indication of a period shorter by a factor of about 8. The first does not match any of the periods given in Table IV for [100] fields very closely, but is of the order of (b), (e), and (f). Neither (b) nor (e) is very sensitive to the size of the form factors and we do not expect errors this large. We would be inclined to guess that the sensitive orbit (f) has been observed. The shorter period agrees nicely with (g) and with no other orbit. These are just the two orbits which remain for high fields in the [100] direction and suggests that at the 200 kG which he used, breakdown has become important.

Condon and Marcus[26] and Condon[27] have studied the de Haas–van Alphen effect in fields up to 30 kG and find results consistent with the low-field surface, though quantitative comparison is not complete.

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Evidence for the ⟨110⟩ Swelling Constant Energy Surface for Heavy Holes in Silicon

H. Miyazawa, K. Suzuki, and H. Maeda
Toshiba Central Research Laboratory, Kawasaki, Japan
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A newly devised experimental technique has revealed that the puzzling weak-field anisotropy of the galvanomagnetic effects in *p*-type silicon above 77°K belongs, according to our classification, to the last of the four possible types for cubic semiconductors. The strange behavior is ascribable to the growth of the ⟨110⟩ swelling energy contour for the heavy-hole band. A brief description is given of the calculation of the nonparabolicity with the recent band parameters and of the calculation of the conductivity tensor for a fictitious energy surface.

SYMMETRY arguments predict that only four types can exist for the anisotropy of the weak-field galvanomagnetic effects in cubic semiconductors.[1] These are listed in Table I, together with the corresponding materials and their band shapes, in accordance with the results thus far established.[2] The similarity of the valence band of silicon and germanium might suggest that *p*-type silicon would belong to the third type, as is the case for *p*-type germanium. Careful measurements of the weak-field magnetoresistance,[3] however, have disclosed that the anisotropy of the former above liquid-nitrogen temperature is not of the third type and that

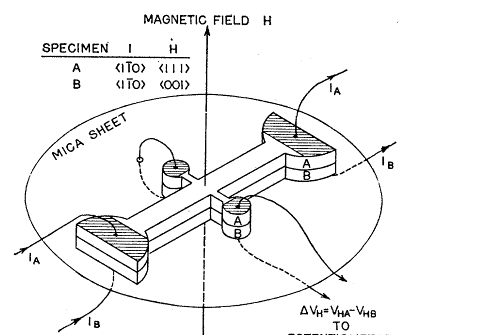

FIG. 1. Principle of the differential method to detect the anisotropy with a low-precision magnet. The merit is in the simultaneous observation of two competing responses within a small space.

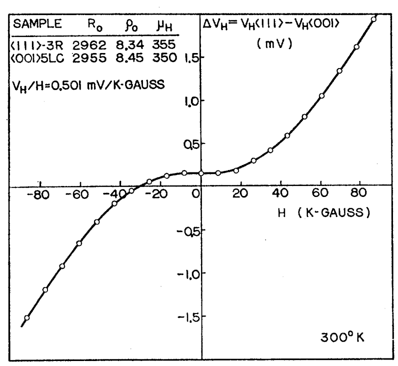

FIG. 2. An experimental result observed by the differential method. The measurement was made with a Bitter-type air core solenoid generating 90-kG maximum field. The result proves that $R_H\langle111\rangle > R_H\langle001\rangle$ at finite fields.

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[2] G. L. Pearson and H. Suhl, Phys. Rev. **83**, 768 (1951); B. Abeles and S. Meiboom, *ibid.* **95**, 31 (1954); M. Shibuya, *ibid.* **95**, 1385 (1954); C. Goldberg, E. N. Adams, and R. E. Davis, *ibid.* **105**, 865 (1957); J. G. Mavroides, and B. Lax, *ibid.* **107**, 1530 (1957); W. M. Bullis, *ibid.* **109**, 292 (1958); W. E. Krag, *ibid.* **118**, 435 (1960); H. Miyazawa and H. Maeda, in *Proceedings of the International Conference on Semiconductor Physics, 1960* (Czechoslovakian Academy of Sciences, Prague, 1961), p. 169, and J. Phys. Soc. Japan **15**, 1924 (1960).
[3] D. Long and J. Myers, Phys. Rev. **109**, 1098 (1958).

TABLE I. Four possible types of anisotropy of weak-field galvanomagnetic effects in cubic semiconductors. Corresponding materials and their band shapes thus far established are also listed.

Type	$\frac{4}{3}$ law[a] R_H	$\frac{4}{3}$ law[a] M_l	1/2/3 law[b] M_t	Example	Band shape
First	+	+	−	n-Ge	⟨111⟩ valley
Second	−	−	+	n-Si	⟨100⟩ valley
Third	+	−	+	p-Ge	$E=\frac{\hbar^2}{2m}\{Ak^2\pm[B^2k^4+C^2(k_x^2k_y^2+k_y^2k_z^2+k_z^2k_x^2)]^{1/2}\}$
Fourth	−	+	−		

[a] Plus signs mean that $|Q\langle 001\rangle_H|$ is the maximum and $|Q\langle 111\rangle_H|$ the minimum and vice versa for negative signs.
[b] Plus sign means that $M_{1\bar{1}0}^{001}$ is the maximum and $M_{1\bar{1}0}^{110}$ the minimum.

the behavior near room temperature is reminiscent of ⟨111⟩ valleys. According to our classification the results suggest that p-type silicon belongs either to the first type or to the fourth type.

To distinguish between the two possibilities the sign of the $\frac{4}{3}$ law for the Hall coefficient at finite weak fields was determined by a new technique, "the differential method." A pair of differently oriented specimens are arranged and connected as shown in Fig. 1. Constant currents, I_A and I_B, are so adjusted that the Hall voltages of the two specimens may completely cancel each other at sufficiently weak fields. As the magnetic field H is increased, the difference of the two Hall voltages ΔV_H should no longer vanish and should exhibit cubic bending up or down with respect to H. The technique is of particular advantage when the available magnetic field is comparatively unstable and/or inhomogeneous.

An experimental result obtained in this way is shown in Fig. 2, which leads us to the decisive conclusion that the $\frac{4}{3}$ law is negative and that p-type silicon belongs to the fourth type at the room temperature. Such experiments were carried out at 300, 195, 90, and 77°K on oriented crystal pairs doped with 1.5×10^{15} cm^{-3} boron.

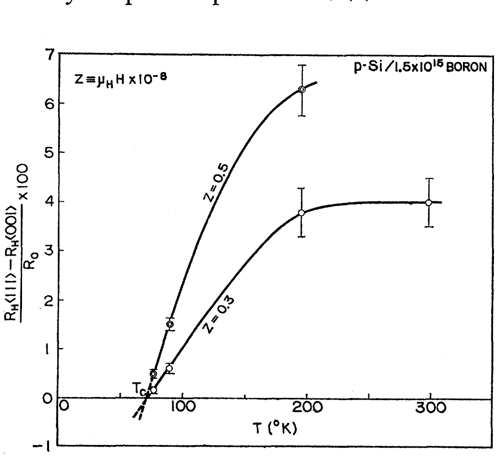

FIG. 3. Temperature dependence of the modulation amplitude for two values of $\mu_H H$, suggesting that sign reversal occurs at T_c.

The modulation amplitude $(R_H^{\langle 111\rangle}-R_H^{\langle 001\rangle})/R_0$ obtained from the data is given in Fig. 3 as a function of temperature for two values of $\mu_H H$. The anisotropy rapidly diminishes below 195°K and shows a trend to change sign near about 70°K. Since the magnetoresistance coefficients also indicate a tendency toward a similar sign reversal at nearly the same temperature,[3] we call this the critical temperature T_c at which the anisotropy laws are transformed from the fourth to the third type. Our expectation is that p-type silicon sufficiently below T_c will behave like p-type germanium. The appearance of the fourth type above T_c is ascribed

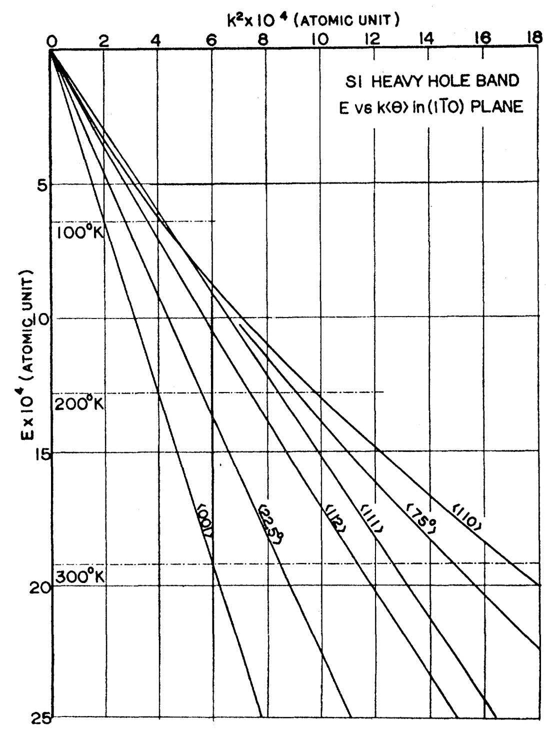

FIG. 4. Nonparabolicity of the heavy-hole band for six directions of $\mathbf{k}$ in the $(1\bar{1}0)$ plane, calculated with band parameters $A=-4.22$, $B=-1.0$, $N=-8.10$ (Stickler *et al.*, Ref. 5), and $\Delta=0.044$ eV (Zwerdling *et al.*, Ref. 6).

TABLE II. Components of the conductivity tensor and the resultant R_H and M_t calculated for the fictitious ⟨110⟩ swelling under the assumption described in the text. Let m^* be the tube mass on square surfaces; then $\mu=(9/10)(e\tau/m^*)$ and $\mu_H=(20/27)\mu$, where μ and μ_H are the conductivity and Hall mobility, respectively.[a]

I	H	S_{11}	S_{12}	R_H	M_t
$\langle 1\bar{1}0\rangle$	$\langle 001\rangle$	$ne\mu\left(1-\frac{1}{2}\frac{\mu_H H}{c}\right)$	$ne\mu\frac{\mu_H H}{c}\left(1-\frac{3}{8}\frac{\mu_H H}{c}\right)$	$\frac{\mu_H}{\mu}\frac{1}{nec}\left(1+\frac{5}{8}\frac{\mu_H H}{c}\right)$	$\frac{1}{2}\frac{\mu_H}{c}$
$\langle 1\bar{1}0\rangle$	$\langle 111\rangle$	$ne\mu\left(1-\frac{1}{\sqrt{3}}\frac{\mu_H H}{c}\right)$	$ne\mu\frac{\mu_H H}{c}$	$\frac{\mu_H}{\mu}\frac{1}{nec}\left(1+\frac{2}{\sqrt{3}}\frac{\mu_H H}{c}\right)$	$\frac{1}{\sqrt{3}}\frac{\mu_H}{c}$

[a] Only low-field expansions are indicated. The results suggest the positive law for M_l, since the same sign is forbidden between M_t and M_l.

to the nonparabolic effect in the valence band. This is based on the following two grounds.

First, our calculation using Kane's theory,[4] the recent inverse mass parameters of Stickler *et al.*,[5] and the spin-orbit splitting value of Zwerdling *et al.*,[6] clearly demonstrates that the energy contour for heavy holes above 120°K has a warped shape with ⟨110⟩ swelling as shown in Figs. 4 and 5, while that for light holes remains almost spherical. The temperature, where $[\partial E/\partial(k_{110}^2)]^{-1}$ crosses over $[\partial E/\partial(k_{111}^2)]^{-1}$, is about 64°K, in rough agreement with the observed T_c. Though a unique assignment of band parameters together with the sign of B, theoretically proposed by Hasegawa[7] and experimentally determined by Hensel and Feher,[8] made it possible for the first time to calculate the nonparabolicity, these parameters, to our surprise, gave rise to a ⟨110⟩ swelling too strong to close around the center of the k space above 150°K.

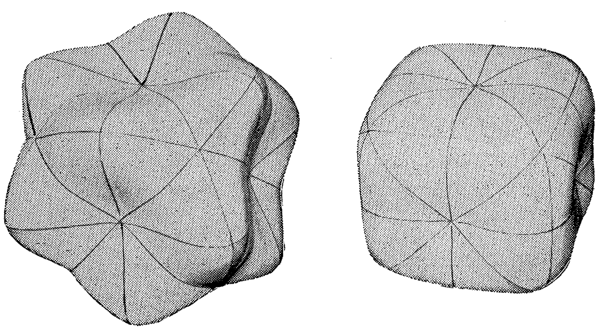

FIG. 5. ⟨110⟩ swelling energy contour for heavy holes at $E=0.026$ eV or 300°K (left). For comparison, the familiar one at the band edge (4°K) is also shown (right, magnified $\times[300/4]^{1/2}$).

Second, to elucidate that the ⟨110⟩ swelling is responsible for the fourth type of anisotropy, the true shape was replaced, for simplicity, by a truncated cube consisting of six squares and eight regular triangles, and the conductivity tensor was calculated by means of McClure's method[9] assuming a Boltzmann distribution and constant relaxation time. The results listed in Table II qualitatively support our view that the true ⟨110⟩ swelling yields the fourth-type anisotropy.

We are grateful to Professor Fukuroi of the Research Institute for Iron, Steel, and other Metals, Tohoku University, for arranging the use of the Bitter air core solenoid.

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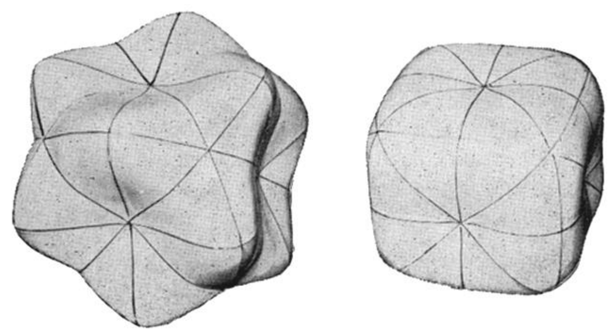

FIG. 5. ⟨110⟩ swelling energy contour for heavy holes at $E=0.026$ eV or 300°K (left). For comparison, the familiar one at the band edge (4°K) is also shown (right, magnified $\times[300/4]^{1/2}$).