PHYSICAL REVIEW VOLUME 131, NUMBER 6 15 SEPTEMBER 1963

Magnetic Field Dependence of the Amplification of Sound by Conduction Electrons

HAROLD N. SPECTOR
Department of Physics, Case Institute of Technology, Cleveland, Ohio
(Received 15 April 1963)

A Boltzmann equation technique is used to calculate the magnetic field dependence of sound, amplified by interaction with conduction electrons in the presence of crossed dc electric and magnetic fields. It is shown that both geometric resonances and cyclotron resonances can be found under conditions of amplification. This occurs when the electron-drift velocity in the crossed fields, v_H, has a component in the direction of propagation of sound which exceeds the sound velocity v_s. The geometric resonances occur under the same conditions as in zero electric field, but the cyclotron resonances are Doppler shifted and occur for $\omega-\mathbf{q}\cdot\mathbf{v}_H=n\omega_c$.

I. INTRODUCTION

RECENT experiments[1–3] have indicated that amplification of sound is possible in semiconductors and semimetals, whenever the drift velocity of the conduction electrons in external fields exceeds the velocity of sound. When we have a dc electric field acting alone, the drift velocity is $\mathbf{v}_d=-(e\tau/m)\mathbf{E}$, while in crossed electric and magnetic fields, the electron drift velocity is $\mathbf{v}_H=c(\mathbf{E}\times\mathbf{H})/H^2$.

A complete theoretical treatment of the amplification in zero dc magnetic field has been given which accounts for the major experimental features of the amplification.[4,5] In the case of finite magnetic field, however, the calculations have been limited to treating the problem in the high-field limit. Dumke and Haering[6] and Hopfield[7] have given phenomenological treatments of the amplification of sound in semimetals and the author[8] has given a like treatment of the amplification in extrinsic semiconductors for crossed electric and magnetic fields. Eckstein[9] has calculated the amplification in the high-field limit as a function of the angle between the drift velocity and the direction of propagation using the Boltzmann equation. However, none of these treatments are valid in the magnetic-field region where geometric resonances and cyclotron resonances occur. It is, therefore, of interest to examine the whole problem of the electron-sound-wave interaction in crossed electric and magnetic fields using a Boltzmann equation treatment which is valid for all magnetic fields in the semiclassical limit.

In Sec. II, we use the model of a free-electron gas developed by Cohen, Harrison, and Harrison[10] for the conduction electrons in a solid, and in general, adopt the formalism used by them. We shall only treat the case of propagation transverse to the external electric and magnetic fields. It is in this case that geometric resonances[11,12] and cyclotron resonances[13] have been observed in the attenuation. In Sec. III, we consider the case of geometric resonances and in Sec. IV, the case of cyclotron resonance. In our concluding section, Sec. V, we give a discussion of the results of our calculation.

II. FORMAL THEORY

A. Constitutive Equation

In the model developed by Cohen, Harrison, and Harrison,[10] the conduction electrons are replaced by the model of a free-electron gas of density N_0. The sound wave of wave vector $\mathbf{q}$ and frequency ω manifests itself as a velocity field, $\mathbf{u}\propto\exp i(\mathbf{q}\cdot\mathbf{r}-\omega t)$, in the background. If we are considering the case of a metal or an extrinsic semiconductor, the electron gas is neutralized by a positive background of the same density N_0. For a semimetal, the background is neutral, and the electrons are neutralized in the absence of the sound wave by an equal number of holes.[14] The formalism developed here can be applied, with some modifications, to either model. The interaction between the electron (and hole) gas and the sound wave can be represented partly by means of a self-consistent internal electromagnetic field and partly by means of a deformation potential. The self-consistent electromagnetic field induced by the passage of the sound wave can be derived from Maxwell's equations. In our case, the latter can be written in the form

$$\mathbf{J}=-\sigma_0\mathsf{B}\cdot\boldsymbol{\mathcal{E}}\,, \tag{2.1}$$

where $\mathbf{J}$ and $\boldsymbol{\mathcal{E}}$ are the total current and electric field accompanying the sound wave and B is the diagonal tensor,

$$\mathsf{B}=i[\beta\mathbf{I}-(\beta+\gamma)\hat{q}\hat{q}]. \tag{2.2}$$

Here, $\hat{q}$ is a unit vector in the direction of propagation,

[1] A. R. Hutson, J. H. McFee, and D. L. White, Phys. Rev. Letters **7**, 237 (1961).
[2] L. Esaki, Phys. Rev. Letters **8**, 4 (1962).
[3] R. W. Smith, Phys. Rev. Letters **9**, 87 (1962).
[4] G. Weinreich, Phys. Rev. **104**, 321 (1956).
[5] H. N. Spector, Phys. Rev. **127**, 1084 (1962).
[6] W. P. Dumke and R. R. Haering, Phys. Rev. **126**, 1974 (1962).
[7] J. J. Hopfield, Phys. Rev. Letters **8**, 311 (1962).
[8] H. N. Spector, Phys. Rev. **130**, 910 (1963).
[9] S. Eckstein (to be published).
[10] M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. **117**, 937 (1960).
[11] R. W. Morse, H. V. Bohm, and J. D. Gavenda, Phys. Rev. **109**, 1394 (1958).
[12] D. H. Reneker, Phys. Rev. **115**, 303 (1959).
[13] B. W. Roberts, Phys. Rev. Letters **6**, 453 (1961).
[14] M. J. Harrison, Phys. Rev. **119**, 1260 (1960).

σ_0 is the dc conductivity, $\gamma=\omega/\omega_p{}^2\tau$, $\beta=(c/v_S)^2\gamma$, and ω_p is the plasma frequency of the electrons.

The electronic current can be obtained from the distribution function in the usual manner;

$$\mathbf{j}_e=-e\int d\mathbf{v}\,\mathbf{v}f, \tag{2.3}$$

where $\mathbf{j}_e$ is the total electronic current. The Boltzmann equation from which the distribution function is determined in the presence of external electric and magnetic field is

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}-\frac{e}{m}\left(\boldsymbol{\mathcal{E}}_s+\mathbf{E}+\frac{\mathbf{v}\times\mathbf{H}}{c}\right)\cdot\frac{\partial f}{\partial \mathbf{v}}=-\frac{(f-f_s)}{\tau}. \tag{2.4}$$

In (2.4), $\mathbf{E}$ is the dc electric field, $\mathbf{H}$ is the dc magnetic field, $\boldsymbol{\mathcal{E}}_S=\boldsymbol{\mathcal{E}}-i\mathbf{qq}\cdot(\mathbf{C}\cdot\mathbf{u}/e\omega)$ is the effective electromagnetic field arising from the passage of the sound wave, and $\mathbf{C}$ is the deformation potential tensor. The distribution function relaxes, in the presence of the sound wave, to an equilibrium distribution which is centered about the impurity velocity. Also, scattering is local and cannot change the electron density. Therefore,

$$f_s(\mathbf{r},\mathbf{v},t)=f_0(\mathbf{v}-\mathbf{u}(\mathbf{r},t),\,E_F(\mathbf{r},t)), \tag{2.5}$$

where $f_0(\mathbf{v},E_F)$ is the equilibrium Fermi distribution, and $E_F(\mathbf{r},t)$ is the Fermi energy chosen to give the correct electron density.

When we are considering the case of a semimetal, we have an equation for the hole distribution function which is identical to (2.4) except that we replace e by $-e$ and the values of the electron mass, relaxation time, and deformation potential by those for the holes. In the following treatment, we shall assume that the masses, relaxation time, and deformation potentials for the holes and the electrons are equal. It has been shown elsewhere[15] that this assumption should not qualitatively effect our final results.

It has been shown that the Boltzmann equation can be solved by a method due to Chambers.[16] This solution is

$$f(\mathbf{r},\mathbf{v},t)=\int_{-\infty}^{t} f_s(\mathbf{r}',\mathbf{v}',t')e^{-(t-t')/\tau}\frac{dt'}{\tau}. \tag{2.6}$$

Expanding the distribution function to first order in $\mathbf{E}$, $\mathbf{u}$, and terms proportional to $\mathbf{u}$, and keeping terms that are of first order in both $\mathbf{E}$ and $\mathbf{u}$, we have

$$f=f_0+f_1{}^0+f_1{}^1. \tag{2.7}$$

Here f_0 is the unperturbed distribution function, $f_1{}^0$ is that part of the perturbed distribution function that is independent of the sound wave, and $f_1{}^1$ is that part of the perturbed distribution function which varies as $\exp i(\mathbf{q}\cdot\mathbf{r}-\omega t)$. Thus, we obtain

$$f_1{}^0=\frac{\partial f_0}{\partial E}\frac{mv_H\omega_c\tau}{1+(\omega_c\tau)^2}[v_y-\omega_c\tau v_x], \tag{2.8a}$$

$$f_1{}^1=-\int_{-\infty}^{t}dt'\,e^{-(t-t')/\tau}\Bigg\{\left[-e\left(\boldsymbol{\mathcal{E}}_s-\frac{m\mathbf{u}}{e\tau}\right)\cdot\mathbf{v}'+\tfrac{2}{3}E_{F^0}\frac{N_1}{N_0\tau}\right]\left[\frac{\partial f_0}{\partial E}+\frac{\partial^2 f_0}{\partial E^2}m\mathbf{v}_H\cdot(\mathbf{v}'-\mathbf{v})\right]$$
$$-\frac{mv_H\omega_c\tau}{1+(\omega_c\tau)^2}\left[\frac{e}{m}(\mathcal{E}_{sy}-\omega_c\tau\mathcal{E}_{sx})\frac{\partial f_0}{\partial E}+e\mathbf{E}_s\cdot\mathbf{v}'\frac{\partial^2 f_0}{\partial E^2}(v_y'-\omega_c\tau v_x')\right]\Bigg\}, \tag{2.8b}$$

where $\mathbf{v}_H=c(\mathbf{E}\times\mathbf{H}/H^2)$ is the drift velocity of the electrons in the crossed electric and magnetic fields. We have chosen our y axis to be in the direction of $\mathbf{E}$ and the z axis to be in the direction of $\mathbf{H}$. From (2.3), we see that (2.8a) will only contribute to the dc current and, thus, can be neglected in considering the electron-sound-wave interaction. The electronic current which is proportional to the sound wave can be obtained from (2.3) and (2.8b). The desired constitutive equation is

$$\mathbf{j}_e=\boldsymbol{\sigma}\cdot\left(\mathcal{E}_s-\frac{m\mathbf{u}}{e\tau}\right)-\mathbf{R}N_1ev_s+\boldsymbol{\Sigma}\cdot\mathcal{E}_s, \tag{2.9}$$

where

$$\boldsymbol{\sigma}=-e^2\int d\mathbf{v}\mathbf{v}\int_{-\infty}^{t}dt'e^{h(t')}\mathbf{v}'\left[\frac{\partial f_0}{\partial E}+\frac{\partial^2 f_0}{\partial E^2}m\mathbf{v}_H\cdot(\mathbf{v}'-\mathbf{v})\right],$$

$$\mathbf{R}=-\frac{2}{3}\frac{E_{F0}}{N_0v_s\tau}\int d\mathbf{v}\mathbf{v}\int_{-\infty}^{t}dt'e^{h(t')}\left[\frac{\partial f_0}{\partial E}+\frac{\partial^2 f_0}{\partial E^2}m\mathbf{v}_H\cdot(\mathbf{v}'-\mathbf{v})\right], \tag{2.10a}$$

$$\boldsymbol{\Sigma}=\frac{e^2v_H\omega_c\tau}{1+(\omega_c\tau)^2}\int d\mathbf{v}\mathbf{v}\int_{-\infty}^{t}dt'e^{h(t')}\left[(\omega_c\tau\hat{x}+\hat{y})\frac{\partial f_0}{\partial E}+\frac{\partial^2 f_0}{\partial E^2}m\mathbf{v}'(\omega_c\tau v_x'-v_y')\right], \tag{2.10b}$$

$$h(t')=i[\mathbf{q}\cdot(\mathbf{r}'-\mathbf{r})-\omega(t'-t)]+(t'-t)/\tau. \tag{2.10c}$$

[15] H. N. Spector, Phys. Rev. **125**, 1192 (1962).
[16] R. G. Chambers, Proc. Phys. Soc. (London) **A65**, 458 (1952); **A238**, 344 (1957).

The equation of continuity relates the nonuniform part of the electron density N_1 to the current along the direction of propagation; i.e., $\hat{q}\mathbf{j}=-N_1ev_s$. Defining a tensor $\mathbf{R}$ by means of the relation

$$\mathbf{R}\cdot\mathbf{j}_e=\mathbf{R}\hat{q}\cdot\mathbf{j}_e, \tag{2.11}$$

we can rewrite (2.9) in the form

$$\mathbf{j}_e=\sigma_0\boldsymbol{\sigma}'(\boldsymbol{\mathcal{E}}_s-(m\mathbf{u}/e\tau))+\sigma_0\boldsymbol{\Sigma}'\cdot\boldsymbol{\mathcal{E}}_s, \tag{2.12}$$

where

$$\boldsymbol{\sigma}'=[\mathbf{I}-\mathbf{R}]^{-1}(\boldsymbol{\sigma}/\sigma_0),\quad \boldsymbol{\Sigma}'=[\mathbf{I}-\mathbf{R}]^{-1}(\boldsymbol{\Sigma}/\sigma_0). \tag{2.13}$$

If we have holes present as in a semimetal, then we can write an identical constitutive equation for the holes with e replacing $-e$ everywhere in (2.10) and (2.12).

B. The Absorption Coefficient

The quantity of interest experimentally in studying the interaction between the sound wave and the electrons is the absorption (or amplification) coefficient α. This coefficient gives the exponential decay (or growth) of sound intensity with distance. The absorption coefficient is the average power density transferred between the sound wave and the electrons (or holes) per unit energy flux, or

$$\alpha=Q/\tfrac{1}{2}\rho|\mathbf{u}|^2v_s, \tag{2.14}$$

where ρ is the density of the material.

In a semimetal, the net power transferred per unit volume is

$$Q=\tfrac{1}{2}\operatorname{Re}\left\{\mathbf{j}_e^*\cdot\boldsymbol{\mathcal{E}}_s^e+\mathbf{j}_h^*\cdot\boldsymbol{\mathcal{E}}_s^h-\frac{N_0m\mathbf{u}^*}{\tau_e}(\langle\mathbf{v}_e\rangle-\mathbf{u}) -\frac{N_0m\mathbf{u}^*}{\tau_h}(\langle\mathbf{v}_h\rangle-\mathbf{u})\right\}, \tag{2.15}$$

where the subscript e denotes quantities associated with the electrons and h denotes those associated with the holes. The self-consistent electric field arising from the electron and hole currents is

$$-\sigma_0\mathbf{B}\cdot\boldsymbol{\mathcal{E}}=\mathbf{J}_e+\mathbf{J}_h. \tag{2.16}$$

Using (2.15) and (2.16) together with the constitutive Eq. (2.12), one can now calculate α for a semimetal. For semimetals, the forces arising from the deformation potential dominate the electrostatic forces for sound frequencies greater than a megacycle. In fact, it is only in the region where the deformation forces are strong, that we have appreciable interaction between the sound wave and the conduction electrons in materials with low-carrier densities such as semimetals and semiconductors.[17] When the deformation forces do dominate the interaction, we find

$$\alpha=(2N_0m/\rho v_S)(\hat{q}\cdot\mathbf{C}\cdot\hat{\mu}/mv_S^2)^2\omega^2\tau\operatorname{Re}\hat{q}\cdot(\boldsymbol{\sigma}'+\boldsymbol{\Sigma}')\cdot\hat{q}, \tag{2.17}$$

where $\hat{\mu}$ is a unit vector in the direction of polarization of the sound wave.

In an extrinsic semiconductor, the net power transferred per unit volume is

$$Q=\tfrac{1}{2}\operatorname{Re}[\mathbf{J}_e^*\cdot\boldsymbol{\mathcal{E}}_s+(m\mathbf{u}^*/e\tau)(\mathbf{J}_e+N_0e\mathbf{u})]. \tag{2.18}$$

The self-consistent field arising from Maxwell's equations in this case is

$$-\sigma_0\mathbf{B}\cdot\boldsymbol{\mathcal{E}}=\mathbf{J}_e+N_0e\mathbf{u}. \tag{2.19}$$

Substituting (2.12) and (2.19) into (2.18), we can now obtain α for an extrinsic semiconductor. Here, too, the interaction is appreciable only when there are strong deformation forces acting.[18] In this case we have

$$\alpha=\frac{N_0m}{\rho v_s\tau}\left(\frac{\hat{q}\cdot\mathbf{C}\cdot\hat{\mu}}{mv_s^2}\right)^2\left(\frac{\omega}{\omega_p}\right)^4\operatorname{Re}\hat{q}\cdot[\boldsymbol{\sigma}'+\boldsymbol{\Sigma}'+\mathbf{B}]^{-1}\cdot\hat{q}. \tag{2.20}$$

Because $\beta>1$ in extrinsic semiconductors for frequencies greater than 1 Mc/sec, we need calculate only the diagonal component of the conductivity tensors in the direction of propagation to determine α in both semiconductors[19] and in semimetals.

C. The Conductivity Tensor

The coefficient of absorption is now specified in terms of the conductivity tensors $\boldsymbol{\sigma}'$ and $\boldsymbol{\Sigma}'$. The present task is to evaluate explicitly the integral expressions (2.10a)–(2.10c). We first note that for arbitrary $g(\mathbf{v})$

$$\int d\mathbf{v}\left(-\frac{\partial f_0}{\partial_E}\right)g(\mathbf{v})=\frac{3}{8}\frac{N_0}{\pi E_F^0}\int d\Omega\, g(\mathbf{v}_F), \tag{2.21a}$$

$$\int d\mathbf{v}\left(-\frac{\partial^2 f_0}{\partial_E^2}\right)g(\mathbf{v})=-\frac{3}{8\pi}\frac{N_0}{E_F^0mv_F^2}\frac{d}{dv}v\times\int d\Omega\, g(\mathbf{v})|_{v=v_F}, \tag{2.21b}$$

so that $g(\mathbf{v})$ need only be evaluated for $v=v_F$.

We also note that in the expressions for the absorption coefficient (2.17) and (2.20), $\boldsymbol{\sigma}$ and $\boldsymbol{\Sigma}$ occur only in the combination $\mathbf{T}=\boldsymbol{\sigma}+\boldsymbol{\Sigma}$, $\mathbf{T}'=\boldsymbol{\sigma}'+\boldsymbol{\Sigma}'$. Since we are interested primarily in phenomena which occur when the electrons can go along an orbit several times

---

[17] Another case where the interaction between the sound wave and the electrons can be quite large, even when the carrier densities are low, occurs when there is a large piezoelectric effect as in CdS.

[18] H. N. Spector, Phys. Rev. **125**, 1880 (1962).

[19] This can be seen by writing out the tensor appearing in (2.20). We have

$$[\boldsymbol{\sigma}'+\boldsymbol{\Sigma}'+\mathbf{B}]_{xx}^{-1}=\frac{1}{\sigma_{xx}'+\Sigma_{xx}'-i\gamma+\Lambda},$$

where $\Lambda=(\sigma_{xy}'+\Sigma_{xy}')(\sigma_{yx}'+\Sigma_{yx}')/\sigma_{yy}+\Sigma_{yy}'+ik$, and we chose the x axis to be in the $\hat{q}$ direction. When $\beta\gg1$, Λ is negligible compared to the other term in the denominator.

before being scattered, we can use the condition $\omega_c\tau \gg 1$ in evaluating the expression for the tensor **T**.

We choose a coordinate system having the x axis in the direction of **q**, the y axis in the direction of **E**, and the z axis in the direction of **H**. In this coordinate system, the relation between (**r**,**v**) and (**r**′,**v**′) is

$$
\begin{aligned}
v_x' &= v_\perp \cos[\omega_c(t'-t)+\phi] - v_H \cos\omega_c(t'-t) + v_H\,,\\
v_y' &= v_\perp \sin[\omega_c(t'-t)+\phi] - v_H \sin\omega_c(t'-t)\,,\\
v_z &= v_{\parallel}\,,\quad v_\perp = v\sin\theta\,,\quad v_{\parallel} = v\cos\theta\,,\quad v_H = cE/H\,,\\
x' &= x + \frac{v_\perp}{\omega_c}\{\sin[\omega_c(t'-t)+\phi] - \sin\phi\} - \frac{v_H}{\omega_c}\sin\omega_c(t'+t) + v_H(t'-t)\,,\\
y' &= y - \frac{v_\perp}{\omega_c}\{\cos[\omega_c(t'+t)+\phi] - \cos\phi\} - \frac{v_H}{\omega_c}[1-\cos\omega_c(t'-t)]\,,\\
z' &= z + v_{\parallel}(t'-t)\,,
\end{aligned}
\tag{2.22}
$$

where $\omega_c = eH/mc$ is the cyclotron frequency and θ and ϕ are the polar angles of **v**.

The integrals that occur in the expressions for the conductivity tensor are evaluated explicitly in the Appendix. Using the results of the Appendix, we find the following expressions for the components of **T** and **R**:

$$
\begin{aligned}
T_{xx} &= \frac{3\sigma_0}{iql}\left\{ -\frac{V_H}{V_F} + \frac{1}{X}\sum_{n,m=-\infty}^{+\infty} \frac{J_m(XV_H/V_F)}{\lambda + i(n-m)\omega_c\tau}\left[(\lambda n g_n)/(X) - \frac{V_H}{V_F}(1-i\omega\tau)J_{2n}'(2X) \right]\right\},\\
R_x &= \frac{\omega_c}{\omega}\sum_{n,m=-\infty}^{+\infty} \frac{J_m(XV_H/V_F)}{\lambda + i(n-m)\omega_c\tau}\left\{ n g_n(X) - \frac{V_H}{V_F}J_{2n}'(2X)\right\},
\end{aligned}
\tag{2.23}
$$

where $X = qV_F/\omega_c$, $\lambda = 1 - i\omega\mu\tau$, and $\mu = 1 - v_H/v_S$. In deriving (2.23), we have dropped terms that are smaller by a factor of the order of $(v_S/v_F)^2$ and $(v_H/v_F)^2$ than the remaining terms. In semimetals and degenerate semiconductors, the ratio of the sound velocity to the Fermi velocity is of order 10^{-2}. The ratio v_H/v_F is also much less than unity for all attainable electric fields in conducting solids. In any case, our linear Boltzmann equation treatment would no longer be valid if the drift velocity of the electrons became larger than the Fermi velocity. The functions $g_n(X)$ are those defined by Cohen, Harrison, and Harrison[10] in their paper.

The expressions (2.23) can be rewritten in the form

$$
\begin{aligned}
T_{xx} &= \frac{3\sigma_0}{(ql)^2}\left\{ -iqV_H\tau + \lambda - \sum_{n,m=-\infty}^{+\infty} \frac{(J_m XV_H/V_F)[(\lambda - im\omega_c\tau)\lambda g_n(X) + iqV_H\tau(1-i\omega\tau)(J_{2n}'/X)(2X)]}{\lambda + i(n-m)\omega_c\tau}\right\},\\
R_x &= \frac{1}{i\omega\tau}1 - \left\{ \sum_{n,m=-\infty}^{+\infty} \frac{J_m(XV_H/V_F)[(\lambda - im\omega_c\tau)g_n(X) + iqV_H\tau(J_{2n}'/X)(2X)]}{\lambda + i(n-m)\omega_c\tau}\right\}.
\end{aligned}
\tag{2.24}
$$

This was done by noting that

$$
\sum_{n=-\infty}^{+\infty} g_n(x) = 1\,,\quad \sum_{m=-\infty}^{+\infty} J_m(Xv_H/v_F) = 1\,,
$$

and

$$
\sum_{m=-\infty}^{+\infty} mJ_m(Xv_H/v_F) = Xv_H/v_F\,.
$$

The latter two relations are derived in the Appendix. It now only remains to evaluate (2.24) in the regions of interest and to calculate the absorption coefficient α.

III. GEOMETRIC RESONANCES

We expect geometric resonances to occur when the phonon wavelength is of the order of the classical orbit radius, i.e., when X is of order unity. In this case, ω_c is greater than ω (by a factor of order v_F/v_S), $X(v_H/v_F) \ll 1$, and if in addition $|\omega_c\tau/\lambda| \gg 1$, we obtain for the conductivity tensors

$$
\begin{aligned}
T_{xx} &= \frac{3\sigma_0}{(ql)^2}\left\{ \lambda[1-g_0(X)] - iqv_H\tau\left[1 - g_1(X) + \frac{(1-i\omega\tau)}{\lambda}\frac{J_0'(2X)}{X}\right]\right\},\\
R_x &= \frac{1}{i\omega\tau}\left\{ 1 - g_0(X) - \frac{iqv_H\tau}{\lambda}\left[\frac{J_0'(2X)}{X} - g_1(X)\right]\right\}.
\end{aligned}
\tag{3.1}
$$

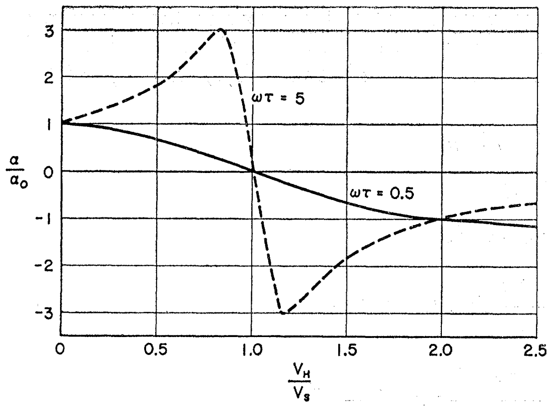

FIG. 1. The ratio of the absorption coefficient at finite dc electric field to that at zero field is shown as a function of V_H/V_S for $X=3$, and for $\omega\tau=0.5$ and $\omega\tau=5$. As $\omega\tau$ increases, the positions of the maxima move towards $V_H/V_S=1$.

If the condition $|\omega_c\tau/\lambda|\gg 1$ is not satisfied, then terms with $|n-m|$ higher than zero enter. That these terms of higher $|n-m|$ tend to wash out the oscillations can be seen from the relation

$$\sum_{n=-\infty}^{+\infty} g_n(X)=1$$

and the slow variation of the frequency denominator with field in the range where $\omega_c\gg\omega$.

The effective conductivity tensor $\mathbf{T}'$ has the following form under the above conditions:

$$T_{xx}'=\frac{3}{(ql)^2}\frac{\lambda[1-g_0(X)]}{\mu+i[1-g_0(X)]/\omega\tau}. \tag{3.2}$$

In obtaining (3.2), we have again dropped a term of order $(v_H/v_F)^2$, and have used the following relationship[20] between the functions $g_0(X)$, $g_1(X)$, and $J_0'(2X)$:

$$g_0(X)+J_0'(2X)/X-g_1(X)=0. \tag{3.3}$$

Using (3.2) in (2.17), we find the following expression for the absorption coefficient of a semimetal:

$$\alpha_j=\frac{6N_0m}{\rho v_s}\left(\frac{C_{xj}}{mv_s^2}\right)^2\frac{\omega^2\tau\mu(v_s/v_F)^2g_0(X)[1-g_0(X)]}{(\omega\mu\tau)^2+[1-g_0(X)]^2}, \tag{3.4}$$

where the subscript j denotes the direction of polarization. Therefore, we see that we have amplification of the sound wave instead of attenuation whenever v_H exceeds v_s. We also note that we have maxima in the amplification and the attenuation at fields such that

$$\frac{v_H}{v_s}=1\pm\frac{[1-g_0(X)]}{\omega\tau}, \tag{3.5}$$

where the upper sign corresponds to amplification and the lower to attenuation. The value of the absorption coefficient at these maxima is

$$\alpha_j=\mp\frac{3N_0m\omega}{\rho v_s}\left(\frac{C_{xj}}{mv_s^2}\right)^2\left(\frac{v_s}{v_F}\right)^2g_0(X), \tag{3.6}$$

and is independent of the relaxation time. The value of the absorption coefficient at these maxima increases linearly with frequency. As the relaxation time, τ, goes to infinity, the maxima in both the amplification and the attenuation occur at $\mu=0$. The behavior of (3.4) as a function of v_H/v_S is shown in Fig. 1 for $X=3$ and for $\omega\tau=0.5$ and $\omega\tau=5$.

In the high-field limit, $X\ll 1$, and we can use the following limiting form for $g_0(X)$:

$$g_0(X)=1-\tfrac{1}{3}X^2. \tag{3.7}$$

Using (3.7) in (3.4), we find that the absorption coefficient of a semimetal in the limit of high magnetic fields is

$$\alpha_j=\frac{2N_0m}{\rho v_s}\omega^2\tau\left(\frac{C_{xj}}{mv_s^2}\right)^2\frac{\mu(\omega_c\tau)^2}{\mu^2(\omega_c\tau)^4+\frac{1}{9}(ql)^2(v_F/v_s)^2}, \tag{3.8}$$

while the maxima in the absorption coefficient occur when

$$\frac{v_H}{v_s}=1\pm\frac{1}{3}\frac{ql}{(\omega_c\tau)^2}\frac{v_F}{v_s}. \tag{3.9}$$

These results agree with those derived by Dumke and Haering[6] and Eckstein[9] in the high-field limit. In the limit of zero dc electric field, (3.4) reduces to Harrison's result.[14]

For degenerate extrinsic semiconductors, we can obtain the absorption coefficient by using (3.2) in (2.20)

$$\alpha_j=\frac{1}{6}\frac{N_0m}{\rho V_S}\frac{\left(\frac{C_{xj}}{mV_S^2}\right)^2\left(\frac{\omega}{\omega_p}\right)^4\left(\frac{V_F}{V_S}\right)^2 q\omega\mu\tau g_0(X)[1-g_0(X)]}{[1-g_0(X)]^2\left[1+\frac{1}{3}\left(\frac{V_F}{V_S}\right)^2\left(\frac{\omega}{\omega_p}\right)^2\right]^2+(\omega\mu\tau)^2\left[1-g_0(X)+\frac{1}{3}\left(\frac{V_F}{V_S}\right)^2\left(\frac{\omega}{\omega_p}\right)^2\right]^2}. \tag{3.10}$$

Here, we also have amplification whenever $v_H/v_S>1$. The maxima in the absorption coefficient occur at fields such that

$$\frac{V_H}{V_S}=1\pm\frac{[1-g_0(X)]}{\omega\tau}\left[\frac{1+\frac{1}{3}(V_F/V_S)^2(\omega/\omega_p)^2}{1+\frac{1}{3}(V_F/V_S)^2(\omega/\omega_p)^2-g_0(X)}\right], \tag{3.11}$$

[20] This relationship can readily be seen from Eq. (A1) in Ref. 10.

where again the upper sign corresponds to amplification and the lower to attenuation. The value of the absorption coefficient at the maxima is

$$\alpha_j = \mp\frac{1}{12}\frac{(N_0m/\rho V_S)(C_{xj}/mV_S^2)^2(\omega/\omega_p)^2 q g_0(X)}{[1-g_0(X)+\frac{1}{3}(V_F/V_S)^2(\omega/\omega_p)^2][1+\frac{1}{3}(V_F/V_S)^2(\omega/\omega_p)^2]},$$

and is again independent of the relaxation time. The behavior of the absorption with dc electric field is essentially the same as in semimetals. In the high-field limit, (3.10)–(3.12) reduce to the expressions previously derived for this case.[5]

IV. CYCLOTRON RESONANCE

In zero dc electric field, we get cyclotron resonance effects when the phonon frequency is of the order of the cyclotron frequency. When we have a net drift velocity along the direction of propagation in the external fields, we expect the frequency at which resonance occurs to be Doppler shifted from ω to $\omega\mu$. In these circumstances, the frequency denominators in the conductivity tensors (2.24) can become small, and the possibility of oscillatory behavior arises. Under this condition, X will be very large, since $X=(v_F/v_s)(\omega/\omega_c)$. Thus, it will be convenient to take the asymptotic form for the tensors in (2.24). The asymptotic forms for the functions $g_n(X)$ and $J_{2n}'(2X)/X$ are[10,21]

$$g_n(X)\approx(1/2X)+O(X^{-3/2})$$
$$(J_{2n}'(2X)/X)\approx O(X^{-3/2}). \qquad (4.1)$$

These expressions are valid only for $X>n$; when n exceeds X, $g_n(X)$ and $J_{2n}'(2X)$ become small. Hence, if we take the asymptotic forms for $g_n(X)$, etc., in evaluating (2.24), we make an error of the form of the final term in the following equation:

$$\sum_{n=-\infty}^{+\infty}\frac{g_n(X)}{\lambda+i(n-m)\omega_c\tau}=\frac{1}{2X}\sum_{n=-\infty}^{+\infty}\frac{1}{\lambda+i(n-m)\omega_c\tau} -O\left[\frac{1}{2X}\sum_{n=X}^{\infty}\frac{2(\lambda-im\omega_c\tau)}{(\lambda-im\omega_c\tau)^2+(n\omega_c\tau)^2}\right]. \qquad (4.2)$$

The last term may be estimated by replacing the summation by an integration over n, and the term is found to be of the order of $1/(ql)^2$, whereas the first term is of order $1/ql$. We are interested in the case where $\omega\tau$ is large; hence, ql will be large, and we can retain only the first term which may be evaluated directly, noting that[22]

$$\sum_{m,n=-\infty}^{+\infty}\frac{J_m(XV_H/V_F)(\lambda-im\omega_c\tau)}{\lambda+i(n-m)\omega_c\tau}=\sum_{m=-\infty}^{+\infty}J_m\left(X\frac{V_H}{V_F}\right)(\lambda-im\omega_c\tau)\sum_{p=-\infty}^{+\infty}\frac{1}{\lambda+ip\omega_c\tau}$$
$$=(1-i\omega\tau)\frac{\pi}{\omega_c\tau}\left[\frac{\omega_c\tau}{\pi\lambda}+\frac{2\lambda\pi}{\omega_c\tau}\sum_{p=1}^{\infty}\frac{1}{(\pi\lambda/\omega_c\tau)+p^2\pi^2}\right]=\frac{(1-i\omega\tau)}{\omega_c\tau}\pi\coth\frac{\pi\lambda}{\omega_c\tau}. \qquad (4.3)$$

We can now obtain the limiting expressions for $\mathbf{T}$ and $\mathbf{R}$

$$T_{xx}=(3\sigma_0/(ql)^2)(1-i\omega\tau)\times[1-(\pi\lambda/2ql)\coth(\pi\lambda/\omega_c\tau)],$$
$$R_x=(1/i\omega\tau)\times[1-(1-i\omega\tau)(\pi/2ql)\coth(\pi\lambda/\omega_c\tau)]. \qquad (4.4)$$

The effective conductivity tensor $\mathbf{T}'$ has the form

$$T_{xx}'=-\frac{3iv_s}{qlv_F}\left[\frac{1-(\pi\lambda/2ql)\coth(\pi\lambda/\omega_c\tau)}{1-(\pi/2ql)\coth(\pi\lambda/\omega_c\tau)}\right] \qquad (4.5)$$

in the region of cyclotron resonance.

Using (4.5) in (2.17), we find that the absorption coefficient of a semimetal can be written as

$$\alpha_j=\frac{3N_0m}{\rho v_s}\omega^2\tau\left(\frac{C_{xj}}{mv_s^2}\right)^2\left(\frac{v_s}{v_F}\right)^2\frac{\mu[A(1-A)-B^2]}{(1-A)^2+B^2}, \qquad (4.6)$$

where

$$A+iB=(\pi/2ql)\coth(\pi\lambda/\omega_c\tau). \qquad (4.7)$$

From (4.7), we can write down the following expressions for the real and imaginary parts of $\coth(\pi\lambda/\omega_c\tau)$:

$$A=\frac{\pi}{2ql}\frac{\tanh(\pi/\omega_c\tau)\sec^2(\omega\mu\pi/\omega_c)}{\tanh^2(\pi/\omega_c\tau)+\tan^2(\omega\mu\pi/\omega_c)},$$
$$B=\frac{\pi}{2ql}\frac{\tan(\omega\mu\pi/\omega_c)\operatorname{sech}^2(\pi/\omega_c\tau)}{\tanh^2(\pi/\omega_c\tau)+\tan^2(\omega\mu\pi/\omega_c)}.$$

From (4.6) and (4.8), we see that we have oscillations in the absorption coefficient as long as $\omega_c\tau>1$. Since the sound frequency has to be of the same order of magnitude as the cyclotron frequency, this means that we require $\omega\tau>1$. When this condition is not satisfied,

[21] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), Vol. 2, p. 1321.

[22] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis* (Cambridge University Press, New York, 1950), 4th ed., p. 136.

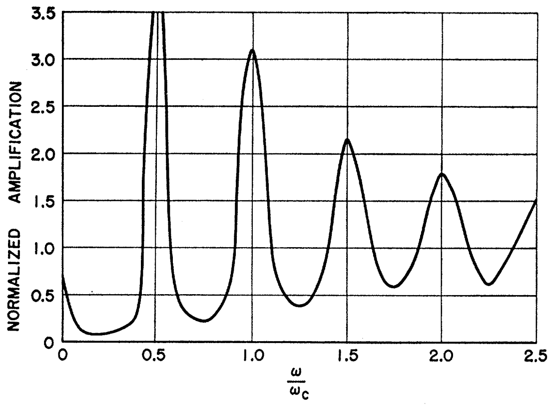

FIG. 2. The normalized absorption coefficient $2\alpha\rho V_S/3N_0m\pi\omega \times(C_{xj}/mV_S^2)^2(V_S/V_F)^3$ is shown as a function of the ratio of the sound frequency to the cyclotron frequency. The product of the sound frequency and the relaxation time, $\omega\tau$, is taken equal to ten and V_H/V_S is taken equal to three. The positions of the maxima are Doppler shifted from their values in zero dc electric field.

the oscillations are damped out. The maxima in the coefficient occur whenever $\omega\mu=n\omega_c$, so that the positions of the cyclotron resonance are Doppler shifted from their value in zero electric field. We also see that we get amplification whenever $v_H>v_s$.

Since we are interested in the case where $ql>1$, we can neglect A and B with respect to unity, and (4.6) reduces to

$$\alpha_j=-\frac{3}{2}\frac{N_0m\pi\omega}{\rho v_s}\left(\frac{C_{xj}}{mv_s^2}\right)^2\left(\frac{v_s}{v_F}\right)^3\mu\,\mathrm{Re}\coth\frac{\pi\lambda}{\omega_c\tau}. \quad (4.9)$$

From (4.9), we see that the amplification is independent of the relaxation time as long as $\omega_c\tau>1$. This expression is displayed in Fig. 2 for $\omega\tau=10$ and $\mu=-2$.

For extrinsic semiconductors, using (4.5), (4.7), and (2.20), we find that the absorption coefficient is

$$\alpha_j=\frac{1}{12}\frac{N_0m\pi v_F}{\rho v_s}\times\frac{(C_{xj}/mv_s^2)^2(\omega/\omega_p)^4q\mu\,\mathrm{Re}\coth(\pi\lambda/\omega_c\tau)}{[1+\frac{1}{3}(v_s/v_F)^2(\omega/\omega_p)^2]^2}, \quad (4.10)$$

where we have neglected terms of higher order in $1/ql$. In this case, as in semimetals, we get Doppler-shifted cyclotron resonances under conditions of amplification when $v_H>v_s$. Both (4.9) and (4.10) reduce to the ordinary cyclotron resonances when $v_H=0$.

V. DISCUSSION

In our calculations, we have found that geometric resonances and cyclotron resonances in the sound-wave intensity can occur under conditions of amplification. This happens when the drift velocity imparted to the conduction electrons in the crossed electric and magnetic fields is greater than the sound velocity. Under these conditions, the conduction electrons can radiate phonons in analogy with the Čerenkov radiation of light in a medium. This analogy has been developed in more detail by Eckstein[9] who pointed out that there would be a resonant transfer of energy between the electrons and the sound wave when the energy denominators which appear in the conductivity tensors vanish. When the phonon frequency is of the same order of magnitude as the cyclotron frequency, this vanishing of the energy denominator leads to cyclotron resonances. The frequency at which this cyclotron resonance occurs is Doppler-shifted from its value in zero dc electric field because the electrons now have an average drift velocity in the direction of propagation. When the cyclotron frequency becomes much larger than the sound frequency, the vanishing of the energy denominators can only occur when $\omega-\mathbf{q}\cdot\mathbf{v}_H=0$. Then, the electrons drifting in the direction of propagation under the influence of the crossed fields are moving in phase with the sound wave and we get a resonant transfer of energy. We see that we get maxima in both the attenuation and the amplification which occur when this condition is satisfied in the limit of infinite relaxation time. When the electrons have a finite relaxation time, the maxima move away from the position of the resonance. When $\omega\tau\ll1$, the resonance is damped and the maxima will not occur at attainable values of the electric field conducting solids. In this case, the amplification increases linearly with dc electric field and resonant behavior is not observed. When $\omega\tau\gg1$, the positions of the maxima move to the position of the resonance. This is similar to what happens to the resonances in the tilt effect.[23]

Another way of seeing how this resonance condition arises is to look at the laws of conservation of energy and momentum in the electron-phonon interaction. These conservation laws are satisfied when the frequency denominators vanish. However, to have the energy of a phonon defined in this interaction, the phonon energy $\hbar\omega$, must be greater than $\hbar/\tau$ because of the uncertainty principle. Rewriting this in terms of the phonon frequency gives us the condition $\omega\tau>1$, for observing the resonance behavior.

The occurrence of geometric resonances and cyclotron resonances in the amplification presents the possibility of studying these effects under more favorable circumstances than is possible at present. For cyclotron resonance, we require $\omega\tau>1$, and at frequencies high enough to satisfy this condition, the attenuation is usually to large to measure anything conveniently. Under conditions of amplification, however, this problem would not arise. We would only get large amplification factors at the points where the resonances occur for $\omega\tau\gg1$.

Because the maximum amplification increases with frequency, we would be able to obtain high-intensity

[23] H. N. Spector, Phys. Rev. 120, 1261 (1960).

acoustic waves in the high-frequency region. In these higher frequency ranges, we can only generate low intensity acoustic waves by other methods.[24]

The mechanism discussed in this paper for amplifying sound waves in a magnetic field can only be applied in semimetals and semiconductors. In metals, the conductivity is too high to obtain the dc electric fields needed to cause v_H to exceed v_s. Also, we would have very large amounts of power to dissipate in metals. In semimetals and semiconductors, on the other hand, the lower carrier densities allow us to obtain the necessary dc electric fields in the material and the amount of power dissipated becomes more reasonable.

ACKNOWLEDGMENTS

This work was begun while the author was at the Hebrew University of Jerusalem during the tenure of a National Science Foundation Postdoctoral Fellowship. It has been supported in part by the Atomic Energy Commission. We would also like to thank Dr. S. Eckstein for interesting conversations.

APPENDIX

The integrals that occur in (2.10a)–(2.10c) are similar but more complicated than the integrals that usually occur when considering the wave number and frequency dependence of the conductivity tensor in magnetic fields.[10] Since the T_{xx} component of the conductivity tensor is the most important component in our calculation, we will evaluate it explicitly. The other components of T and R can be evaluated in a similar fashion.

Using the condition $\omega_c\tau\gg1$, and the equations of motion of the electrons, (2.22), we can write $T_{xx}=\sigma_{xx}+\Sigma_{xx}$ in the form

$$T_{xx}=-e^2\int d\mathbf{v}\, v_x\int_0^\infty ds\, e^{-\lambda s/\tau}\exp\left[i\frac{X}{v_F}(v_y'-v_y)\right]\times\left[(v_x'-v_H)\frac{\partial f_0}{\partial E}+mv_H\frac{\partial^2 f_0}{\partial E^2}v_x'\left(\frac{v_y'}{\omega_c\tau}-v_x\right)\right],\quad\text{(A1)}$$

where we have used the change of variable, $s=t-t'$. We can rewrite the product, $v_x'\exp(iXv_y'/v_F)$, as

$$v_x'\exp(iXv_y'/v_F)=-(1/iq)[(d/ds)+iqv_H]\exp(iXv_y'/v_F)\quad\text{(A2)}$$

and integrate (A1) by parts. This together with the relation

$$v_y'\exp(iXv_y'/v_F)=-iv_F(d/dX)\exp(iXv_y'/v_F)\quad\text{(A3)}$$

allows us to do the integration over s in (A1). We now obtain

$$T_{xx}=-e^2\int d\mathbf{v}\, v_x\left\{\frac{1}{iq}\frac{\partial f_0}{\partial E}\left[1-\frac{\lambda}{\tau}e^{-iXv_y/v_F}\mathcal{g}(\theta,\phi)\right]+\frac{mv_H}{iq}\frac{\partial^2 f_0}{\partial E^2}\left[(v_y/\omega_c\tau)-v_x-\frac{1}{iq\tau}+e^{-iXv_y/v_F}\times\left(\frac{\lambda}{iq\tau}+(1-i\omega\tau)\left(i\frac{v_F}{\omega_c\tau}\frac{d}{dX}+v_x\right)\right)\mathcal{g}(\theta,\phi)\right]\right\},\quad\text{(A4)}$$

where

$$\mathcal{g}(\theta,\phi)=\int_0^\infty ds\, e^{-\lambda s/\tau}\exp-i\frac{X}{v_F}\times[v\sin\theta\sin(\omega_c s-\phi)-v_H\sin\omega_c s].\quad\text{(A5)}$$

This can be evaluated by noting that [21]

$$\exp iz\sin\psi=\sum_{n=-\infty}^{+\infty}e^{in\psi}J_n(z).\quad\text{(A6)}$$

Then we have

$$\mathcal{g}(\theta,\phi)=\tau\sum_{n,m=-\infty}^{+\infty}\frac{e^{in\phi}J_n(Xv\sin\theta/v_F)J_m(Xv_H/v_F)}{\lambda+i(n-m)\omega_c\tau}.\quad\text{(A7)}$$

The integration over the angular coordinates θ, ϕ can be done using the same kind of relations. We end with the functions $g_n(X)$, and their derivatives after using (2.21a)–(2.21b). Using the definition of the functions, $g_n(X)$, we find that

$$g_n'(X)=(1/X)[J_{2n}(2X)-g_n(X)]$$
$$g_n''(X)=(2/X)[J_{2n}'(2X)-g_n'(X)].\quad\text{(A8)}$$

The use of these relations allows us to write T_{xx} in the form (2.23). We can evaluate the integrals that occur in the other components of the conductivity tensor in an analogous manner.

It can be seen directly that the summation,

$$\sum_{m=-\infty}^{+\infty}J_m\left(X\frac{v_H}{v_F}\right)=1,$$

can be obtained from (A6), by setting $\psi=0$. The summation,

$$\sum_{m=-\infty}^{+\infty}mJ_m\left(X\frac{v_H}{v_F}\right)=X\frac{v_H}{v_F},$$

can be done by taking the derivative with respect to ψ, of both sides of (A6), and then setting $\psi=0$.

[24] H. E. Bommel and K. Dransfeld, Phys. Rev. Letters **1**, 234, (1958); **2**, 298 (1959); **3**, 83 (1959). E. H. Jacobsen, *ibid.* **2**, 249 (1959); N. S. Shiren, *ibid.* **6**, 168 (1961).