

is not a serious disadvantage, however, since most nuclides satisfy this requirement. The second method has the advantage that assignments are unambiguous for all resonances since fluctuations and variations caused by the spin of the capture state are suppressed.

#### ACKNOWLEDGMENT

The author gratefully acknowledges the encouragement and suggestions of Dr. L. M. Bollinger. It is also a pleasure to acknowledge an informative discussion with Dr. D. Kurath.

## Nuclear Spectroscopy of Bi<sup>210</sup> with Stripping Reactions\*

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(Received 8 November 1962; Revised manuscript received 9 May 1963)

Forty-four single neutron states are located in Bi<sup>210</sup> with Bi<sup>209</sup>(*d,p*) reactions. These are compared with the levels predicted from the *j-j* coupling shell model. Of the ten levels predicted from a coupling of the *h*<sub>9/2</sub> proton and the *g*<sub>9/2</sub> neutron, nine are observed including the ground state. One of them is assumed to be composed of two close-lying levels, thus, accounting for all the ten levels. Proton angular distributions and total cross section for exciting these ten levels suggest that they have a very pure (*h*<sub>9/2</sub>)<sub>p</sub>(*g*<sub>9/2</sub>)<sub>n</sub> configuration. A comparison of the relative cross sections for exciting these ten levels with the 2*J*+1 statistical factor gives consistent spin values for these levels. Angular distributions of protons from several prominent groups in Bi<sup>209</sup>(*d,p*) reactions are compared with those from Pb<sup>208</sup>(*d,p*) reactions, identifying the (*d*<sub>5/2</sub>), (*s*<sub>1/2</sub>), and (*d*<sub>3/2</sub>) neutron states in Bi<sup>210</sup>. Effect of configuration mixing is discussed. The level density above 2.5-MeV excitation is found to be small compared to the shell-model prediction.

#### INTRODUCTION AND EXPERIMENTAL

FROM shell-model considerations Bi<sup>210</sup> has a particularly simple configuration, a proton in the *h*<sub>9/2</sub> state and a neutron in *g*<sub>9/2</sub> state outside a doubly closed shell of protons and neutrons. The low-energy states in Bi<sup>210</sup> will be determined by the single-particle levels available to the outer neutron and proton. In recent months a great deal of information on the location of neutron states in the *N*=126–184 shell has been obtained<sup>1,2</sup> as a result of (*d,p*) reactions on Pb<sup>208</sup>. All the seven single-particle (s.p.) states in this shell are located. Starting from these s.p. states one can very easily build up the possible shell-model states in Bi<sup>210</sup>, provided the corresponding proton levels are also known. Unfortunately, only a few such proton states are known<sup>3</sup> in Bi<sup>209</sup>. Apart from the ground state of Bi<sup>209</sup>, which is *h*<sub>9/2</sub>, it has been established that the first excited state at 0.90 MeV is *f*<sub>7/2</sub>. Several well-separated states are known in Bi<sup>209</sup> from (*n,n'*) and (*p,p'*) reactions.<sup>3</sup> These include the above-mentioned *f*<sub>7/2</sub> state. The next level occurs at 1.60 MeV. From shell-model predictions this should be the *i*<sub>13/2</sub> proton state. Throughout our discussions in the present work we have assumed the 1.60-

MeV state as *i*<sub>13/2</sub>. As will be evident later, the (*d,p*) reaction analysis in Bi<sup>209</sup> suggests that this level is not an odd-parity state (e.g., *p*<sub>3/2</sub> or *f*<sub>5/2</sub>). If it were so, there would have been more states in Bi<sup>210</sup> excited in (*d,p*) reactions near the (*d*<sub>5/2</sub>)<sub>n</sub> group.

It is apparent that in Bi<sup>209</sup>(*d,p*) reactions only those states in Bi<sup>210</sup> are excited which have a large (*h*<sub>9/2</sub>)<sub>p</sub>(*l*<sub>*j*</sub>)<sub>n</sub> admixture. *l* and *j* stand for the orbital angular momenta and the total angular momenta, respectively, for the seven neutron states *g*<sub>9/2</sub>, *i*<sub>11/2</sub>, *j*<sub>15/2</sub>, *d*<sub>5/2</sub>, *s*<sub>1/2</sub>, *g*<sub>7/2</sub>, and *d*<sub>3/2</sub>, in the *N*=126–184 shell. So, corresponding to a particular *j*-neutron state in Pb<sup>209</sup>, one should observe in Bi<sup>209</sup>(*d,p*) reactions states with angular momenta ranging from  $|j-\frac{9}{2}|$  to  $j+\frac{9}{2}$ . Since the neutron binding energy should be of the same order in Pb<sup>209</sup> as in Bi<sup>210</sup>, the above-mentioned levels should be observed approximately at the same *Q* values as in Pb<sup>208</sup>(*d,p*). Over and above these neutron states, one should expect to excite several other states in Bi<sup>210</sup>, where the proton may be in the excited states *f*<sub>7/2</sub>, *i*<sub>13/2</sub>, etc. This becomes possible because of strong configuration mixing between the near lying [(*h*<sub>9/2</sub>)<sub>p</sub>(*l*<sub>*j*</sub>)<sub>n</sub>]<sub>JII</sub> and [(*f*<sub>7/2</sub>)<sub>p</sub>(*l'*<sub>*j'*</sub>)<sub>n</sub>]<sub>JII</sub> states. A study of such states will lead us to understand the extent of configuration mixing in Bi<sup>210</sup>.

Another helpful feature of the (*d,p*) reactions in this region at our available deuteron energy (14.8 MeV) is the marked difference in the angular distribution of the emitted protons at forward angles ( $\theta < 40^\circ$ ) for the different *l* values of the captured neutrons. (See, for example, Fig. 3 in Ref. 1). Thus, a comparison of the

\* Supported by the Office of Naval Research and the National Science Foundation.

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<sup>1</sup> P. Mukherjee and B. L. Cohen, Phys. Rev. **127**, 1284 (1962).

<sup>2</sup> J. R. Erskine and W. W. Buechner, Bull. Am. Phys. Soc. **7**, 360 (1962).

<sup>3</sup> *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences-National Research Council, Washington 25, D. C., 1960).

angular distributions of protons in both Pb<sup>208</sup>(*d,p*) and Bi<sup>209</sup>(*d,p*) reactions will be helpful to identify the *l* values of the captured neutrons. Such angular distribution studies should also indicate the extent of configuration mixing between states like  $[(h_{9/2})_p(lj)_n]_{J\pi}$  and  $[(h_{9/2})_p(l'j')_n]_{J\pi}$ . As for example, above 1.5-MeV excitations several 4<sup>-</sup> and 5<sup>-</sup> states in Bi<sup>210</sup> will be described by such wave functions as

$$\psi = a[(h_{9/2})_p(d_{5/2})_n] + b[(h_{9/2})_p(s_{1/2})_n] + d[(h_{9/2})_p(g_{7/2})_n] + e[(h_{9/2})_p(d_{3/2})_n],$$

together with other basic states like  $(f_{7/2})_p(i_{11/2})_n$ ,  $(f_{7/2})_p(d_{5/2})_n$ , etc., which do not affect the angular distribution. If the mixing is large (i.e., *a*, *b*, *d*, and *e* are comparable in magnitude), the observed angular distribution should be a superposition of the distributions for three *l* values, 0, 2, and 4. The mixing between *s*<sub>1/2</sub> and *d*<sub>3/2</sub> could be very easily studied since the low-angle behavior of the proton angular distributions is markedly different for these two *l* values.

In view of all these considerations a careful study of the Bi<sup>209</sup>(*d,p*) reactions was undertaken. In Ref. 1 we have already reported several states in Bi<sup>210</sup> excited in (*d,p*) reactions. The present data, taken at a somewhat better resolution (30 → 35 keV), indicate that some of the levels are close doublets, in agreement with the MIT work.<sup>4</sup> The over-all conclusions reached in the Indiana work,<sup>5</sup> although limited by poor resolution, are found to be essentially correct. We have also found several weakly excited states around 1.7-MeV excitation, which are not found in the MIT work. These are presumably the positive parity states corresponding to  $(h_{9/2})_p(j_{15/2})_n$  configuration. Because of the higher angular momentum (*l*=7) involved in the capture of the neutron, these states are expected to be very weakly excited at an incident deuteron energy of 8 MeV. Perhaps the most interesting part of the present report is the study of the angular distribution of the protons leading to several well-separated states in Bi<sup>210</sup>. It has been found that the extent of configuration mixing is not as large as might be expected from the shell model, even for some states as high as 2.5 MeV from the ground state.

A thin target of Bi (0.9 mg/cm<sup>2</sup>) was prepared by evaporation of Bi on 0.2 mg/cm<sup>2</sup> Au backing. This was exposed to the 14.8-MeV deuteron beam from the 120-cm-diam cyclotron of the University of Pittsburgh. The experimental arrangement has been described elsewhere.<sup>6</sup> The protons were analyzed by a 60° wedge magnetic spectrograph and were detected by emulsion plates placed along the focal plane of the magnet. Aluminum absorbers of adequate thickness were placed

TABLE I. Energy levels of Bi<sup>210</sup> excited by Bi<sup>209</sup>(*d,p*) reactions.

Level No.	Energy (MeV) <sup>a</sup>	Relative yields <sup>a</sup> at 60°	Configuration
1	0	6	$(h_{9/2})_p(g_{9/2})_n$
2	0.041	2	
3	0.266	42	
4	0.329	25	
5	0.356	9	
6	0.428	68	
7	0.499	22	
8	0.547	25	
9	0.580	35	
10	0.631	5	$(h_{9/2})_p(i_{11/2})_n$
11	0.905	7	$+(f_{7/2})_p(g_{9/2})_n$
12	1.177	7	
13	1.322	11	
14	1.369	9	
15	1.465	12	
16	1.521	4	
17	1.575	19	$(h_{9/2})_p(d_{5/2})_n$
18	1.706	7	$(h_{9/2})_p(j_{15/2})_n$
19	1.738	5	
20	1.778	15	
21	1.835	5	
22	1.916	56	$(h_{9/2})_p(d_{5/2})_n$
23	1.974	290	$+(f_{7/2})_p(i_{11/2})_n$
24	2.029	140	
25	2.086	94	
26	2.111	90	
27	2.167	61	
28	2.228	69	
29	2.518	200	$(h_{9/2})_p(s_{1/2})_n$
30	2.570	278	
31	2.619	26	$(h_{9/2})_p(g_{7/2})_n$
32	2.716	53	$+(h_{9/2})_p(d_{3/2})_n$
33	2.746	59	$+(f_{7/2})_p(d_{5/2})_n$
34	2.829	148	
35	2.904	14	
36	2.988	66	
37	3.014 <sup>b</sup>	69	
38	3.039 <sup>b</sup>	65	
39	3.066 <sup>b</sup>	46	
40	3.098 <sup>b</sup>	77	
41	3.131	252	
42	3.160 <sup>b</sup>	55	
43	3.187 <sup>b</sup>	137	
44	3.228	106	

<sup>a</sup> The energies are correct to within 15 keV and the probable error in the relative yields is 10%, with the exception of the weaker groups where the error is as large as 30%.

<sup>b</sup> Due to the nature of graphical analysis there are considerable uncertainties in the energy and intensity values of these levels.

in front of the emulsion plates which allowed only the protons to be registered on the plates. Exposures were made for laboratory scattering angles of 9°, 10°, 15°, 20°, 25°, 30°, 35°, and 60°. The presence of C<sup>12</sup> and O<sup>16</sup> was disturbing in all low-angle runs, but in most cases the four contaminant proton groups due to these impurities were well separated from the proton groups in Bi<sup>209</sup>(*d,p*) reactions. The subtraction of the background from Au was straightforward.

## RESULTS AND DISCUSSIONS

A typical spectrum of Bi<sup>209</sup>(*d,p*) is shown in Figs. 1(a) and (b). Altogether 44 levels are found, which are also numbered in Fig. 1. The energies and relative yields for

<sup>4</sup> J. R. Erskine, W. W. Buechner, and H. A. Enge, Phys. Rev. **128**, 720 (1962).

<sup>5</sup> G. B. Holm, J. R. Burwell, and D. W. Miller, Phys. Rev. **118**, 1247 (1960).

<sup>6</sup> B. L. Cohen, S. Mayo, and R. E. Price, Nucl. Phys. **20**, 360 (1960).

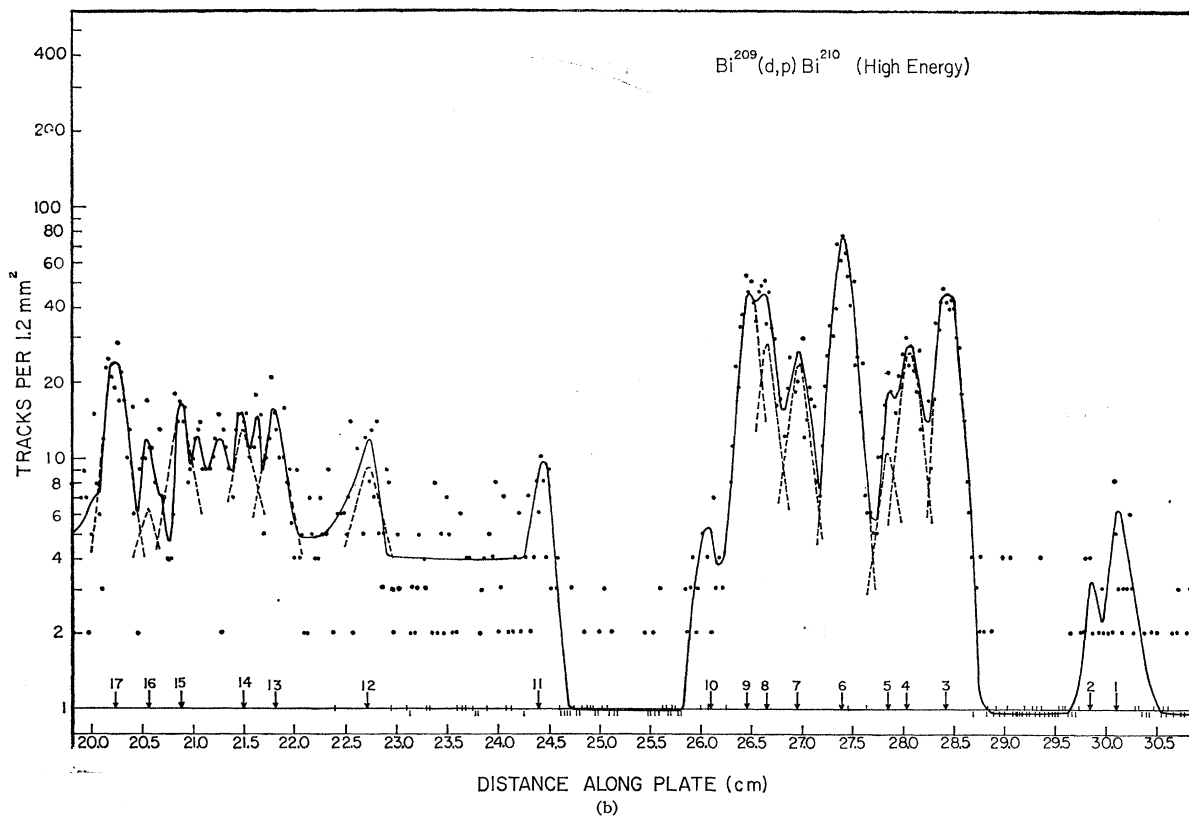
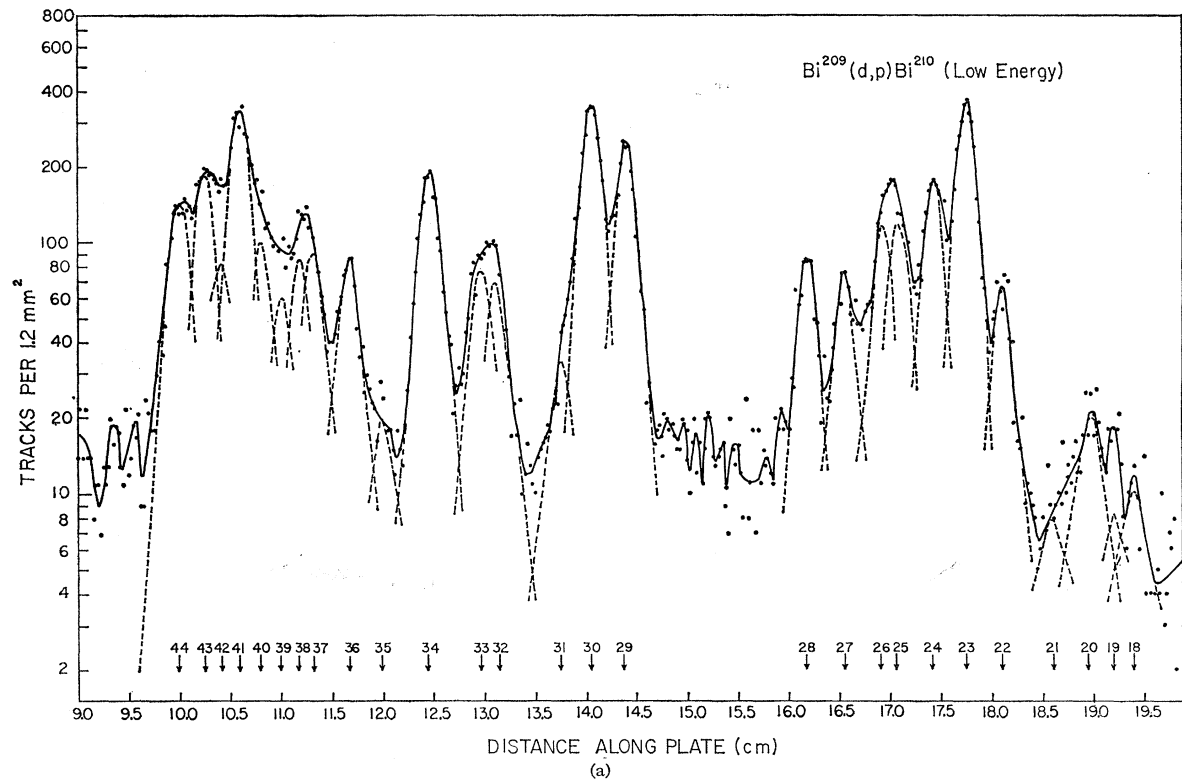


FIG. 1. (a) Low-energy part of the proton energy spectrum in  $\text{Bi}^{209}(d,p)\text{Bi}^{210}$  reactions for a scattering angle of  $60^\circ$ . The dashed curves show the analyzed spectrum. The proton groups are numbered, and the arrows below these numbers indicate the peak positions. (b) High-energy part of the proton spectrum in  $\text{Bi}^{209}(d,p)\text{Bi}^{210}$  reactions for a scattering angle of  $60^\circ$ . The experimental points for 0 and 1 ordinate readings are represented by a different symbol for the sake of clarity. The proton groups are numbered as in Fig. 1(a).

these levels are listed in Table I for a scattering angle of 60°. In order to understand these levels we have constructed the possible zeroth-order level scheme of Bi<sup>210</sup> from *j-j* coupling shell-model considerations. This is shown in Fig. 2. These levels are constructed just by adding the single-proton and single-neutron energies from Bi<sup>209</sup> and Pb<sup>209</sup>, respectively. The possible spins (*J*) and parities ( $\Pi$ ) are also indicated for each level. The encircled spin states are not expected to be excited in (*d,p*) reactions, as they have very little *h*<sub>9/2</sub> proton state admixture. However, this is true only if the Bi<sup>209</sup> ground state is a very pure *h*<sub>9/2</sub> proton state. In view of the double-shell closure at Pb<sup>208</sup> we do expect the ground state of Bi<sup>209</sup> to be a pure *h*<sub>9/2</sub> state. But it should be mentioned that there must be some seniority 3 admixture in the ground-state wavefunction of Bi<sup>209</sup> to explain its anomalous magnetic moment.<sup>7</sup> According to Fig. 2, a group of ten levels are expected near the ground state. We observe nine of them, which we expect to belong to the configuration (*h*<sub>9/2</sub>)<sub>p</sub>(*g*<sub>9/2</sub>)<sub>n</sub>. The ground-state spin of Bi<sup>210</sup> has been measured directly,<sup>8</sup> and found to be 1<sup>-</sup>. As will be seen presently, the relative cross section for exciting this state in (*d,p*) reactions also supports this. The next state at 0.047 MeV is also known<sup>9</sup> from the beta decay of Pb<sup>210</sup>. Its spin is most probably 0<sup>-</sup> as indicated by the *M1* character of the 0.047-MeV gamma ray depopulating it. Its low intensity has made it difficult to identify in (*d,p*) reactions with the present resolution, but its presence was obvious in all the spectra available to us. The next excited state at 0.266 MeV is definitely the 9<sup>-</sup> metastable state of Bi<sup>210</sup>, which decays by alpha emission. The relative cross section for exciting this state is also in nice agreement with the spin value.

If we assume that the effect of configuration mixing is small for the levels in the (*h*<sub>9/2</sub>)<sub>p</sub>(*g*<sub>9/2</sub>)<sub>n</sub> state, then the relative intensities of the proton groups should be proportional to the statistical weight factor  $2J+1$  of the respective levels.<sup>4</sup> The most probable admixture should be (*h*<sub>9/2</sub>)<sub>p</sub>(*i*<sub>11/2</sub>)<sub>n</sub>. From an extrapolation of the data in the distorted-wave Born approximation (DWB) calculation (see, for example, Fig. 5 of Ref. 1) we know that at  $Q=+2$  MeV, the cross section of *i*<sub>11/2</sub> neutron capture in (*d,p*) reactions is about 1/15th of that of *g*<sub>9/2</sub> neutron capture. Thus, the presence of (*h*<sub>9/2</sub>)<sub>p</sub>(*i*<sub>11/2</sub>)<sub>n</sub> admixture in the ground-state configuration can be detected by comparing the total (*d,p*) cross section for exciting all the ten levels in Bi<sup>210</sup> with the ground-state excitation cross section in Pb<sup>209</sup>. At 60° we find that the former is 2.1 mb/sr, while the latter is 2.4 mb/sr. From the DWB calculation we estimate that for  $l=4$ , the cross section goes down with  $Q$  as  $(1.4)^{-Q}$ . So the above figures support the hypothesis that the observed group near the ground state has an almost pure (*h*<sub>9/2</sub>)<sub>p</sub>(*g*<sub>9/2</sub>)<sub>n</sub> configuration. We have, of course, completely ignored

<sup>7</sup>H. Noya, A. Arima, and H. Horie, Suppl. Progr. Theoret. Phys. (Kyoto) No. 8, 33 (1958); N. Freed and L. S. Kisslinger, Nucl. Phys. 25, 611 (1961).

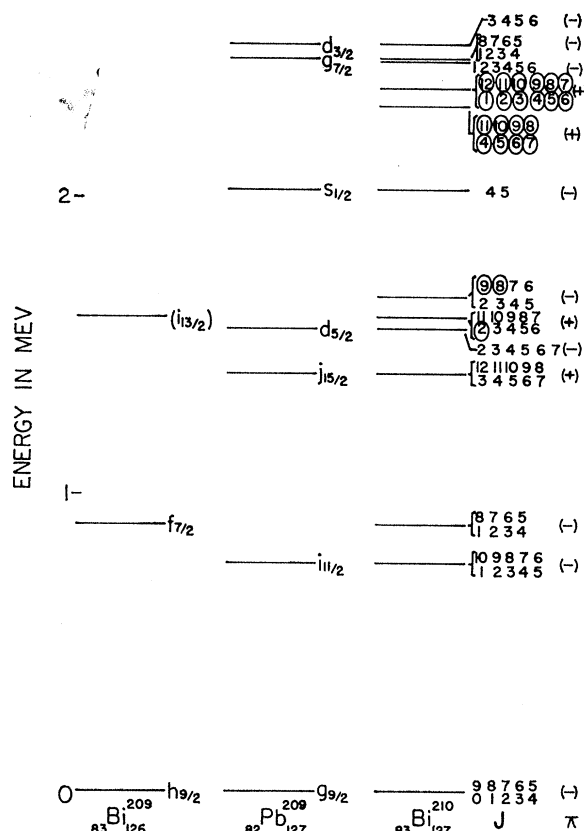


FIG. 2. Known single-proton states in Bi<sup>209</sup> and single-neutron states in Pb<sup>209</sup>, together with the possible *j-j* coupled states in Bi<sup>210</sup> in zeroth order. The encircled spin (*J*) states are not expected to be excited in (*d,p*) reactions.

the possibility of other admixtures like (*h*<sub>9/2</sub>)<sub>p</sub>(*d*<sub>5/2</sub>)<sub>n</sub> or (*h*<sub>9/2</sub>)<sub>p</sub>(*s*<sub>1/2</sub>)<sub>n</sub>, whose presence would evidently increase the proton yield in the ground-state group. But these states are considerably above the ground state, so their admixture will be very small.

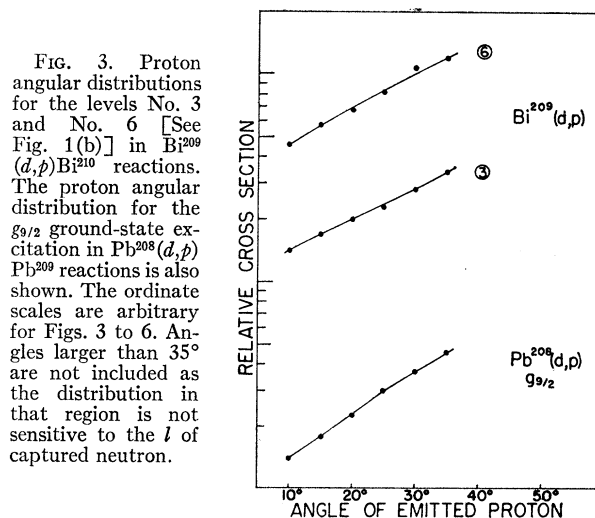


FIG. 3. Proton angular distributions for the levels No. 3 and No. 6 [See Fig. 1(b)] in Bi<sup>209</sup> (*d,p*)Bi<sup>210</sup> reactions. The proton angular distribution for the *g*<sub>9/2</sub> ground-state excitation in Pb<sup>209</sup> (*d,p*)Pb<sup>209</sup> reactions is also shown. The ordinate scales are arbitrary for Figs. 3 to 6. Angles larger than 35° are not included as the distribution in that region is not sensitive to the *l* of captured neutron.

TABLE II. Comparison of the proton yields in the ground-state band of  $\text{Bi}^{209}(d,p)$  with  $2J+1$  factors.

Level No.	Energy ( $E$ ) (MeV)	Intensity <sup>a</sup> ( $I$ )	$J$	$(2J+1)^a (1.4)^b$
1	0.000	2.4	1	2.61
2	0.041	0.8	0	0.87
3	0.266	18.0	9	18.05
4	0.329	10.7	5 (6)	10.72
5	0.356	4.0	2	4.88
6	0.428	29.1	4,7 (3,8)	24.07
7	0.499	8.8	3 (4)	7.24
8	0.547	11.1	6 (5)	13.60
9	0.580	15.1	8 (7)	17.96

<sup>a</sup> Total strength is 100.

Additional evidence for an almost pure  $g_{9/2}$  neutron configuration of the ground-state group comes from angular distribution studies. The angular distributions of protons for the two well-resolved levels are shown in Fig. 3. The proton angular distribution for the  $g_{9/2}$  ground-state excitation in  $\text{Pb}^{208}(d,p)$  reactions is also included in the figure for comparison. The identical angular distributions in the two cases, with almost the same  $Q$  values, support our observation that the ground-state group in  $\text{Bi}^{210}$  is almost pure  $(h_{9/2})_p(g_{9/2})_n$ .

It now seems reasonable to search for suitable  $J$  values for the excited states to give consistent proton yields for the various levels. This is done in Table II for a scattering angle of  $60^\circ$ . It is evident that our choice of  $J$ 's gives reasonable yield ratios, although in some cases an alternative spin can also be found within our experimental uncertainty. In comparing the observed yields with the  $(2J+1)$  factors we have taken into account the dependence of the cross section on  $Q$ . This is represented by the factor  $(1.4)^b$  in column 5 of Table II. The numerical factor 1.4 is found, as already mentioned, from DWB calculations. Thus, the spins of the first three states are in nice agreement with the known data. From  $\text{Bi}^{209}(n,\gamma)$  reactions two prominent gamma rays of energies 4.065 and 4.170 MeV are known.<sup>3</sup> Since in such reactions the neutron is captured predominantly in  $S$  wave, this excites  $4^-$  and  $5^-$  states in  $\text{Bi}^{210}$  at about 4.6 MeV. The two gamma rays, 4.065 and 4.170 MeV, are probably feeding the 0.547- and 0.428-MeV states, respectively, of the ground-state group. The probable spin ( $5^-$  or  $6^-$ ) of the 0.547-MeV state is consistent with a strong gamma branching to this state from a  $5^-$  level at about 4.6 MeV. The large proton yield for the 0.428-MeV level suggests that it is composed of two close-lying levels, and from Table II we see that  $J=4$  and 7 (or 3 and 8) give the correct proton yield for this composite group. The 4.170-MeV gamma ray is presumably exciting the  $4^-$  state.

It will be instructive to compare the observed levels near the ground state with the energy levels calculated from the  $j-j$  coupling shell model. The latter were calculated by Newby and Konopinski<sup>8</sup> using a finite-

<sup>8</sup> N. Newby, Jr., and E. J. Konopinski, Phys. Rev. **115**, 434 (1959).

range central potential as residual neutron-proton interaction. As reported in Ref. 5, such calculations give the spin sequence of the ten states, starting from the ground state, as 0, 1, 9, 2, 3, 7, 5, 4, 6, and 8. This may be compared with our spin identifications: 1, 0, 9, 5 (6), 2, 4 (3), 7 (8), 3 (4), 6 (5), and 8 (7). In the theoretical spectrum<sup>5</sup> there is a gap of about 0.4 MeV between the  $0^-$  state and the  $1^-$  first excited state. The nonexistence of such a gap in the actual spectrum and the reversal of the  $1^-$  and  $0^-$  states perhaps indicate that the residual neutron-proton interaction is not purely central. It will be interesting to revise such shell-model calculations taking into account the tensor interaction between the neutron and the proton. Newby and Konopinski<sup>8</sup> have indicated that the tensor interaction does alter the  $0^-$ ,  $1^-$  sequence. Our conclusion that the  $1^-$  ground state of  $\text{Bi}^{210}$  is pure  $(h_{9/2})_p(g_{9/2})_n$  will have important consequences for the beta spectrum of RaE ( $\text{Bi}^{210}$ ). As indicated by Newby and Konopinski<sup>8</sup> the spectrum parameter  $\xi$  should be about  $+1$ . But a pure  $(h_{9/2})_p(g_{9/2})_n$   $1^-$  state gives  $\xi = -\frac{1}{10}$ , while a  $(h_{9/2})_p(i_{11/2})_n$   $1^-$  state gives  $\xi = +1$ . One possibility will be that there is such a  $1^-$  state belonging to the  $(i_{11/2})_n$  configuration.<sup>9</sup> If we ignore configuration mixing, the  $(d,p)$  cross section for exciting such a state will be about 1/15th of that of exciting the observed ground state. Thus, it will be extremely difficult to locate such a level in our experiments.

From the low yields of the next seven proton groups (Nos. 10 to 16) we conclude that these levels belong to the  $(h_{9/2})_p(i_{11/2})_n$  configuration. Because of their low intensities it was not possible to study their angular distributions. Referring back to Fig. 2, we see that there should be 18 levels having an  $i_{11/2}$  neutron configuration, arising as a result of configuration mixing between  $(h_{9/2})_p(i_{11/2})_n$  and  $(f_{7/2})_p(g_{9/2})_n$  states. We have, thus, failed to locate many possible states in this region. This may be partly due to energy resolution, and partly because of low intensities of such proton groups.

The level No. 17 in Table I does not seem to belong to the  $(i_{11/2})_n$  group. It has an appreciable intensity up to a scattering angle of  $9^\circ$ , unlike other weak levels in the  $(i_{11/2})_n$  group. Unfortunately, the angular distribution study for this level becomes impossible below  $25^\circ$  due to the presence of the contaminant proton group corresponding to the 0.87-MeV excitation in  $\text{O}^{17}$ . But from its yields at  $60^\circ$ ,  $35^\circ$ ,  $30^\circ$ ,  $25^\circ$ , and  $9^\circ$ , we have concluded that the captured neutron is in a  $d$  state. This

<sup>9</sup> Recently, R. M. Spector and R. J. Blin-Stoyle [Phys. Letters **1**, 118 (1962)] have shown that the phenomenological value of the spectrum parameter  $\xi$  ( $A$  in their notation) should lie between  $-0.4$  and  $-1.3$ . With a finite-range Gaussian potential they found the wave function of the lowest  $1^-$  state in  $\text{Bi}^{210}$  to be  $0.963(h_{9/2})_p(g_{9/2})_n + 0.264(h_{9/2})_p(i_{11/2})_n - 0.049(f_{7/2})_p(g_{9/2})_n$ . This gives a magnetic moment in excellent agreement with the measured magnetic moment of RaE, but yields  $\xi = 0.27$ . However, admixture of such states as due to excitation of an  $h_{11/2}$  proton to an  $h_{9/2}$  state (core excitation) changed  $\xi$  drastically, and values ranging from  $-0.73$  to  $-1.37$  could be found depending on the separation energy of  $h_{9/2}$  and  $h_{11/2}$  states. Such admixtures are known to be present in the  $\text{Bi}^{209}$  ground-state wave function.

is most probably the  $2^-$  state belonging to the  $(h_{9/2})_p(d_{5/2})_n$  configuration. A similar conclusion has also been reached for this level in Ref. 4.

Levels 18 to 21 are most likely the positive-parity levels belonging to the  $(h_{9/2})_p(j_{15/2})_n$  configuration. The  $Q$  value of this group also closely corresponds to the  $Q$  value for the  $j_{15/2}$  level in  $\text{Pb}^{209}$ . These are not observed in Ref. 4, probably because at 8-MeV incident deuteron energy they are very weakly excited. From Fig. 1(a) it is obvious that the present resolution is not sufficient to detect all ten positive-parity levels produced in Fig. 2.

From shell-model considerations the next group of seven levels (Nos. 22 to 28) should belong to the  $(h_{9/2})_p(d_{5/2})_n$  configuration. The angular distributions of protons exciting these levels are shown in Fig. 4, together with the angular distribution for exciting the  $d_{5/2}$  neutron state in  $\text{Pb}^{209}$ . The identical nature of the angular distribution indicates that all the seven levels should belong primarily to an  $l=2$  neutron configuration. From Fig. 2 we see that an  $(h_{9/2})_p(d_{5/2})_n$  configuration gives six possible states of spins 2 to 7, and odd parity. Of these we have already identified the  $2^-$  level [No. 17 in Fig. 1(b)]. Thus, we are observing eight levels instead of the six predicted. This is not unexpected due to configuration mixing between  $(h_{9/2})_p(d_{5/2})_n$  and  $(f_{7/2})_p(i_{11/2})_n$  states. In Fig. 2 we thus have six more negative-parity states with spins ranging from 2 to 7. Since these two sets of levels lie very close together, there will be considerable mixing between them. So in  $(d,p)$  reactions they will be almost equally excited. It is obvious that we have failed to detect all the twelve levels here because of our limitation on energy resolution. However, an improvement in resolution by a factor of 4 in Ref. 4 indicates only one more weak

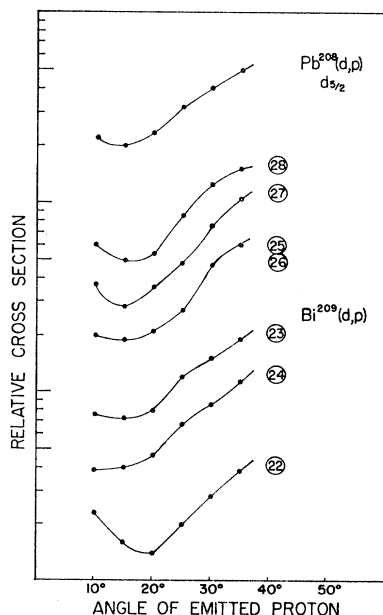


FIG. 4. Proton angular distributions for the levels Nos. 22 through 28 [See Fig. 1(a)] in  $\text{Bi}^{210}(d,p)\text{Bi}^{210}$  reactions. The proton angular distribution for exciting the  $d_{5/2}$  state in  $\text{Pb}^{209}(d,p)\text{Pb}^{209}$  is also included for comparison.

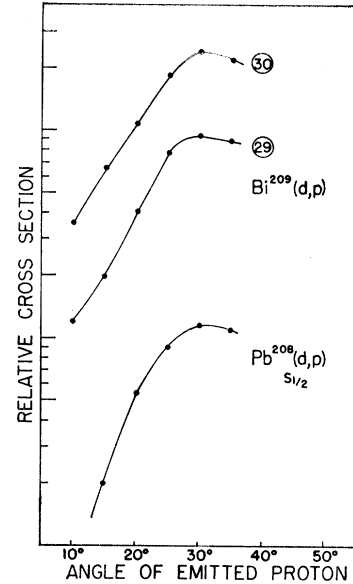


FIG. 5. Comparison of the proton angular distributions for levels Nos. 29 and 30 [See Fig. 1(a)] in  $\text{Bi}^{209}(d,p)\text{Bi}^{210}$ , with that of exciting the  $s_{1/2}$  state in  $\text{Pb}^{209}(d,p)\text{Pb}^{209}$  reactions.

level, that between levels No. 26 and No. 27. Because of its abnormally large yield the level No. 23 may be composite.

We can now examine the possibility that the 1.60-MeV level in  $\text{Bi}^{209}$  is a negative-parity state, e. g.,  $p_{3/2}$  or  $f_{5/2}$ . It is evident that this would give rise to more negative-parity levels in  $\text{Bi}^{210}$  in the  $(d_{5/2})_n$  neutron band. But the observed level density in this energy region does not encourage such an assumption. Such negative-parity states probably lie above 2 MeV in  $\text{Bi}^{209}$ .

The angular distributions of proton groups corresponding to levels No. 29 and No. 30 are shown in Fig. 5, which also includes the proton angular distribution for the  $s_{1/2}$  level in  $\text{Pb}^{209}$ . The two sets of angular distributions are remarkably similar. Even the characteristic dip in the angular distribution pattern at  $\theta=35^\circ$  is found for these two levels in  $\text{Bi}^{210}$ . These are definitely the  $4^-$  and  $5^-$  levels belonging to the  $(h_{9/2})_p(s_{1/2})_n$  configuration. According to the usual coupling rules<sup>10</sup> the  $4^-$  level should lie lower. This is supported by the relative intensities for exciting the two levels. But the most interesting observation is the almost pure  $(s_{1/2})_n$  configuration for these two states. One should expect to observe measurable  $(d_{5/2})_n$ ,  $(d_{3/2})_n$ , and  $(g_{7/2})_n$  admixtures in these two states, since, from Fig. 2, the  $4^-$  and  $5^-$  states are distributed among all these configurations. A comparison of Fig. 4 and 5 shows that the presence of  $(d_{5/2})_n$  state in levels Nos. 29 and 30 could be very easily detected to within 10%. The DWB calculation, as well as the angular distribution for exciting the  $d_{5/2}$  and  $d_{3/2}$  states in  $\text{Pb}^{209}$ , indicate<sup>1</sup> that the general behavior for the  $l=2$  angular distribution pattern remains unaltered in this energy range. So we conclude that there is little (less than 25%)  $d$ -state

<sup>10</sup> D. Kurath, in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960).

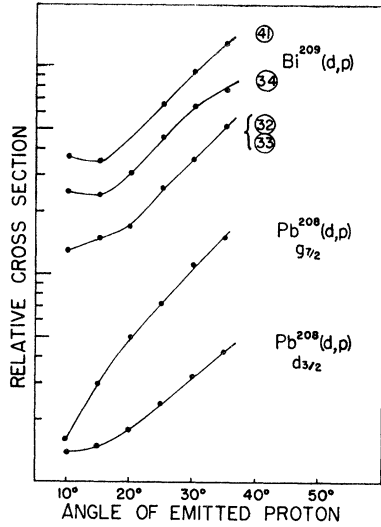


FIG. 6. Proton angular distributions for exciting the levels Nos. 32-32, 34, and 41 [See Fig. 1(a)] in  $\text{Bi}^{209}(d,p)\text{Bi}^{210}$  reactions together with the same for exciting the  $g_{7/2}$  and  $d_{3/2}$  states in  $\text{Pb}^{209}(d,p)\text{Pb}^{209}$  reactions.

neutron admixture in levels Nos. 29 and 30. In the Moshinsky notation<sup>11</sup> this means that

$$\begin{aligned} & \langle 05\frac{1}{2}, \frac{9}{2}; 22\frac{1}{2}, \frac{5}{2}; J | V(r) | 05\frac{1}{2}, \frac{9}{2}; 30\frac{1}{2}, \frac{1}{2}; J \rangle \\ & \ll \epsilon(s_{1/2}) - \epsilon(d_{5/2}) = 0.47 \text{ MeV}, \end{aligned} \quad (1)$$

where  $J$  is 4 or 5,  $V(r)$  stands for the neutron-proton residual interaction, and the  $\epsilon$ 's are s. p. energies. Similarly,

$$\begin{aligned} & \langle 05\frac{1}{2}, \frac{9}{2}; 22\frac{1}{2}, \frac{3}{2}; J | V(r) | 05\frac{1}{2}, \frac{9}{2}; 30\frac{1}{2}, \frac{1}{2}; J \rangle \\ & \ll \epsilon(d_{3/2}) - \epsilon(s_{1/2}) = 0.49 \text{ MeV}. \end{aligned} \quad (2)$$

However, evaluation of the matrix element (1) for  $J=4$  shows<sup>12</sup> that this nondiagonal term is  $+0.352$  MeV for the parameters used by Newby and Konopinski.<sup>8</sup> So, according to the perturbation estimate, the  $(f_{7/2})_p(d_{5/2})_n$  and  $(h_{9/2})_p(s_{1/2})_n$  admixtures should be comparable in the  $4^-$  state. Our observation perhaps

<sup>11</sup> M. Moshinsky, Nucl. Phys. **13**, 104 (1959).

<sup>12</sup> The matrix element has been evaluated by T. S. Kuo, using  $V(r) = [\alpha_{33}^{33}p + \alpha_{11}^{11}p + \alpha_{13}^{13}p + \alpha_{31}^{31}p]V_0(r)$ , where  $[ST]_p$  are projection operators in spin space and isotopic spin space and  $V_0(r) = e^{-r^2/r_0^2}$ ,  $r_0 = 1.85$  F,  $\alpha_{31} = -60$  MeV, and  $\alpha_{11} = \alpha_{33} = 0$ .

indicates that such pure central  $n$ - $p$  interaction may not be adequate in the spectroscopy of  $\text{Bi}^{210}$ .

The last group of levels [from 31 to 44 in Fig. 1(a)] should belong to neutron configurations  $g_{7/2}$  and  $d_{3/2}$ . As is apparent in Fig. 2, there will be 18 levels in this group due to configuration mixing between the  $(f_{7/2})_p(d_{5/2})_n$ ,  $(h_{9/2})_p(g_{7/2})_n$ , and  $(h_{9/2})_p(d_{3/2})_n$  states. Since in the zeroth order these three degenerate levels are very close together, the residual neutron-proton interaction will split them over a wide range. This is evident from the 0.61-MeV separation between the levels Nos. 44 and 31 as compared to the 0.05-MeV separation between the  $4^-$  and  $5^-$  levels in the  $(h_{9/2})_p(s_{1/2})_n$  configuration. In Fig. 6 the angular distribution of protons for some of the well-separated levels are shown. Also listed are the proton angular distributions for exciting the  $g_{7/2}$  and  $d_{3/2}$  states in  $\text{Pb}^{209}$ . Because of lack of resolution and the presence of the  $\text{C}^{13}$ (3.09-MeV level) contaminant group, it was not possible to study the angular distributions for most of the levels. Still the presence of  $d_{3/2}$  neutron configuration is clearly indicated in Fig. 6.

It is expected that the level structure of  $\text{Bi}^{210}$  is actually more complicated for energies above 2.5 MeV. Over and above the 18 negative-parity states in Fig. 2, there should be 14 more negative-parity states belonging to the  $(f_{5/2})_p(g_{9/2})_n$ ,  $(p_{3/2})_p(g_{9/2})_n$ ,  $(p_{1/2})_p(g_{9/2})_n$ , and  $(f_{7/2})_p(s_{1/2})_n$  configurations. There should also be states due to core excitation. Of the 14 states we have managed to identify, many are presumably composite. But it seems very surprising that an improvement in resolution by a factor of 4 has not indicated<sup>4</sup> any other levels in this region.

#### ACKNOWLEDGMENTS

The author is grateful to B. L. Cohen, N. Austern, E. Baranger, I. Dutt, and W. W. Daehnick for many helpful discussions. He thanks T. Kuo for calculating the matrix elements using the IBM-7070 computer of the University of Pittsburgh. He is also thankful to G. Fodor for target preparation, and to the members of the cyclotron staff and plate scanning group for their excellent cooperation.