Spin, Hyperfine Structure, and Nuclear Magnetic Dipole Moment of O^{15}†

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The nuclear spin and hfs splitting of $O^{15}(\tau_{1/2}=124$ sec) in the $^3P_2$ atomic ground state have been determined by the atomic beam magnetic resonance method. O^{15} was produced in the reaction $N^{14}(d,n)O^{15}$ by allowing a 5-MeV deuteron beam from the Columbia Van de Graaff accelerator to impinge upon a gaseous N_2 target. The radioactive gas flowed continuously from the target chamber into the microwave discharge source of the atomic beam apparatus. A rotating-wheel deposition detector specially suited for short-lived radioactive atoms was used to detect the beam. The spin is found to be $\frac{1}{2}$, in agreement with the shell-model prediction. The hfs splitting of O^{15} in the $^3P_2$ state is observed to be 1037.23 ± 0.07 Mc/sec, and the nuclear magnetic dipole moment of O^{15} is deduced to be 0.7189 ± 0.0008 nm.

A. INTRODUCTION

THE magnetic dipole moments of mirror nuclei are of unusual interest in nuclear physics, especially when the moments of both members of a mirror pair have been determined. This is because of the "mirror" theorem,[1] according to which the meson contributions to the magnetic moments of a pair of mirror nuclei are equal and opposite. Thus, in the sum of the moments the net meson current contribution vanishes. A comparison of the sum of experimentally determined moments with the sum of the moments calculated from a theoretical model may, therefore, shed more light on the problems of nuclear structure than a similar comparison of individual moments.

The mirror theorem was first applied to the nuclear moments of He^3 and H^3. It is now accepted that the primary contributions to the deviations of the nuclear moments of He^3 and H^3 from the moments of the neutron and proton, respectively, arise from meson exchange currents.[1,2] It can be seen from Table I that these deviations are very nearly equal and opposite, in accord with the mirror theorem. A similar situation applies for heavier mirror pairs, although, in general, even the sum of the moments of a mirror pair will deviate considerably from that computed on a single-particle model, since the nuclei cannot be described by pure single-particle wave functions. Nevertheless, for the mass numbers $A=15$, 17, 39, and 41, where from the point of view of the shell model, there are closed shells plus or minus one nucelon, the single-particle model should yield more accurate predictions of the sum of the magnetic moments. This is also the case for nuclei in $p_{1/2}$ states.[3]

Using the atomic beam magnetic resonance method, we have verified that the nuclear spin of O^{15} is $\frac{1}{2}$ by observation of weak-field Zeeman transitions in the $^3P_2$ atomic ground state. (See Figs. 1 and 2.) We have measured the zero-field hfs splitting, $\Delta\nu(O^{15}, {}^3P_2)$, from which the dipole coupling constant, $A(O^{15},{}^3P_2)$, is found after applying a small second-order hfs correction. The nuclear magnetic dipole moment of O^{15}, $\mu(O^{15})$, is then determined to within an uncertainty caused by the O^{15}–O^{17} hyperfine anomaly from a knowledge of $A(O^{15},{}^3P_2)$, $A(O^{17},{}^3P_2)$,[4] and $\mu(O^{17})$.[5] The zero-field splitting in O^{15} was obtained by measuring the frequency of the transition $(F=\frac{5}{2}, m_F=\frac{1}{2}) \leftrightarrow (F=\frac{3}{2}, m_F=-\frac{1}{2})$ in a weak magnetic field and applying the Zeeman correction. In order to make the Zeeman correction, the magnetic field was determined by observing the two-quantum Zeeman transition, $(m_J=1) \leftrightarrow (m_J=-1)$, in the metastable $^3S_1$ state of the helium atom.[6]

A description of the experimental method is given in Sec. B. Following this is a summary of results in Sec. C and a brief disucssion of their significance in Sec. D.

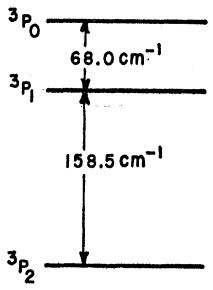

FIG. 1. Fine structure of the $(1s^2,2s^2,2p^4)$ ^3P oxygen atom.

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[1] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1954), Chap. 9.

[2] N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951).

[3] D. Kurath, Phys. Rev. **124**, 552 (1961).

[4] S. Harvey (private communication). See also, R. A. Kamper, K. R. Lea, and C. D. Lustig, Proc. Phys. Soc. (London) **B70**, 897 (1957).

[5] We have used the value of $\mu(O^{17})$ given in N. F. Ramsey, *Molecular Beams* (Oxford University Press, London, 1956), p. 173. Ramsey has corrected the result of F. Alder and F. C. Yu, Phys. Rev. **81**, 1067 (1951) for diamagnetic shielding and for an improved value of the proton moment.

[6] Previous reports on this research are: E. D. Commins and H. R. Feldman, Bull. Am. Phys. Soc. **6**, 73 (1961); H. R. Feldman and E. D. Commins, *ibid.* **7**, 476 (1962).

TABLE I. Some properties of odd-A mirror nuclei. Ar^{37}, Ca^{41}, and Sc^{43} are radioactive and decay to Cl^{37}, K^{41}, and Ca^{43}, respectively. Parentheses ( ) around spin assignments indicate that the spin has not been measured directly. All data are obtained from D. Strominger, J. M. Hollander, and G. T. Seaborg, Rev. Mod. Phys. **30**, 585 (1958), except where a different source is indicated by footnote.

Parent	Daughter	Decay	Maximum energy (MeV)	Half-life	Parent spin parity	Daughter spin parity	Parent moment (nm)	Daughter moment (nm)
n	p	β^-	0.78	13 min	1/2+	1/2+	−1.91	2.79
H^3	He^3	β^-	0.018	12 yr	1/2+	1/2+	2.98	−2.12
Be^7	Li^7	E.C.		53 day	(3/2−)	3/2−		3.26
C^{11}	B^{11}	β^+	0.98	20 min	3/2−	3/2−	[a]	2.69
N^{13}	C^{13}	β^+	1.19	10 min	1/2−	1/2−	−0.32[b]	0.70
O^{15}	N^{15}	β^+	1.68	120 sec	1/2−	1/2−	0.72[c]	−0.28
F^{17}	O^{17}	β^+	1.76	66 sec	(5/2+)	5/2+		−1.89
Ne^{19}	F^{19}	β^+	2.2	18.5 sec	1/2+	1/2+	(−)1.89[d]	2.63
Na^{21}	Ne^{21}	β^+	2.5	22 sec	(3/2+)	3/2+		−0.66
Mg^{23}	Na^{23}	β^+	2.95	12 sec	(3/2+)	3/2+		2.22
Al^{25}	Mg^{25}	β^+	3.2	7.6 sec	(5/2+)	5/2+		−0.86
Si^{27}	Al^{27}	β^+	3.6	4.1 sec	(5/2+)	5/2+		3.64
P^{29}	Si^{29}	β^+	3.9	4.5 sec	(1/2+)	1/2+		±0.56
S^{31}	P^{31}	β^+	4.3	3 sec	(1/2+)	1/2+		1.13
Cl^{33}	S^{33}	β^+	4.2	2.8 sec	(3/2+)	3/2+		0.64
Ar^{35}	Cl^{35}	β^+	4.96	1.8 sec	(3/2+)	3/2+		0.82
K^{37}	Ar^{37}	β^+	5.1	1.2 sec				
Ca^{39}	K^{39}	β^+	5.5	1.0 sec	(3/2+)	3/2+		0.39
Sc^{41}	Ca^{41}	β^+	4.94	0.87 sec	(7/2−)	7/2−		−1.60[e]
Ti^{43}	Sc^{43}	β^+						

[a] See Ref. 16.
[b] M. Posner, J. Snider, A. Bernstein, and D. R. Hamilton, Phys. Rev. Letters **7**, 173 (1961).
[c] Result from this paper.
[d] E. D. Commins and D. A. Dobson, Phys. Rev. Letters **10**, 347 (1963).
[e] E. Brun, J. J. Kraushaar, W. L. Pierce, and W. J. Veigele, Phys. Rev. Letters **9**, 166 (1962).

B. EXPERIMENTAL METHOD

The experimental problems were divided into those associated with: (1) production of O^{15} and its transport in a suitable form from the production site to the experimental apparatus; (2) formation of an *atomic* beam of O^{15}; (3) detection of rf transitions induced in the atoms of the beam.

1. Production of O^{15}

O^{15} was produced in the reaction $N^{14}(d,n)O^{15}$. The 5-MeV deuteron beam from the Columbia Van de Graaff accelerator was allowed to impinge upon a gas target containing about 150 Torr of N_2 and a few Torr of NO. The target assembly is illustrated in Fig. 3. The bombardment chamber was a tantalum-lined, nickel tube 20 cm long with an inside diameter of 1.25 cm. The target gas flowed continuously through the bombardment chamber, entering from a storage tank through $\frac{1}{4}$-in.- copper tubing and leaving through a needle valve which served to reduce the pressure to a few Torr. After passing through the needle valve, the gas flowed through about twenty feet of $\frac{1}{4}$-in.-copper tubing into the source of the atomic-beam apparatus. The target gas was separated from the vacuum in the Van de Graaff accelerator by a molybdenum foil about 0.00063 cm thick, on the target side of which platinum had been evaporated. Other foil thicknesses, materials (W, Ta, Ni, stainless steel, Al), and coatings (none, C, Au, Ag) were tried, but no other type of foil was as durable at operating deuteron beam currents. The thinnest molybdenum foil obtainable was used in order to minimize loss of deuteron energy and scattering of the deuteron beam. The platinum coating served to reduce chemical corrosion of the foil which took place at the high temperature caused by the beam. Early in the experiment many foil ruptures occurred due to local heating at the site of penetration of the concentrated deuteron beam. This difficulty was greatly reduced by installing a "beam wobbler," shown to the left of the foil in Fig. 3. It consists of a pair of electrodes,

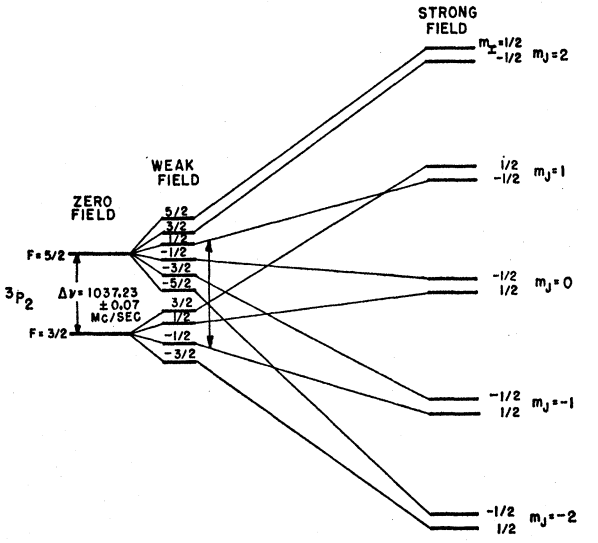

FIG. 2. Schematic representation of the hyperfine structure and Zeeman effect of the $^3P_2$ ground state of the O^{15} atom. The double arrows indicates the $\Delta F=1$ transition observed.

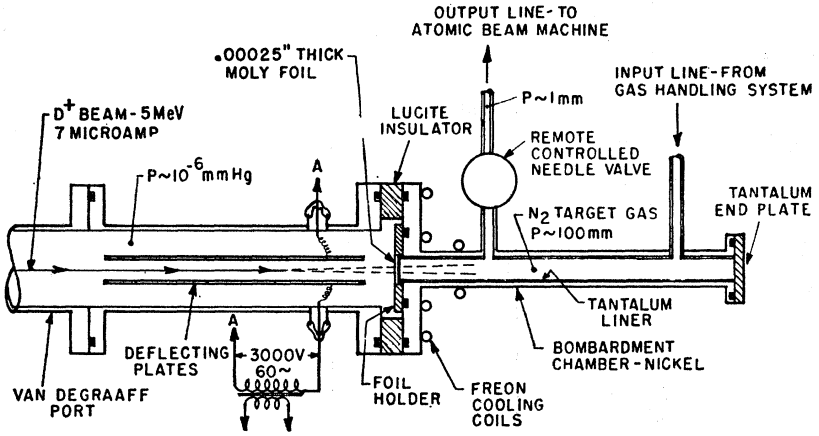

FIG. 3. Target assembly.

with faces parallel to the beam and to each other, to which an alternating voltage was applied in order to spread the (time-averaged) deuteron beam over a larger area of the foil. The tantalum liner in the bombardment chamber served to minimize the neutron background from deuterons striking the walls. The small admixture of NO in the target gas was found to be necessary if a substantial portion of the O^{15} was not to be lost before reaching the beam apparatus. A plausible explanation of this requirement is that the O^{15}, formed as a free atom, is likely to stick to a wall unless there is sufficient density of some component in the gas with which it can combine to form a stable molecule. The reaction $NO+O \rightarrow NO_2$ may be presumed to have taken place in the bombardment chamber and to have prevented the O^{15} atoms from being lost to the walls.

We make an order of magnitude estimate of the rate of production of O^{15} by assuming a mean cross section, $\sigma \cong 10^{-25}$ cm^2 for the reaction $N^{14}(d,n)O^{15}$ over the range of energy the deuterons have as they traverse the target (roughly 4.5–3.5 MeV). We find

$$N = \ln\sigma(I/e) \approx 10^9 \text{ sec}^{-1},$$

where N is the number of O^{15} nuclei produced per second, L is the length of the bombardment chamber (20 cm), n is the density of N^{14} nuclei in the target gas (10^{19} cm^{-3}), I is the deuteron beam current (7 μA), e is the charge on the deuteron (1.6×10^{-19} C). The above result is consistent with our observations.

2. Dissociation of O^{15}-Bearing Molecules and Formation of the Atomic Beam

Molecules bearing O^{15} (presumably mostly NO_2 with some NO and O_2) were carried continuously out of the bombardment chamber along with the target gas to the atomic beam source. A Geiger counter was situated in the connecting tube so that the flux of radioactive gas could be measured. This counting rate was used to normalize the atomic beam intensity during the experiment. The atomic beam source consisted of a quartz discharge tube 0.55 cm in diameter with a flat closed end, in which was ground a rectangular source slit 0.005 cm wide and 0.5 cm high. The pressure of the gas flowing through the discharge tube was 1 to 2 Torr. The last 4 cm of the tube were situated at a voltage antinode of a rectangular resonant cavity, as shown in Fig. 4. The cavity was excited at 2460 Mc/sec by a magnetron. The microwave discharge thereby maintained in the flowing gas caused dissociation of molecules bearing O^{15}. We estimate that about 10% of the O^{15} present in the discharge was rendered atomic. The same discharge tube source assembly was used to excite metastable helium atoms to form the calibrating beam for magnetic field measurements. Its design is similar to that employed by Davis *et al.*[7]

3. The Atomic-Beam Apparatus

The atomic-beam magnetic resonance apparatus was of the conventional "flop-in" type. Specifically, with reference to Fig. 2, it was designed to refocus at the detector plane only those atoms which had undergone a transition in the "C" field such that the sign of their high magnetic field magnetic dipole moment was reversed $(m_J=+1) \leftrightarrow (m_J=-1)$. The fractional solid angle of acceptance by the detector aperture for the total beam was about 10^{-5}.

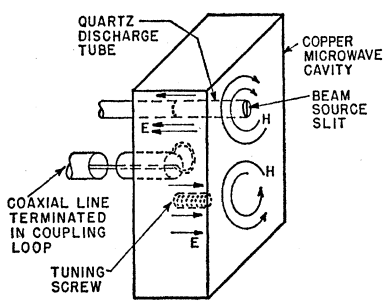

FIG. 4. Microwave discharge source.

[7] L. Davis, B. T. Feld, C. W. Zabel, and J. R. Zacharias, Phys. Rev. **76**, 1076 (1949).

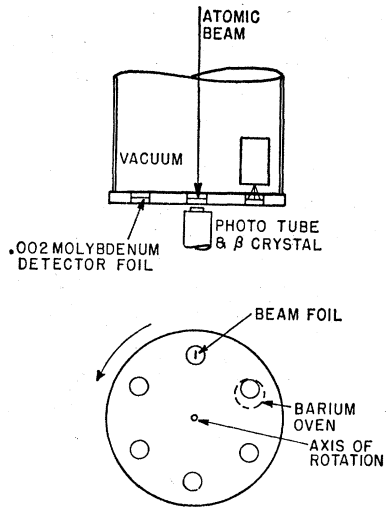

FIG. 5. O^{15} deposition detector.

The only novel feature of the apparatus was the detector. Because of the short half-life of O^{15} it was necessary to design a detector which would enable us to observe decay of O^{15} nuclei *in situ*, that is, without withdrawing detector buttons from the vacuum system, as is a more usual procedure in radioactive atomic beam work.[8] The following design proved, after many trials, to be an efficient and reliable one. A brass disk formed the end plate of the detector chamber. It could be rotated on a vacuum seal about an axis 6 cm below the beam, as shown in Fig. 5. Six equally spaced slots were milled radially in the disk with their centers on a circle of radius 6 cm, and over these were sealed molybdenum foils of thickness 0.005 cm. The disk was rotated so that each foil in turn was placed in the path of the atomic beam. When all six foils had been used so that foil one was again moved into position to receive the beam, the number of O^{15} nuclei deposited on it had been reduced by a factor of about 500 due to the short half-life ($\tau_{1/2}=124$ sec) of O^{15}. With one foil in position to receive the beam, the foil next in line was situated opposite an oven containing metallic barium. The oven could be heated to evaporate barium on the adjacent foil at a controlled rate. When this foil was rotated into the beam position, it possessed a thin, fresh coat of barium which acted as a very efficient sticking surface for the incoming atoms. Our data are consistent with a sticking coefficient very close to 1. For equal normalized beam intensities, as determined by the Geiger counter in the source flow line, there was usually no variation of the counting rate, beyond statistical fluctuations, which depended upon which of the six foils was being used. In addition to barium, we tried calcium, potassium, and magnesium, but none equalled barium for reliability and sensitivity. A conventional plastic scintillation crystal-photomultiplier unit was placed immediately behind the beam foil outside the vacuum envelope and remained in position as the foils were rotated. Positrons emitted in the direction of the scintillator by O^{15} atoms decaying on the barium surface penetrated the barium layer and the molybdenum foil with relatively small energy loss and produced scintillations in the plastic. The resulting pulses were fed through a pulse amplifier and into a scaler in the usual way.

The total count during the ith foil exposure, Q_i, was the sum of three contributions:

$$Q_i=S_i(\nu)+B_i+C_i. \tag{1}$$

$S_i(\nu)$ was the part of Q_i resulting from the decay of O^{15} atoms which were refocused due to resonance reorientation in the rf field of frequency ν. B_i was the part of Q_i resulting from the decay of unrefocused O^{15} which reached the detector despite the presence of a stop-wire blocking the direct beam and despite differential pumping of the magnet chamber. C_i was the part of Q_i resulting from radiation which originated in the Van de Graaff accelerator and target. C_i also included a very small contribution due to all other background radiation.

For each foil exposure, i, two quantities were measured: Q_i and the counting rate in the source flow line, G_i. Every third or fourth foil exposure was taken with rf off so that for these exposures $S_0(\nu)=0$ and $Q_0=B_0+C_0$, where we have used the subscript, 0, to indicate that the rf was off. During the measurements of the Zeeman resonances the value of $S_i(\nu)$ for a given exposure was found by taking

$$S_i(\nu)=Q_i-(B_i+C_i)_{\text{int}}, \tag{2}$$

where $(B_i+C_i)_{\text{int}}$ is the value of (B_i+C_i) found by interpolation between the values of Q_0. The normalized signal $S'(\nu)$ was then given by

$$S_i'(\nu)=S_i(\nu)/G_i. \tag{3}$$

This method of handling the data has the disadvantage of neglecting the values of G measured when the rf was off. Consequently, the following changes were made in handling the data on the $\Delta F=1$ transition:

(1) It was assumed that the value of B_i was proportional to G_i,

$$B_i=KG_i. \tag{4}$$

This assumption seemed plausible since the incoming gas had only a very small radioactive component. We could, therefore, expect that the fraction of radioactive gas scattered into the detector was independent of G_i. K, then, was assumed to depend only upon the conditions in the source discharge and the vacuum system.

(2) An approximate value of K for any given run was found from

$$\bar{Q}_0=\bar{C}+K\bar{G}, \tag{5}$$

where $\bar{Q}_0$ was the average of all values of Q_0 measured during the run, $\bar{C}$ was a single value of C measured before any radioactive gas was allowed to flow into the

[8] J. P. Hobson, J. C. Hubbs, W. A. Nierenberg, and H. B. Silsbee, Phys. Rev. **96**, 1450 (1954).

apparatus, and $\bar{G}$ was the average of all values G_i and G_0.

(3) C_0 was found for each foil exposure with rf off from

$$Q_0 = KG_0 + C_0.$$

C_i was then obtained for each foil exposure with rf on by interpolation between the values of C_0.

(4) The values of $S_i(\nu)$ were then found from

$$S_i(\nu) = Q_i - KG_i - C_i, \tag{6}$$

and the normalized values of the signal, $S_i'(\nu)$, were obtained from Eq. (3).

Efforts were made to minimize B and C by careful design of the vacuum system and shielding of the detector, but they could not be reduced below a rather troublesome minimum. Typically, for normal gas flow and for a deuteron beam current of 7 μA at 5 MeV, B and C were each approximately 500 counts in 3 min, the usual "exposure time" for each foil. With deflection magnets off and stop-wire removed, the total beam was approximately 20 000 counts in 3 min. However, the number of counts due to refocused beam, $S(\nu)$, arising from the hfs resonance, was always much smaller than B and C. In most of our observations $S(\nu)$ was about 100 counts in 3 min at the peak of the resonance.

The above procedure of interpolation served to correct for slow changes in deuteron beam intensity, radioactive gas flow rate, etc., but could not correct for rapid changes, and these introduced fluctuations in $S'(\nu)$ which were larger than those expected on purely statistical grounds. These fluctuations resulted in a signal-to-noise ratio barely greater than one.

C. RESULTS

Observation of low-field Zeeman transitions provided verification that the nuclear spin of O^{15} is $\frac{1}{2}$. Two-quantum transitions were induced between the states $(F=\frac{5}{2}, m_F=\frac{1}{2})$ and $(F=\frac{5}{2}, m_F=-\frac{3}{2})$ and also between the states $(F=\frac{3}{2}, m_F=\frac{3}{2})$ and $(F=\frac{3}{2}, m_F=-\frac{1}{2})$ (see Fig. 2). The observed result:

$$\nu(F=\tfrac{5}{2}) \approx 0.6\nu(\mathrm{He},{}^3S_1)$$
$$\nu(F=\tfrac{3}{2}) \approx 0.9\nu(\mathrm{He},{}^3S_1)$$

is consistent only with spin assignment $I=\frac{1}{2}$. Figure 6 shows a typical set of data.

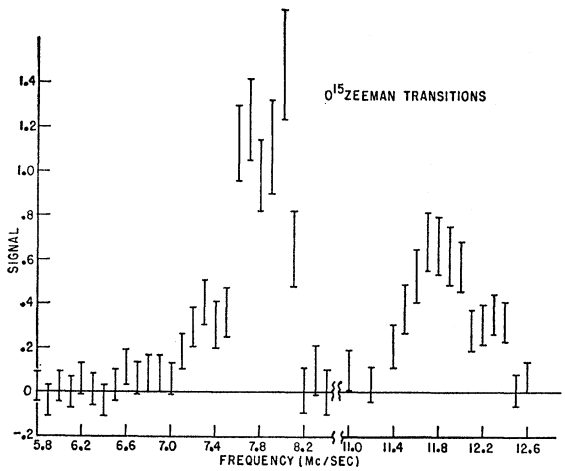

FIG. 6. Typical set of data on the Zeeman transitions in the $^3P_2$ ground state of O^{15}. The error flags indicate statistical uncertainty only.

Observations of the $F=\frac{5}{2}$ Zeeman resonance were next carried out in a magnetic field of about 80 G in order to obtain an estimate of $\Delta\nu$ from the quadratic Zeeman effect. In this field the electronic angular momentum and nuclear spin of O^{15} are partially decoupled. The energy levels of the $F=\frac{5}{2}$, $^3P_2$ state may be expressed by the formula:

$$W(\tfrac{5}{2},m_F) = W_0 + \frac{g_J(\mathrm{O},{}^3P_2)}{g_J(\mathrm{He},{}^3S_1)} m_F \nu_{\mathrm{He}} + \frac{\Delta\nu}{2}(1 - 0.8 m_F x + x^2)^{1/2}, \tag{7}$$

where W_0 is a constant, ν_{He} is the measured He^4 resonance frequency, and

$$x \equiv [g_J(\mathrm{O},{}^3P_2) - g_I(\mathrm{O}^{15})](\mu_0 H)/(h\Delta\nu) = \frac{[g_J(\mathrm{O},{}^3P_2) - g_I(\mathrm{O}^{15})]}{g_J(\mathrm{He},{}^3S_1)\Delta\nu} \nu_{\mathrm{He}}.$$

For the present purposes we can use the approximate value for $g_I(O^{15})$ given by:

$$g_I(\mathrm{O}^{15}) \approx \frac{2}{5} \frac{g_I(\mathrm{O}^{17})}{A(\mathrm{O}^{17},{}^3P_2)} \Delta\nu. \tag{7a}$$

The values of $A(O^{17},{}^3P_2)$, $g_I(O^{17})$, $g_J(O,{}^3P_2)$, and $g_J(He,{}^3S_1)$ are known.[4,5,9] All gyromagnetic ratios (including nuclear ones) are defined with a minus sign, as is usual for electronic gyromagnetic ratios. The estimate obtained, based on three measurements of the O^{15} transition, was $\Delta\nu(O^{15}) = 1040 \pm 20$ Mc/sec.

A search was then made for the transition $(F=\frac{5}{2}, m_F=\frac{1}{2}) \leftrightarrow (F=\frac{3}{2}, m_F=-\frac{1}{2})$ at weak magnetic field. This is the only $\Delta F=1$ transition resulting in a refocused beam in our apparatus (see Fig. 2). After the $\Delta F=1$ resonance was found, 33 observations of it were made at various magnetic fields up to about 8 G. Corroboration of our identification of the line was obtained by confirming the expected dependence of the resonant frequency on the magnetic field. Figure 7 shows one of the better sets of data taken on this transition.

In order to plot data taken at different magnetic

[9] C. Drake, A. Lurio, V. W. Hughes, and J. A. White, Bull. Am. Phys. Soc. 3, 7 (1958); H. E. Radford and V. W. Hughes, *ibid.* 3, 8 (1958).

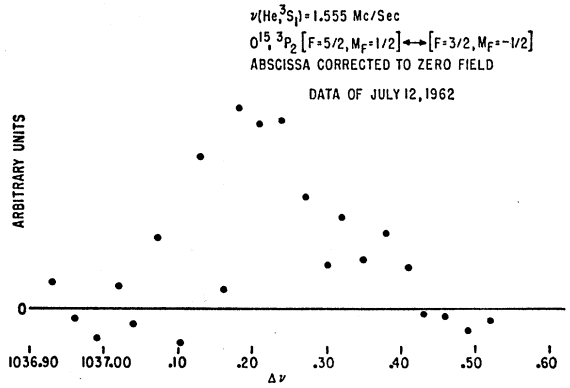

FIG. 7. A good set of data on the $\Delta F=1$ transition in the $^3P_2$ ground state of O^{15}.

fields on the same graph (although the data of Fig. 7 were all taken at the same field), the field-dependent part of the transition frequency was subtracted from all data before plotting. Thus, the abscissa in Figs. 7 and 8 is defined as the applied frequency minus the field-dependent part of the O^{15} transition frequency (which was known from the measurement of the He, $^3S_1$ Zeeman resonance). The ordinate is $S'(\nu)$ (see Sec. B).

Usually the signal-to-noise ratio was too small for the data to yield unambiguous results unless several foil exposures were made at each frequency. Figure 8 shows a plot of all the data taken on the $\Delta F=1$ transition. The measurements within frequency intervals corresponding to a 20 kc/sec spread in $\Delta\nu$ were averaged. The linewidth taken from Fig. 8 is consistent with the linewidth we usually observed in the He transition. The broadening is due to the severely inhomogeneous field present in our "C" magnet. The result of these measurements is

$$\Delta\nu(O^{15},{}^3P_2)=1037.23\pm0.07 \text{ Mc/sec}.$$

Our stated error has been extracted by inspection from Fig. 8. It is about 5 times the statistical error one would obtain by assigning a resonant frequency to each series of points traversing the line.

The magnetic dipole hyperfine interaction constant, $A(O^{15},{}^3P_2)$, may be expressed as

$$A(O^{15},{}^3P_2)=[\Delta\nu(O^{15},{}^3P_2)+\Delta']\cdot\tfrac{2}{5}, \tag{8}$$

where Δ' is a second-order hfs interaction correction (between the $F=\frac{3}{2}$ levels of the $^3P_2$ and $^3P_1$ states) (see Figs. 1 and 2). Δ' is calculated from second-order perturbation theory and is given simply by

$$\Delta'=-\frac{|M|^2}{W(^3P_1)-W(^3P_2)}, \tag{9}$$

where $W(^3P_1)-W(^3P_2)$ is the fine-structure splitting between the $^3P_2$ and $^3P_1$ states, and M is the matrix element of the magnetic dipole hyperfine interaction between the $F=\frac{3}{2}$ levels of the $^3P_1$ and $^3P_2$ states. The minus sign in Eq. (9) arises from the assumption that the $F=\frac{5}{2}$ level is more energetic than the $F=\frac{3}{2}$ level in the $^3P_2$ state $[A(^3P_2)>0]$. M is found from

$$\begin{aligned}M&=\langle J,I,F|H_{\text{dipole}}|J-1,I,F\rangle\\&=-a_J'\times\tfrac{1}{2}[(F+J-I)(F+I-J+1)\\&\qquad\times(F+I+J+1)(I+J-F)]^{1/2},\end{aligned} \tag{10}$$

where a_J' is defined by Bessis *et al.*[10] and is found from

$$\frac{a_2'(O^{15})}{a_2'(O^{17})}\approx\frac{g_I(O^{15})}{g_I(O^{17})}. \tag{11}$$

Using $a_2'(O^{17})=-126.6\pm1.4$ Mc/sec as measured at Oxford[4] and the value of $g_I(O^{15})$ obtained from Eq. (7a), we obtain $\Delta'=-46$ kc/sec. If we calculate M directly, following the method of Trees[11] and using Hartree-Fock wave functions[12] to obtain $\langle r^{-3}\rangle_{\text{av}}$, we obtain the value, $\Delta'=-60$ kc/sec, in reasonable agreement with the above. Using the value, $\Delta'=-46$ kc/sec, Eq. (8) yields

$$A(O^{15},{}^3P_2)=414.87\pm0.03 \text{ Mc/sec}.$$

Neglecting effects due to hyperfine anomaly, we obtain $\mu(O^{15})$ from the equation

$$\frac{\mu(O^{15})}{I(O^{15})A(O^{15},{}^3P_2)}=\frac{\mu(O^{17})}{I(O^{17})A(O^{17},{}^3P_2)}. \tag{12}$$

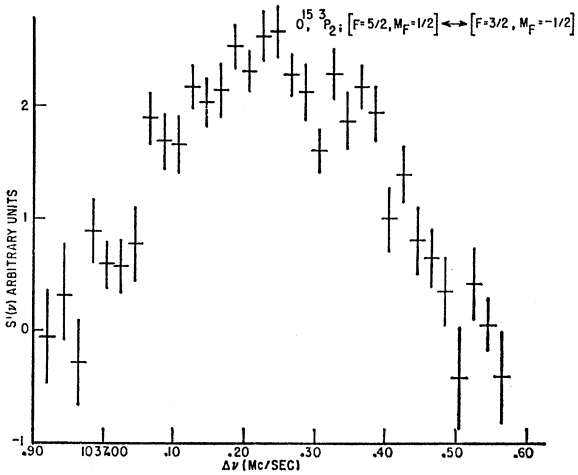

FIG. 8. A plot of all data taken on the $\Delta F=1$ transition in the $^3P_2$ ground state of O^{15}. The horizontal bars indicate the range of measured frequency over which the data were averaged to obtain a given point on the graph. The vertical bars indicate the statistical uncertainty. The frequency at which each observation was made was corrected for Zeeman effect prior to averaging and plotting so that results obtained at different external magnetic fields could be displayed on one graph.

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The result is

$$\mu(O^{15})=0.7189 \text{ nm},$$

where we have used $\mu(O^{17})=-1.89370\pm0.00009$ nm[5] and $A(O^{17},{}^3P_2)=-218.569\pm0.010$ Mc/sec.[4] The major source of uncertainty in this value of $\mu(O^{15})$ is a possible $O^{15}-O^{17}$ hyperfine anomaly. We have reason to think that this hyperfine anomaly should not be exceptionally large, however. The primary contribution to hyperfine anomaly, as discussed by Bohr and Weisskopf[13] arises from the fact that the magnetic dipole moments of the two nuclei may result in different proportion from the orbital and intrinsic moments of their nucleons. In our case, this contribution is zero to the extent that the neutrons do not have any orbital moment and the eight protons do not have any net orbital angular momentum.[14] Some hfs anomaly should be present due to the different distributions of intrinsic moment in O^{15} and O^{17}, as well as to the fact that there may be a small orbital moment which contributes a different fraction of the total moment in the two nuclei. The hfs anomaly due to different distributions of nuclear charge (Breit-Rosenthal effect[15]) which modify the electronic wave functions differently at small radii is always less than a part in 10^4 and should be particularly small in the case of $O^{15}-O^{17}$ since the closed shell of eight protons is a very stable configuration. There is an additional reason to expect the hyperfine anomaly to be small, for the atomic electronic configuration is $(1s^2,2s^2,2p^4)$ so that in the nonrelativistic limit the (unpaired) electron density at the nucleus is zero. In view of these facts, an upper limit of 0.1% on the $O^{15}-O^{17}$ hyperfine anomaly would seem conservative. We then arrive at the value

$$\mu(O^{15})=0.7189\pm0.0008 \text{ nm}.$$

D. DISCUSSION

From Table I it can be seen that, at present, the magnetic moments of both members of a mirror pair are known in four cases: $A=3$, 13, 15, and 19.[16] These results, together with the corresponding Schmidt values, are summarized in Table II. If we assume the validity of the mirror theorem with respect to mesonic contributions to the magnetic moments, we can explain a discrepancy between the sum of the experimental moments and the sum of the Schmidt moments only by concluding that the nuclear wave function is not a pure single-particle shell-model wave function or by concluding that the Schmidt moments are not correctly calculated

TABLE II. Magnetic moments (in nm) of mirror pairs of nuclei. (μ_E is the experimental moment, μ_S the Schmidt single-particle moment.) References for the moments are the same as in Table I.

Nucleus		μ_E	μ_S	$(\mu_E-\mu_S)$
He^3		−2.127	−1.913	−0.214
H^3		2.979	2.793	0.186
	Sums	+0.852	+0.880	−0.028
C^{13}		0.702	0.638	0.064
N^{13}		−0.321	−0.264	−0.057
	Sums	+0.381	+0.374	+0.007
N^{15}		−0.283	−0.264	−0.018
O^{15}		0.719	+0.638	+0.080
	Sums	+0.436	+0.374	+0.062
F^{19}		2.627	2.793[a]	−0.166
Ne^{19}		−1.886	−1.913[a]	+0.027
	Sums	+0.741	+0.880	−0.139

[a] Upon the assumption that the odd nucleon is in an $s_{1/2}$ state.

single-particle moments. Jensen and Mayer have made the suggestion[17] that the single-particle moment should be modified, when the unpaired nucleon is charged, due to the large spin-orbit interaction in nuclei. Drell and Walecka[18] have listed estimates of this effect for N^{15} and F^{19} made by Jensen and Mayer[17] and also by Marty.[19] In both cases the effect has the appropriate sign and the right order of magnitude for bringing the sum of the calculated moments of the $A=15$ and $A=19$ mirror pairs into agreement with the sum of the experimentally observed moments. These estimates are based on the assumption of a pure $p_{1/2}$ state for $A=15$ and a pure s state for $A=19$. There is strong evidence, however, that the F^{19} nucleus does not have such a simple wave function,[20,21] and analysis of the case $A=19$ is thereby complicated. Also, if we wish to explain the deviation of the sum of the $A=15$ moments, we must explain at the same time the fact that the sum of the $A=13$ moments shows almost no deviation, whereas the Jensen-Mayer effect should contribute approximately equally in N^{13} and N^{15}. In addition, there is evidence[22] that the single-particle shell model is not a very good approximation even for nuclei close to the "doubly magic" O^{16}. Consequently, it seems to us that these deviations, none larger than 0.14 nm, are too small to be susceptible to useful analysis at this stage in our knowledge of nuclear structure.

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Optical Double Resonance in Radioactive Atoms: Spin and Nuclear Moments of the Ground State of Cd^{109}†

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The optical double-resonance technique has been employed for the study of the Zeeman effect and hyperfine structure of the $(5s5p)\ ^3P_1$ state of 470-day Cd^{109}. The nuclear spin, I, magnetic hyperfine interaction constant, A, and quadrupole interaction constant, B, are: $I=\frac{5}{2}$, $A=-1$, 148.6 ± 2.0 Mc/sec, and $B=-167.3\pm2.0$ Mc/sec. If nuclear structure and quadrupole antishielding corrections are neglected, the corresponding nuclear moments are $\mu=-0.8286(15)\mu_N$ and $Q=+0.78(10)$ b. These values are discussed in terms of the configuration mixing model of Arima and Horie. A number of the problems encountered in the application of the double resonance technique to radioactive atoms are discussed. The sensitivity of the method is limited by the shot noise in the instrumentally scattered light. In the case of the present apparatus, Zeeman resonances are detectable with 2×10^6 cadmium atoms in the vapor phase.

I. INTRODUCTION

THE optical double-resonance technique as suggested by Brossel and Kastler[1] and Brossel and Bitter[2] is well suited for the study of the Zeeman effect and hyperfine structure of the optically excited states of radioactive atoms. Such studies are useful for the determination of nuclear moments that do not produce an interaction in the normal state of the atom. For example, the double-resonance technique has found considerable application in the determination of the quadrupole moments of the alkali atoms.[3–12] Here we report in detail[13] on the application of this method to 470-day Cd^{109}. This isotope and the other Group II elements are characterized by diamagnetic $(ns)^2\ ^1S_0$ electronic ground states that exhibit no hfs. The determination of the nuclear moments of these elements by the well-known radioactive atomic-beam method would be difficult at best. With the double-resonance technique, one can excite and study these atoms in the paramagnetic $(nsnp)\ ^3P_1$ state. This state exhibits both a magnetic-dipole and an electric-quadrupole interaction. The method is also clearly applicable to many other elements. This is the first report on a series of double-resonance measurements that we have in progress on the cadmium and zinc isotopes. For this reason, many details and general relations are given that will be of interest to the entire series of measurements.

With important exceptions, many of the atomic-beam studies that have been made of nuclear moments have been restricted to the case of odd-proton nuclei simply because such atoms exhibit paramagnetic ground states. The application of the double-resonance method to even-proton elements such as the alkaline earths and the rare gases will provide complimentary data on odd-neutron nuclei. It is particularly useful to obtain the magnetic and quadrupole moments for a series of isotopes of a given element so that the effect of adding

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