by the electrons during this time can be measured from the avalanche photographs. Using the observed drift velocities the corresponding values of E/P_0 are calculated (P_0 is the final cloud-chamber pressure corrected to 296°K and, hence, corresponds to the argon gas density used by other observers). These results along with results of other observers are shown in Fig. 3. The present data indicate that in the range of E/P_0 values used (7.9 to 10.4 V/cm mm Hg) k_0 is independent of E/B_0 and has a value of 3.04×10^5 cm^2/V sec mm Hg. Herring[6] measured drift velocities with E/P_0 values up to 6 V/cm mm of Hg. Herring's data is in excellent agreement with Nielsen's[7] data over Nielsen's range of E/P_0, i.e., E/P_0 values of less than four. Nielsen's data is usually considered to give the best drift velocity information for electrons in pure argon. The present mobility measurements appear to agree quite well with this data if its range is extended, although they are from 10 to 15% higher than the mobilities measured by Errett.[8] The higher mobilities measured by Riemann[9] can be understood in the light of Errett's measurements of the effect of water vapor on electron mobilities in argon; Errett showed a sizable increase in electron mobilities in argon containing a very small admixture of water vapor. The fact that the present measurements made using tank argon fractionally distilled in the vapor source agree well with results in purified argon is not surprising. Bortner, Hurst, and Stone,[10] using tank argon fractionally distilled, using liquid nitrogen as the coolant, measured electron mobilities that agreed quite well with Nielsen's measurements. In the present measurements the nitrogen and oxygen impurities are estimated to be less than 0.1% which according to the measurements of Errett would produce an increase in the mobility measurements over that in the pure gas of 5% or less in the E/P_0 range used.

ACKNOWLEDGMENT

I should like to thank Professor Carl E. Nielsen for suggesting the problem considered in this paper.

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Lattice Thermal Conductivity of Disordered Semiconductor Alloys at High Temperatures*

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The high-temperature thermal conductivity of a disordered semiconductor alloy is derived using the Klemens-Callaway theory. It is assumed that the reciprocal relaxation times depend on frequency ω as ω^4 for strain and mass point defects and as ω^2 for normal and umklapp three-phonon anharmonic processes. The thermal conductivity is expressed in terms of the lattice parameters and mean atomic weights of the alloy and its constituents. Agreement is obtained between theory and published experimental data on Ge-Si alloys at temperatures 300–1200°K, and on (Ga,In)As alloys at 300°K, using the value 2.5 for the ratio of umklapp to normal relaxation times. It is found that the large thermal resistivity of Ge-Si alloys is predominantly due to mass defect scattering, whereas that of (Ga,In)As alloys is mainly due to strain scattering.

I. INTRODUCTION

SOLID solution alloys are well suited for studying the effects of imperfections on the transport of heat by lattice waves. The present investigation is concerned with semiconductor alloys, in particular Ge-Si alloys and III-V compound alloys, because extensive experimental and theoretical data, relating to the thermal conductivities of these systems, are already available. The choice of the temperature range is motivated by the general interest in these materials for high-temperature thermoelectric devices.

The simple phenomenological model of thermal conductivity, developed by Klemens[1–4] and Callaway[5,6], is used in the present work. An alternative treatment of thermal conductivity is by the variational method of Ziman,[7] or modification thereof by Tavernier.[8]

The high-temperature limit of the theory was applied

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* Work Supported by the U. S. Naval Bureau of Ships.

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by Klemens[4] to the thermal conductivities of Ge-Si alloys measured by Steele and Rosi.[9] The thermal resistance of these materials was shown to be due to scattering of phonons by mass fluctuations and anharmonic phonon-phonon scattering. A recent investigation[10] revealed that the previously published values of thermal conductivities[9] at 300°K are too large by a factor of about 1.6. To obtain fit between theory and the new experimental data it was necessary to assume that there is appreciably more anharmonic scattering in the alloys than in the pure components, Ge and Si. It was suggested[10,11] that this effect is due to five-phonon processes, resulting from simultaneous three-phonon anharmonic processes and two-phonon point defect processes.

There are two kinds of three-phonon processes, N-processes, in which crystal momentum is conserved, and U-processes, in which total momentum is changed by a reciprocal lattice vector. Klemens[12] suggested that N-processes, which were neglected in his earlier work,[4] contribute substantially to the thermal resistance even at high temperatures and he estimated that they may account for the aforementioned discrepancy between theory and experiment.

Following this suggestion, the high-temperature theory is extended to include N-processes, using the formalism of Callaway.[5] An expression is derived for the thermal resistivity of a disordered alloy in which the ratio of scattering rates due to N-processes to that due to U-processes is introduced as an adjustable parameter. Agreement between theory and experiment for Ge-Si alloys is obtained over a wide range of compositions and temperatures using a single value of this parameter.[13]

The other subject treated is scattering of phonons due to lattice strains. An alloy whose components have widely differing lattice constants usually contains a large concentration of highly strained regions. These scatter phonons effectively. A simple, heuristic, elastic continuum treatment of such effects is presented here. The results are in good agreement with published data on (Ga, In)As alloys.[14]

II. THE VIRTUAL CRYSTAL

The alloys are assumed to be a random mixture of atoms, with different masses and volumes, arranged in a lattice. To calculate the thermal conductivity, use is made of the customary artifice, in which the disordered lattice is replaced by an ordered virtual crystal and the disorder is treated as a perturbation. The phonons are scattered as a result of the disorder perturbation and the anharmonicity of the virtual crystal. In this section are discussed the parameters of the virtual crystal, the effects of disorder are treated in Sec. III.

The parameters of the virtual crystal are derived using the following physical argument. Heat is carried primarily by the acoustical phonons of the nondispersed (large group velocity) part of the spectrum. These phonons are related to the elastic continuum properties of the alloy. Therefore, the virtual crystal must have the density and elastic constants of the alloy. Thus, the atomic weight M of the virtual crystal is given by

$$M=\sum x_i M_i, \qquad (1)$$

where M_i and x_i are the atomic weight and fractional concentration of the component i of the alloy, respectively.

In most cases Vegard's law,

$$\delta=\sum x_i \delta_i', \qquad (2)$$

is a good approximation for the atomic volume, δ^3, of the alloy. The quantity δ_i' in Eq. (2) is the cube root of the atomic volume of component i of the alloy when it is in its own lattice. For diamond and zinc-blende lattices $\delta=a/2$, where a is the lattice parameter.

In the case of an alloy of compounds, M_i and $(\delta_i')^3$ in Eqs. (1) and (2) are defined as the mean atomic weight and volume of the component compound i.

Keyes[15] observed that the elastic constants, c_{ik}, of the column IV, III-V, and II-VI covalent crystal systems depend on the atomic volumes only. The relationship is given by

$$c_{ik}\delta^4 \approx \text{const}, \qquad (3)$$

where the constant assumes a different value for each of the above systems. The assumption is made here that Eq. (3) is also valid when the atomic volume is changed by pressure or by alloying.

Steigmeier[16] determined the unknown Debye temperatures of the III-V compounds using the elastic constants computed from Eq. (3). Since $\theta \sim \delta^{1/2} c_{ik}^{1/2} M^{-1/2}$, Eq. (3) leads to the relation

$$\theta M^{1/2} \delta^{3/2} = \beta, \qquad (4)$$

where β is nearly constant within a given covalent crystal system.

The anharmonicity constant γ is defined by

$$\gamma=-d(\ln\theta)/d(\ln\delta^3). \qquad (5)$$

Substituting Eq. (4) in Eq. (5) yields $\gamma=\frac{1}{2}$. The volume dependence of θ, given by Eq. (4), has been verified for germanium by Daniels,[17] who derived the value of $\gamma=0.49$ from the measured volume dependence of the elastic constants. For silicon, however, the value of γ derived in this manner,[17] is 0.25.

No explicit calculations of phonon-phonon scattering

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have been made so far. It is assumed that three-phonon N- and U-processes can be characterized by relaxation times τ_N and τ_U given by [4-6]

$$\tau_N^{-1}=B_1\omega^2\,, \tag{6}$$

$$\tau_U^{-1}=B_2\omega^2\,, \tag{7}$$

where

$$B_1/B_2=\alpha\,, \tag{8}$$

ω is the phonon frequency and B_1 and B_2 are independent of ω and the ratio α is independent of T. All higher order phonon processes are neglected. Furthermore, it is assumed that the phonons have the isotropic nondispersed Debye velocity $v=k\hbar^{-1}(6\pi^2)^{-1/3}\theta\delta$.

Leibfried and Schlömann using a variational method,[18] derived an expression for the lattice thermal resistivity W_p due to three phonon anharmonic scattering,

$$W_p=10\pi^3 3^{-1}4^{-1/3}\hbar^3 k^{-3}m^{-1}\gamma_1{}^2 TM^{-1}\delta^{-1}\theta^{-3}\,, \tag{9}$$

where T is the absolute temperature, m is the unit atomic mass and the quantity γ_1 is of the order of 2, and is related to the anharmonicity of the lattice. Although this formula is based on a highly idealized model, it gives the correct dependence[3,19] of the thermal resistivity on the parameters T, M, δ and θ. A similar formula was derived by Dugdale and McDonald[20] using a dimensional argument. The assumption is made here that Eq. (9) is valid for the virtual crystal as well.

III. DISORDER SCATTERING

The disorder of the lattice is taken into account as follows. An atom of the virtual crystal is replaced by an atom of the alloy. This atom acts as a virtual impurity and scatters phonons. In general, the virtual impurity atom differs from the atoms of the virtual crystal in its mass, in its size and in the coupling forces to its neighbors. Furthermore, as a result of anharmonicity the coupling forces are modified by the 'misfit' strain field in the neighborhood of the impurity. For a general treatment of the subject see the review paper by Carruthers.[21]

Klemens treated the case of point-defect scattering by an impurity in a simple cubic lattice with only nearest neighbor forces acting. The relaxation time $\tau_{P,i}{}^{-1}$ for this process, given by Klemens,[2] is

$$\tau_{P,i}{}^{-1}=\omega^4\delta^3\Gamma_i/4\pi v^3\,, \tag{10}$$

where

$$\Gamma_i=x_i[(\Delta M_i/M)^2 +2((\Delta G_i/G)-2\times 3.2\gamma(\Delta\delta_i/\delta))^2]\,, \tag{11}$$

characterizes the scattering cross section of the impurity atom i, δ_i is the radius of the impurity atom in the host lattice, G_i is an average stiffness constant of the nearest neighbor bonds from impurity to host lattice, G is the corresponding quantity for the host atoms, $\Delta G_i=G_i-G$, $\Delta\delta_i=\delta_i-\delta$, $\Delta M_i=M_i-M$ and γ is an average anharmonicity of the bonds.

The scattering can also be treated in terms of the elastic continuum "sphere-in-hole" model.[22] This formulation has the advantage of leading to a more natural definition of G_i and δ_i than that used in the impurity model of Klemens.

The argument runs as follows: A sphere with radius δ representing the virtual crystal atom is cut in the virtual crystal matrix. A sphere with radius δ_i' and mass M_i representing the impurity atom is introduced. Matrix and impurity are treated as continuous isotropic media which are constrained to touch along a sphere of intermediary radius δ_i. The radius of the impurity in its own lattice, δ_i', and its radius in the host lattice, δ_i, are related by the expression[22]

$$\Delta\delta_i/\delta=[(\delta_i'-\delta)/\delta]\mu/(1+\mu)\,, \tag{12}$$

where

$$\mu=(1+\nu)G_i/2G(1-2\nu)\,, \tag{13}$$

G and ν are the bulk modulus and Poisson ratio of the matrix, respectively, and G_i is the bulk modulus of the impurity sphere.

In a real crystal, the Poisson ratio ν is anisotropic and does not have a uniquely defined average value.[23] For the purpose of the present work the value $\nu=c_{12}/(c_{11}+c_{12})$, corresponding to the isotropic case, is used. Since this quantity is nearly the same for Ge, Si, and the III-V compounds its average value $\nu=0.3$ is used. From Eqs. (12) to (14) it follows that $G_i/G\approx 1$ and the quantity $\mu/(1+\mu)\approx 0.62$. This value is used hereon.

Rayleigh[24] has treated the scattering of sound waves in a gas by spherical obstacles. Ziman[25] used a simple physical argument to extend the Rayleigh formula to the case of the isotropic elastic continuum. The expression he obtained differs from Eq. (11) in that the strain term, proportional to $\Delta\delta_i/\delta$, vanishes and in a slightly different coefficient of the term $\Delta G_i/G$. The strain term vanishes for the isotropic continuum because the displacement field around the impurity sphere has a vanishing dilation. To obtain finite strain scattering

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[19] D. S. Beers, G. D. Cody, and B. Abeles, in *Proceedings of the International Conference on the Physics of Semiconductors, Exeter, 1962* (The Institute of Physics and the Physical Society, London, 1962), p. 41.

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[23] H. B. Huntington, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1958), Vol. 7, p. 316.

[24] Lord Rayleigh, *Theory of Sound* (Macmillan and Company Ltd., London, 1929), 2nd ed., Vol. 2, p. 284.

[25] Ref. 7, p. 222.

anisotropy must be taken into account. The model used here is already so simplified that a rigorous calculation is not warranted and the assumption is made that Eq. (11) describes the macroscopic situation.

To express $\Delta G_i/G$ in terms of $\Delta\delta_i/\delta$ use is made of Eq. (3), with the result,

$$\Delta G_i/G = -4\Delta\delta_i/\delta\,. \tag{14}$$

Substituting Eqs. (12), (14), and $\gamma=\frac{1}{2}$ in Eq. (11) results in

$$\Gamma_i = x_i\{(\Delta M_i/M)^2+\epsilon[(\delta-\delta_i')/\delta]^2\}\,, \tag{15}$$

where $\epsilon=39$. In general, ϵ should be regarded as a phenomenological, adjustable parameter.

The disorder scattering relaxation time τ_P is given by

$$\tau_P^{-1} = \omega^4\delta^3\Gamma/4\pi v^3\,, \tag{16}$$

where

$$\Gamma = \Sigma_i\Gamma_i \tag{17}$$

is the disorder parameter. In performing the summation it is implicitly assumed that multiple scattering can be neglected.

In the case of a mixture of two kinds of atoms, A and B, Eq. (17) becomes

$$\Gamma = x(1-x)[(\Delta M/M)^2+\epsilon(\Delta\delta/\delta)^2]\,, \tag{18}$$

where

$$\Delta M = M_A - M_B\,, \tag{19}$$

$$\Delta\delta = \delta_A' - \delta_B'\,, \tag{20}$$

and

$$M = xM_A + (1-x)M_B\,. \tag{21}$$

IV. THERMAL CONDUCTIVITY

The expression for the thermal resistivity W is derived by substituting in Callaway's[5] Eqs. (16) and (19)–(21) the expressions τ_N, τ_U and τ_P given by Eqs. (6), (7), and (16). In the limit of high temperatures, $\theta/T\leq 1$, one obtains by simple integration[13,26]

$$\frac{W}{W_p} = (1+(5/9)\alpha)\left[\frac{\tan^{-1}U}{U} + \frac{(1-(\tan^{-1}U/U))^2}{[(1+\alpha)/5\alpha]U^4-\frac{1}{3}U^2+1-(\tan^{-1}U/U)}\right]^{-1}, \tag{22}$$

where

$$U_0^2 = (\tfrac{9}{2}\pi)^{1/3}\pi^2\hbar k^{-2}\delta\Gamma\theta^{-1}W_p^{-1}\,, \tag{23}$$

$$U^2 = U_0^2(1+(5/9)\alpha)^{-1}\,, \tag{24}$$

and the quantities B_1 and B_2 in Eqs. (6) and (7) have been expressed in terms of the ratio α and in terms of W_p, the thermal resistance in the absence of point defect scattering, and the velocity has been expressed in terms of the Debye temperature. The quantity W/W_p, given by Eq. (22) is plotted in Fig. 1 as a function of the parameter U_0 for several values of α.

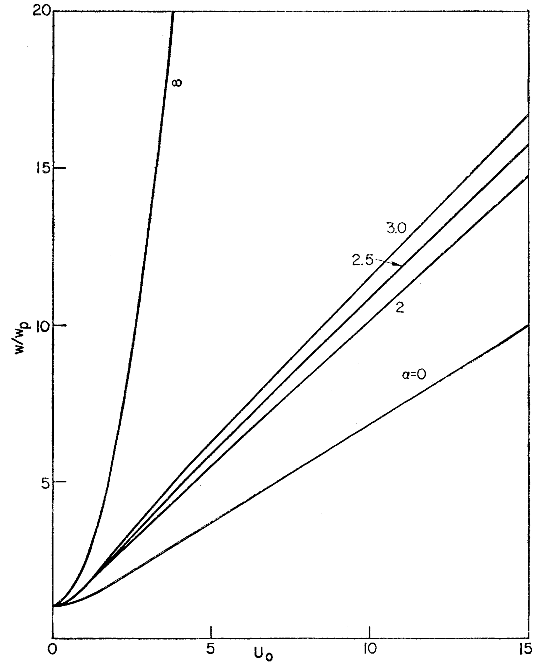

FIG. 1. The ratio of thermal resistivity of a disordered alloy, W, to that of the alloy in absence of disorder scattering (virtual crystal) W_p as a function of the disorder parameter U_0. The solid curves were computed from Eq. (22) for several values of the parameter $\alpha=B_1/B_2$.

In the limit of vanishing N-processes, $\alpha=0$, Eq. (22) reduces to the expression given by Klemens[4] and Callaway and Von Baeyer.[6] In the case of weak point-defect scattering, $U_0\ll 1$, expansion of Eq. (22) results in

$$W/W_p = 1+U_0^2\,3^{-1}\left(1+2\alpha+\frac{25}{21}\alpha^2\right)(1+(5/9)\alpha)^{-2} \tag{25}$$

which for $\alpha=0$ reduces to the expression given by Ambegaokar.[27] In the limit of strong point-defect scattering, $U_0\gg 1$, the second term in the denominator of Eq. (22) can be neglected, and for $\alpha=9/5$ the expression reduces to the one of Klemens.[12]

For interpreting experimental data it is convenient to eliminate the Debye temperature in Eqs. (9) and (24) using Eq. (4). The result is

$$W_p = 1.75\times 10^{-1}\gamma_1^2\beta^{-3}M^{1/2}\delta^{7/2}T \quad \mathrm{W^{-1}\,deg\,cm}\,. \tag{26}$$

and

$$U = 8.69\times 10^6(1+(5/9)\alpha)^{-1/2}\gamma_1^{-1}\beta\Gamma^{1/2}\delta^{-1/2}T^{-1/2} \tag{27}$$

In the limit of strong point-defect scattering, $U_0\gg 1$, expanding Eq. (22) and substituting Eqs. (26) and (27) results in

$$\begin{aligned} W = {} & 9.67\times 10^5(1+(5/9)\alpha)^{1/2}\gamma_1\beta^{-2}\Gamma^{1/2}M^{1/2}\delta^3T^{1/2} \\ & +7.08\times 10^{-2}(1+(5/9)\alpha)\gamma_1^2\beta^{-3}M^{1/2}\delta^{7/2}T \\ & \qquad \mathrm{W^{-1}\,deg\,cm}\,. \end{aligned} \tag{28}$$

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[26] J. E. Parrott, Proc. Phys. Soc. (London) **81**, 726 (1963), derived independently the identical expression.

[27] V. Ambegaokar, Phys. Rev. **114**, 488 (1959).

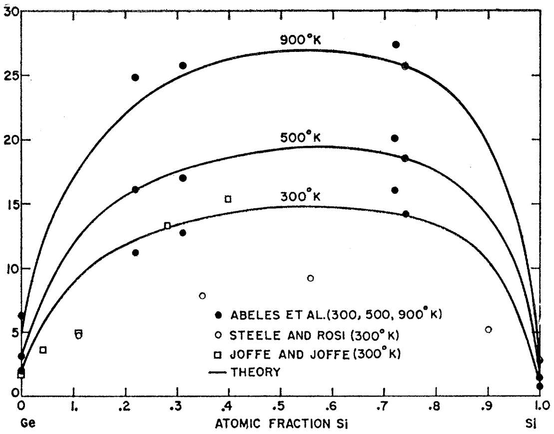

FIG. 2. Thermal resistivities of Ge-Si alloys as function of composition, at 300, 500, and 900°K. The solid curves were computed from Eqs. (18), (22), (26), and (27) with $\alpha=2.5$, $\beta=1.52\times10^{-8}$ deg cm$^{3/2}$, $\gamma_1=1.77$ and $\epsilon=39$. The experimental points are taken from the works of Joffe and Joffe (Ref. 29), Steele and Rosi (Ref. 9) and Abeles *et al.* (Ref. 30).

Equations (18), (22), (26), (27), and (28) express the lattice thermal conductivity of an alloy and its components in terms of their atomic weights, their lattice constants, the temperature, and the three adjustable parameters: The ratio α of 'N' to 'U' scattering rates, the anharmonicity parameter, γ_1 and the strain parameter, ϵ. These parameters, as well as the parameter β appearing in Eq. (4), are nearly constant within a particular covalent crystal system and do not vary appreciably from system to system. The above equations are an extension of the one of Keyes[28] which relates the lattice thermal conductivities of covalent crystals to their atomic weights, their lattice parameters and their melting points.

V. Ge-Si ALLOYS AND III-V COMPOUNDS ALLOYS

The high-temperature thermal resistivities of Ge-Si alloys, published by three different groups of workers, are plotted in Fig. 2. Included are the room-temperature measurements of Joffe and Joffe,[29] and Steele and Rosi[9] and measurements at 300, 500, and 900°K by Abeles, Beers, and Cody.[30] The results of Joffe and Joffe[29] are in good agreement with those of the third group, but the values of Steele and Rosi are smaller by a factor of 1.6. This discrepancy may be due to a photon contribution to the thermal conductivity in their[9] high electrical resistance samples. The measurements on Ge-Si alloys by Toxen[31] are not included because they were made at low temperatures.

The theoretical curves for W in Fig. 2 were computed using Eqs. (18), (22), (27), and published[32] values of δ. The value of β was determined by substituting in Eq. (4) the Debye temperatures[19] of Ge and Si, computed from the elastic constants. The resulting values of β, for Ge and Si, differ by less than 1% and their mean value $\beta=1.52\times10^{-8}$ deg cm$^{3/2}$ was used. The value of $\epsilon=39$, estimated on the basis of the impurity model of Klemens, was used in Eq. (18). The resulting strain contribution to Γ is only about 10% of the mass contribution. The values of γ_1 and α were determined by fitting the theoretical curve to the measured thermal resistivities. The best fit is obtained for $\gamma_1=1.77$ and $\alpha=2.5$.

The thermal resistivities of (Ga,In) As and In(As,P) alloys were measured by Abrahams *et al.*[14] and Bowers *et al.*,[33] respectively. These systems are particularly interesting, because, on the basis of differences in lattice parameters one expects strain scattering to be appreciably stronger in (Ga, In)As than in In(As,P).

The theoretical curve for (Ga,In)As, plotted in Fig. 3, was determined in a manner similar to that of the Ge-Si alloys. The lattice parameters were calculated from Eq. (2). The values of β, computed from Eq. (4) using the Debye temperatures of[19] GaAs and[34] InAs differ by 8% and their mean value $\beta=1.33\times10^{-8}$ deg cm$^{3/2}$ was used. The value of $\alpha=2.5$, determined on Ge-Si alloys, was assumed. This assumption is justified by the argument that the quantity α is determined by a

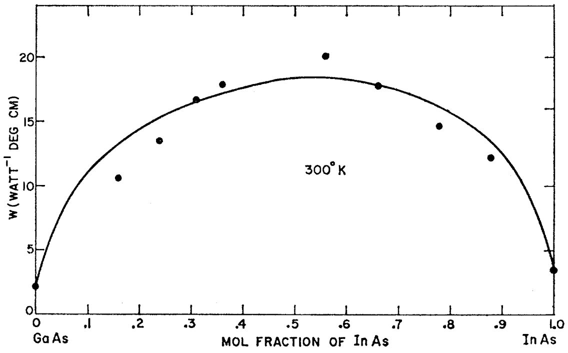

FIG. 3. Thermal resistivities of (Ga,In)As alloys as function of composition at 300°K. The solid curve was computed from Eqs. (18), (22), (26), and (27) with $\alpha=2.5$, $\beta=1.33\times10^{-8}$ deg cm$^{3/2}$, $\gamma_1=1.75$ and $\epsilon=45$. The experimental points are from the work of Abrahams, Braunstein, and Rosi. (Ref. 14), with the exception of the GaAs value which is taken from J. Blanc, R. H. Bube, and L. R. Weisberg [Phys. Rev. Letters **9**, 252 (1962)].

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[28] R. W. Keyes, Phys. Rev. **115**, 564 (1959).

[29] A. V. Joffe and A. F. Joffe, Izv. Akad. Nauk SSSR Ser. Fiz. **20**, 65 (1956).

[30] Included in Fig. 2 are measurements on specimens T-1810, GS-8, JPD-7, and S-1142 (Ref. 10), specimen GS-68 (Ref. 19) and specimen GS-9, containing 78% Ge and 22% Si (unpublished). The specimens designated (see Refs. 10 and 19) DS-4, D-75, D-171, and JPD-12 are not included in Fig. 2 because the first three samples had appreciable conduction of heat by photons and in the third, heavily doped sample, phonons were scattered by electrons. The remaining specimens were doped sufficiently to eliminate photon conduction but not doped sufficiently to give rise to appreciable phonon-electron scattering. The increase in W due to phonon electron scattering in these specimens is believed to be less than 10%. The thermal resistivities were corrected for a small electronic contribution to the thermal conductivity which amounted to a few percent.

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[32] E. R. Johnson and S. M. Christian, Phys. Rev. **95**, 560 (1954).

[33] R. Bowers, J. E. Bauerle, and A. J. Cornish, J. Appl. Phys. **30**, 1050 (1959).

[34] D. Gerlich (to be published).

ratio of areas of 'N' and 'U' scattering surfaces and thus is essentially a geometric property of the Brillouin zone. It is reasonable to assume that III-V compounds with mass ratios close to unity have similarly shaped phonon spectra as Ge and Si and, therefore, should have similar values of α. The quantities γ_1, and ϵ were used as adjustable parameters to obtain the best fit between theory and experiment. The curve in Fig. 3 was computed for $\gamma_1=1.75$, $\epsilon=45$. In contrast to the case of Ge-Si alloys, here the strain contribution to Γ is about three times as large as the mass fluctuation contribution. The agreement between the phenomenological value, $\epsilon=45$, and that computed from the model of Klemens, $\epsilon=39$, is surprisingly good.

The same numerical values of α, β, γ_1, and ϵ were used for In(As,P) as for (Ga,In)As. The computed curves and experimental results are compared in Fig. 4. In this case the strain contribution to Γ is about one half that of the mass fluctuation contribution. The agreement becomes worse the richer the alloy is in InP. This may be related to the fact that InP has a mass ratio of 3.7 and probably has a vibrational spectrum differing from the semiconductors with mass ratio close to unity.

VI. DISCUSSION

The model used in this work is essentially that of an isotropic elastic continuum. The only use made of the discreteness of the lattice is in the assumption of anharmonic U-processes. The fact that an elastic continuum model explains fairly well the lattice thermal conductivity is not surprising since heat is carried primarily by the phonons of the nondispersed part of the vibrational spectrum.

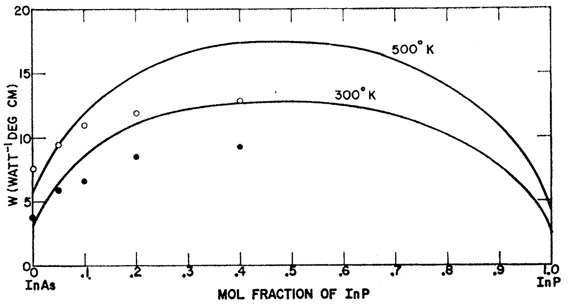

FIG. 4. Thermal resistivities of In(As,P) alloys as function of composition at 300 and 500°K. The solid curves were computed from Eqs. (18), (22), (26), and (27) using the same values of the parameters α, β, γ_1, and ϵ as for (In,Ga)As alloys. The experimental points are from the work of Bowers, Bauerle, and Cornish (Ref. 33).

One can narrow down the identity of the phonons carrying the heat to the longitudinal acoustic (LA). Of the six phonon branches in germanium and silicon,[35] the LA phonons are least dispersed, have the highest group velocity and are nearly isotropic. One expects similar behavior for the phonons of the III-V compounds. Because of this property, the number of three-phonon N- and U-processes in which LA phonons can participate is much smaller than the corresponding number for the optical (O) phonons and the highly dispersed and anisotropic transverse acoustical (TA) phonons. Thus, there is a group of phonons, mainly LA, with a long mean-free path, which carry most of the heat energy. The rest of the phonons, mainly O and TA phonons, have a short mean-free path and act as a heat reservoir for the LA phonons.

The assumptions made here for the relaxation times τ_N can be justified for the case of strong point-defect scattering. Point defects scatter selectively TA phonons and all short-wave phonons so that only long-wave LA phonons contribute to heat conduction. Herring[36] has shown that for long-wave LA phonons Eq. (6) applies. Moreover, if N scattering is stronger than U scattering, then the thermal resistance of the disordered alloy is not sensitive to the choice of τ_U; for then point-defect scattering combined with three phonon N process is more effective in destroying phonon momentum than three phonon U processes.

Five-phonon processes and anharmonic three-phonon processes result in the same temperature dependence of thermal conductivity[10,11] so that they cannot be separated easily. In the present work, five-phonon processes were neglected.

In view of the simplifying assumptions made, the good agreement obtained between theory and experiment may be somewhat fortuitous. In particular, the assumption implicit in Eq. (17) requires further examination.

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