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Magnetic Moment of Negative Muons*

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The magnetic moment of negative muons bound in atoms of carbon, oxygen (in water), magnesium (metallic and in MgH_2), silicon, and sulfur has been measured with a precision ranging from 3×10^{-5} in carbon to 1.6×10^{-4} in sulfur. The measured moment is corrected for relativistic effects, diamagnetism, and nuclear polarization before being compared to the moment of the positive muon. The two moments are found to be equal to 3×10^{-4}, where the major uncertainty is due to Knight shift. The relativistic, diamagnetic, nuclear, and solid-state shifts are large enough compared to the statistical and systematic errors to make this technique usable for the investigation of these effects.

I. INTRODUCTION

THIS article, the second of two[1] dealing with muon magnetic moments, reports the extension of the technique for measuring these moments to negative muons bound in various atoms. Such a measurement is of interest for several reasons:

(a) A measurement of the negative muon magnetic moment and comparison with the positive muon would test the prediction, from *TCP* invariance, that the two muons, as particle and antiparticle, must have equal magnetic moments.

(b) Since the apparent magnetic moment is modified by the environment of the muon, the muon can be used as a probe. In particular, one might expect to observe nuclear effects, since the negative muon comes to rest in a Bohr orbit whose radius, because of the large muon mass, is comparable to nuclear radii for high Z.

With these two goals in view, measurements were made on muons stopped in graphite, water, magnesium, silicon, and sulfur. Graphite, where environmental effects were expected to be small, served to measure the moment. The experiments with the other materials were intended as investigations of environmental effects. All the targets used have zero-spin nuclei, to avoid depolarization through hyperfine interaction.[2]

Section II gives a brief description of those details in which this experiment differs from the one on positive muons, as described in Ref. 1. Section III contains the experimental results and a discussion of the environmental effects which modify the apparent magnetic moment.

II. EXPERIMENT

The experimental arrangement is described in detail in HMSP. The only difference is a shorter electron acceptance gate because of the shorter μ^- lifetimes.

The precision of the μ^- results is substantially less than that of the μ^+ results because of shorter lifetimes and lower asymmetry. It is worth reviewing certain features of the experimental technique to see exactly how the precision depends on lifetime and asymmetry.

As is explained in HMSP, the precession of the longitudinally polarized muon spin about a vertical magnetic field (of 13.4 kG), together with the asymmetric decay of the muon, produces a periodic variation, at the precession frequency, in the time distribution of decay electrons emitted in a fixed laboratory direction. This periodic variation (at 178 Mc/sec) takes the form of a sinusoid superimposed on the usual exponential decay. The experimental technique consists of comparing the frequency of this sinusoid with that

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[1] The first article is: D. P. Hutchinson, J. Menes, G. Shapiro, and A. Patlach, preceding article, Phys. Rev. **131**, 1351 (1963); and will, henceforth, be denoted by HMSP.

[2] E. Lubkin, Phys. Rev. **119**, 815 (1960).

of an oscillator oscillating at a nearby frequency by watching the relative phase of the two frequencies vary during the muon lifetime. This change in relative phase over time is expressed in HMSP in terms of a quantity called "$\alpha_{\text{early}}-\alpha_{\text{late}}$" and several plots of ($\alpha_{\text{early}}-\alpha_{\text{late}}$) against magnetic field are given in Fig. 1. The intercept, where $\alpha_{\text{early}}=\alpha_{\text{late}}$, corresponds to the field where the precession frequency equals the oscillator frequency.

The difference in phase between two frequencies ω_1 and ω_2 is simply $T(\omega_1-\omega_2)$, where T is the time. Hence, the quantity $\alpha_{\text{early}}-\alpha_{\text{late}}$ will, for a fixed difference between oscillator and precession frequencies, be proportional to the time available for observation, that is, the μ^- lifetime. This is reflected in the slopes of the curves in Fig. 1. Since it is the phase which is measured in this experiment, it becomes clear that, other things being equal, the precision of the frequency measurement will be directly proportional to the lifetime of the muon in the target in question.

The shorter lifetime has another effect in that the number of decay electrons is reduced as the capture process takes over. The relative statistical errors are proportional to (number of events)$^{-1/2}$, and hence to (lifetime)$^{-1/2}$. The total effect of the lifetime is to make the precision of the final results proportional to (lifetime)$^{3/2}$.

The precision with which one can measure the phase of the sinusoidal component of the decay electron time distribution depends directly on the amplitude of the sinusoid, which is the apparent decay asymmetry. Since there is a depolarization of $\frac{5}{6}$ as the μ^- cascades into the atomic 1S state, the error on the phase measurement will be six times larger than in a similar measurement on positive (free) muons. This effect is very evident in comparing the error flags in Fig. 1 with those in similar graphs in HMSP, which represent roughly equal running time.

Since all the targets used have spin-zero nuclei, there is no further depolarization due to hyperfine interaction. If the muon beam were entirely polarized, the asymmetry averaged over the decay electron spectrum would be 0.3/6=0.05. The asymmetry actually observed during the run was 0.02–0.03. This 50% reduction in asymmetry was also observed in the runs on μ^+. Half of this loss is accounted for by beam polarization, large solid angle subtended by the electron telescope, and electronic time resolution. We have no satisfactory explanation for the rest of this loss in asymmetry. It may be due to the large magnetic field (13.4 kG) which distorts the solid angle.

III. RESULTS AND DISCUSSION

The experimental results are summarized in Table I in terms of the proton frequency (that is, the magnetic field) at which the muon precession frequency equals four times the crystal oscillator frequency. The results are also given in terms of the departure of the apparent

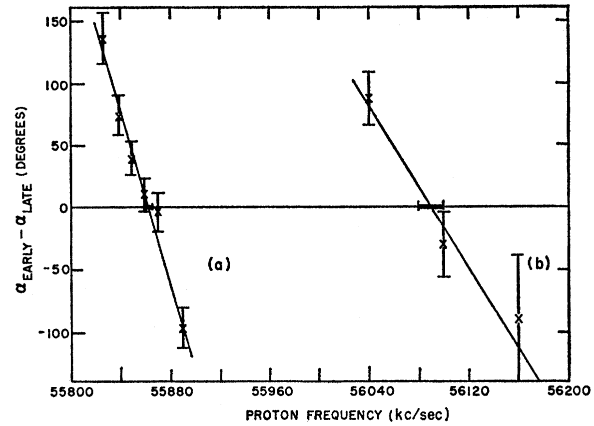

FIG. 1. ($\alpha_{\text{early}}-\alpha_{\text{late}}$) versus magnetic field in proton kc/sec. The intercept, where $\alpha_{\text{early}}=\alpha_{\text{late}}$, gives the field at which the precession frequency equals the oscillator frequency. (a) μ^- in graphite, reactor grade. Lifetime $\sim$2.0 μsec. (b) μ^- in sulfur. Lifetime $\sim$0.6 μsec.

g factor of the negative muon from that of the positive muon, $(g^--g^+)/g^+$. The proton frequency quoted for the positive muon corresponds to $f_\mu/f_p=3.18338 \pm 0.00004$, and is corrected for chemical shift.[1,3] The remainder of Table I gives several corrections which should be applied to the measured moment of the negative muon before comparing it with the positive muon.

These corrections are all affected by the fact that the muon is observed in the ground state of a mesonic atom whose radius, neglecting finite nuclear size effect, is r_0m_e/Zm_μ; where r_0=Bohr radius, m_e, m_μ=mass of electron and muon, Z=charge of host nucleus. In magnesium, this radius is 2×10^{-12}, compared to a nuclear radius of about 3.5×10^{-13}.

The radiative corrections $\alpha/2\pi+O(\alpha^2)+\cdots$ do not enter directly into the comparison, since they are assumed to be the same for both signs of the muon.

The corrections to be discussed here are due to (A) relativistic effect; (B) diamagnetic shielding by electrons; (C) Coulomb effect on radiative correction; (D) nuclear polarization; and (E) metallic and chemical shifts.

A. Relativistic Effect

The largest correction to the magnetic moment is due to the kinetic energy of the bound muon.[4] For a muon in a 1S state, the g factor becomes $g(1-\frac{4}{3}\int F^2dr)$, where F is the small component of the radial wave function of the muon. Since the magnitude of the small component is proportional to v/c, the modified g factor can be expressed as $g(1-2\langle T\rangle/3mc^2)$, for $\langle T\rangle\ll mc^2$. $\langle T\rangle$ is the expectation value of the kinetic energy of the

[3] N. F. Ramsey, *Nuclear Moments* (John Wiley & Sons, Inc., New York, 1953), p. 71.
[4] H. Margenau, Phys. Rev. **57**, 383 (1940); G. Breit, Nature **122**, 649 (1928).

TABLE I. Experimental results and theoretical corrections to measured moment of negative muon.

				Theoretical corrections ($\times 10^4$)				
Z	Stopping material	Proton frequency at resonance (kc/sec)	$(g^- - g^+)/g^+$ ($\times 10^4$)	Relativistic effect (Ref. 5)	Coulomb effect on radiative correction (Ref. 5)	Nuclear polarization (Ref. 5)	Diamagnetic shielding (Ref. 7)	Total
μ^+	(Corrected for diamagnetism)	55 821.1±0.7	0.0±0.13					
6	Graphite, Reactor grade	55 863.4±1.7	−7.6±0.3	−6.29	−0.08	+0.04	−1.99±0.1	−8.32±0.1
6	Graphite, 99.995% pure, 300°K	55 860.5±3.1	−7.1±0.6	−6.29	−0.08	+0.04	−1.99±0.1	−8.32±0.1
6	Graphite, 99.995% pure, 77°K	55 865.7±2.7	−8.0±0.5	−6.29	−0.08	+0.04	−1.99±0.1	−8.32±0.1
8	Oxygen in water	55 873.7±5.7	−9.4±1.0	−11.04	−0.13	+0.12	−3.25±0.2	−14.30±0.2
12	Magnesium, metal	55 968.5±3.7	−26.4±0.7	−23.79	−0.29	+0.53	−6.29±0.3	−29.74±0.3
12	Magnesium, in MgH_2	55 985.6±3.7	−29.6±0.7	−23.79	−0.29	+0.53	−6.29±0.3	−29.74±0.3
14	Silicon 300Ω cm, n type	56 023.5±6.0	−36.3±1.1	−31.72	−0.40	+0.90	−7.95±0.4	−39.17±0.4
16	Sulfur	56 090.0±9.1	−48.2±1.6	−40.35	−0.51	+1.4	−9.70±0.5	−49.16±0.5

muon. For a point nucleus, $2\langle T\rangle/3mc^2$ reduces to $\frac{1}{3}\alpha^2Z^2$.

The quantity $\frac{4}{3}\int F^2dr$ has been calculated by Ford, Hughes, and Wills[5] using muon wave functions derived from a nuclear charge distribution of the form

$$\rho(r)=Ze/4\pi r_1^3N_0\times\begin{cases}1-\frac{1}{2}e^{-n(1-x)}, & x<1,\\ \frac{1}{2}e^{-n(x-1)}, & x\leq 1,\end{cases}$$

where $x=r/r_1$ and N_0=normalization. The radius r_1 and surface thickness parameter n are obtained from electron scattering experiments.

Table II gives both $\frac{4}{3}\int F^2dr$ and $\frac{1}{3}\alpha^2Z^2$ for the targets used. The difference between the two, which is due to finite nuclear size, is greater than the statistical error for $Z\geq 12$. For sulfur ($Z=16$), the difference is three times the statistical error. A simple perturbation calculation shows that the first-order change in the muon energy level due to a finite nucleus is proportional to r_1^2/a^2, where r_1=nuclear radius and a=radius of muon orbit. Hence, if one considers this experiment to be a measurement of nuclear radii, the results in sulfur determine the nuclear radius to 15–20%. An increase in the sensitivity of this experiment by a factor of five, which is not out of the question, would make this technique competitive with other methods of measuring nuclear radii. Furthermore, this experiment measures the effect of the finite nucleus on the kinetic energy of the muon, and thus sees the nucleus in a different way than do mu mesonic x-ray and electron-scattering experiments.

The $\frac{4}{3}\int F^2dr$ numbers are the ones that are quoted under "Relativistic effect" in Table I.

TABLE II. Relativistic correction to g factor.

Z	$10^4\times\frac{4}{3}\int F^2dr$ (finite nucleus) Ref. 5	$10^4\times\frac{1}{3}\alpha^2Z^2$ (point nucleus)
6	6.29	6.39
8	11.04	11.36
12	23.79	25.56
14	31.72	34.79
16	40.35	45.44

B. Diamagnetic Shielding by Electrons

The next largest correction results from diamagnetic shielding at the nucleus by the electrons of the host atom.[3,6] The effect can be visualized as a Larmor precession by the electrons which acts to reduce the field inside the electron current loops. If one computes the fractional change in the field at the nucleus in this classical way, one obtains

$$\Delta H(0)/H_{\text{ext}}=-(e/3mc^2)\int_0^\infty\frac{\rho(r)}{r}dr=(e/3mc^2)v(0),$$

where $\rho(r)$=radial charge density and hence $v(0)$ =electrostatic potential at the nucleus due to all the electrons.

A more complete treatment[3] gives, in addition to the above diamagnetic term, a second-order paramagnetic term. This term arises from virtual transitions to excited electronic states. This paramagnetism has not been included in our corrections. It depends very strongly on the lowest excited levels available to the electrons, since, as a second-order process, it has an energy denominator. In the crystalline environment provided by all of the targets, except water, there is a

[5] K. Ford, V. W. Hughes, and J. G. Wills, Phys. Rev. **129**, 194 (1963).

[6] W. E. Lamb, Jr., Phys. Rev. **60**, 817 (1941).

good deal of symmetry and hence one may expect relatively few low-lying electronic levels. Furthermore, since it is the outermost (valence) electrons that are responsible for low-lying levels, the effect is very difficult to calculate because it is precisely these electrons whose configurations are most affected by the presence of neighbors in the crystal. For these reasons, the second-order paramagnetism has been neglected.

On the other hand, the $v(0)$ term, because of its r^{-1} dependence, is produced primarily by the core electrons and is very little affected by the electronic state. Table III gives $v(0)$ in atomic units for the atoms N, N^-, O^+, O, O^-. As can be seen, $v(0)$ depends mainly on Z. Dickinson[7] has calculated $\Delta H(0)/H_{\text{ext}}$, using Hartree wave functions, for a large number of neutral and ionized atoms. Since the bound muon makes the nuclear charge effectively $Z-1$, Dickinson's values for $Z-1$ are quoted in Table I under "Diamagnetic shielding."

Since the nucleus together with the bound muon has an effective charge of $Z-1$, one might expect that the host atom would have $Z-1$ electrons. Z is even in all the targets used and hence there would be an odd electron, which means an unpaired spin. Such an unpaired spin would create a field of roughly (Bohr magneton)/($\frac{1}{2}$ interatomic distance)$^3 \sim 10^5$ G. The largest deviation from the magnetic moment of a free muon was seen in sulfur and could be described as a change in the local field of about (48×10^{-4}) (13.4 kG)$\sim$65 G. This clearly rules out any unpaired electron spin in the vicinity of the muon, especially since most of the change in the moment can be accounted for otherwise.

It should be noted that the magnitude of the diamagnetic correction is sufficient for this experimental method to provide a determination of the absolute shielding.

C. Effect of Coulomb Field on Radiative Correction

Although the radiative correction is the same for positive and negative muons, it is modified if the muon is in a bound state. The external (Coulomb) field acts as a perturbation in the radiative process with the result that $\alpha/2\pi$ becomes $(\alpha/2\pi)(1-52\langle V\rangle/15mc^2)$.[8] $\langle V\rangle$ is the expectation value of the potential energy of the muon in the Coulomb field. Numerical calculations have been made[5] and are quoted in Table I under Coulomb effect on radiative correction. The effect is quite small compared to the experimental errors.

D. Nuclear Polarization

Another small correction arises from the virtual excitation of the nucleus to an excited state by the magnetic field, and consequent interaction of the excited nucleus with the muon. Ford, Hughes, and Wills[5] relate this effect to magnetic dipole transition rates and then use a closure approximation to evaluate it. Their results are quoted in Table I. This correction is significant only for the higher Z targets.

[7] W. C. Dickinson, Phys. Rev. **80**, 563 (1950).
[8] E. H. Lieb, Phil. Mag. **46**, 311 (1955).

TABLE III. The potential at the nucleus due to all the electrons $v(0)$ versus charge of the nucleus and state of ionization.

Charge of nucleus and state of ionization	$v(0)$ in atomic units Ref. 7
7	18.32
7^-	18.70
8^+	21.61
8	22.26
8^-	22.72

E. Metallic Shifts

The effect of the magnetic susceptibility of the bound electrons on the field at the nucleus has been discussed in Sec. B. For reasons given there, the susceptibility of the outer (valence) electrons is not significant. This is not the case, however, if the valence electrons are free, as in a metal. Although the bulk susceptibility of the degenerate electron gas in a metal is quite small (10^{-5}–10^{-6}), the peaking of the free electron wave functions at the lattice sites (the nuclei) can produce shifts in the magnetic field at the nuclei which are much greater than the bulk shifts. These shifts, the Knight shifts,[9] have been measured in a large number of substances, including magnesium[10] (where it is 11.1×10^{-4}) and silicon[11] (1.8×10^{-4}) but not graphite.

There are several factors in the present situation which make the published Knight-shift measurements inapplicable. First, the host atom with its muon is a $Z-1$ impurity in the lattice and one would expect the free electron wave function to be smaller than usual in its vicinity, giving a smaller Knight shift. Second, there may be radiation damage. The energy required to eject an atom from a lattice point is of the order of 20 eV,[12] and there are several hundred keV available when the muon is captured into a $1S$ state. Since annealing times are of the order of seconds,[13] there is, thus, a high probability that the μ-mesonic atom will be situated interstitially, further perturbing the electron wave function.

There is a further complication in the case of silicon. Since the Knight shift is proportional to the free electron density at the lattice sites, it depends on the number of free electrons. In silicon, the number of free

[9] W. D. Knight, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1956), Vol. II, p. 93.
[10] T. J. Rowland, *Progress in Materials Science* (Pergamon Press, Inc., New York, 1961), Vol. 9, p. 14.
[11] H. E. Weaver, Jr., Phys. Rev. **89**, 923 (1953).
[12] H. Brooks, J. Appl. Phys. **30**, 1118 (1959).
[13] W. L. Brown, W. M. Augustyniak, and T. R. Waite, J. Appl. Phys. **30**, 1258 (1959).

TABLE IV. Knight shifts and conductivities of typical metals.

Z	Knight shift $\times 10^4$	Conductivity $[10^6(\Omega\ \text{cm})^{-1}]$	Knight-shift reference
3 (lithium)	3.4	0.11	(9)
11 (sodium)	10.5	0.23	(9)
12 (magnesium)	11.1	0.22	(10)
13 (aluminum)	16.2	0.34	(9)
29 (copper)	24.0	0.6	(9)
6 (graphite)	?	0.0012	

electrons is a function of both doping and temperature, neither of which is specified in the published measurement. Our silicon sample was 300 Ω cm n type, with a free electron density of about 10^{12}–10^{13}/cc at room temperature. This is 10 orders of magnitude below the density of free electrons in a metal. The susceptibility *per electron* in this silicon should be greater than in a metal by a factor of about 200, because at the low density of 10^{12}/cc, the electron gas is not degenerate. Hence all electrons, rather than only the fraction kT/E_F, can be paramagnetic. E_F, the Fermi energy, is typically 5 eV in a metal. Nevertheless, the density of electrons is so low that, even with a factor of 200, there can be no significant Knight shift.

The Knight shift in graphite is unknown. However, it is worth making an estimate because of the importance of the graphite results in the comparison of μ^+ and μ^- magnetic moments. Table IV gives some typical Knight shifts for low and medium Z. The shift varies fairly regularly with Z, without very much dependence on the number of valence electrons. Therefore, if graphite were a typical metal, one could estimate a value of roughly 6×10^{-4} for its Knight shift. Actually, the conductivity of graphite is lower by about a factor of 200 than that of typical metals (see Table IV). The Knight shift is proportional to the cube root of the number of free electrons, provided the free electrons form a degenerate gas.[14] If one then assumes that the lower conductivity of graphite is due primarily to a lower number of free electrons, then the Knight shift should be reduced by a factor $(200)^{1/3}\sim6$. On the basis of this argument, one estimates the Knight shift in graphite to be about 1×10^{-4}–2×10^{-4}. In the present experiment there are complicating factors because the mu-mesonic atom is a $Z-1$ impurity and because of radiation damage, as discussed above.

There is, of course, no Knight shift in water or sulfur. The total of all the corrections (not including Knight shift) is given in the last column of Table I for each target and is to be compared with the experimental results under "$(g^- - g^+)/g^+$."

The graphite result is in reasonable agreement with prediction. A Knight shift of 0.8×10^{-4} would make the agreement perfect. However, because of uncertainty in the Knight shift, one must make a more conservative claim on the agreement, perhaps 3×10^{-4}. Since the graphite results are used to check on the equality of μ^- and μ^+ magnetic moments, one can claim that the moments are equal to 3×10^{-4}.

The data on oxygen indicate a paramagnetic shift of 5×10^{-4}. It should be pointed out that there can be no shift due to muons stopping in the hydrogen. Negative muons that stop in hydrogen are transferred to impurities (oxygen, in this case) in times of the order of 10^{-10} sec for concentrations corresponding to that of oxygen in water.[15] Hence, all the muon precession (and decay) is in oxygen. The 5×10^{-4} discrepancy is presumably due to some chemical effect. Paramagnetic chemical shifts of this magnitude have been observed in the nuclear resonance of nitrogen in various compounds.[16] Nitrogen is relevant here because an oxygen nucleus with a bound nuon is chemically a nitrogen nucleus. These shifts are relative shifts, due to different electronic configurations in the various compounds, and are ascribed to the second-order paramagnetism mentioned in Sec. B above. The discrepancy seen in this experiment corresponds to the shift between an isolated nitrogen atom and a nitrogen atom in whatever environment the μ-oxygen system finds itself during the muon lifetime. The shift measured here is, thus, an absolute shift and, hence, this experimental technique may have application in the study of chemical shifts.

The magnesium data show a significant discrepancy between magnesium metal and magnesium in a nonconducting compound (MgH_2). The MgH_2 data agree with the predicted result, whereas the metallic form shows a paramagnetic shift of 3.4×10^{-4}. The published Knight shift of 11.1×10^{-4} is not seen, presumably for the reasons given above.

The silicon and sulfur data are in fair agreement with prediction. The silicon result shows a small discrepancy while the sulfur result is within experimental error. We have no ready explanation for the discrepancy in silicon. The silicon and sulfur results together are a substantial verification of the larger corrections. The finite nucleus effect on the relativistic correction is required to obtain the agreement seen. The diamagnetic shielding correction is also necessary, and one may estimate that the results verify this correction to perhaps 20%.

IV. CONCLUSIONS

The results of this experiment indicate that the magnetic moments of positive and negative muons are equal to three parts in 10^4, with the major uncertainty due to the Knight shift. The relativistic correction has been substantially verified and the effect of finite

[14] This can easily be seen. If the number of free electrons is n, the number that are close enough to the Fermi surface to be paramagnetic is nkT/E_{Fermi}. Now $n\propto p_{\text{Fermi}}^3\propto E_{\text{Fermi}}^{3/2}$, so that the number of paramagnetic electrons is proportional to $n/n^{2/3}=n^{1/3}$. This argument is valid as long as $kT\ll E_{\text{Fermi}}$.

[15] M. Schiff, thesis, University of Chicago, Report 351, 1961 (to be published).

[16] J. A. Pople, W. G. Schneider, and H. J. Bernstein, *High Resolution Nuclear Magnetic Resonance* (McGraw-Hill Book Company, Inc., New York, 1959), p. 313.

nuclear size seen. The absolute diamagnetic shielding has been seen, perhaps for the first time, and verified to about 20%. Solid-state and chemical effects have apparently been seen, but their meaning in this experiment is not clear. Until these latter effects have been elucidated, either theoretically or by further measurements, they will serve to obscure a closer study of nuclear effects. It may be that the solid state and chemical shifts will become a subject for investigation in their own right, the muon serving as a tool for probing them.

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Asymptotic Invariants in Gravitational Radiation Fields

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Asymptotic integral invariants are constructed for the electromagnetic and gravitational fields. The integrals are taken over closed two-dimensional surfaces embedded in a null hypersurface. In the absence of incoming radiation, the asymptotic behavior of the electromagnetic field F^{ab} and the Riemann tensor R^{abcd} is such that the integrals formed with these quantities are independent of the particular space-like surface of integration, as long as it lies in the same null hypersurface. Therefore, the integrals are related to the multipole structure of the charge distribution and the matter distribution, respectively. This relationship is shown explicitly for the electromagnetic field and for the linearized gravitational field. It follows that energy radiation as determined by the Einstein pseudotensor depends on the existence of a type III asymptotic behavior of the Riemann tensor. Finally, the asymptotic conditions are formulated under which the superpotential $U_m{}^{ns}$ will also lead to asymptotically invariant integrals. It is pointed out that the linearized gravitational field with retarded potentials satisfies these conditions as do the asymptotic solutions for the Einstein field equations $R_{ab}=0$, which have been constructed by Bondi and Newman. The significance of this result for the interpretation of the Bondi metric is discussed.

1. INTRODUCTION

THE classification of the Riemann tensor for an Einstein space constructed by Petrov[1] was given its preliminary physical interpretation by Pirani[2] who identified certain of the special Petrov classes with the existence of gravitational radiation. In the following years, a distinction was drawn between the pure gravitational radiation field, corresponding to plane waves in the electromagnetic field, and an asymptotic gravitational field which may result from a matter distribution.[3–5] Thus, the existence of a pure gravitational radiation field leads to one of the algebraically special Petrov classes, in accord with Pirani, whereas a field with explicit sources belongs to the most general Petrov class and may become algebraically special at large distances. The purpose of this paper is to describe an additional tool, namely, asymptotically invariant integrals, for investigating the physical significance of vacuum gravitational fields, $G_{ab}=0$, particularly those containing radiation.

There have been two different approaches to the study of the asymptotic gravitational field, one looking at the properties of the Riemann tensor and the other examining the asymptotic behavior of the metric tensor. The Petrov classification has been shown to be related to the existence of preferred null directions at each point of space-time.[5–7] In fact, when the Riemann tensor is algebraically special, there always exists a congruence of shear-free null geodesics.[8–10] Sachs[5] used the properties of null geodesic congruences to discuss the propagation of the Riemann tensor along the null rays. From the explicit distance dependence in the algebraically

* The major portion of this research was performed while the author was on leave at Kings College, University of London, as a National Science Foundation Senior Post-Doctoral Fellow.

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