$Be^9(Li^7,Li^8)Be^8$ Reaction in a Coulomb-Distorted Wave Approximation*

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The angular distributions of the $Be^9(Li^7,Li^8)Be^8$ reaction, measured between 2 and 4 MeV by Norbeck *et al.*, are compared with a theory for the differential cross section which has been derived for (d,p) reactions at low bombarding energies on heavy nuclei, trivially modified to apply to the reaction under consideration. The theory uses Coulomb wave functions to describe the relative motion in the initial and final states of the system. The only parameter is a normalization constant which is determined by comparison with experiment at one energy. The theory is in good agreement with experiment at forward angles, and it is suggested that refinements in experimental technique may cause better agreement at back angles. Results for the classical theory of the reaction as given by Allison are also presented for comparison.

INTRODUCTION

THE angular distribution for the reaction $Be^9(Li^7,Li^8)Be^8$ has been measured by Norbeck *et al.*[1] at bombarding energies between 2 and 4 MeV. It was subsequently suggested by Allison[2] that the mechanism of the reaction can be visualized in a classical manner, by combining pure Coulomb scattering with a probability for neutron pickup by the Li^7 projectile during the scattering, and a simple equation for the angular distribution of the reaction was proposed. The theory was based on several features of the reaction: (1) Bombarding Be^9 with 2-MeV Li ions corresponds to $\eta_i=3.54$ and $\eta_f=3.10$, where η_i and η_f are the initial and final Coulomb parameters for the reaction. In general,

$$\eta=ZZ'e^2/\hbar v\,, \tag{1}$$

where Ze and $Z'e$ are the electric charges of the initial or product pair of nuclei, and v is the corresponding initial or final relative velocity of the pair. Since η is also equal to one half the ratio of the classical distance of closest approach to the (reduced) de Broglie wavelength of the system's reduced mass, its values are sufficiently large in this case to give validity to the classical picture of the interacting particles moving in hyperbolic orbits. (2) The binding energy of the last neutron in Be^9 is small (1.63 MeV) which results in a nonnegligible probability of finding the neutron at distances quite far from the "Be^8 core." (3) The Q value of the reaction is small (0.37 MeV), and the mass of the neutron is small in comparison with the mass of the Li^7 projectile.

Although the classical theory is appealing in its simplicity, it is apparently of limited applicability precisely because of those rather special conditions on which it is based. It is the intent of this note to show that the theory of the $Be^9(Li^7,Li^8)Be^8$ reaction can be put on a quantum mechanical basis by making use of an approximation which has been developed for use in interpreting (d,p) reactions at low bombarding energies on heavy nuclei.[3–5] The approximation requires only that the values of the initial and final Coulomb parameters for the reaction be substantially larger than unity, and thus suggests its applicability to a larger class of reactions than was possible with the classical theory.

THEORETICAL DISCUSSION

In the case of deuteron bombardment under a condition corresponding to $\eta\gg1$, an expression has been derived for the stripping amplitude of a (d,p) process by means of first-order perturbation theory, whose use is known to be justified under this condition.[3] The resulting matrix element, involving the interaction potential between the neutron and proton in the deuteron, is just what would be used in a distorted-wave Born approximation if it were assumed that the sole distortion of the deuteron and proton waves was due to a pure Coulomb field. It was further shown[3,4] that the matrix element can be approximated by (neglecting spin)

$$F\propto\frac{1}{2\pi}\frac{\mu_f}{\hbar^2}\int[\psi_{\mathbf{k}_f}{}^{(-)}(\mathbf{r})\Phi_n(\mathbf{r})]^*\psi_{\mathbf{k}_i}{}^{(+)}(\mathbf{r})d\mathbf{r}\,, \tag{2}$$

where the proportionality constant depends only on properties of the internal wave function for the deuteron and μ_f is the reduced mass of the proton-residual nucleus final configuration. $\psi_{k_i}{}^{(+)}$, $\psi_{k_f}{}^{(-)*}$ are Coulomb wave functions describing in the center-of-mass system the incoming deuteron and outgoing proton, with associated wave numbers $\mathbf{k}_i$ and $\mathbf{k}_f$, and are given explicitly by

$$\begin{aligned}\psi_{\mathbf{k}_i}{}^{(+)}&=\Gamma(1+i\eta_i)\exp(-\tfrac{1}{2}\pi\eta_i+i\mathbf{k}_i\cdot\mathbf{r})\\&\quad\times{}_1F_1[-i\eta_i,1,i(k_ir-\mathbf{k}_i\cdot\mathbf{r})],\\\psi_{\mathbf{k}_f}{}^{(-)*}&=\Gamma(1+i\eta_f)\exp(-\tfrac{1}{2}\pi\eta_f-i\mathbf{k}_f\cdot\mathbf{r})\\&\quad\times{}_1F_1[-i\eta_f,1,i(k_fr+\mathbf{k}_f\cdot\mathbf{r})].\end{aligned} \tag{3}$$

Φ_n is the bound-state wave function of the transferred neutron in the residual nucleus. Outside the range R of

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[1] E. Norbeck, J. M. Blair, L. Pinsonneault, and R. J. Gerbracht, Phys. Rev. **116**, 1560 (1959).
[2] S. K. Allison, Phys. Rev. **119**, 1975 (1960).
[3] K. A. Ter-Martirosian, Zh. Eksperim. i Teor. Fiz. **29**, 713 (1955) [translation: Soviet Phys.—JETP **2**, 620 (1956)].
[4] L. C. Biedenharn, K. Boyer, and M. Goldstein, Phys. Rev. **104**, 383 (1956).
[5] W. D. Barfield, B. M. Bacon, and L. C. Biedenharn, Phys. Rev. **125**, 964 (1962).

the nuclear forces in the residual nucleus Φ_n is proportional to $h_L^{(1)}(ik_nr)Y_L^m(\theta,\phi)$ for capture of the neutron with $L\hbar$ units of angular momentum, and magnetic quantum number m. $k_n=(2\mu_n|E_n|/\hbar^2)^{1/2}$, where μ_n, E_n are the reduced mass and binding energy of the neutron in the residual nucleus.

For the special case of $L=0$, we approximate the integral in Eq. (2) by

$$\int_0^\infty r^2dr\int d\Omega\,\psi_{\mathbf{k}_i}{}^{(+)}\psi_{\mathbf{k}_f}{}^{(-)*}h_0{}^{(1)*}, \tag{4}$$

which can be calculated in closed form.[3] For $\eta\gg1$ the Coulomb wave functions are exponentially small in the region $r\leq R$ and the exterior solution $h_0^{(1)}$ for the neutron wave function may be extended in to the origin with negligible error.[5] The result for the differential cross section for the proton emerging at the center-of-mass angle θ is

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{L=0}\propto\frac{k_f}{k_i}|F|^2$$
$$=A_0N(E)(1-x)^{-2}|{}_2F_1(-i\eta_i,\,-i\eta_f,\,1;\,x)|^2, \tag{5}$$

where

$$x=\frac{-4k_ik_f\sin^2(\tfrac{1}{2}\theta)}{k_n^2+(k_i-k_f)^2}$$

and ${}_2F_1$ is the hypergeometric function. $N(E)$ is independent of θ, and contains most of the energy dependence of the cross section:

$$N(E)=\frac{k_f\eta_i\eta_f\exp[-2\pi\eta_f-2(\phi_i\eta_i-\phi_f\eta_f)]}{k_i[1-\exp(-2\pi\eta_i)][1-\exp(-2\pi\eta_f][k_n^2+(k_i-k_f)^2]^2},$$

where

$$\tan\phi_i=\frac{2k_ik_n}{k_i^2-k_f^2-k_n^2},\quad 0\leq\phi_i\leq\pi,$$
$$\tan\phi_f=\frac{2k_fk_n}{k_i^2-k_f^2+k_n^2},\quad 0\leq\phi_f\leq\pi.$$

All remaining factors independent of angle and energy have been combined in the single normalization constant A_0.

To apply this result to the $Be^9(Li^7,Li^8)Be^8$ reaction we replace the deuteron and proton by Be^9 and Be^8, respectively, changing only the wave numbers and Coulomb parameters to correspond to the new situation. While the theory nominally will then describe the differential cross section for the scattering of Be^8 at an angle θ with respect to the incident direction of Be^9, this is, of course, exactly equal in the center-of-mass system to the differential cross section for scattering Li^8 at the angle θ, where θ is now measured with respect to the incident direction of Li^7, and the latter conforms to the manner in which the experiment was done. The formal replacement should be accompanied by several comments. The essential fact used in the derivation of Eq. (2) is that the range r_0 of the nuclear potential binding the neutron in the deuteron projectile is much less than the classical distance of closest approach $r_{\min}$ of projectile and target. For the Be^9 nucleus the corresponding r_0 is still only a few fermis, while $r_{\min}$ for a Li^7-Be^9 collision at 2 MeV is 15 F. Secondly, Eq. (5) pertains to a neutron initially in an S state captured into an S state, whereas the ground-state transition under consideration involves the transfer of a neutron having $L=1$ in both initial and final states. This difference should not be a serious defect, however, as it has been shown[3–5] that under conditions of large η, the angular distribution is not sensitive to the L value of the captured nucleon, in contrast to the situation which exists at higher bombarding energies.

It will be observed that the structure of expression (4) is not consistent with the reciprocity theorem for nuclear reactions. That is, if one considers the inverse reaction, $Li^8(Be^8,Be^9)Li^7$, it is known from very general principles that the angular dependence of its cross section must be exactly the same as the original reaction. But the previous formalism is applicable to the inverse reaction also, and it is evident that the angular dependence of the cross section will be again given by expression (4), except that the value of k_n which occurs is now related to the binding energy of the neutron in Be^9 rather than in Li^8. This circumstance is completely analogous to the situation which exists in the plane-wave Born approximation for stripping: while it can be shown[6] that the matrix element which corresponds to the interaction being taken in the initial channel is equivalent to that obtained from considering the interaction in the final channel, the results obtained from the two approaches differ because of the additional approximations which are made in the evaluation of the matrix element in both cases. From a practical standpoint this difficulty is not of much consequence in the particular application under consideration. Equation (5) does not seem to depend very sensitively on k_n, and the two values are sufficiently close that the predicted differential cross section is changed by less than 15% at all angles and energies, if one k_n is replaced by the other.

The hypergeometric function in Eq. (5) was evaluated through its power series expansion for $|x|<1$, and the appropriate analytic continuation formula for $|x|>1$,[7]

[6] E. Guerjuoy, Phys. Rev. **91**, 645 (1953).

[7] A. Erdelyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions* (McGraw-Hill Book Company, Inc., New York, 1955), Vol. I, p. 108.

making use of the Control Data Corporation 1604 computer at the University of Minnesota.[8] The results were compared with those from an asymptotic expression for $|{}_2F_1|^2$ given in Ref. 3, and no significant differences between the two were noted. There has recently appeared an approximate expression for (2) which may be more convenient to use than (5) as it does not involve the hypergeometric function.[9]

An interesting alternative to the approach utilized in this paper has been advanced by Greider.[10] Whereas our method is equivalent to the conventional one which regards the incoming projectile wave as becoming distorted by the Coulomb field of the target and considers the nuclear interaction of the neutron bound in the projectile as the "mechanism" for the reaction, Greider formulates the problem in such a fashion that the Coulomb field is directly associated with the reaction mechanism. The latter method is presumably applicable to any reaction in which the Coulomb interaction clearly dominates. In an exact calculation the two methods must, of course, be equivalent. Greider shows that his approach gives good agreement with a neutron exchange reaction involving N^{14} ions at 32-MeV bombarding energy. The present calculation shows that it is possible to obtain substantial agreement with heavy-ion neutron exchange reactions with the more conventional approach as well.[11]

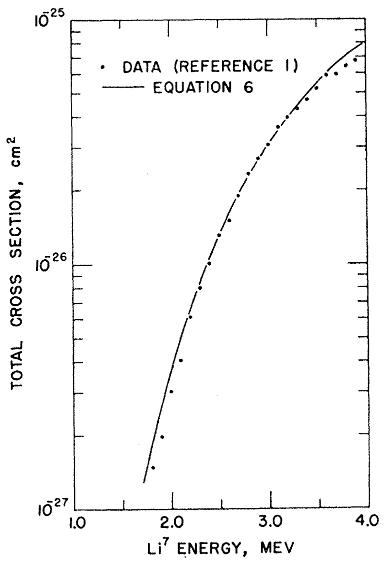

FIG. 1. The excitation function for the $Be^9(Li^7,Li^8)Be^8$ reaction. The curve is the theoretical total cross section Eq. (6) normalized at 3 MeV (lab energy) to the experimental data of Ref. 1.

EXPERIMENTAL COMPARISON

The normalization constant A_0 was determined by numerically integrating the theoretical differential cross section (5) to obtain a theoretical total cross section σ,

$$\sigma = 2\pi \int_0^{\pi} \left(\frac{d\sigma(\theta)}{d\Omega}\right)_{L=0} \sin\theta d\theta\,, \tag{6}$$

which was then normalized to the experimentally determined total yield curve for the reaction. Figure 1 shows the theoretical curve for σ normalized at 3 MeV to the experimental data.

The angular distributions obtained in Ref. 1 were presented as relative yields in laboratory coordinates. In order to compare these results with Eq. (5) it was first necessary to integrate numerically the experimental angular distributions and compare the result with the experimental total yield curve for the purpose of assigning absolute values to the experimental points. The resulting absolute laboratory differential cross sections were then converted to center-of-mass differential cross sections by the usual methods. Figures 2–4 show typical comparisons of the experimental data and theory. The results of the classical theory of Allison are also presented here for comparison. The classical theory is expected to most valid at forward angles and low energies; consequently the normalization for the latter was deter-

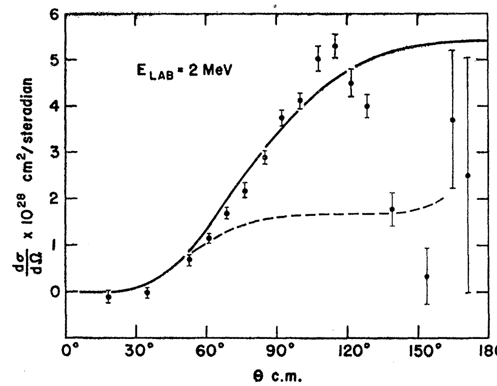

FIG. 2. The angular distribution for the $Be^9(Li^7,Li^8)Be^8$ reaction at 2 MeV (lab energy) in center-of-mass coordinates. Experimental points are from Ref. 1. The solid curve is Eq. (5). The dashed curve is the classical result from Ref. 2.

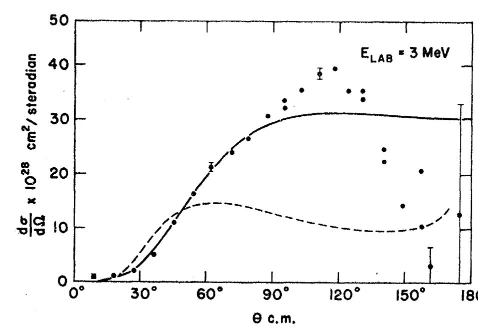

FIG. 3. The angular distribution for the $Be^9(Li^7,Li^8)Be^8$ reaction at 3 MeV (lab energy) in center-of-mass coordinates. For explanation of curves refer to caption of Fig. 2.

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[8] Dr. R. K. Hobbie and L. Pinsonneault assisted in the programming of the hypergeometric function.
[9] R. H. Lemmer, Nucl. Phys. **39**, 680 (1962).
[10] K. R. Greider, Phys. Rev. Letters **9**, 392 (1962).
[11] The author is grateful to Professor M. Bolsterli for several helpful discussions of the method of Greider.

mined by arbitrarily normalizing to the forward angle experimental data at 2 MeV.

The apparent disagreement between the quantum mechanical theory and experiment at back angles is greatly exaggerated by the very large conversion factor which enters in the transformation from laboratory to center-of-mass coordinates. To emphasize this fact we have re-expressed in Fig. 5 the data and theoretical curves of Fig. 2 in laboratory rather than center-of-mass coordinates. Because the detailed behavior of the theoretical result at back angles tends to be obscured when expressed in laboratory coordinates, the majority of the results have been presented in center-of-mass coordinates.

The possibility cannot be excluded that better agreement could result with improvement of the experimental techniques. The rather thick target which was used (approximately 400 keV for a 2-MeV Li ion) suggests the possibility that some of the Li^8 was not produced with enough kinetic energy to enable it to emerge from the target and be detected. For reactions induced with 2-MeV Li^7 ions, the energy of Li^8 changes from 680 to 40 keV as the laboratory angle of observation changes from 70° to 180°. In addition, although the above analysis assumes that Li^8 is produced in its ground state, it is energetically possible for first-excited-state production to occur as well, and the experiment did not separate these two contributions to the cross section. It has been estimated that first excited state Li^8 contributes between 10 and 15% to the yield at forward angles at 3.3 MeV.[12] It would be desirable to eliminate as far as possible these several experimental uncertainties to better evaluate the degree of success of the theory.

CONCLUSION

It is natural to speculate that Eq. (5) may have some applicability in reactions involving transfer of charged particles, as well as the case of neutron transfer for which it was specifically developed. This is in analogy with the neglect of the proton's charge in the Butler theory of deuteron stripping, which formally puts (d,p) and (d,n) reactions on the same footing. A study of several (Li^7,He^6) reactions is being planned at the University of Minnesota Van de Graaff Laboratory which it is hoped will test this hypothesis.

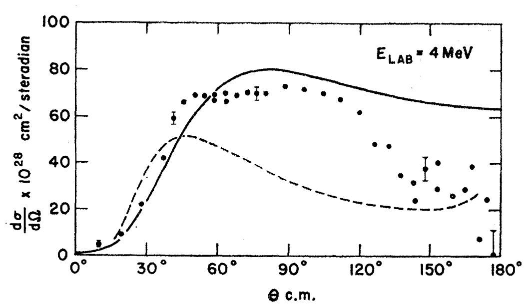

FIG. 4. The angular distribution for the $Be^9(Li^7,Li^8)Be^8$ reaction at 4 MeV (lab energy) in center-of-mass coordinates. For explanation of curves refer to caption of Fig. 2.

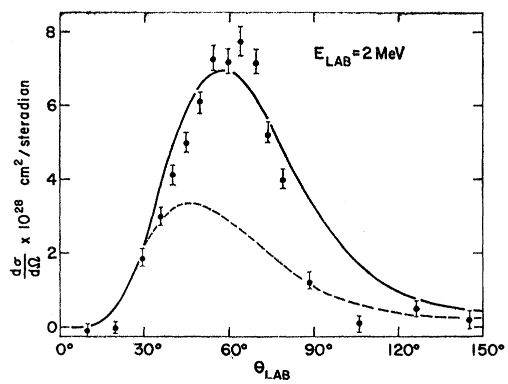

FIG. 5. The angular distribution for the $Be^9(Li^7,Li^8)Be^8$ reaction at 2 MeV (lab energy) in laboratory coordinates. For explanation of curves refer to caption of Fig. 2.

It should be noted that the tendency to backward peaking exhibited by the theoretical cross section in Figs. 2–4 is a persistent feature of Eq. (5) under large formal changes in the Q value and binding energy of the captured neutron. This suggests that caution should be exercised in qualitative interpretations of angular distributions of reaction products resulting from low energy Li bombardment. If such interpretations are guided by considerations which are true at higher energies (for example, associating a forward peak in an angular distribution with a fragment from the projectile), they may be entirely without basis under conditions of η larger than unity.

Previous attempts[13,14] to calculate angular distributions of Li-induced reactions have used the Butler-type plane wave approach, even though its appropriateness at the low energies considered is questionable. It is hoped that the present calculation will encourage attempts to apply DWBA methods, or possibly the method of Greider, to the increasing number of measured angular distributions of reactions induced by Li ions.

ACKNOWLEDGMENTS

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[12] J. M. Blair (private communication).

[13] J. J. Leigh, Phys. Rev. **123**, 2145 (1961).

[14] M. El Nadi and H. Sherif, Nucl. Phys. **28**, 331 (1961).