takes place as the coherence is broken. Inductive coupling even with $\gamma \ll 1$ is already better in that the coherence is not artificially broken by demanding more current than the superconducting film can supply. But with $\gamma \ll 1$, the film is forced to make quantum jumps to other values of A_0 at essentially the maximum current points (as established experimentally by Mercereau and Crane[7]) and most of the depairing regime is never entered. Only with the sandwiches can we achieve $\gamma > 6$ and, as a result of blocking available momentum space by the unpaired electrons, reduce the energy gap gradually to zero. For this case it is easy to see that the free energy rises monotonically as a function of A_{ex} to its normal state value at $A_{ex} = 1.36\, A_s$, and that a second-order transition does indeed take place. (Actually, the film never really enters the normal state but remains, for $A_{ex} > 1.36\, A_s$, as explained above, at the threshold between the normal state and one of the completely depaired superconducting states.) For $\gamma < 6$, the free energy rises monotonically above the normal-state value into a metastable region. Although each individual quantum state exhibits a cusp in the plot of free energy as A_{ex}, the return branch is inaccessible and the film follows the envelope of the cusps (constant free energy) as soon as a cusp is reached.

For $\gamma > 6$, tunneling from the outside film of the sandwich should provide a convenient check on the expected monotonic decrease of the energy gap in the depairing regime. The results of detailed computations of tunneling current versus voltage (I vs V) will be presented elsewhere and it suffices here to describe them qualitatively. It is important to distinguish between the energy gap parameter Δ, which is a true measure of the superconducting coherence, and the "gap" in the I vs V curve, which is of less fundamental significance. This is clear for $A = A_s$ where depairing has not yet started, but the I versus V curve has nonvanishing I for all values of V (varying as $V^{3/2}$ for $V \approx 0$). For A somewhat greater than A_s there are already enough low-energy unpaired electrons that the I versus V curve is considerably filled in at low voltages, although $\Delta \neq 0$.

Note added in proof. Because of interaction of the electrons with the film surfaces, depairing and the decrease in the gap already begin to occur at low fields as calculated by Douglass[9] from the Ginzburg-Landau theory. Consequently, the function $f(x)$ describing the $-J$ versus A current characteristic has a broader maximum than shown in Fig. 1. But the above general remarks concerning the effect of self-inductance remain unchanged. In particular, the first-order transition found by Douglass is a result only of geometry, and becomes second order for a sandwich.

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Determination of the Size of the "Necks" in Fermi Surfaces of Even-Valence Metals: Application to Lead*

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A technique is described which permits a precise measurement of features in the magnetoresistance which are directly related to the Fermi surface "neck" size of *even*-valence metals. The axis of the sample is tipped through a precisely known angle, by means of a goniometer arc, causing the disappearance of cusp-like troughs which indicate the boundaries of the particular features. The dimensions of these features are determined for Pb within uncertainties an order of magnitude smaller than those of previous measurements. The neck size for Pb, as calculated within the framework of Gold's free-electron model for the Fermi surface, is $0.29b$, where b is the period of the reciprocal lattice.

I. INTRODUCTION

The existence of open (multiply connected) trajectories or orbits on the Fermi surfaces of metals and their directions in momentum space can be easily determined by means of high-field magnetoresistance measurements.[1] For monovalent metals, this measurement yields, within the framework of a reasonable model for the Fermi surface such as the nearly free electron model, a very precise calipering of the size of the necks (usually the intersections of the Fermi surface with the Brillouin zone boundaries).[2] With even valence metals, only a very crude estimate of the neck size can be made from data obtained with the conventional techniques. The primary purpose of this communication is to demonstrate the feasibility of a magnetoresistance technique

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[2] M. G. Priestley, Phil. Mag. **5**, 111 (1960).

which enables, with the assumption of a suitable model, a precise determination of the neck dimension in the open portions of the Fermi surfaces of even-valence metals. Pb was chosen for this investigation because its Fermi surface is perhaps the best known of the even-valence metals having open surfaces.[3–5] The uncertainty in the dimensions of the interesting features in the magnetoresistance of Pb is reduced by these data by nearly an order of magnitude.

II. THEORETICAL BACKGROUND

About certain high-symmetry directions perpendicular to planes in which the Fermi surface is open in two or more intersecting directions, there exist two-dimensional regions, i.e., solid angles of applied magnetic field H, in which the magnetoresistance varies as $H^2 \cos^2 \alpha$ for all current directions, where α is the angle between the open orbit and the current.[1] Chambers[6] has designated these open orbits as Type B, in contrast to the open orbits found in one-dimensional areas of applied field. If the latter orbits appear for a complete rotation of 360°, they are called Type A; if they appear for less than 360°, Type A'.[7] The $H^2 \cos^2 \alpha$ law holds in all three regions.

For monovalent metals, such as the noble metals,[8,9] magnetoresistance saturates with field except in regions where open orbits are found. Very sharp peaks in magnetoresistance are observed when the edges of the regions are crossed by the applied field, unless α is close to 90°. In particular, when Type-B regions are traversed by the applied field, the distance between the two peaks observed is directly proportional to the diameter of the "neck" or intersection of the Fermi surface at the (111) face of the Brillouin zone.[2] The dimension calipered by this method is parallel to the principal direction about which the Type-B region appears, and can be quite easily measured to 1% or better.[10] This process can be visualized by imagining the direction of the applied magnetic field to be tipped in an arbitrary direction away from the principal axis, which is perpendicular to a plane containing two or more intersecting directions in which the Fermi surface is multiply connected, until the section of the Fermi surface intercepted by the plane of the field just becomes closed.[6] The angle at which this occurs constitutes the dimension of the Type-B region in the direction the field was tipped and is proportional to the dimension of the neck parallel to the principal direction and the distance between the intersections of the open directions. In certain low-index directions, larger angles of tip are permitted, giving rise to Type-A' open orbits.[5,7] Type-A' regions ("whiskers") have been observed by Alekseevskii and Gaidukov (AG) in Pb,[5] and Alekseevskii *et al.* in Sn,[11] and would appear to give rise to the peaks Klauder and Kunzler[9] associated with tertiary and quaternary open orbits in Cu. These features of the magnetoresistance measurement in monovalent metals made it appealing for examining changes of the detailed topology of the Fermi surface with hydrostatic pressure.[10]

The situation is radically different for metals possessing even valence magnetoresistance character.[12] This group seems to contain all the remaining elements with open Fermi surfaces. Here a compensation, due to equal hole and electron Fermi surface areas, leads to magnetoresistance varying as H^2 over the entire stereogram of applied field. Sharp cusp-like troughs appear when open orbits are present due to the $H^2 \cos^2\alpha$ rule. There is no apparent feature designating the edges of the Type-B regions and, therefore, the corresponding neck dimension. Alekseevskii and Gaidukov (AG)[5] attempted to measure these regions in Pb by examining a large number of samples and noting when the cusp-like troughs disappeared in various directions. They were able to estimate the dimensions of the Type-A' regions within an error of about ±2 deg. Their estimate of the size of the Type-B regions by noting the disappearance of the (111) troughs is suspect as will be shown below. This method is tedious and impractical for a precise determination of neck size such as needed in a pressure experiment.

III. EXPERIMENTAL TECHNIQUE

A method has been developed which allows the measurement, under optimum conditions, of the edges of a Type-B or Type-A' region to ±0.05 deg of arc. The method relies on the fact that the direction of an open orbit is given by the intersection of the plane perpendicular to the applied field and the plane perpendicular to the principal direction about which a Type-B or Type-A' region appears. Therefore, there will be a unique field direction (that when the plane containing $\mathbf{j}$ the current direction and the applied magnetic field direction passes through the principal direction) when $\alpha = 90°$. See Fig. 1.

If the angle between the current and the axis of rotation is changed so that the field sweeps through several portions of the region, a cusp-like trough will be observed until the sweep no longer intersects the region. Figure 2 displays a series of such sweeps for a Pb crystal. The boundary, in this case the end of a Type-A' orbit, can be determined to ±0.04 deg from similar

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[7] R. G. Chambers (private communication).

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[11] N. E. Alekseevskii, Yu. P. Gaidukov, I. M. Lifshitz, and V. G. Peschanskii, Zh. Eksperim. i Teor. Fiz. **39**, 1201 (1960) [translation: Soviet Phys.—JETP **12**, 837 (1961)].

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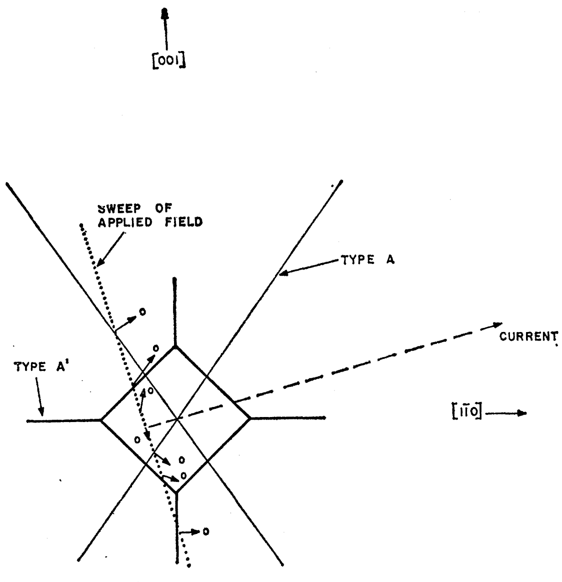

FIG. 1. Partial stereogram for Pb showing direction of open orbits (0) at various directions of applied magnetic field. Diamond shaped area is a Type-*B* region. The current is in the plane of the paper.

sweeps. This particular sequence was chosen to show, for comparison, sweeps over a larger series of angles of tip than normally recorded.

The essential portion of the experimental apparatus consists of one arc of an x-ray goniometer connected directly to a rotational counter graduated in 0.01 revolution so that the sample can be tipped through ±25° relative to the plane of rotation of the applied magnetic field. The goniometer screws used tipped the sample about one degree per revolution.

The samples were $\frac{1}{16}$-in.×$\frac{1}{16}$-in.×$\frac{1}{2}$-in. parallelepipeds spark-cut from a large crystal obtained from Virginia Institute for Scientific Research. The ratio of the resistance at room temperature to that at 4°K for these samples was of the order of 10^4. The resistance at 4°K was determined by extrapolation of the magnetoresistance observed in the center of a Type-*B* region to zero field. The data were taken at 25 kG and 4°K and were displayed continuously as in Fig. 2 on an xy recorder where the ordinate was the potential drop across the sample and the abscissa, the angular displacement of the applied field.

The samples were mounted on the goniometer so that the principal axis through a Type-*B* region was as nearly perpendicular as possible to the axis of tip, and with the current direction $\mathbf{j}$ perpendicular to both this principal axis and to the axis of tip. Sample orientations were determined initially to ±2 deg by standard back reflection x-ray techniques. With samples Pb *B*-3, -4, and -5, the orientations were then adjusted to better than ±0.5 deg by the spacing of known symmetry features in the magnetoresistance (usually widely spaced Type-*A* troughs). This uncertainty in the orientation affects the dimensions of the regions only as the cosine of a very small angle since *both* extremities of a region were measured.

IV. RESULTS

Our magnetoresistance data are in complete agreement with the general features observed by AG. They found Type-*B* regions surrounding the [110] directions with Type-*A'* regions protruding from the Type-*B* regions in the (110) and (100) planes. The Type-*A* regions are found in the (111) planes. This situation is depicted in the partial stereogram in Fig. 1.

The orientations and the distances of tip from the center of the Type-*B* region of the features we observed are given in Table I. The estimated uncertainty of each determination is given and varies somewhat with the angle between the current and the pertinent orbit (in the case of Type-*A'* regions), noise level, and proximity of other features. Troughs due to the Type-*A* and Type-*A'* orbits were observed even within the Type-*B* regions, so that the Type-*B* cusp for some orientations was sandwiched in a converging interval of a few degrees between Type-*A* and Type-*A'* troughs. The failure of these troughs to disappear can be attributed to the dependence of the "constant" of proportionality in the $H^2 \cos^2\alpha$ rule on the fraction of the Fermi surface available for a particular orbit. The Type-*B* orbits would seem to occupy a relatively smaller proportion of the surface.

The technique employed by AG to determine the size of the Type-*B* region was to observe the disappearance of the cusps in the (111) planes (the Type-*A* troughs) as the plane of the field approached the [110] direction. This will give a crude estimate at best be-

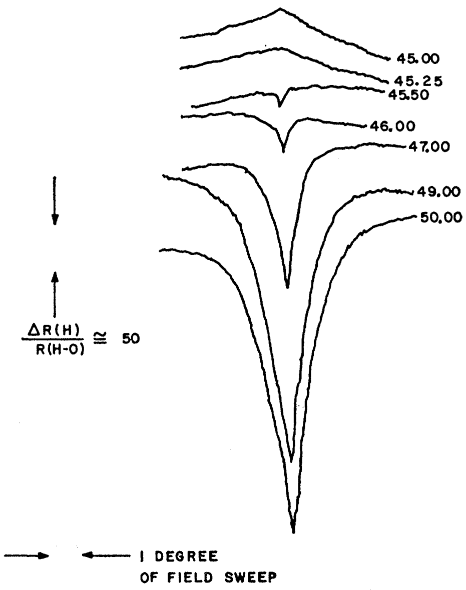

FIG. 2. Reproduction of a recorder tracing of the magnetoresistance of Pb as the plane of the field is tipped through the end of the Type-*A'* region in the (001) plane. Traces labeled in degrees of tip.

TABLE I. Sample orientations and observed extremities of features of interest.

Sample	Orientation (current direction)	Angle of tip[a] of extremities of features observed (in deg)
Pb *B*-1	82° from [011] in (100)	±15.85±0.15 Type *A′* (100)
Pb *B*-2	89° from [011] 3° from [100]	±15.5±0.2 Type *A′* (110)
Pb *B*-3	37° from [100] in (011)	±12.25±0.2 Type *A′* (110), ±2.5±0.1 Type *A′* (100),[b] +10.8±0.15 Type *B*[b]
Pb *B*-4	18.5° from [100] in (011)	±6.25±0.15 Type *B*, ±14.15±0.1 Type *A′* (110)
Pb *B*-5	66° from [100] in (011)	±6.22±0.15 Type *B*, ±14.68±0.1 Type *A′* (100)

[a] Values of tip have been corrected for cases when feature extremity was not perpendicular to axis of goniometer.
[b] This region centered 30° from axis of goniometer.

cause, as mentioned above and as shown in Fig. 3, these troughs do not disappear until they merge with the Type-*B* cusp at a considerable distance within the Type-*B* region.

The dimensions of the various regions measured in the planes indicated are compared with those given by AG in Table II. The notation is such that the direction denoted [001] is perpendicular to the plane denoted (001). The Type-*B* regions are found to be square within experimental uncertainty. This finding is not incompatible with Gold's free electron surface.[4] However, a prediction of the relative size of the features observed does depend very critically upon exactly how the tubes forming this surface are joined, since these tubes must be quite thick to support open orbits. The Type *A′* (110) would appear to depend the least upon the exact nature of these intersections, so the dimension of this region was used to calculate the tube diameter. This was accomplished by using an "ice cube tray" model which has dimensions only parallel to [110]. The value for the diameter of the tube obtained under these assumptions is 0.29*b*, where *b* is the period of the reciprocal lattice in the [001] direction ($b=4\pi/a$, $a=4.9$ Å).

TABLE II. Comparison of the dimensions of the Type-*A′* and Type-*B* regions obtained in this work with those of AG.

Region	Plane in which measured	This work[a]	AG[a]
Type *A′*	(100)	31.7±0.3	30±3
Type *A′*	(110)	31.0±0.4	30±3
Type *B*	(110)	16.3±0.3	14±2
Type *B*	(100)	16.3±0.3	20±2

[a] Distance between extremities in deg.

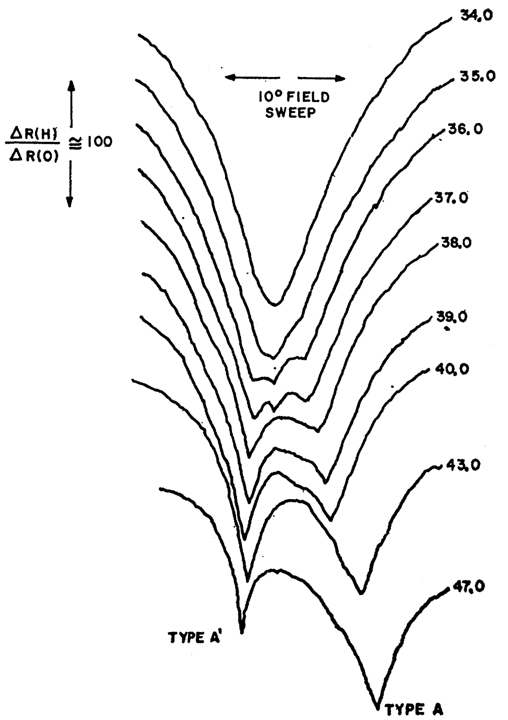

FIG. 3. A reproduction of a recorder tracing of the magnetoresistance of Pb showing the existence of the Type-*A* and Type-*A′* troughs within the Type-*B* region. The boundary of the Type-*B* region is between 38.0 and 39.0 deg.

It is not relevant to compare this dimension with that obtained by AG for their [111] topology as such radically different models would be expected to have very different dimensions. It is of interest, however, to compare our result with that obtained by AG from their Hall measurements. This calculation involves only the minimum diameters of the tubes.[1] Since the arms of Gold's model do not all lie in the (110) plane, the effective diameter, i.e., the depth of the (110) planes capable of supporting the open orbit, must be used in this comparison. This effective diameter is 0.135*b* as compared with 0.16*b* obtained by AG from their Hall data. They do not assign an error to this value but the graph displaying their data (Fig. 3 of Ref. 5) shows that the Hall emf is a very peaked function in the [110] direction, differing in equivalent crystallographic directions by at least 10%. It is likely that the disparity between the two determinations is within their experimental uncertainties. It is probably worth emphasizing that the major uncertainty in determining the neck size with our technique lies in the model, not in the measurement, so the method is especially well suited to examining changes in the topology such as might be expected with alloying or pressure.

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