Multiple Scattering of Electrons and Positrons*

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Multiple scattering of 2650 electron and 1887 positron tracks was studied in nitrogen by cloud-chamber techniques. The theory of Williams with limiting angles due to Bethe best describes the multiple scattering of electrons and positrons in the momentum range between 1000 and 6000 G-cm. The theory of Molière is less satisfactory, underestimating the multiple scattering above 2000 G-cm by an appreciable amount.

Analysis of positron-electron differences suggests that electron multiple scattering exceeds that for positrons by, at most, a few percent. This is in agreement with theoretical predictions of Nigam, Sundaresan, and Wu, and Mohr. No evidence is found for the very large differences claimed in most other experimental work.

INTRODUCTION

A CONSIDERABLE amount of both theoretical and experimental work on multiple scattering has been done to determine the dependence on the kind and momentum of the scattered particle and on the several parameters describing the scattering medium. Differences exist among the theories[1] as to momentum and charge dependence. Some of these differences are accounted for by the different approximations made in reducing this very complex problem to a point where numerical predictions can be made.

Experimental work on multiple scattering was at first done mainly in thin foils and with the exception of some experiments[2] often did not agree well with any of the theoretical interpretations. More recent work has generally been performed in gases and nuclear emulsions where the effects of large-angle single scattering could be removed by visual observations. Groetzinger, Berger, and Ribe[3] examined the tracks of 132 electrons from a P^{32} source for multiple scattering as a function of momentum. Of these, 108 above 2000 G-cm were fitted to a smooth curve by the Gauss least-squares method. No definite conclusion could be reached from their results, as they agreed fairly well with theories of Molière, Snyder, and Scott and also Williams as modified by Bethe. Hisdal,[4] on the other hand, using an emulsion technique, measured scattering of electrons at 0.59 MeV and found a distribution 40% narrower than that predicted by Molière. Also Cusack and Stott,[5] using a cloud chamber, found a distribution narrower than predicted around 0.4 MeV. The difference between positron-electron multiple scattering has been worked out by Mohr[6] and Nigam, Sundaresan, and Wu.[7] These calculations indicate small but finite differences. Experimental work[8] on the positron-electron difference has been inconclusive. It is interesting to note that, except for those of Cusack and Stott,[8] all the experiments found differences considerably larger than the predictions of Mohr and Nigam.

The present work describes the multiple scattering of electrons and positrons in nitrogen. Two aspects of this problem are examined in detail. These are the momentum dependence of multiple scattering for electrons and positrons separately, and the differences between the electron and positron dependence as a function of momentum.

APPARATUS AND METHOD USED

To carry out the experiment two major pieces of apparatus were constructed, an automatic low-turbulence Wilson chamber with associated magnet coils, controls and camera, and a scanner-comparator for analyzing the 35 mm films and making coordinate measurements.

The Wilson chamber was capable of running with little attention for long enough to complete one hundred feet of film at a time. The chamber was filled with nitrogen at about 1000 mm of mercury and water-ethanol mixture to provide the tracks. An As^{74} source was used, supplying both the positrons and the electrons, the distinction being made later visually from the direction of curvature. The camera took one view only, so that the scattering measured was the projection in a plane.

A number of accidental high-energy cosmic-ray tracks were included in the measurements. Of these, twelve had radii of curvature of 20 m or more. The

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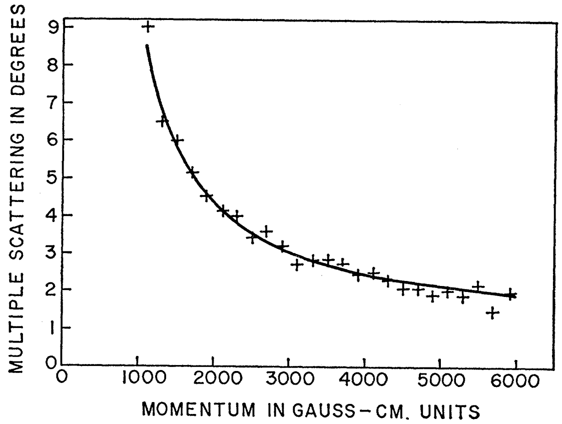

FIG. 1. Best-fit curve to electron scattering data for 2 cm path in nitrogen at 760 mm pressure, 20°C.

computed multiple scattering in all these cases was no more than would result from measurement error on a straight line or circle. The effect of turbulence was, therefore, considered negligible.

The radial variation of the magnetic field was computed from the formulas of Foss[9] and the expansion coefficients used in the subsequent data reduction.

The scanner-comparator was based on those at Columbia University's Nevis Cyclotron Laboratory. However, Coleman Digitizers were used to measure the coordinates, which were then punched automatically by the IBM 026 key punch, together with film and frame identification and a number indicating whether a positron track, electron track, or other information was being recorded.

DATA REDUCTION

Because of the inherently statistical nature of the problem it was necessary, to achieve a reasonable accuracy, that measurements be made on a very large number of tracks. To handle this large quantity of data it was necessary to perform most of the calculations on an IBM 7090 computer.

The data reduction was performed in a number of steps. The first program computed the probable momentum and multiple scattering for each track and recorded the output on cards, one per track. In addition, this program was designed to check input and output for possible errors, noting these on the simultaneous printed output. The output cards not rejected here were used as input to the succeeding steps. The scattering for each track was then adjusted to that for 25°C, 760 mm pressure and 2 cm path length and the tracks grouped into a number of momentum ranges and a weighted mean value found for each. These composite points were then used as a basis for finding the best-fit curves in the Gaussian least-squares sense. The program used could not easily use the large number of original points. Bethe's criterion for multiple scattering was used in selecting tracks for measurements. This meant that the tracks contained no visible large-angle single scattering, larger than 0.1 rad.

[9] M. H. Foss, Tech. Rept. No. 2, Carnegie Institute of Technology, Task Order 1 NR 025-035 (unpublished).

EXPERIMENTAL RESULTS, ERRORS, AND CONCLUSIONS

Results

Close to 5000 tracks were measured as described above and the resulting data processed by computers. Altogether 4537 tracks were used, of which 2650 were of electrons and 1887 were of positrons. The combined length of all the tracks was 33 846 cm. The tracks used in the final analysis have a minimum of 4 angle measurements and an average of between 6 and 7 angles.

Experimental results are shown in Figs. 1 and 2 for the electrons and positrons, respectively. Each point represents the mean for a number of tracks in a momentum range of 200 G-cm. The abscissa is the weighted mean of the momentum for each of the contributing tracks. The weighting factor is the number of angles measured in each track. The ordinate, similarly, is the weighted mean of the average scattering for each track. The smooth curves in the Figures are best fits,[10] in the Gaussian least-squares sense, to the experimental points.

The points were fitted to a function of the form

$$S=(a_0+a_2x^{-2}+a_4x^{-4})^{1/2},$$

where S = scattering in deg/2 cm path at 76 cm, 25°C; $x=H\rho$ in G-cm. The weighting factor, W_i, used in finding the smooth curve is

$$W_i=\frac{N_i}{\sigma_i^2},$$

where N_i = number of tracks in ith momentum range and σ_i = standard deviation of the points in the ith momentum range about their mean value.

The curve-fitting program gives several kinds of statistical information besides the coordinates of points on the best-fit curve. It includes the standard deviation of the predicted mean so that one is able to attach a certain amount of significance to differences between the experimental curves and the theories and, more accurately, to differences between the electron and positron cases. In the latter case, due to equal treatment of the two kinds of track, any systematic error affects both equally and differences are not expected to be changed drastically.

[10] W. E. Deming, *Statistical Adjustment of Data* (John Wiley & Sons, Inc., New York, 1943). A. Hald, *Statistical Theory with Engineering Applications* (John Wiley & Sons, Inc., New York, 1952). R. H. Moore and R. K. Zeigler, Los Alamos Scientific Laboratory Report, (PTD) LA-2367, TID-4500, 1959, 15th ed. (unpublished).

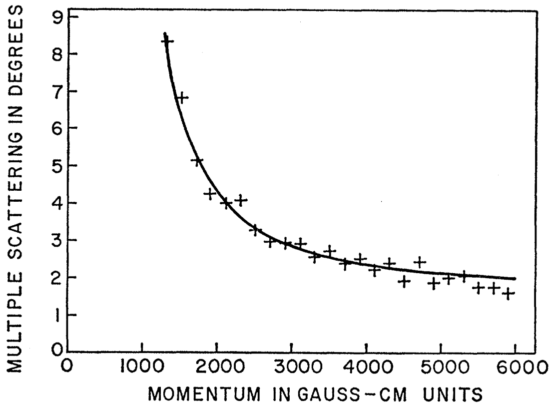

FIG. 2. Best-fit curve to positron scattering data for 2 cm path in nitrogen at 760 mm pressure, 20°C.

Figure 3 shows both the electron and positron curves together with a number of theoretical curves for electrons. Figure 4 has been drawn with a view to interpreting any electron-positron difference that may be significant. Two sets of points are given. The crosses together with the scale on the left show the difference between electron and positron scattering as a function of $H\rho$ in G-cm. As may also be seen from the curves in Fig. 4, for low momentum this difference is large and negative. Between 2200 and 3800 G-cm the difference is positive and at the higher momenta, small but again negative. The set of points marked by circles are to be associated with the same abscissae as the other points but the ordinate scale is given on the right-hand side. The significance of the negative difference turns out to be low and their existence is probably largely due to the choice of the fitted function and also to a certain extent may be due to chance because of small statistics.

Errors

There are two groups of errors to be considered. The first of these is the effect of experimental error and the second is the statistical error due to the finite amount of statistics, quite independent of the accuracy of the measurements.

The sources of experimental error are varied. Errors due to loss of energy due to ionization, turbulence in the chamber, photography, the stage linearity, fluctuations in the coil current and the radial variations of magnetic field from the central value were considered in details. Their contributions to error were found to be either negligible or small and taken into account. The error in momentum due to multiple scattering was estimated to be in the neighborhood of $\pm10\%$. The Digitizer resolution is given as ±1 Digitizer unit, corresponding to approximately ±0.02 mm in the chamber. The largest source of error turns out to be operator error. Two classes may be considered, procedural mistakes and inaccurate measurements. It was found possible to construct the computer program so that any definable procedural mistake could be detected and removed. The second class of error may only be estimated from measurements and was found to be of the order of $+1\%$. All points used in the calculations were corrected individually for this effect. The presence of alcohol and water vapor in the nitrogen increases the effective Z of the scattering medium by an average of 0.7%. The curves as given are for $Z=7.05$ rather than $Z=7$.

The effect of the finite statistics is much more important than the measurement errors. The data points (multiple scattering vs momentum) were grouped into 200 G-cm intervals so that the mean momentum error is ±50 G-cm (in addition to those described above). We, thus, have a distribution of multiple scattering values for each of a number of momentum values, and for both positrons and electrons. The distributions are approximately Gaussian and a mean value and σ may be found for each. From the differences between the mean values for positrons and electrons and the values of σ for each distribution in each momentum range we may estimate the probability that such a difference could occur by chance alone. This has been done and the results plotted in Fig. 4.

It is seen that for four consecutive momentum points between 2500 and 3100 G-cm there is less than a 5% probability of any one difference being due to chance. Between 2000 and 4000 G-cm at all points the probability is less than 20%.

This is the basis of the claim that a small difference exists, although there are a number of difficulties here not all unfavorable to the conclusions. First, the chance probability figure does not give the most probable value of the difference, nor does it give automatically the probable error of the difference. On the other hand, estimates of error from the cosmic-ray measurements described above are in agreement with the order of magnitude of the chance probabilities in Fig. 4.

Second, no over-all figure can be given for the difference at all points considered together although a weighted mean may be found over the significant range of 4%.

In favor of the difference being real is the fact that

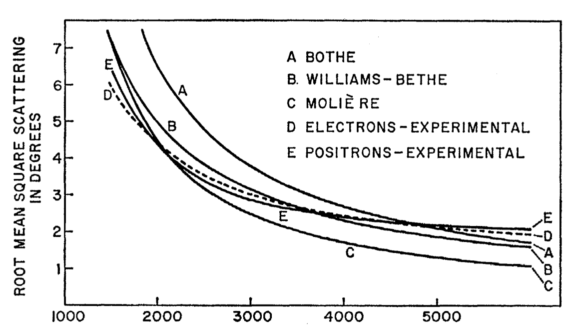

FIG. 3. Comparison of experimental and theoretical curves in 2 cm nitrogen at 760 mm, 20°C.

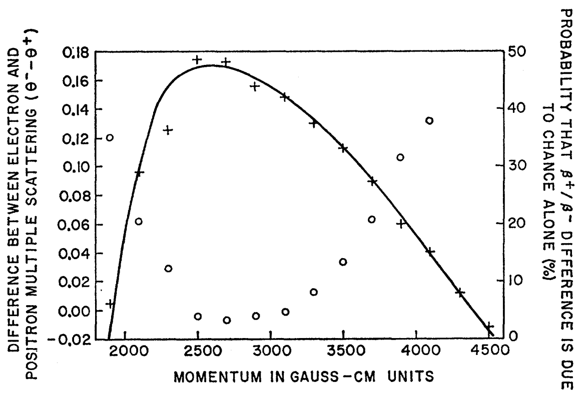

FIG. 4. Electron-positron difference and significance curves.

the probability of a chance difference for such a succession of momentum values is enormously smaller than for the best single point.

Conclusions

The analysis of the results falls into two parts which are discussed separately. First, is the agreement between the experimental data and the several theories as to the variation in magnitude with momentum. Second, the positron-electron differences are examined for significance and agreement with theoretical predictions.

A qualitative examination of the experimental and theoretical curves given in Fig. 3 shows a close agreement with the predictions of Williams, using the limits of Bethe. Molière's theory for momenta above 2000 G-cm gives a much lower value than the experimental data. To check on this intuitively found agreement, the data were examined statistically. For each experimental point the deviation from theory was found and compared with the experimental standard deviation.

For any given theoretical value of the parameter, the disagreement with the mean experimental value may be found in terms of the standard deviation, whence a probability that the agreement is due to chance or not may be computed.

In the two-dimensional case here this has been carried over to find disagreement at each point in terms of standard deviation. In Table I the results of this analysis are given for electrons. It is seen that the Williams-Bethe curve differs from the data by around two or three standard deviations above 2500 G-cm. In this same range the Molière curve varies between ten and nineteen standard deviations away from the data.

On the basis of these figures it is clear that the Williams-Bethe theory is a much better description of the experimental results than is the Molière theory. However, the fit of the data to the Williams-Bethe theory is by no means perfect. It should be noted that the theories of Molière and Snyder and Scott are more accurate than that of Williams and Bethe. Molière uses a more exact quantum-mechanical solution of the single scattering problem, with a potential that is the sum of three exponential potentials. Snyder and Scott employ a different approach to the problem. Starting from an integral diffusion equation, they achieve an exact solution and a numerical integration which gives results similar to those of Molière. It is not quite clear why the present results and also those of Hisdal and Cusack and Stott do not agree better with these more accurate theories.

TABLE I. Deviation of best-fit experimental curves from Molière and Williams-Bethe theory.

$H\rho$ (G-cm)	$\lvert\Delta_{\text{Molière}}\rvert$	$\lvert\Delta_{\text{Molière}}\rvert/\sigma$	$\lvert\Delta_{\text{W-B}}\rvert$	$\lvert\Delta_{\text{W-B}}\rvert/\sigma$
2100	0.07	1.35	0.47	8.32
2300	0.25	4.85	0.34	6.63
2500	0.39	7.99	0.24	4.92
2700	0.47	10.53	0.17	3.76
2900	0.53	12.81	0.12	2.86
3100	0.58	15.16	0.07	1.68
3300	0.62	17.05	0.03	0.73
3500	0.64	18.24	0.00	0.00
3700	0.66	19.09	0.04	1.05
3900	0.67	18.75	0.05	1.51
4100	0.68	18.09	0.08	2.03
4300	0.68	16.87	0.09	2.23
4500	0.69	15.79	0.11	2.49
4700	0.70	14.78	0.13	2.70
4900	0.71	13.88	0.14	2.71
5100	0.71	12.97	0.16	2.93
5300	0.72	12.32	0.18	3.07
5500	0.72	11.57	0.19	3.07
5700	0.73	11.22	0.21	3.25
5900	0.73	10.52	0.22	3.17

ELECTRON-POSITRON DIFFERENCES

The electron-positron differences are given in Table II as percentages of the experimental best-fit electron scattering. By a method similar to that described in the previous section, one is able to attribute numerical significance to these differences. The standard deviation used here is the sum of those for electrons and positrons separately. The result is given in Table II, both as differences in units of combined standard deviation and as a probability of such a difference being due to chance. The latter is also plotted in Fig. 4 so that it may be compared directly with the actual differences at this point.

TABLE II. Electron-positron difference.

Mean $H\rho$ (G-cm)	Percentage difference $[100\times(\theta^- - \theta^+)/\theta^-]$	Differences in standard deviations	Percentage probability of chance difference
1900	+0.00	0.04	35
2100	+2.29	0.83	20
2300	+3.28	1.17	12
2500	+4.92	1.75	4
2700	+5.22	1.85	3
2900	+5.04	1.76	4
3100	+5.02	1.67	5
3300	+4.63	1.41	8
3500	+4.19	1.12	13
3700	+3.48	0.82	21
3900	+2.40	0.49	31
4100	+1.66	0.30	38

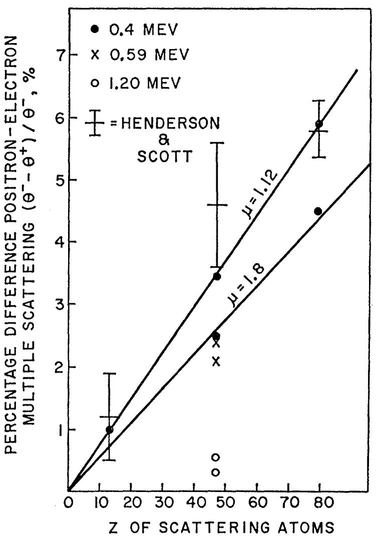

FIG. 5. Predicted values based on Nigam theory.

A significant difference is immediately apparent, although the actual values may be prejudiced somewhat by the choice of the function to which the Gaussian fit was made. A direct calculation may be made from the experimental points.

Let n_i^+ and n_i^- be the number of measurements in the ith momentum range, s_i^+ and s_i^- the mean multiple scattering angle, σ_i^+ and σ_i^- the standard deviations of the measurements about their mean. The superscripts refer to positrons and electrons, respectively.

In each range, i, the weighing factor, W_i, is found where

$$W_i = \frac{n_i^+}{(\sigma_i^+)^2} \times \frac{n_i^-}{(\sigma_i^-)^2}.$$

The mean percentage difference between the electron and positron cases, D, is found from

$$D = \frac{\sum_i W_i (s_i^- - s_i^+) s_i^-}{\sum_i W_i} \times 100.$$

The result is $D = 3.4\%$.

The mean error of each distribution has been estimated at $\pm 1.3\%$ so that we have the small but finite difference

$$D = (3.4 \pm 2.6)\%.$$

At this point, a comparison may be made against the theoretical predictions of Nigam *et al.* They have not given a general expression, but have rather used their method to compute values to check against experiments by Henderson and Scott.[11] The figures given in their Table IV have been plotted in Fig. 5. Points are plotted for three energies, and at 0.4 MeV for gold, silver, and aluminum. Each, in turn, is given for two values of μ, an undetermined parameter in their theory. The higher value of each pair is for $\mu = 1.12$, the lower for $\mu = 1.8$. The experimental results of Henderson and Scott are also shown. For $\mu = 1.12$ and 0.4 MeV a linear relationship appears to exist which suggests a 0.5% difference might be expected for nitrogen.

The experimental value for the mean difference over a range of momentum between 2000 and 6000 G-cm of $(3.4 \pm 2.6)\%$ is not inconsistent with the projected value from Nigam's theory.

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Finally the use of the computer facilities of the Los Alamos Scientific Laboratory, Los Alamos, New Mexico for the majority of the calculations performed is gratefully acknowledged.

[11] C. Henderson and A. Scott, Proc. Phys. Soc. (London) **A70**, 188 (1957).