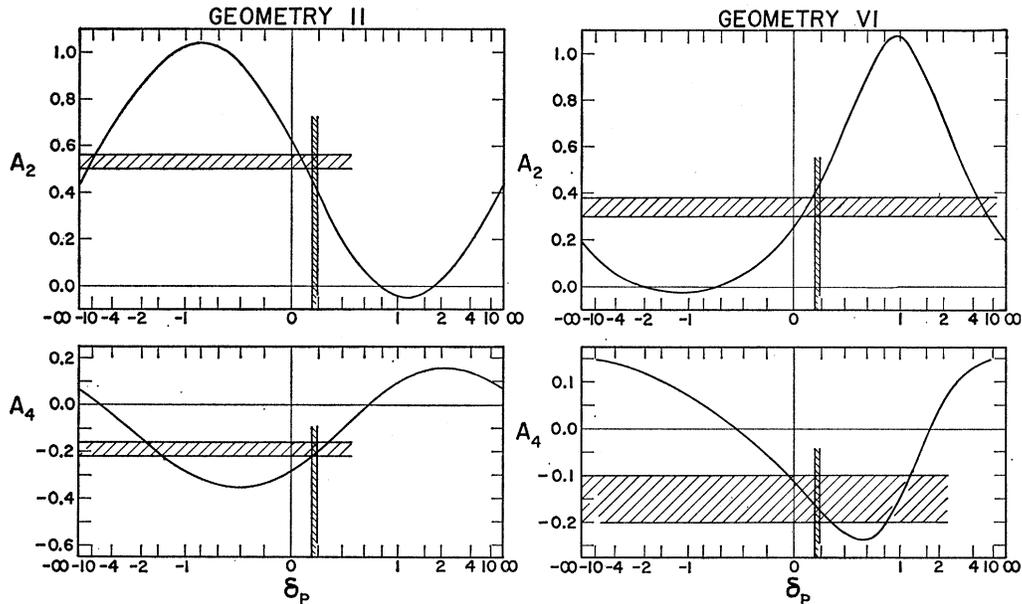


FIG. 8. Diagrams of the Geometry II and VI triple-correlation results for the Res. $\rightarrow 3.41 \rightarrow 1.27$ cascade at the 1509-keV resonance. The 3.41-MeV level is assumed here to be $7/2+$. In this case it is seen that the triple-correlation results (horizontal bands) and the angular-distribution results (vertical bands) are consistent with the theoretical curves. A similar situation holds for the Geometry I and IV results which are not shown.

Geometry II measurement was performed the second time simultaneously with a Geometry II measurement of the Res. $(\frac{5}{2}+) \rightarrow 1.27(\frac{3}{2}+) \rightarrow \text{G.S.}(\frac{1}{2}+)$ cascade which theoretically cannot yield an $A_4 \neq 0$. No significant A_4 term was observed in the latter cascade, but an A_4 coefficient agreeing with the original result was obtained for the cascade through the 3.41-MeV level. The results of the second measurements for the other geometries also agreed with the result of the original measurements within statistics. The results quoted represent the weighted averages of the separate measurements.

One is led, therefore, to question the existence of the 3.41-MeV ground-state transition. One possibility is that the 3.41-MeV peak in the spectrum taken in coincidence with the 5.34-MeV primary gamma ray (see I, Fig. 10) may be a sum effect caused by the 1.27- and 2.15-MeV members of the cascade originating from the 3.41-MeV level being simultaneously detected. In order to check this possibility, spectra similar to that shown in I were obtained by observing the pulses in a 5-in.-diam \times 5-in.-long crystal, located at a distance D and at 90° relative to the incoming beam, which were in coincidence with pulses passed by a "window" over the 5.34-MeV peak in the spectrum of an 8-in.-diam \times 8-in.-long crystal located at 270° and 4.75 in. from the target. Coincidence spectra were obtained for distances D between 1.25 and 12 in. The ratio R of the intensities of the 3.41- and 2.15-MeV peaks in each spectrum was measured and is shown in Fig. 9 for the various values of D . It is apparent that for D less than about 4 in. sum effects may be significant, but for $D > 4$ in., however, there is no

tendency for R to vanish as expected if the 3.41-MeV peak was caused entirely by a sum effect.

A second possibility, which cannot be excluded by the present measurements, is that the 3.41-MeV peak is caused by a weak primary transition to a level near 5.3-MeV excitation which decays to the ground state. A third and reasonable possibility is that the "3.41-MeV" peak in the coincidence spectrum is instead 3.51 MeV and arises from a weak, competing cascade through the 3.51-MeV level. An attempt was made to remove this ambiguity by alternately obtaining coincidence spectra at the 1324-keV resonance, which is known to decay strongly by a cascade through the 3.51-MeV level, and at the 1509-keV resonance. The spectra at the 1324-keV resonance provided a calibration point at 3.51-MeV which could be compared with the weak peak near that energy in the 1509-keV coincidence spectrum. Because of the low intensity of the transition in question, however, and insufficient detector resolution, a definite determination could not be made.

If one assumes that the peak is caused by a competing cascade through the 3.51-MeV level, then it seems a rather striking coincidence that in seven resonances (including the six studied by Broude *et al.*) such competition between the cascades through the 3.41- and 3.51-MeV levels occur with very closely the same primary branching ratio. In conclusion, the correlation measurements discussed above definitely establish an assignment of $\frac{7}{2}+$ for the 3.41-MeV level, but the possibility of an $M3$ or $M3-E4$ ground-state decay mode being observed, while not excluded by the data to date, is considered unlikely.

3.51-MeV Level

Cascades through this level were identified in I as originating from the resonance levels at $E_p=1204, 1324, 1392,$ and 1489 keV. This cascade is strongest at the 1324 -keV ($\frac{5}{2}+$) resonance where it accounts for about 28% of the radiation from the resonance level. As discussed in I, a previously unreported decay mode of the 3.51 -MeV level to either the 1.27 - or to the 2.23 -MeV level (or perhaps both), in addition to the known ground-state mode, was identified by means of coincidence measurements at the 1324 -keV resonance. The ambiguity in the level involved (1.27 or 2.23) in the cascade originating from the 3.51 -MeV level is due to the relative positions of the 3.51 -, 2.23 -, and 1.27 -MeV levels which precludes a determination by the usual energy and coincidence measurements.

The angular distribution results at each resonance are consistent with the $\frac{3}{2}+$ assignment of Broude *et al.* for the 3.51 -MeV level made from angular correlation measurements at the 760 -keV resonance. A conflicting report²⁴ of $\frac{1}{2}+$ from a study of data at the same resonance is in disagreement with results of the present work. Strong anisotropies, for example, were observed in the $3.51 \rightarrow$ G.S. transition (Tables III, IV), and the observed distribution $W(\theta)=1-(0.03\pm 0.04)P_2+(0.03\pm 0.05)P_4$ of the primary transition to the 3.51 -MeV level from the 1324 -keV resonance disagrees with the expected distribution $W(\theta)=1+0.57P_2-0.57P_4$ for a $\frac{5}{2}+ \rightarrow \frac{1}{2}+$ transition. For a $\frac{5}{2}+ \rightarrow \frac{3}{2}+$ transition the observed $W(\theta)$ leads to the unique value $\delta_p=-0.18 \pm 0.02$ for the primary transition at the 1324 -keV resonance which will be used in the determination of secondary mixing ratios in the following discussion.

Triple correlations of the Res. ($\frac{5}{2}+$) \rightarrow 3.51 ($\frac{3}{2}+$) \rightarrow G.S. ($\frac{1}{2}+$) cascade were measured at the 1324 -keV reso-

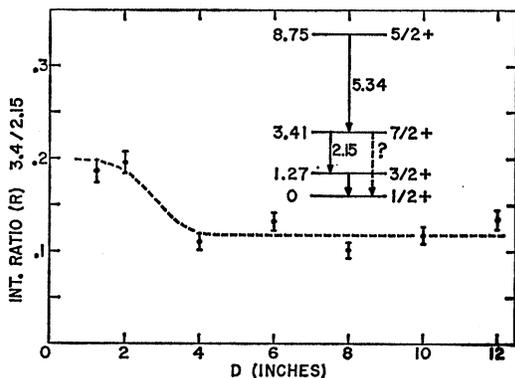


FIG. 9. Ratio R of observed intensities of the gamma-ray peak near 3.4 MeV and the 2.15 -MeV peak in a spectrum in coincidence with the 5.34 -MeV gamma ray at the 1509 -keV resonance. The detector positions are discussed in the text. The variation of R with the movable detector distance D indicates that the 3.4 -MeV peak is not simply a sum effect. The dotted line is merely a visual estimate of the true trend of the data.

²⁴ L. Simons, Nucl. Phys. **10**, 215 (1959).

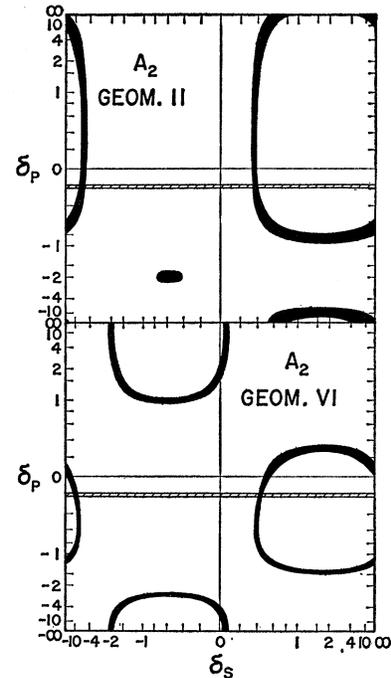


FIG. 10. Contour plots of Geometry II and VI triple-correlation results for the Res. \rightarrow $3.51 \rightarrow$ G.S. cascade at the 1324 -keV resonance. The possible values of δ_s are seen to be approximately 0.4 and -7.5 . The horizontal band is derived from angular-distribution data.

nance with the following results:

$$\text{Geometry II: } W(\theta)=1+(0.16\pm 0.05)P_2,$$

$$\text{Geometry VI: } W(\theta)=1+(0.28\pm 0.04)P_2.$$

These results are indicated in Fig. 10 which shows the consistency of the measurements with the values $\delta_s=0.41\pm 0.02$ or -7.5 ± 1.4 for the decay of the 3.51 -MeV level to the ground state. The values reported by Broude *et al.* were $\delta_s=0.30$ or -5.3 . No errors were stated. As in the decay of the 3.13 -MeV level, it can be shown²⁵ that no choice between these two values of δ_s can be made by means of direction-correlation measurements alone.

The coincidence spectra obtained in the course of the above triple-correlation measurements contained, in addition to the 3.51 -MeV gamma ray, the 1.3 - and 2.2 -MeV gamma rays from the cascade which originates at the 3.51 -MeV level. The correlations of these two gamma rays were also extracted from the data with the following results:

$$1.3 \text{ MeV, Geometry II: } W(\theta)=1-(0.42\pm 0.05)P_2,$$

$$1.3 \text{ MeV, Geometry VI: } W(\theta)=1-(0.31\pm 0.05)P_2,$$

$$2.2 \text{ MeV, Geometry II: } W(\theta)=1+(0.04\pm 0.06)P_2,$$

$$2.2 \text{ MeV, Geometry VI: } W(\theta)=1+(0.01\pm 0.06)P_2.$$

If it is assumed the cascade proceeds through the 1.27 -MeV level, then for $\delta_p=-0.18$ and a $\frac{5}{2}+ \rightarrow \frac{3}{2}+ \rightarrow \frac{3}{2}+$ sequence the Geometry II and VI results for the 2.2 -MeV gamma ray are consistent with each other. The

²⁵ G. I. Harris (to be published).

weighted values of δ_s for the assumed $3.51 \rightarrow 1.27$ transition would be $\delta_s = -0.22 \pm 0.04$ or $|\delta_s| \geq 11.4$. On the other hand, if it is assumed the cascade proceeds through the 2.23-MeV level, then for $\delta_p = -0.18$ and a $\frac{5}{2}^+ \rightarrow \frac{3}{2}^+ \rightarrow \frac{5}{2}^+$ sequence the values of δ_s for the $3.51 \rightarrow 2.23$ transition as derived from the above data would be as follows:

$$\text{Geometry II: } \delta_s = 0.38 \pm 0.08 \quad \text{or} \quad 11 \pm 5,$$

$$\text{Geometry IV: } \delta_s = 0.23 \pm 0.06 \quad \text{or} \quad |\delta_s| \geq 19.$$

These values of δ_s from the two geometries are seen to be separated by about two standard errors which cannot be considered sufficiently significant to rule out an assumed cascade through the 2.23-MeV level. Since, however, the results obtained by assuming a cascade through the 1.27-MeV level were self-consistent to well within one standard deviation, it is considered most probable that the 3.51-MeV level cascades through the 1.27- rather than the 2.23-MeV level. The presence of cascades through both levels would, of course, alter the above analysis and would complicate further attempts to determine the nature of the cascade decay of the 3.51-MeV level by means of angular-correlation measurements.

During the course of the present study a question arose regarding the possibility that the 3.51-MeV level may be a closely spaced doublet. Before the results of the triple-correlation measurements were available, an analysis was made of the intermediate-radiation-unobserved, angular-distribution results for the Res. $\rightarrow 3.51 \rightarrow$ G.S. cascade at the 1204-, 1324-, 1392-, and 1489-keV resonances which yielded mutually inconsistent values of δ_s for the $3.51 \rightarrow$ G.S. transition. It was also at this time that the presence of a cascade decay from the 3.51-MeV level was first noted at the 1324-keV resonance. Subsequently, it was reported that calculations based upon the weak-coupling vibrational model predicted a $\frac{3}{2}^+, \frac{1}{2}^+$ doublet near 3.5-MeV excitation in P²¹.²⁶ These observations, plus the presence of a weak suggestion of a double peak corresponding to the 3.51-MeV level in the high-resolution, inelastic-scattering data of Endt and Paris,²⁷ were considered sufficient to require further consideration of the possibility of a doublet.

In spite of these earlier experimental suggestions and the interesting theoretical prediction, it is now considered that the presently existing data do not support the presence of a doublet for the following reasons: (1) More recent triple-correlation measurements (discussed earlier in this section) lead to accurate values of δ_s for the $3.51 \rightarrow$ G.S. transition which disagree with the values extracted from the earlier intermediate-radiation-unobserved angular-distribution results at the same resonance. This casts doubt upon any possible significance of the observation of mixing ratios in mutual

disagreement at several resonances. A re-examination of the earlier angular-distribution data shows that in the spectra of the resonances considered, uncertainties in the determination of gamma-ray intensities are greatest near the 3.5-MeV region. It is in just this region of the singles pulse-height spectra that the accumulation of errors in the stripping analysis has the greatest effect. This uncertainty does not appear in the triple-correlation results which are derived from coincidence spectra. (2) The observation of a cascade decay from the 3.51-MeV level at the 1324-keV resonance, and not in earlier work or at other resonances in the present work, can probably be attributed to the fact that the 3.51-MeV level is rather weakly excited at all except the 1324-keV resonance. The identification of this cascade is complicated in all cases because the gamma-ray energies involved are almost identical with the energies of transitions from the 2.23- and 1.27-MeV levels which are independently excited at most resonances. (3) The absence of significant $P_4(\cos\theta)$ terms in the primary transitions to the 3.51-MeV level from the $\frac{5}{2}^+$ resonances at 1204, 1324, and 1392 keV indicates that a hypothetical $\frac{1}{2}^+$ member of a doublet near 3.5 MeV is not being significantly excited by direct primary transitions. The coincidence measurements which showed the existence of the cascade decay of the 3.51-MeV level indicate that both the cascade and direct ground-state transitions from the level are in coincidence with a direct primary. These arguments do not exclude the presence of a doublet, but do indicate however that present data can be explained by a single $\frac{3}{2}^+$ level at 3.51 MeV, and that a $\frac{1}{2}^+$ member, if present, is not being excited.

Finally, it is worth mentioning that the angular-correlation data are not consistent with an assumption that the Res. $\rightarrow 3.51 \rightarrow$ G.S. cascade at the 1324-keV resonance is, instead, a cascade which proceeds through a $J = \frac{3}{2}, \frac{3}{2},$ or $\frac{5}{2}$ level near 5-MeV excitation. The latter cascade would involve about the same gamma-ray energies and, thus, could not be excluded by ordinary coincidence measurements.

4.19-MeV Level

The result of angular distribution measurements at the 1177-, 1392-, and 1400-keV resonances are consistent with the $\frac{3}{2}^+$ assignment of Broude *et al.* for the 4.19-MeV level. They obtained the value $\delta_s = -0.25 \pm 0.03$ or -1.8 ± 0.4 for the $4.19 \rightarrow 1.27$ transition. No independent measurements of δ_s were obtained in the present study because of the weak excitation of the 4.19-MeV level at all three resonances.

4.26-MeV Level

Cascades through this level are prominent features of the decay systems of the 1204- and 1324-keV resonances. The interpretation of angular distribution measure-

²⁶ V. K. Thankappan, Phys. Letters 2, 122 (1962).

²⁷ P. M. Endt and C. H. Paris, Phys. Rev. 106, 764 (1957).

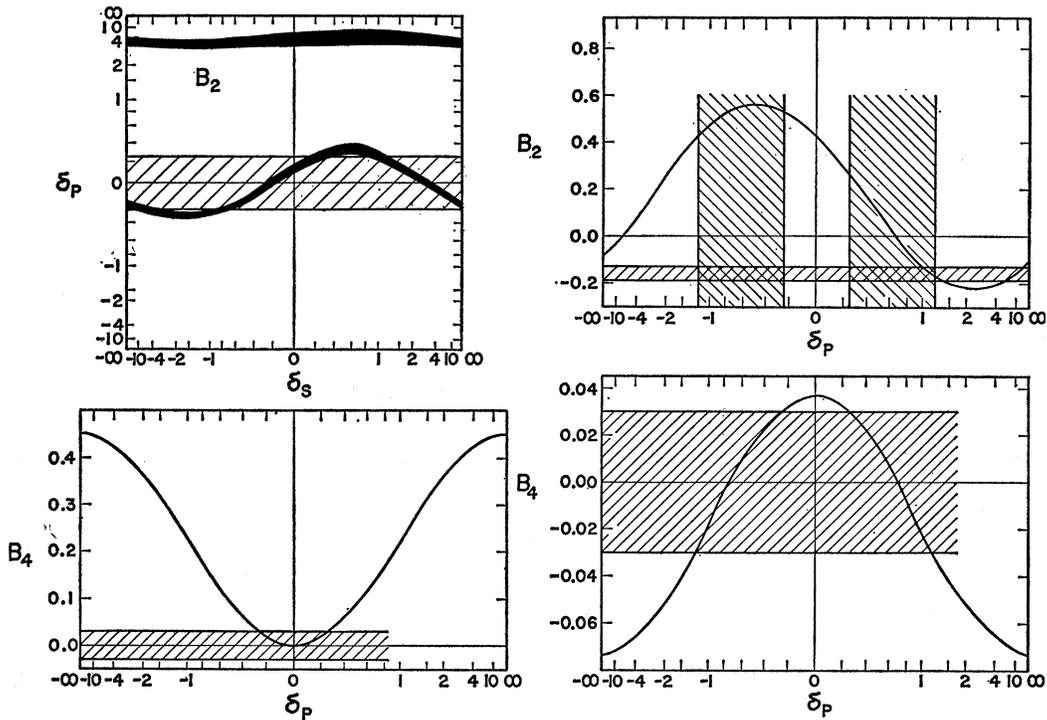


FIG. 11. Diagrams illustrating the results of angular-distribution measurements on the Res. \rightarrow 4.26 \rightarrow G.S. cascade at the 1204-keV resonance. The theoretical curves for the B_2 and B_4 coefficients are derived as discussed in the text. Since the primary and secondary members of the cascade were not experimentally resolved, the curves represent appropriately averaged coefficients for the angular distribution of the combination of the two members. The two diagrams on the left assume $3/2+$ for the 4.26-MeV level, and those on the right assume $5/2+$. These data, combined with the observed distribution of the 4.26 \rightarrow 1.27 transition, lead to the assignment of $3/2+$ to the 4.26-MeV level. The sign of δ_s should be reversed from that shown in the upper-left diagram.

ments is complicated at both resonances by the fact that the 4.26-MeV secondary is not experimentally resolved from the primary to the 4.26-MeV level (see I). The best data were obtained at the 1204-keV resonance and are used in the following discussion.

The observed distribution of the 4.2-MeV peak is actually a measurement of a weighted average of those for the 4.19-MeV primary and the 4.26-MeV secondary transition. As shown in I, the 4.26-MeV level decays to the ground state and to the 1.27-MeV level with relative intensities 84% and 16%, respectively. The distribution of the 4.2-MeV combination peak can, therefore, be written as

$$W(\theta) = 1 + B_2 P_2 + B_4 P_4,$$

where B_2 and B_4 are of the form

$$B_2 = 0.54A_2(1) + 0.46A_2(2),$$

$$B_4 = 0.54A_4(1) + 0.46A_4(2).$$

The A_2 and A_4 are obtained from the *Tables of Angular-Distribution Coefficients*,¹⁶ and the (1) and (2) refer to the primary and secondary radiation, respectively. Experimentally, $B_2 = -0.16 \pm 0.03$ and $B_4 = 0.00 \pm 0.03$. The resonance level is $5/2+$, so if the 4.26-MeV level were $1/2+$ it would require $B_2 = 0.31$ and $B_4 = -0.31$. If the level were $1/2-$, the minimum value of B_2 is $+0.16$, at

which minimum, $B_4 = -0.69$. Hence, $J = 1/2$ is excluded. $J \geq 7/2$ need not be considered because of the strong ground-state decay of the 4.26-MeV level. If $J^\pi = 3/2-$, B_2 and B_4 would be -0.40 and 0 , respectively. For $5/2-$, the minimum value of B_2 is 0.33 , at which minimum, $B_4 = 0.08$. The only remaining possibilities are $3/2+$ and $5/2+$.

The experimental values of B_2 and B_4 are shown in Fig. 11 for the two possibilities $3/2+$ and $5/2+$ for the 4.26-MeV level. The coefficients for the $5/2+$ case depend only upon δ_p since the secondary transition would be pure $E2$ by our initial assumptions. The experimental value of B_4 limits δ_p to the values indicated by the vertical bands in the B_2 coefficient diagram. A solution is seen to exist for $\delta_p \approx 1.1$. As shown in Table II, the 2.99-MeV, 4.26 \rightarrow 1.27 transition has the distribution $W(\theta) = 1 + (0.8 \pm 0.2)P_2$. For $\delta_p \approx 1.1$ and $5/2+$ assignment for the 4.26-MeV level, the maximum theoretical value of A_2 for this transition is 0.33 which occurs at $\delta_s = 1.33$. Hence, the data are in disagreement with $5/2+$ for the 4.26-MeV level.

For $3/2+$ the coefficient B_2 depends upon both δ_p and δ_s , but the B_4 coefficient depends only upon δ_p . By inspection of the B_4 diagram for the $3/2+$ case (Fig. 11), it is seen that δ_p is limited by experiment such that $|\delta_p| \leq 0.26$. This region is indicated in the B_2 contour diagram by the horizontal band. The experimental B_2 is

indicated in this diagram by the darkly shaded, curved bands. It is evident that statistically significant solutions exist for all δ_s . Furthermore, for the 2.99-MeV transition there is agreement with $\frac{3}{2}+$ for $|\delta_p| \leq 0.26$ and $0.10 \leq \delta_s \leq 2.8$. It is concluded, therefore, that $J^\pi = \frac{3}{2}+$ for the 4.26-MeV level. The results at the 1324-keV resonance are consistent with this conclusion.

It was mentioned in I that the "2.99-MeV" gamma ray is not likely to arise from a decay of the 3.13-MeV level. This statement is supported by the observed angular distribution of the gamma ray. The resonance level and the 3.13-MeV level are $\frac{5}{2}+$ and $\frac{3}{2}+$, respectively. From this information and the observed $W(\theta)$ it can be determined that, if the gamma ray arose from the 3.13-MeV level, the required values are $|\delta_p| \leq 0.5$ and $0.82 \leq \delta_s \leq 6.1$. However, it has already been shown that $\delta_s = 0.22$ or -3.1 for the decay of the 3.13-MeV level. The possibility that the 2.99-MeV gamma ray arises from a decay of the 4.19-MeV level to the 1.27-MeV level can be eliminated by a similar argument.

4.43-MeV Level

The angular distribution of the 4.21-MeV primary transition to this level was measured at the 1392-keV resonance with the result $W(\theta) = 1 + (0.12 \pm 0.05)P_2$. This result is in agreement with any of $\frac{3}{2}+$, $\frac{5}{2}+$, $\frac{7}{2}+$, and $\frac{9}{2}-$ for the 4.43-MeV level.

5.01-MeV Level

A relatively strong transition to this level occurs at the 1489-keV ($\frac{3}{2}+$) resonance. From pulse-height spectra taken at this resonance it was shown (I) that the 5.01-MeV level decays to the ground state and to the 1.27-MeV level with relative intensities 70% and 30%, respectively. The primary transition and the $5.01 \rightarrow 1.27$ transition both have an energy of about 3.75 MeV at this resonance. The angular distribution of the resulting 3.75-MeV peak in the spectrum can be analyzed in a manner analogous to that discussed above for the 4.26-MeV level. In this case,

$$B_k = 0.77A_k(1) + 0.23A_k(2), \quad (k=2, 4).$$

Experimentally,

$$B_2 = -0.27 \pm 0.04 \quad \text{and} \quad B_4 = -0.05 \pm 0.06.$$

If the 5.01-MeV level were $\frac{1}{2}-$, $\frac{3}{2}-$, or $\frac{5}{2}-$, the expected values would be $B_2 = -0.39$, $+0.32$, or -0.14 , respectively, and $B_4 = 0$ in each case. $J \geq \frac{7}{2}$ need not be considered because of the strong transition to the ground state. Hence, J^π is $\frac{1}{2}+$, $\frac{3}{2}+$, or $\frac{5}{2}+$ for the 5.01-MeV level. For appropriately chosen mixing ratios, an analysis of the data using diagrams similar to those in Fig. 11 shows that all three remaining assignments are possible. The correlation is found to be relatively insensitive in each case to values of δ_s as expected, but the possible values of δ_p are strictly limited by the data. For

$\frac{1}{2}+$ they are $\delta_p = -0.08 \pm 0.03$ or 2.2 ± 0.2 ; for $\frac{3}{2}+$, $\delta_p = 0.60 \pm 0.07$ or $2.4 \leq \delta_p \leq 11$; and for $\frac{5}{2}+$, $-0.45 \leq \delta_p \leq 0.03$ or $|\delta_p| \geq 14$. Considering the small possible values of δ_p for the $\frac{1}{2}+$ and $\frac{5}{2}+$ cases, it would seem that a $\frac{3}{2}+$ assignment is less likely than the other two. On the other hand, $\frac{5}{2}+$ may be considered unlikely because of the observation of a direct primary to the 5.01-MeV level from the 620-keV ($\frac{1}{2}+$) resonance by Broude *et al.* Hence, the 5.01-MeV level is most likely $\frac{1}{2}+$, however, $\frac{3}{2}+$ is definitely not excluded by the data. These results are consistent with those of Hoogenboom who finds $J = \frac{1}{2}$ or $\frac{3}{2}+$ for the 5.01-MeV level.²⁸

SUMMARY AND DISCUSSION

The total angular momentum and the parity for several resonance states and of the intermediate states at 3.41 and 4.26 MeV in P^{31} have been determined by the measurement of angular correlations in the $\text{Si}^{30}(p,\gamma)P^{31}$ reaction. Previous assignments for several states in P^{31} have been confirmed. Some emphasis has been placed in this study upon the determination of $M1-E2$ mixing ratios for electromagnetic transitions in P^{31} . It has been possible in many cases to determine unique values of $E2/M1$ amplitude ratios from the angular distribution data alone, and in other cases by a combination of angular-distribution and triple-correlation measurements. The level assignments and $E2/M1$ ratios for both primary and secondary transitions are summarized in Table XII.

It can be seen that the ratios, where uniquely determined, for primary transitions are generally small as expected. The ratios for primary transitions to the 3.13-MeV level from the $\frac{3}{2}+$ resonances at 1400- and 1489-keV are, however, unusually large. From the measurements of Broude *et al.* at the $\frac{3}{2}+$, 500-, and 1000-keV (983-keV on our scale) resonances, it can be seen that the $E2$ component in the transitions to the 3.13-MeV level must also be unusually large at these resonances. It will be interesting to see if an unusually large $E2$ component occurs at other resonances in this reaction.

It was discovered during the course of this work that the form of the angular-correlation functions rigorously exclude the removal of ambiguities in the secondary $E2/M1$ ratios from the $\frac{3}{2}+$, 3.13-, and 3.51-MeV levels by any combination of angular-distribution and triple-correlation measurements for the reaction used. This was, in fact, found to be true, in general, for cases in which the intermediate state has $J_2 \leq \frac{3}{2}$.^{25,29} The reason for this limitation is that the co-

²⁸ P. M. Endt and C. Van der Leun, Nucl. Phys. 34, 1 (1962).

²⁹ Note added in proof. After the acceptance of this paper for publication, a preprint of a similar study of P^{31} by H. Van Rinsvelt and P. B. Smith was received by the authors. They have independently discovered this theorem and present a rigorous proof in an Appendix. They stress that the limitation imposed by the theorem strictly applies only to ground-state transitions from the decaying level, i.e., to transitions which are not followed up in a cascade.

TABLE XII. Summary of primary and secondary $E2/M1$ amplitude ratios (δ_p and δ_s) for gamma transitions in P³¹. The indicated errors are those which correspond to the standard deviations in the angular-distribution and triple-correlation measurements. Some of the indicated values are weighted averages of several determinations.

Initial State		Final State	δ_p	δ_s
Res. (keV)	E_x (MeV) ^a	(MeV)		
	1.27(3/2+)	G.S.(1/2+)		-0.28±0.02
	3.13(3/2+)	G.S.(1/2+)		0.22±0.02 or -3.0±0.2
	3.29(5/2+)	1.27(3/2+)		0.52±0.05
	3.51(3/2+)	G.S.(1/2+)		0.41±0.02 or -7.5±1.4
		1.27(3/2+) ^b		-0.22±0.04 or $ \delta_s \geq 11.4$
	4.19(5/2+) ^c	1.27(3/2+)		-0.25±0.03 or -1.7±0.4
	4.26(3/2+)	1.27(3/2+)		0.10 ≤ δ_s ≤ 2.8
1177(7/2+)	8.42	2.23(5/2+)	-0.03±0.02	
		3.29(5/2+)	-0.05±0.02	
		4.19(5/2+)	-0.11±0.03	
1204(5/2+)	8.45	1.27(3/2+)	0.02±0.03	
		2.23(5/2+)	-0.16±0.07	
		3.29(5/2+)	-0.01±0.05	
		3.51(3/2+)	-0.11±0.03	
		4.26(3/2+)	$ \delta_p \leq 0.26$	
1324(5/2+)	8.57	1.27(3/2+)	-0.01±0.01	
		2.23(5/2+)	0.09±0.05	
		3.13(3/2+)	0.04±0.05	
		3.51(3/2+)	-0.18±0.02	
		4.26(3/2+)	-0.60 ≤ δ_p ≤ 0.05	
1392(5/2+)	8.64	1.27(3/2+)	0.04±0.02	
		2.23(5/2+)	0.03±0.04	
		3.29(5/2+)	0.13±0.06	
		3.51(3/2+)	-0.09±0.09	
		4.19(5/2+)	≈ -0.6	
1400(3/2+)	8.65	1.27(3/2+)	-0.09±0.02	
		2.23(5/2+)	0.28±0.05 or 1.85±0.20	
		3.13(3/2+)	3.7 ± 0.3	
		3.29(5/2+)	-0.10±0.06 or 5.6 ≤ δ_p ≤ 14.3	
		4.19(5/2+)	0.29 ≤ δ_p ≤ 1.8	
1480.5(3/2+)	8.73	G.S.(1/2+)	-0.04±0.01 or 1.91±0.05	
1482(3/2)	8.73	G.S.(1/2+)	-0.02±0.02 or 1.83±0.07 ^d	
1489(3/2+)	8.74	1.27(3/2+)	-0.01±0.03 or -3.6±0.4	
		2.23(5/2+)	0.21±0.04	
		3.13(3/2+)	-4.0 ± 1.5	
		3.29(5/2+)	0.08±0.05	
1509(5/2+)	8.75	1.27(3/2+)	-0.25±0.02	
		2.23(5/2+)	-0.09±0.05	
		3.41(7/2+)	0.13±0.02	

^a The excitation energy E_x for the resonance levels are based upon the Q value of 7.286 MeV for the Si³⁰(p, γ)P³¹ reaction.
^b Assumes the cascade decay of the 3.51-MeV level involves the 1.27-MeV level and not the 2.23-MeV level. See discussions of 3.51-MeV level in text.
^c Values of δ_s for this transition are from Ref. 7.
^d If the parity of the 1482-keV resonance is odd then $\delta_p = 0$.

efficients (A_{KMN}/A_{000})(δ_p, δ_s) of the angle functions $P_K^N(\theta_1)P_M^N(\theta_2) \cos N\phi$ in the general expansion (geometry unspecified) of the triple correlations either all have the same (quadratic) dependence upon δ_s , or are independent of δ_s . The functional dependence upon δ_s is also the same as that for the corresponding intermediate-unobserved angular distribution case. These ambiguities can, in general, of course, be removed by a combination of direction-correlation and polarization measurements, as has been done, for example, for the 1.27($\frac{3}{2}+$) → G.S.($\frac{1}{2}+$) transition discussed in this paper. A comparison of the polarization results of McCallum²⁰ with the angular correlation results of the present work lead to the unique value of δ_s shown in Table XII. It is now evident that direction-correlation measurements alone are not sufficient to uniquely determine the δ_s values for transitions from the 3.13, 3.51 and other $\frac{3}{2}+$ levels of P³¹.

The initial assumption in this work that multipolarity

mixings are of the form $M1-E2$ rather than $E1-M2$ have led to the result that the weak $J = \frac{7}{2}$ resonance at $E_p = 1177$ keV has even parity and, hence, is formed by g -wave protons. Since the g -wave penetrability is in this case smaller by a factor of about 10^3 than that for the d -wave protons which form neighboring $\frac{3}{2}+$ or $\frac{5}{2}+$ resonances, whereas that for f -wave protons which correspond to odd parity is only about 24 times smaller, an odd parity for the 1177-keV resonance would seem more likely. The data, however, contradict an odd-parity assignment unless perhaps $E1-M2$ combinations are important in this case or the "stripping" analysis of the angular-distribution spectra contained a systematic error. In that event the 1177-keV resonance would correspond to the lowest energy, odd-parity state in P³¹ yet observed.

As mentioned earlier, detailed calculations of level positions and electromagnetic transitions in P³¹ have been performed elsewhere using the framework of the

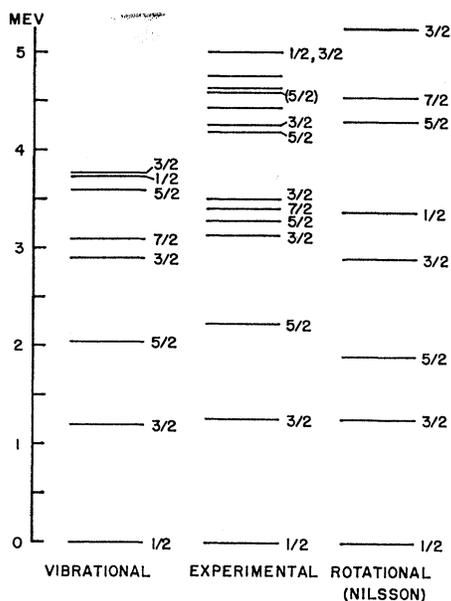


FIG. 12. Comparison of the experimental level scheme of P^{81} with the results of calculations based upon the weak-coupling vibrational model (Ref. 26) and the Nilsson (rotational) model [L. L. Green (private communication)]. No odd-parity levels have been observed and none are predicted in the excitation-energy range shown. The $5/2$ assignment for the 4.59-MeV level is from Ref. 28 in which brackets are used to indicate that the assignment is uncertain.

unified model. Calculations based upon a weak-coupling vibrational model^{5,26} have met with some success in reproducing experimental level positions. A particular success would seem to be the prediction of a $7/2^+$ level below 4 MeV which is identified at 3.41-MeV in the present work. The model also, however, predicts a $1/2^+$ level below 4 MeV which is not observed. Earlier speculations that the 3.51-MeV level may be a $3/2^+$, $1/2^+$ doublet are not supported by more recent data. The only

definite experimental indication so far of a $1/2^+$ bound state other than the ground state is the observation that the 5.01-MeV level is either $1/2^+$ or $3/2^+$, where $1/2^+$ is the more likely. The decay properties and lifetimes of low-lying levels calculated from this model are found to be in qualitative agreement with observation for an appropriate choice of the free parameter k of the model.²⁶

Early calculations of level positions from the Nilsson (rotational) model appeared to reproduce the positions of the first 6 excited states accurately, but predicted the 3.41-MeV level to be $1/2^+$.⁷ More recent calculations have revised considerably the level scheme predicted by the Nilsson model, and the agreement with observation is now not so satisfactory.⁴ The experimental level scheme is compared with the latest results of both models in Fig. 12. The better agreement for the weak-coupling model may indicate that in the language of a unified model of nuclear structure in the region $A \approx 30$, the fairly well established rotational effects near $A = 25$ may be becoming less important and, in fact, giving way to a vibrational mechanism in the formation of excited states. It is evident that further experimental and theoretical study of the region between $A = 30$ and $A = 40$ where major neutron and proton shells are filled at Ca^{40} would be interesting.

ACKNOWLEDGMENTS

The authors are indebted to D. V. Breitenbecher who participated in all phases of the measurements at the Aeronautical Research Laboratories, to Carl Peck and W. A. Anderson for their assistance in the measurements and checking of angular-correlation calculations, and to others mentioned in paper I who also assisted with work discussed here. We also take pleasure in acknowledging the many useful discussions with Dr. F. W. Prosser at the University of Kansas, and the comments and suggestions of Dr. H. E. Gove and others at Chalk River.