Three-Body Leptonic Decays of the K Mesons*

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A treatment is given of the three-body leptonic decays of the K mesons on the basis of a phenomenological strangeness-changing weak-interaction Hamiltonian and an analysis is presented of the relevant decay form factors in terms of simple assumptions regarding the groups of eigenstates which are strongly coupled to the $K\pi$ system. The possibility of a violation of time-reversal invariance arising from the "out of phaseness" of the strangeness-changing $\Delta T=\frac{3}{2}$ and $\Delta T=\frac{1}{2}$ currents in H_{weak} is considered. Throughout the discussion, the available experimental data are used to extract as much information as possible about the parameters which are important theoretically.

I. INTRODUCTION

THE three-body leptonic decays of the K mesons afford valuable insight into several fundamental features of the strangeness-changing weak interactions; in particular, questions associated with the space-time and with the isospin structure of the strangeness-changing currents can, in principle, be answered on the basis of a sufficiently detailed analysis of the decays. In the ensuing discussion we present a treatment of the problem from the point of view of a phenomenological strangeness-changing weak-interaction Hamiltonian and attempt to clarify the implications of the already available experimental information.[1,2]

II. FORM FACTORS IN K^+ LEPTONIC DECAY

We begin by writing the appropriate transition matrix element for the weak-interaction-induced process $K^+\to\pi^0+l^++\nu_l$ ($l^+\equiv e^+$ or μ^+)

$$\begin{aligned}\langle\nu_l l^+\pi^0|H_{\text{weak}}(0)|K^+\rangle&=\left\langle\nu_l l^+\pi^0\left|\frac{G}{\sqrt{2}}l_\lambda(0)[s_\lambda^{(V)}(0)+s_\lambda^{(A)}(0)]\right|K^+\right\rangle\\&=\frac{G}{\sqrt{2}}(\bar{u}_{\nu_l}\gamma_\lambda(1+\gamma_5)u_l^*)\langle\pi^0|s_\lambda^{(V)}(0)|K^+\rangle,\end{aligned}\tag{1}$$

where

$$\begin{aligned}&2(4E_KE_\pi)^{1/2}\langle\pi^0|s_\lambda^{(V)}(0)|K^+\rangle=[f_+(q^2)Q_\lambda+f_-(q^2)q_\lambda];\\&Q_\lambda\equiv p_{K;\lambda}+p_{\pi;\lambda};\quad q_\lambda\equiv p_{K;\lambda}-p_{\pi;\lambda}=p_{l;\lambda}+p_{\nu_l;\lambda};\\&\gamma_\lambda=\gamma_\lambda^\dagger,\quad\gamma_{1,2,3}=\gamma_{1,2,3}^*,\quad\gamma_4=-\gamma_4^*,\quad\gamma_5\equiv\gamma_1\gamma_2\gamma_3\gamma_4=\gamma_5^\dagger=-\gamma_5^*,\end{aligned}\tag{2a}$$

and

$$\begin{aligned}&q^2\equiv q_\lambda q_\lambda=-m_K^2-m_\pi^2+2m_KE_\pi=-m_l^2-2E_lE_{\nu_l}[1-(p_l/E_l)\cos(l,\nu_l)],\\&(\bar{u}_{\nu_l}\gamma_\lambda(1+\gamma_5)u_l^*)[f_+(q^2)Q_\lambda+f_-(q^2)q_\lambda]\\&\qquad=i\{m_l(\bar{u}_{\nu_l}(1-\gamma_5)u_5^*)[f_+(q^2)-f_-(q^2)]+2m_K(\bar{u}_{\nu_l}\gamma_4(1+\gamma_5)u_l^*)f_+(q^2)\},\end{aligned}\tag{2b}$$

[$\mathbf{p}_K=0$ in Eq. (2b)] and where the strangeness-changing current $s_\lambda^{(V)}(0)$ rather than the strangeness-changing current $s_\lambda^{(A)}(0)$ contributes because π and K are taken to have the same intrinsic parity. As indicated by our notation, we assume a $V-A$ type strangeness-changing H_{weak} of the same form and coupling constant for μ as for e; such an assumption is consistent with the observed energy spectra in $K^+\to\pi^0+e^++\nu_e$, $K_2^0\to\pi^\mp+e^\pm+\nu_e$ and with the idea of "$\mu-e$ universality" provided that we also assume that the form factors $f_\pm(q^2)$ vary relatively slowly with q^2 or, equivalently, with E_π. This last assumption appears reasonable since the $f_\pm(q^2)$ are analytic functions of q^2 except for a cut given by $(-q^2)\geq(m_K+m_\pi)^2$, while the physical region for q^2 corresponds to $m_l^2\leq(-q^2)\leq(m_K-m_\pi)^2=0.33(m_K+m_\pi)^2=0.53m_K^2$—thus, we can write

$$f_\pm(q^2)=f_\pm(0)\left(1+\lambda_\pm\left[\frac{-q^2}{m_K^2}\right]+\cdots\right),\tag{3}$$

with $|\lambda_\pm|\lesssim1$.

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[1] Some of the material in this paper has been presented by one of the authors (H.P.) in lectures at the Bergen International School of Physics during June, 1962 (unpublished).
[2] A discussion similar to ours in general outlook has been given by J. D. Jackson and R. L. Schult, Technical Report No. 43, Physics Department, University of Illinois, Urbana, Illinois (unpublished). Many references to the literature, both experimental and theoretical, are given in this paper.

We proceed to decompose the form factors $f_{\pm}(q^2)$ into a sum of terms each of which is associated with a definite group of isospin, angular momentum, and parity eigenstates which couple strongly to

$$|\bar{\pi}^0K^+\rangle = -(\tfrac{1}{3})^{1/2}|T=\tfrac{1}{2},\, T^{(3)}=\tfrac{1}{2};\, -p_\pi,\, p_K\rangle + (\tfrac{2}{3})^{1/2}|T=\tfrac{3}{2},\, T^{(3)}=\tfrac{1}{2};\, -p_\pi,\, p_K\rangle.$$

These groups of eigenstates are necessarily $(0^+,\frac{1}{2})$, $(1^-,\frac{1}{2})$, $(0^+,\frac{3}{2})$, $(1^-,\frac{3}{2})$ (in a notation J^P,T) since $\langle\pi^0|s_{1,2,3}^{(V)}(0)|K^+\rangle$ and $\langle\pi^0|s_4^{(V)}(0)|K^+\rangle$ transform like a vector and like a scalar, respectively, under three-dimensional rotations in momentum space. A kinematic analysis then shows that[3]

$$f_{\pm}(q^2) = -(\tfrac{1}{3})^{1/2} f_{\pm,1/2}(q^2) + (\tfrac{2}{3})^{1/2} f_{\pm,3/2}(q^2),$$

$$f_{+,T}(q^2) = f_{1^-,T}(q^2), \quad f_{-,T}(q^2) = f_{0^+,T}(q^2) + \frac{m_K^2 - m_\pi^2}{q^2} f_{1^-,T}(q^2), \tag{4}$$

$$2(4E_KE_\pi)^{1/2}\langle\text{vac}|s_\lambda^{(V)}(0)|T, T^{(3)}=\tfrac{1}{2};\, -p_\pi, p_K\rangle = [f_{+,T}(q^2)Q_\lambda + f_{-,T}(q^2)q_\lambda],$$

where, since $f_{-,T}(0) < \infty$, we must have

$$f_{0^+,T}(q^2) = -[(m_K^2 - m_\pi^2)/q^2] f_{1^-,T}(0) + g_{0^+,T}(q^2),$$
$$g_{0^+,T}(q^2) = g_{0^+,T}(0)(1 + \lambda_{\pm,T}[-q^2/m_K^2] + \cdots); \quad g_{0^+,T}(0) < \infty. \tag{5}$$

Writing

$$f_{\pm,T}(q^2) = f_{\pm,T}(0)(1 + \lambda_{\pm,T}[-q^2/m_K^2] + \cdots) \tag{6}$$

we have, from Eqs. (3) and (4),

$$\lambda_{\pm} = \frac{\lambda_{\pm,1/2} f_{\pm,1/2}(0) - \sqrt{2}\lambda_{\pm,3/2} f_{\pm,3/2}(0)}{f_{\pm,1/2}(0) - \sqrt{2} f_{\pm,3/2}(0)} = \lambda_{\pm,1/2} \frac{1 - \sqrt{2}(\lambda_{\pm,3/2}/\lambda_{\pm,1/2})\alpha_{\pm}}{1 - \sqrt{2}\alpha_{\pm}}; \quad \alpha_{\pm} \equiv f_{\pm,3/2}(0)/f_{\pm,1/2}(0) \tag{7}$$

and

$$\xi \equiv \frac{f_-(0)}{f_+(0)} = \frac{f_{-,1/2}(0) - \sqrt{2} f_{-,3/2}(0)}{f_{+,1/2}(0) - \sqrt{2} f_{+,3/2}(0)} = \xi_{1/2} \frac{1 - \sqrt{2}\alpha_-}{1 - \sqrt{2}\alpha_+}; \quad \xi_{1/2} \equiv f_{-,1/2}(0)/f_{+,1/2}(0). \tag{8}$$

Until further notice (see Sec. VI below) we assume our H_{weak} invariant under time reversal so that the various form factors that we introduce [$f_{\pm}(q^2)$ in Eqs. (2), $f_{\pm}^{(0)}(q^2)$ and $\bar{f}_{\pm}^{(0)}(q^2)$ in Eq. (33) below] are all relatively real for q^2 in the physical region.

We now express the branching ratio B of the $K^+ \to \pi^0 + \mu^+ + \nu_\mu$ and $K^+ \to \pi^0 + e^+ + \nu_e$ decay modes in terms of the parameters ξ, $\lambda_{\pm}$. A straightforward calculation yields[4]

$$B \equiv \frac{\Gamma(K^+ \to \pi^0 + \mu^+ + \nu_\mu)}{\Gamma(K^+ \to \pi^0 + e^+ + \nu_e)} = 0.65 + 0.13\xi + 0.019\xi^2 + 0.007\lambda_+ + 0.033\xi(\lambda_+ + \lambda_-) + 0.012\xi^2\lambda_- + \cdots, \tag{9}$$

where the Γ's are rates for the indicated decay modes and where the contribution of $f_-(q^2)q_\lambda$ to the e^+ decay mode ($\sim m_e$) is neglected. Experimentally, we have,[5]

$$B_{\text{exp}} = 0.96 \pm 0.15 \tag{10}$$

and it is our first problem (see next section) to determine whether the formula for B in Eq. (9) with reasonable *a priori* theoretical values of ξ, $\lambda_{\pm}$ can be made to reproduce the B_{exp} of Eq. (10).

In concluding this section we calculate the form factors associated with the divergence of the (strangeness-changing polar-vector) current $s_\lambda^{(V)}(x)$. Thus, using Eqs. (2) and (4), we get

$$[2(4E_KE_\pi)^{1/2}]\left\langle \pi^0 \left| \left\{\frac{\partial s_\lambda^{(V)}(x)}{i\partial x_\lambda}\right\}_{x=0} \right| K^+ \right\rangle = q_\lambda[f_+(q^2)Q_\lambda + f_-(q^2)q_\lambda]$$
$$= (m_\pi^2 - m_K^2) f_+(q^2) + q^2 f_-(q^2) = (\tfrac{1}{3})^{1/2}[q^2 f_{0^+,1/2}(q^2)] + (\tfrac{2}{3})^{1/2}[q^2 f_{0^+,3/2}(q^2)], \tag{11}$$

so that "asymptotic conservation" of the current $s_\lambda^{(V)}(x)$ corresponds to

$$\lim_{q^2\to\infty}[q^2 f_{0^+,T}(q^2)] = 0, \tag{12}$$

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[3] S. W. MacDowell, Phys. Rev. **116**, 1047 (1959).
[4] See Ref. 2; also A. Fujii and M. Kawaguchi, Phys. Rev. **113**, 1156 (1959); and Ref. 16.
[5] B. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters **7**, 346 (1961).

whence, remembering Eq. (5),

$$-(m_K^2-m_\pi^2)f_{1^-,T}(0)+\lim_{q^2\to\infty}[q^2g_{0^+,T}(q^2)]=0. \quad (13)$$

Equation (13) shows that one can write an unsubtracted dispersion relation for $g_{0^+,T}(q^2)$,

$$g_{0^+,T}(q^2)=g_{0^+,T}(0)\left(1+\lambda_{0^+,T}\left[\frac{-q^2}{m_K^2}\right]+\cdots\right)=\int_{(m_K+m_\pi)^2}^{\infty}\frac{\rho_{0^+,T}(m^2)d(m^2)}{m^2+q^2+i\epsilon};$$

$$\int_{(m_K+m_\pi)^2}^{\infty}\rho_{0^+,T}(m^2)d(m^2)=(m_K^2-m_\pi^2)f_{1^-,T}(0), \quad (14)$$

where $\rho_{0^+,T}(m^2)=-(1/\pi)\,\mathrm{Im}g_{0^+,T}(-m^2)$. For future reference we note that Eq. (14) gives

$$g_{0^+,T}(0)=\frac{m_K^2-m_\pi^2}{\langle m^2\rangle_{0^+;\,T}}f_{1^-,T}(0), \quad \lambda_{0^+,T}=m_K^2\langle m^2\rangle_{0^+;\,T}/\langle m^4\rangle_{0^+;\,T}, \quad (15)$$

with

$$\frac{1}{\langle m^n\rangle_{0^+;\,T}}\equiv\int_{(m_K+m_\pi)^2}^{\infty}\rho_{0^+;\,T}(m^2)\frac{d(m^2)}{m^n}\Big/\int_{(m_K+m_\pi)^2}^{\infty}\rho_{0^+;\,T}(m^2)d(m^2). \quad (16)$$

III. ANALYSIS OF K^+ LEPTONIC DECAY FORM FACTORS

We proceed to study the connection between the form factors $f_{1^-,1/2}(q^2)$, $f_{0^+,1/2}(q^2)$, $f_{1^-,3/2}(q^2)$, $f_{0^+,3/2}(q^2)$, and the corresponding $(1^-,\frac{1}{2})$, $(0^+,\frac{1}{2})$, $(1^-,\frac{3}{2})$, $(0^+,\frac{3}{2})$ groups of eigenstates which are strongly coupled to the $K\pi$ system. Two such eigenstates in the $(1^-,\frac{1}{2})$ group are now known experimentally; these are the well established "K^* meson" with $m_*=880$ MeV and the recently reported "K^{**} meson" with $m_{**}=730$ MeV.[6] We can therefore display explicitly the K^*, K^{**} pole terms in $f_{1^-,1/2}(q^2)$ and write

$$f_{1^-,1/2}(q^2)=\frac{c_*m_*^2}{m_*^2+q^2}+\frac{c_{**}m_{**}^2}{m_{**}^2+q^2}+g_{1^-,1/2}(q^2), \quad g_{1^-,1/2}(q^2)=g_{1^-,1/2}(0)\left(1+\lambda_{1^-,1/2}\left[\frac{-q^2}{m_K^2}\right]+\cdots\right), \quad (17)$$

where the residues $c_*m_*^2$ and $c_{**}m_{**}^2$ are proportional to products of appropriate effective coupling constants, i.e., $c_*m_*^2\sim(g_{l\nu_lK^*}/G)\cdot(g_{K^*\pi K})$ and $c_{**}m_{**}^2\sim(g_{l\nu_lK^{**}}/G)\cdot(g_{K^{**}\pi K})$, while $g_{1^-,1/2}(q^2)$ contains the contribution to $f_{1^-,1/2}(q^2)$ of all the $(1^-,\frac{1}{2})$ eigenstates other than K^*, K^{**}. Equations (17) and (5) yield

$$f_{0^+,1/2}(q^2)=-\frac{(m_K^2-m_\pi^2)}{q^2}(c_*+c_{**}+g_{1^-,1/2}(0))+g_{0^+,1/2}(q^2), \quad (18)$$

while Eqs. (17), (18), and (4) give

$$f_{+,1/2}(q^2)=\frac{c_*m_*^2}{m_*^2+q^2}+\frac{c_{**}m_{**}^2}{m_{**}^2+q^2}+g_{1^-,1/2}(0)(1+\lambda_{1^-,1/2}[-q^2/m_K^2]+\cdots), \quad (19)$$

$$f_{-,1/2}(q^2)=-\left(\frac{m_K^2-m_\pi^2}{m_*^2}\right)\frac{c_*m_*^2}{m_*^2+q^2}-\left(\frac{m_K^2-m_\pi^2}{m_{**}^2}\right)\frac{c_{**}m_{**}^2}{m_{**}^2+q^2}-\left(\frac{m_K^2-m_\pi^2}{m_K^2}\right)\lambda_{1^-,1/2}g_{1^-,1/2}(0)+\cdots+g_{0^+,1/2}(q^2), \quad (20)$$

which exhibits the contribution of the K^*, K^{**} pole terms to $f_{+,1/2}(q^2)$, $f_{-,1/2}(q^2)$.

We now assume that these K^*, K^{**} pole terms dominate the expressions for $f_{+,1/2}(q^2)$, $f^-_{,1/2}(q^2)$ in Eqs. (19), (20); we can then write, remembering also Eqs. (6) and (8),

$$f_{+,1/2}(q^2)\cong c_*m_*^2/(m_*^2+q^2)+c_{**}m_{**}^2/(m_{**}^2+q^2),$$

$$f_{+,1/2}(0)\cong c_*+c_{**}, \quad (21)$$

$$\lambda_{+,1/2}\cong\frac{c_*m_K^2/m_*^2+c_{**}m_K^2/m_{**}^2}{c_*+c_{**}}=\frac{0.31c_*+0.45c_{**}}{c_*+c_{**}},$$

[6] For recent data on the K^* see W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Rev. Letters **9**, 330 (1962); G. A. Smith, J. Schwartz, D. H. Miller, G. R. Kalbfleisch, R. W. Huff, O. I. Dahl, and G. Alexander, *ibid.* **10**, 138 (1963). For the K^{**} see G. Alexander, G. R. Kalbfleisch, D. H. Miller, and G. R. Smith, *ibid.* **8**, 447 (1962).

and

$$f_{-,1/2}(q^2)\cong-\left(\frac{m_K^2-m_\pi^2}{m_*^2}\right)\left(\frac{c_*m_*^2}{m_*^2+q^2}\right)-\left(\frac{m_K^2-m_\pi^2}{m_{**}^2}\right)\left(\frac{c_{**}m_{**}^2}{m_{**}^2+q^2}\right),$$

$$f_{-,1/2}(0)\cong-\left(\frac{m_K^2-m_\pi^2}{m_*^2}\right)c_*-\left(\frac{m_K^2-m_\pi^2}{m_{**}^2}\right)c_{**}=-(0.29c_*+0.42c_{**}),$$

$$\lambda_{-,1/2}\cong\left[\left(\frac{m_K^2-m_\pi^2}{m_K^2}\right)c_*\frac{m_K^2}{m_*^2}+\left(\frac{m_K^2-m_\pi^2}{m_{**}^2}\right)c_{**}\frac{m_K^2}{m_{**}^2}\right]\Big/\left[\left(\frac{m_K^2-m_\pi^2}{m_*^2}\right)c_*+\left(\frac{m_K^2-m_\pi^2}{m_{**}^2}\right)c_{**}\right]$$

$$=\frac{0.31c_*+0.65c_{**}}{c_*+1.45c_{**}}, \quad (22)$$

while

$$\xi_{1/2}\equiv\frac{f_{-,1/2}(0)}{f_{+,1/2}(0)}=-\left(\frac{0.29c_*+0.42c_{**}}{c_*c_{**}}\right)\cong-\lambda_{+,1/2}\cong\lambda_{-,1/2}, \quad (23)$$

where the last (approximate) equality corresponds to a numerical value of c_{**}/c_* used below. Since $|\lambda_{\pm,1/2}|$, like $|\lambda_\pm|$, is expected to be less than unity [see Eqs. (3), (6), and (7)] a similar restriction must hold for $|\xi_{1/2}|$ and this in turn puts a lower bound on the possible value of $|c_*+c_{**}|$. Clearly, the situation under discussion here will not be modified essentially if additional $(1^-,\frac{1}{2})$ "K^{***} mesons," "K^{****} mesons," $\cdots$, are found.

[We may also note that if m_*^2, m_{**}^2 were in fact $\gg(m_K-m_\pi)^2$ and m_K^2, we would have $f_{+,1/2}(q^2)\cong c_*+c_{**}=\text{const}$, $f_{-,1/2}(q^2)\cong 0$. These are just the values of $f_{+1/2}(q^2)$, $f_{-,1/2}(q^2)$ appropriate to the case where

$$s_\lambda^{(V)}(x)=\{s_\lambda^{(V)}(x)\}_0\equiv i^{-1}\sqrt{2}\{\pi(x)^\dagger[\partial K(x)/\partial x_\lambda]-[\partial\pi(x)^\dagger/\partial x_\lambda]K(x)\},$$

with $\pi(x)^\dagger$ and $K(x)$ field operators which create a (physical) π and destroy a (physical) K, respectively (with neglect of source terms giving rise to (physical) particle creation and destruction one has: $\lim_{m_K\to m_\pi}\partial\{s_\lambda^{(V)}(x)\}_0/\partial x_\lambda=0$). Thus, any appreciable deviation of $f_{-,T}(q^2)/f_{+,T}(q^2)$ from zero is an indication of a corresponding appreciable deviation of $s_\lambda^{(V)}(x)$ from $\{s_\lambda^{(V)}(x)\}_0$.]

We proceed to discuss the various experimentally testable results which may be deduced from Eqs. (1)–(9), (21)–(23). Let us, in addition to the K^*, K^{**} pole dominance assumptions made above, also assume that the $(1^-,\frac{3}{2})(0^+,\frac{3}{2})$ groups of eigenstates contribute relatively little to the $f_\pm(q^2)$—it is clear that this last assumption is just the $\Delta T=\frac{1}{2}$ rule and corresponds to neglect of the $f_{\pm,3/2}(0)$ or the $\alpha_\pm$ in Eqs. (8) and (7). These equations then yield

$$\xi\cong\xi_{1/2};\quad \lambda_\pm\cong\lambda_{\pm,1/2}, \quad (24)$$

so that, substituting Eqs. (24) and (23) into Eq. (9),

$$B=0.65-0.12\lambda_++0.02\lambda_+^2-0.01\lambda_+^3+\cdots. \quad (25)$$

Thus, remembering that $|\lambda_+|\lesssim 1$ on the basis of available data on the π^0 energy spectrum in $K^+\to\pi^0+e^++\nu_e$,[7] we obtain, taking $\lambda_+=-1$,

$$B=0.80. \quad (26)$$

This value, which corresponds to $\xi\cong\xi_{1/2}\cong\lambda_{-,1/2}\cong-\lambda_{+,1/2}\cong\mp\lambda_\pm\cong 1$ [Eqs. (24) and (23)], differs from the experimental value in Eq. (10) by 1 standard deviation; according to Eqs. (24) and (23), $\xi\cong 1$ corresponds to

$$\left(\frac{c_{**}}{c_*}\right)=(g_{l\nu_lK^{**}}g_{K^{**}\pi K}m_*^2)/(g_{l\nu_lK^*}g_{K^*\pi K}m_{**}^2)=-0.91,$$

which is certainly not unreasonable *a priori*. It is, therefore, seen that the available data on the branching ratio B and on the shape of the π^0 energy spectrum in $K^+\to\pi^0+e^++\nu_e$ do not exclude the assumptions (I): that the K^*, K^{**} pole terms dominate the contribution to the $f_{\pm,1/2}(q^2)$ of the $(1^-,\frac{1}{2})$, $(0^+,\frac{1}{2})$ groups of eigenstates and (II): that the $(1^-,\frac{3}{2})$ and $(0^+,\frac{3}{2})$ groups of eigenstates make a relatively small contribution to the $f_\pm(q^2)$. With regard to the shape of the muon energy spectrum in $K^+\to\pi^0+\mu^++\nu_\mu$, two apparently conflicting experimental results have been reported: that of Brown *et al.*[7] which is not inconsistent with $\xi\cong 1$, $|\lambda_\pm|\cong 1$ [Eqs. (23)–(24)] and that of Dobbs *et al.*[8] which appears to favor $\xi\cong-9$, $|\lambda_\pm|\lesssim 1$ and so, while consistent with Eqs. (9), (10), contradicts

[7] J. L. Brown, J. A. Kadyk, G. H. Trilling, R. T. Van de Walle, B. P. Roe, and D. Sinclair, Phys. Rev. Letters **8**, 450 (1962).

[8] J. M. Dobbs, K. Lande, A. K. Mann, K. Reibel, F. J. Sciulli, H. Uto, D. H. White, and K. K. Young, Phys. Rev. Letters **8**, 295 (1962); see also A. M. Boyarski, E. C. Loh, L. Q. Niemela, D. M. Ritson, R. Weinstein, and S. Ozaki, Phys. Rev. **128**, 2998 (1962).

Eqs. (24), (23) (see, however, Sec. VI, below). Thus, any future confirmation of the result of Dobbs *et al.*[8] rather than that of Brown *et al.*[7] would imply the invalidity of either or both of the assumptions (I), (II) stated above. It is also worth mentioning that if the K^{**} meson does not exist, i.e., if $c_{**}=0$, Eqs. (23) and (24) yield:

$$-\xi \cong -\xi_{1/2}=0.29 \cong \lambda_{\pm,1/2} \cong \lambda_{\pm} \tag{27}$$

which, with Eq. (9), gives

$$B \equiv \Gamma(K^+ \to \pi^0+\mu^+ +\nu_\mu)/\Gamma(K^+ \to \pi^0+e^+ +\nu_e)=0.61\,. \tag{28}$$

This value differs by $2\frac{1}{2}$ standard deviations from the experimental value in Eq. (10), so that it would seem as if the existence of the K^{**} meson with $c_{**}\approx -c_*$ is required for the viability of our assumptions (I), (II).

We now abandon assumption (I) (i.e., abandon the K^*, K^{**} pole dominance assumption) but retain assumption (II). Then from Eqs. (20), (19), (6), (5),

$$\xi_{1/2} \equiv \frac{f_{-,1/2}(0)}{f_{+,1/2}(0)} = \left(-\frac{m_K^2-m_\pi^2}{m_*^2}c_* - \frac{m_K^2-m_\pi^2}{m_{**}^2}c_{**} - \frac{m_K^2-m_\pi^2}{m_K^2}\lambda_{1^-,1/2}g_{1^-,1/2}(0)+g_{0^+,1/2}(0)\right)\Big/ [c_*+c_{**}+g_{1^-,1/2}(0)]\,,$$

$$\lambda_{+,1/2} = \left(c_*\frac{m_K^2}{m_*^2}+c_{**}\frac{m_K^2}{m_{**}^2}+\lambda_{1^-,1/2}g_{1^-,1/2}(0)\right)\Big/[c_*+c_{**}+g_{1^-,1/2}(0)]\,, \tag{29}$$

$$\lambda_{-,1/2} = \left[\left(\frac{m_K^2-m_\pi^2}{m_*^2}\right)c_*\frac{m_K^2}{m_*^2}+\left(\frac{m_K^2-m_\pi^2}{m_{**}^2}\right)c_{**}\frac{m_K^2}{m_{**}^2}-\lambda_{0^+,1/2}g_{0^+,1/2}(0)\right]\Big/ \left[\left(\frac{m_K^2-m_\pi^2}{m_*^2}\right)c_*+\left(\frac{m_K^2-m_\pi^2}{m_{**}^2}\right)c_{**}+\left(\frac{m_K^2-m_\pi^2}{m_K^2}\right)\lambda_{1^-,1/2}g_{1^-,1/2}(0)-g_{0^+,1/2}(0)\right],$$

so that, for a numerical estimate, a relationship between $g_{0^+,1/2}(q^2)$ and $f_{1^-,1/2}(q^2)$ is obviously needed. Now precisely such a relationship is provided by the assumption of "asymptotic conservation" of the current $s_\lambda^{(V)}(x)$—assumption (I′)—which results in Eqs. (15), (16) above. Substituting these equations into Eq. (29) and using also Eq. (17), we have

$$\xi_{1/2} = \left[-\left(\frac{m_K^2-m_\pi^2}{m_*^2}\right)c_*-\left(\frac{m_K^2-m_\pi^2}{m_{**}^2}\right)c_{**}-\left(\frac{m_K^2-m_\pi^2}{m_K^2}\right)\lambda_{1^-,1/2}g_{1^-,1/2}(0)+\frac{m_K^2-m_\pi^2}{\langle m^2\rangle_{0^+,1/2}}(c_*+c_{**}+g_{1^-,1/2}(0))\right]\Big/ [c_*+c_{**}+g_{1^-,1/2}(0)]\,,$$

$$\lambda_{+,1/2} = \left(c_*\frac{m_K^2}{m_*^2}+c_{**}\frac{m_K^2}{m_{**}^2}+\lambda_{1^-,1/2}g_{1^-,1/2}(0)\right)\Big/[c_*+c_{**}+g_{1^-,1/2}(0)]\,, \tag{30}$$

$$\lambda_{-,1/2} = \left[\left(\frac{m_K^2-m_\pi^2}{m_*^2}\right)c_*\frac{m_K^2}{m_*^2}+\left(\frac{m_K^2-m_\pi^2}{m_{**}^2}\right)c_{**}\frac{m_K^2}{m_{**}^2}-\frac{m_K^2(m_K^2-m_\pi^2)}{\langle m^4\rangle_{0^+,1/2}}(c_*+c_{**}+g_{1^-,1/2}(0))\right]\Big/ \left[\left(\frac{m_K^2-m_\pi^2}{m_*^2}\right)c_*+\left(\frac{m_K^2-m_\pi^2}{m_{**}^2}\right)c_{**}+\left(\frac{m_K^2-m_\pi^2}{m_K^2}\right)\lambda_{1^-,1/2}g_{1^-,1/2}(0)-\frac{(m_K^2-m_\pi^2)}{\langle m^2\rangle_{0^+,1/2}}(c_*+c_{**}+g_{1^-,1/2}(0))\right],$$

whence it is seen that

$$\xi_{1/2} \cong \left\{-\lambda_{+,1/2}+\frac{m_K^2-m_\pi^2}{\langle m^2\rangle_{0^+,1/2}}\right\} \approx |\lambda_{-,1/2}|\,, \tag{31}$$

analogous to Eq. (23). Since we retain assumption (II), Eq. (24) holds; thus, a result for B similar to that in Eq. (26) and not inconsistent with $\xi\cong 1$, $|\lambda_\pm|\cong 1$[7] may be anticipated from Eq. (9)—it remains to be added that according to Eqs. (24), (30), and (31), $\xi\cong 1$, $|\lambda_\pm|\cong 1$[7] correspond to values for c_{**}/c_*, $g_{1^-,1/2}(0)/c_*$, $\lambda_{1^-,1/2}$, $m_K^2\langle m^2\rangle_{0^+,1/2}/\langle m^4\rangle_{0^+,1/2}$, which are not unreasonable *a priori*. On the other hand, any future confirmation of $\xi\cong -9$, $|\lambda_\pm|\lesssim 1$[8] [which is consistent with Eqs. (9) and (10)] will contradict Eqs. (24), (30), and (31) and so requires either abandonment of assumption (II), [i.e., of Eq. (24)] or of assumption (I′) [i.e., of Eqs. (15), (16) which with Eq. (29) lead to Eqs. (30), (31)]. In particular, abandonment of assumption (I′) and retention of assumption (II) corresponds to the joint validity of Eqs. (24) and (29) from which one may obtain $\xi\cong 9$, $|\lambda_\pm|\lesssim 1^8$ by assuming, for example, $g_{0^+,1/2}(0)\approx -9g_{1^-,1/2}(0)$; $g_{1^-,1/2}(0)\gg c_*, c_{**}$; $|\lambda_{1^-,1/2}|\approx|\lambda_{0^+,1/2}|\lesssim 1$; conversely, retention of assumption (I′) or of assumption (I) and abandonment of assumption (II) leads to the joint validity of Eqs. (30), (31), or Eq. (23) together with Eqs. (8), (7) (with α_+, α_- no longer both $\ll 1$) and also permits $\xi\cong -9$, $|\lambda_\pm|\lesssim 1^8$ even though $\xi_{1/2}\cong 1$, $|\lambda_{\pm,1/2}|\lesssim 1$. This is accomplished by use of Eqs. (8), (7) with

$$(1-\sqrt{2}\alpha_-)/(1-\sqrt{2}\alpha_+)\cong -9/\xi_{1/2}\cong -9.$$

IV. ANALYSIS OF K^0 LEPTONIC DECAY FORM FACTORS

In this section we investigate the weak-interaction induced processes: $K^0\to\pi^-+l^++\nu_l$, $\bar{K}^0\to\pi^-+l^++\nu_l$, or, equivalently, $K_1{}^0\to\pi^-+l^++\nu_l$, $K_2{}^0\to\pi^-+l^++\nu_l$. We have, analogous to Eqs. (1)–(8):

$$\langle\nu_l l^+\pi^-|H_{\text{weak}}(0)|K^0 \text{ or } \bar{K}^0\rangle=\frac{G}{\sqrt{2}}(\bar{u}_{\nu_l}\gamma_\lambda(1+\gamma_5)u_l{}^*)\langle\pi^-|s_\lambda{}^{(V)}(0)|K^0\text{ or }\bar{K}^0\rangle;$$

$$2(4E_KE_\pi)^{1/2}\langle\pi^-|s_\lambda{}^{(V)}(0)|K^0\rangle=[f_+{}^{(0)}(q^2)Q_\lambda+f_-{}^{(0)}(q^2)q_\lambda];$$

$$2(4E_KE_\pi)^{1/2}\langle\pi^-|s_\lambda{}^{(V)}(0)|\bar{K}^0\rangle=[\bar{f}_+{}^{(0)}(q^2)Q_\lambda+\bar{f}_-{}^{(0)}(q^2)q_\lambda];$$

$$f_\pm{}^{(0)}(q^2)=f_\pm{}^{(0)}(0)(1+\lambda_\pm{}^{(0)}[-q^2/m_K{}^2]+\cdots);\quad \bar{f}_\pm{}^{(0)}(q^2)=\bar{f}_\pm{}^{(0)}(0)(1+\bar{\lambda}_\pm{}^{(0)}[-q^2/m_K{}^2]+\cdots);$$

$$|\pi^+K^0\rangle=(\tfrac{2}{3})^{1/2}|T=\tfrac{1}{2}, T^{(3)}=\tfrac{1}{2}; -p_\pi, p_K\rangle+(\tfrac{1}{3})^{1/2}|T=\tfrac{3}{2}, T^{(3)}=\tfrac{1}{2}; -p_\pi, p_K\rangle;$$

$$|\pi^+\bar{K}^0\rangle=|T=\tfrac{3}{2}, T^{(3)}=\tfrac{3}{2}; -p_\pi, p_K\rangle; \tag{33}$$

$$f_\pm{}^{(0)}(q^2)=(\tfrac{2}{3})^{1/2}f_{\pm,1/2}(q^2)+(\tfrac{1}{3})^{1/2}f_{\pm,3/2}(q^2);\quad \bar{f}_\pm{}^{(0)}(q^2)=\phi_{\pm,3/2}(q^2);$$

$$\langle\nu_l,l^+,\pi^-|H_{\text{weak}}(0)|K_1{}^0\text{ or }K_2{}^0\rangle=\frac{G}{\sqrt{2}}(\bar{u}_{\nu_l}\gamma_\lambda(1+\gamma_5)u_l{}^*)\langle\pi^-|s_\lambda{}^{(V)}(0)|K_1{}^0\text{ or }K_2{}^0\rangle;$$

$$2(4E_KE_\pi)^{1/2}\langle\pi^-|s_\lambda{}^{(V)}(0)|K_1{}^0\text{ or }K_2{}^0\rangle=[f_+{}^{(1),(2)}(q^2)Q_\lambda+f_-{}^{(1),(2)}(q^2)q_\lambda];$$

$$f_\pm{}^{(1),(2)}(q^2)=f_\pm{}^{(1),(2)}(0)(1+\lambda_\pm{}^{(1),(2)}[-q^2/m_K{}^2]+\cdots);$$

$$f_\pm{}^{(1),(2)}(q^2)=\frac{1}{\sqrt{2}}\{(\tfrac{2}{3})^{1/2}f_{\pm,1/2}(q^2)+(\tfrac{1}{3})^{1/2}f_{\pm,3/2}(q^2)\pm\phi_{\pm,3/2}(q^2)\};$$

$$\xi^{(1),(2)}\equiv\frac{f_-{}^{(1),(2)}(0)}{f_+{}^{(1),(2)}(0)}=\xi_{1/2}\frac{[1+(1/\sqrt{2})\alpha_-](1\pm x_-)}{[1+(1/\sqrt{2})\alpha_+](1\pm x_+)};$$

$$\xi_{1/2}\equiv f_{-,1/2}(0)/f_{+,1/2}(0);\quad \alpha_\pm\equiv f_{\pm,3/2}(0)/f_{\pm,1/2}(0);\quad x_\pm\equiv\frac{\phi_{\pm,3/2}(0)}{(\tfrac{2}{3})^{1/2}f_{\pm,1/2}(0)+(\tfrac{1}{3})^{1/2}f_{\pm,3/2}(0)}=(\tfrac{3}{2})^{1/2}\frac{(\phi_{\pm,3/2}(0)/f_{\pm,3/2}(0))}{1+(\tfrac{1}{2})^{1/2}\alpha_\pm}\alpha_\pm;$$

$$\lambda_\pm{}^{(1),(2)}=\frac{\lambda_{\pm,1/2}f_{\pm,1/2}(0)+(\tfrac{1}{2})^{1/2}\lambda_{\pm,3/2}f_{\pm,3/2}(0)\pm(\tfrac{3}{2})^{1/2}\bar{\lambda}_\pm{}^{(0)}\phi_{\pm,3/2}(0)}{f_{\pm1/2}(0)+(\tfrac{1}{2})^{1/2}f_{\pm,3/2}(0)\pm(\tfrac{3}{2})^{1/2}\phi_{\pm,3/2}(0)}$$

$$=\left(\frac{\lambda_{\pm,1/2}}{1\pm x_\pm}\right)\left(\frac{1+(\tfrac{1}{2})^{1/2}(\lambda_{\pm,3/2}/\lambda_{\pm,1/2})\alpha_\pm}{1+(\tfrac{1}{2})^{1/2}\alpha_\pm}\right)\pm\left(\frac{\bar{\lambda}_\pm{}^{(0)}x_\pm}{1\pm x_\pm}\right).$$

It is to be noted that $\phi_{\pm,3/2}(q^2)=f_{\pm,3/2}(q^2)$ only if $\langle\text{vac}|s_\lambda{}^{(V)}(0)|T=\tfrac{3}{2}, T^{(3)}=\tfrac{3}{2}; -p_\pi, p_K\rangle=\langle\text{vac}|s_\lambda{}^{(V)}(0)|T=\tfrac{3}{2}, T^{(3)}=\tfrac{1}{2}; -p_\pi, p_K\rangle$, i.e., only if $s_\lambda{}^{(V)}(0)$ is a sum of terms each of which transforms under rotations in isospace as a spinor of definite rank[9]; also the $\Delta S=\Delta Q$ rule corresponds to $\phi_{\pm,3/2}(q^2)\ll 1$, while the $\Delta T=\tfrac{1}{2}$ rule implies that $f_{\pm,3/2}(q^2)\ll 1$ and $\phi_{\pm,3/2}(q^2)\ll 1$, so that $|\alpha_\pm|\ll 1$, $|x_\pm|\ll 1$.

We next set down the expression for the branching ratio $B^{(1),(2)}$ of the $K_{1,2}{}^0\to\pi^-+\mu^++\nu_\mu$ and $K_{1,2}{}^0\to\pi^-+e^++\nu_e$ decays. We have, analogous to Eq. (9):

$$B^{(1),(2)}\equiv\Gamma(K_{1,2}{}^0\to\pi^-+\mu^++\nu_\mu)/\Gamma(K_{1,2}{}^0\to\pi^-+e^++\nu_e)=0.65+0.13\xi^{(1),(2)}$$
$$+0.019[\xi^{(1),(2)}]^2+0.007\lambda_+{}^{(1),(2)}+0.033\xi^{(1),(2)}(\lambda_+{}^{(1),(2)}+\lambda_-{}^{(1),(2)})+0.012[\xi^{(1),(2)}]^2\lambda_-{}^{(1),(2)}+\cdots. \tag{34}$$

[9] R. Behrends and A. Sirlin, Phys. Rev. Letters **5**, 476 (1960); Phys. Rev. **121**, 324 (1961); Phys. Rev. Letters **8**, 221 (1962).

Experimentally[10]

$$B_{\mathrm{exp}}^{(2)}=0.79\pm0.19\,,$$

and this is consistent with the prediction of Eq. (34) for $\xi^{(2)}\cong\xi_{1/2}\cong\lambda_{-,1/2}\cong-\lambda_{+,1/2}\cong\mp\lambda_{\pm}^{(2)}\cong1$; it will be recalled [Eqs. (24)–(26) *et seq.*) that this set of values of $\xi^{(2)}$, $\lambda_{\pm}^{(2)}$ follows from the assumption (II), i.e., the $\Delta T=\frac{1}{2}$ rule ($\alpha_{\pm}\ll1$, $x_{\pm}\ll1$), applied to Eq. (33) and from the use of assumption (I), i.e., the K^*, K^{**} pole dominance, which leads to Eq. (23). In contradistinction to the situation with K^+ no measurement of the shape of the muon energy spectrum from K_2^0 has as yet been reported so that it is at present not possible to say whether $\xi^{(2)}\cong1$, $|\lambda_{\pm}^{(2)}|\cong1$ is consistent with such a spectrum.

We proceed to a discussion of the ratio R of the decay rates of $K^+\to\pi^0+e^++\nu_e$ and $K_2^{(0)}\to\pi^-+e^++\nu_e$. We have from Eqs. (1)–(4), and Eq. (33):

$$R\equiv\frac{\Gamma(K^+\to\pi^0+e^++\nu_e)}{\Gamma(K_2^0\to\pi^-+e^++\nu_e)}\cong\left(\frac{f_{+,1/2}(0)-\sqrt{2}f_{+,3/2}(0)}{f_{+,1/2}(0)+(\frac{1}{2})^{1/2}f_{+,3/2}(0)-(\frac{3}{2})^{1/2}\phi_{+,3/2}(0)}\right)^2=\left(\frac{1-\sqrt{2}\alpha_+}{1+(\frac{1}{2})^{1/2}\alpha_+}\frac{1}{1-x_+}\right)^2;$$
$$\alpha_+=\sqrt{2}\left(\frac{1\mp|R^{1/2}|(1-x_+)}{2\pm|R^{1/2}|(1-x_+)}\right),\tag{35}$$

where we have taken $f_{+,1/2}(q^2)$, $f_{+,3/2}(q^2)$, $\phi_{+,3/2}(q^2)\cong f_{+,1/2}(0)$, $f_{+,3/2}(0)$, $\phi_{+,3/2}(0)$ and neglected the terms in $f_{-,1/2}(q^2)q_\lambda$, $f_{-,3/2}(q^2)q_\lambda$, $\phi_{-,3/2}(q^2)q_\lambda$ which are all $\sim m_e$. The quantity

$$x_+=\left(\frac{3}{2}\right)^{1/2}\left(\frac{\phi_{+,3/2}(0)}{f_{+,3/2}(0)}\right)\left(\frac{\alpha_+}{1+(\frac{1}{2})^{1/2}\alpha_+}\right)$$

[Eq. (33)] is essentially the ratio of the transition amplitude for $\bar{K}^0\to\pi^-+e^++\nu_e(\Delta S=-\Delta Q)$ to that for $K^0\to\pi^-+e^++\nu_e(\Delta S=\Delta Q)$ and has recently been determined as[11]

$$(x_+)_{\mathrm{exp}}=0.50\pm0.15\,,\tag{36}$$

while the reported experimental value for R is[12]

$$R_{\mathrm{exp}}=\frac{(4.1\pm0.4)\times10^6}{(3.0\pm0.5)\times10^6}=1.4\pm0.3\,,\tag{37}$$

only a little more than 1 standard deviation away from the value $R=1$ predicted by the $\Delta T=\frac{1}{2}$ rule (which corresponds to $|\alpha_{\pm}|\ll1$, $|x_{\pm}|\ll1$). It is also to be noted that the assumption $(\phi_{+,3/2}(0)/f_{+,3/2}(0))=\sqrt{3}$ yields $x_+=(3/\sqrt{2})\times\alpha_+/(1+(\frac{1}{2})^{1/2}\alpha_+)$ and so $R=1$ [Eqs. (33) and (35)]. On the other hand, substitution of the numerical values of $(x_+)_{\mathrm{exp}}$ [Eq. (36)] and R_{exp} [Eq. (37)] into Eq. (35), yields[13]

$$\left(\frac{f_{+,3/2}(0)}{f_{+,1/2}(0)}\right)=\alpha_+=\begin{Bmatrix}0.22\\1.7\end{Bmatrix},$$
$$\left(\frac{\phi_{+,3/2}(0)}{f_{+,3/2}(0)}\right)=(\tfrac{2}{3})^{1/2}\left(\frac{1+(\frac{1}{2})^{1/2}\alpha_+}{\alpha_+}\right)x_+=\begin{Bmatrix}2.2\\0.53\end{Bmatrix},\tag{38}$$

so that both the $\Delta T=\frac{1}{2}$ rule and the $\Delta S=\Delta Q$ rule appear to be violated; also $[\phi_{+,3/2}(0)/f_{+,3/2}(0)]$ seems to differ appreciably from unity, i.e., the values of $\langle\mathrm{vac}|s_\lambda^{(V)}(0)|T=\frac{3}{2},T^{(3)};-p_\pi,p_K\rangle$ seem to be significantly different for $T^{(3)}=\frac{1}{2}$ and for $T^{(3)}=\frac{3}{2}$. We may, therefore, decompose the strangeness-changing current $s_\lambda(0)=s_\lambda^{(V)}(0)$

[10] D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamota, Phys. Rev. Letters 7, 255 (1961).

[11] R. P. Ely, W. M. Powell, H. White, M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, O. Fabbri, F. Farini, C. Filippi, H. Huzita, G. Miari, U. Camerini, W. F. Fry, and S. Natali, Phys. Rev. Letters 8, 132 (1962). See also G. Alexander *et al.*, Ref. 12.

[12] G. Alexander, S. P. Almeida, and F. S. Crawford, Phys. Rev. Letters 9, 69 (1962); see also Ref. 10, and D. Neagy, E. O. Okonov, N. I. Petrov, A. M. Rosanova, and V. A. Rusakov, *ibid.* 6, 552 (1961).

[13] The numerical values for $[f_{+,3/2}(0)/f_{+,1/2}(0)]$ and $[\phi_{+,3/2}(0)/f_{+,3/2}(0)]$ in Eq. (38) and the conclusions we have drawn from them have been previously given by Jackson and Schult in Ref. 2; obviously, these values are uncertain by at least 25%. The two possible values of $[f_{+,3/2}(0)/f_{+,1/2}(0)]$ [and so of $(\phi_{+,3/2}(0)/f_{+,3/2}(0)]$ correspond to the amplitudes for $K^+\to\pi^0+e^++\nu_e$ and $K_2^0\to\pi^-+e^++\nu_e$ being, respectively, in phase and 180° out of phase.

$+s_\lambda^{(A)}(0)$ into a sum of three terms

$$s_\lambda(0)=\sum_{\Delta T=1/2,3/2}\ \sum_{0<\Delta T^{(3)}\leq\Delta T}\{s_\lambda(0)\}_{\Delta T,\Delta T^{(3)}},\tag{39}$$

where

$$\begin{aligned}2(4E_KE_\pi)^{1/2}\langle\text{vac}|\{s_\lambda(0)\}_{\Delta T,\Delta T^{(3)}}|T,T^{(3)};-p_\pi,p_K\rangle&=2(4E_KE_\pi)^{1/2}\langle\text{vac}|\{s_\lambda^{(V)}(0)\}_{\Delta T,\Delta T^{(3)}}|T,T^{(3)};-p_\pi,p_K\rangle\\&=\delta_{\Delta T,T}\delta_{\Delta T^{(3)},T^{(3)}}\{f_{+;\Delta T,\Delta T^{(3)}}(q^2)Q_\lambda+f_{-;\Delta T,\Delta T^{(3)}}(q^2)q_\lambda\}\end{aligned}\tag{40}$$

and where, in general, none of the $\{s_\lambda(0)\}_{\Delta T,\Delta T^{(3)}}$ transform under rotations in isospace as spinors of definite rank. In terms of our previous notation (see Eqs. (4), (33)] we have

$$f_{\pm,1/2}(q^2)=f_{\pm;1/2,1/2}(q^2);\quad f_{\pm,3/2}(q^2)=f_{\pm;3/2,1/2}(q^2);\quad \phi_{\pm;3/2}(q^2)=f_{\pm;3/2,3/2}(q^2),\tag{41}$$

with [Eq. (38)]

$$f_{+;3/2,1/2}(0)/f_{+;1/2,1/2}(0)\neq0;\quad f_{+;3/2,3/2}(0)/f_{+;1/2,1/2}(0)\neq0;\quad f_{+;3/2,3/2}(0)/f_{+;3/2,1/2}(0)\neq1.\tag{42}$$

Finally, substitution of Eq. (38) for α_+ into Eq. (8) gives

$$\xi=\xi_{1/2}\frac{1-\sqrt2\alpha_-}{1-\sqrt2\alpha_+}=\xi_{1/2}(1-\sqrt2\alpha_-)\begin{Bmatrix}1.4\\-0.71\end{Bmatrix},\tag{43}$$

whence, it is seen that $\alpha_-\cong\alpha_+$ if $\xi_{1/2}\cong1$ [Eq. (31)—assumption (I′) or Eq. (23)—assumption (I)] and $\xi\cong1$ (Brown *et al.*[7]) while $\alpha_-=\begin{Bmatrix}5.3\\-8.4\end{Bmatrix}$ if $\xi_{1/2}\cong1$ and $\xi\cong-9$ (Dobbs *et al.*[8]). Thus, any future confirmation of this $\xi\cong-9$ result, together with retention of assumption (I′) or assumption (I) (i.e., $\xi_{1/2}\cong1$), will lead to the conclusion that $\alpha_-=\begin{Bmatrix}5.3\\-8.4\end{Bmatrix}$ is appreciably greater numerically than $\alpha_+=\begin{Bmatrix}0.22\\1.7\end{Bmatrix}$ (see discussion at end of Sec. III and also in Sec. VI).

V. FORM FACTORS IN Ξ LEPTONIC DECAY

We would now like to draw some further implications of Eq. (39). This equation predicts that transition matrix elements of $s_\lambda(0)$ associated with weak-interaction-induced processes other than $K\to\pi+l+\nu_l$, e.g., transition matrix elements of $s_\lambda(0)$ associated with $\Sigma\to n+l+\nu_l$ or with $\Xi\to\Sigma+l+\nu_l$, will be characterized by values which in general depend not only on ΔT, but for a given ΔT, also on $\Delta T^{(3)}$. In particular, let us treat the processes:

$$\begin{aligned}&\Xi^-\to\Sigma^0+e^-+\bar\nu_e,\\&\Xi^0\to\Sigma^++e^-+\bar\nu_e,\\&\Xi^0\to\Sigma^-+e^++\nu_e.\end{aligned}\tag{44}$$

The processes: $\Sigma^-\to n+e^-+\bar\nu_e$, $\Sigma^0\to p+e^-+\bar\nu_e$, and $\Sigma^+\to n+e^++\nu_e$ can be treated in a wholly similar way.

We begin by writing the analogs of Eqs. (1)–(8) and (33),

$$\begin{aligned}\langle\bar\nu_ee^-\Sigma^0|H_{\text{weak}}(0)|\Xi^-\rangle&=\frac{G}{\sqrt2}(\bar u_e\gamma_\lambda(1+\gamma_5)u_{\nu_e}{}^*)\langle\Sigma^0|\{s_\lambda^{(V)}(0)+s_\lambda^{(A)}(0)\}^\dagger|\Xi^-\rangle,\\\langle\bar\nu_ee^-\Sigma^+|H_{\text{weak}}(0)|\Xi^0\rangle&=\frac{G}{\sqrt2}(\bar u_e\gamma_\lambda(1+\gamma_5)u_{\nu_e}{}^*)\langle\Sigma^+|\{s_\lambda^{(V)}(0)+s_\lambda^{(A)}(0)\}^\dagger|\Xi^0\rangle,\\\langle\nu_ee^+\Sigma^-|H_{\text{weak}}(0)|\Xi^0\rangle&=\frac{G}{\sqrt2}(\bar u_{\nu_e}\gamma_\lambda(1+\gamma_5)u_e{}^*)\langle\Sigma^-|s_\lambda^{(V)}(0)+s_\lambda^{(A)}(0)|\Xi^0\rangle,\end{aligned}\tag{45}$$

where

$$\begin{aligned}\langle\Sigma^0|\{s_\lambda^{(V),(A)}(0)\}^\dagger|\Xi^-\rangle&=\mp\langle\bar\Sigma^0|s_\lambda^{(V),(A)}(0)|\bar\Xi^+\rangle\\&=\mp\bar u_\Sigma\left\{F_{V,A}{}^{(0)}(q^2)\gamma_\lambda+F_{M,T}{}^{(0)}(q^2)\sigma_{\lambda\kappa}\left(\frac{q_\kappa}{m_n}\right)+F_{S,P}{}^{(0)}(q^2)\left(\frac{q_\lambda}{m_n}\right)\right\}(1,\gamma_5)u_\Xi\\&\cong\mp F_{V,A}{}^{(0)}(q^2)(\bar u_\Sigma\gamma_\lambda(1,\gamma_5)u_\Xi),\end{aligned}$$

$$\langle\Sigma^+|\{s_\lambda^{(V),(A)}(0)\}^\dagger|\Xi^0\rangle=\mp\langle\bar{\Sigma}^-|s_\lambda^{(V),(A)}(0)|\bar{\Xi}^0\rangle$$
$$=\mp\bar{u}_\Sigma\left\{F_{V,A}^{(+)}(q^2)\gamma_\lambda+F_{M,T}^{(+)}(q^2)\sigma_{\lambda\kappa}\left(\frac{q_\kappa}{m_n}\right)+F_{S,P}^{(+)}(q^2)\left(\frac{q_\lambda}{m_n}\right)\right\}(1,\gamma_5)u_\Xi$$
$$\cong\mp F_{V,A}^{(+)}(0)(\bar{u}_\Sigma\gamma_\lambda(1,\gamma_5)u_\Xi),\quad (46)$$
$$\langle\Sigma^-|s_\lambda^{(V),(A)}(0)|\Xi^0\rangle=\bar{u}_\Sigma\left\{F_{V,A}^{(-)}(q^2)\gamma_\lambda+F_{M,T}^{(-)}(q^2)\sigma_{\lambda\kappa}\left(\frac{q_\kappa}{m_n}\right)+F_{S,P}^{(-)}(q^2)\left(\frac{q_\lambda}{m_n}\right)\right\}(1,\gamma_5)u_\Xi$$
$$\cong F_{V,A}^{(-)}(q^2)(\bar{u}_\Sigma\gamma_\lambda(1,\gamma_5)u_\Xi).$$

Then since

$$|\Sigma^0\bar{\Xi}^+\rangle=(\tfrac{1}{3})^{1/2}|T=\tfrac{1}{2},T^{(3)}=\tfrac{1}{2};-p_\Sigma,p_\Xi,-s_\Sigma,s_\Xi\rangle+(\tfrac{2}{3})^{1/2}|T=\tfrac{3}{2},T^{(3)}=\tfrac{1}{2};-p_\Sigma,p_\Xi,-s_\Sigma,s_\Xi\rangle$$
$$|\Sigma^+\bar{\Xi}^0\rangle=-(\tfrac{2}{3})^{1/2}|T=\tfrac{1}{2},T^{(3)}=\tfrac{1}{2};-p_\Sigma,p_\Xi,-s_\Sigma,s_\Xi\rangle+(\tfrac{1}{3})^{1/2}|T=\tfrac{3}{2},T^{(3)}=\tfrac{1}{2};-p_\Sigma,p_\Xi,-s_\Sigma,s_\Xi\rangle\quad (47)$$
$$|\bar{\Sigma}^+\Xi^0\rangle=|T=\tfrac{3}{2},T^{(3)}=\tfrac{3}{2};-p_\Sigma,p_\Xi,-s_\Sigma,s_\Xi\rangle,$$

we have ($x=V,A,M,T,S,P$)

$$F_x^{(0)}(q^2)=(\tfrac{1}{3})^{1/2}f_{x;1/2,1/2}(q^2)+(\tfrac{2}{3})^{1/2}f_{x;3/2,1/2}(q^2),$$
$$F_x^{(+)}(q^2)=-(\tfrac{2}{3})^{1/2}f_{x;1,2,1/2}(q^2)+(\tfrac{1}{3})^{1/2}f_{x;3/2,1/2}(q^2),\quad (48)$$
$$F_x^{(-)}(q^2)=f_{x;3/2,3/2}(q^2),$$

so that

$$\frac{\Gamma(\Xi^-\to\Sigma^0+e^-+\bar{\nu}_e)}{\Gamma(\Xi^0\to\Sigma^++e^-+\bar{\nu}_e)}\cong\frac{(F_V^{(0)}(0))^2+3(F_A^{(0)}(0))^2}{(F_V^{(+)}(0))^2+3(F_A^{(+)}(0))^2}$$
$$=\frac{(f_{V;1/2,1/2}(0)+\sqrt{2}f_{V;3/2,1/2}(0))^2+3(f_{A;1/2,1/2}(0)+\sqrt{2}f_{A;3/2,1/2}(0))^2}{(-\sqrt{2}f_{V;1/2,1/2}(0)+f_{V;3/2,1/2}(0))^2+3(-\sqrt{2}f_{A;1/2,1/2}(0)+f_{A;3/2,1/2}(0))^2},$$
$$\frac{\Gamma(\Xi^0\to\Sigma^-+e^++\nu_e)}{\Gamma(\Xi^0\to\Sigma^++e^-+\bar{\nu}_e)}\cong\frac{(F_V^{(-)}(0))^2+3(F_A^{(-)}(0))^2}{(F_V^{(+)}(0))^2+3(F_A^{(+)}(0))^2}\quad (49)$$
$$=\frac{(f_{V;3/2,3/2}(0))^2+3(f_{A;3/2,3/2}(0))^2}{(-\sqrt{2}f_{V;1/2,1/2}(0)+f_{V;3/2,1/2}(0))^2+3(-\sqrt{2}f_{A;1/2,1/2}(0)+f_{A;3/2,1/2}(0))^2},$$

with $(F_A^{(0)}(0)/F_V^{(0)}(0))$, $(F_A^{(+)}(0)/F_V^{(+)}(0))$, $(F_A^{(-)}(0)/F_A^{(-)}(0))$ determinable, for example, from a study of the Σ energy spectra in $\Xi^-\to\Sigma^0+e^-+\nu_e$, $\Xi^0\to\Sigma^++e^-+\nu_e$, $\Xi^0\to\Sigma^-+e^++\nu_e$.

We proceed to consider the relationship among the various $f_{x;T;T^{(3)}}$. By analogy with Eqs. (38)–(42) we expect violation of both the $\Delta T=\frac{1}{2}$ rule and the $\Delta S=\Delta Q$ rule, i.e., we expect

$$f_{x;3/2,1/2}(0)/f_{x;1/2,1/2}(0)\neq0,\quad f_{x;3/2,3/2}(0)/f_{x;1/2,1/2}(0)\neq0.\quad (50)$$

We also anticipate, again on the basis of analogy with Eqs. (38)–(42), that the value of

$$\langle\text{vac}|s_\lambda(0)|T=\tfrac{3}{2},T^{(3)};-p_\Sigma,p_\Xi,-s_\Sigma,s_\Xi\rangle$$

is different for different $T^{(3)}$, i.e., we anticipate

$$f_{x;3/2,3/2}(0)/f_{x;3/2,1/2}(0)\neq1.\quad (51)$$

VI. TIME-REVERSAL INVARIANCE?

We have so far assumed that all our form factors are relatively real for momentum transfers in the physical region. Therefore [see Eqs. (7), (8), (33), and (41)],

$$\alpha_\pm\equiv f_{\pm;3/2,3/2}(0)/f_{\pm;1/2,1/2}(0);\quad \xi\equiv f_-(0)/f_+(0);$$
$$\xi=\xi_{1/2}\frac{1-\sqrt{2}\alpha_-}{1-\sqrt{2}\alpha_+};\quad \xi_{1/2}\equiv f_{-;1/2,1/2}(0)/f_{+;1/2,1/2}(0);\quad (52)$$
$$x_\pm\equiv\frac{f_{\pm;3/2,3/2}(0)}{(\tfrac{2}{3})^{1/2}f_{\pm;1/2,1/2}(0)+(\tfrac{1}{3})^{1/2}f_{\pm;3/2,1/2}(0)}=\left(\frac{3}{2}\right)^{1/2}\left(\frac{f_{\pm;3/2,3/2}(0)}{f_{\pm;3/2,1/2}(0)}\right)\frac{\alpha_\pm}{1+(\tfrac{1}{2})^{1/2}\alpha_\pm};$$

are all real. If, however, $\{s_\lambda\}_{\Delta T=1/2,\Delta T^{(3)}}$ and $\{s_\lambda\}_{\Delta T=3/2,\Delta T^{(3)}}$ are "out of phase" in the sense that the strangeness-changing

$$H_{\text{weak}}=\left[\frac{G}{\sqrt{2}}l_\lambda s_\lambda\right]+\text{H.c.}=\frac{G}{\sqrt{2}}l_\lambda\sum_{\Delta T^{(3)}=1/2}\{s_\lambda\}_{1/2,\Delta T^{(3)}}+\frac{G}{\sqrt{2}}l_\lambda\sum_{\Delta T^{(3)}=1/2,3/2}\{s_\lambda\}_{3/2,\Delta T^{(3)}}+\text{H.c.}=H_{\text{weak};\,1/2}+H_{\text{weak};\,3/2}$$

is not invariant under time reversal[14] (e.g., $H_{\text{weak};1/2}\to H_{\text{weak};1/2}$, $H_{\text{weak};3/2}\to -H_{\text{weak};3/2}$ under time reversal) the $f_{\pm;3/2,\Delta T^{(3)}}(q^2)$ and $f_{\pm;1/2,\Delta T^{(3)}}(q^2)$ form factors need no longer be relatively real (even for q^2 in the physical region) and we would in general expect $\alpha_\pm\neq\alpha_\pm^*$, $x_\pm\neq x_\pm^*$, $\xi\neq\xi^*$ (but $\xi_{1/2}=\xi_{1/2}^*$). Equations (9) and (35) are then modified to read[15]

$$B\equiv\frac{\Gamma(K^+\to\pi^0+\mu^++\nu_\mu)}{\Gamma(K^+\to\pi^0+e^++\nu_e)}=0.65+0.13(\text{Re}\xi)+0.019[(\text{Re}\xi)^2+(\text{Im}\xi)^2]$$
$$R\equiv\frac{\Gamma(K^+\to\pi^0+e^++\nu_e)}{\Gamma(K_2{}^0\to\pi^-+e^++\nu_e)}=\left|\frac{1-\sqrt{2}\alpha_+}{1+(\frac{1}{2})^{1/2}\alpha_+}\frac{1}{1-x_+}\right|^2. \tag{53}$$

Thus, using the value of B_{exp} in Eq. (10), we find that the allowed values of Reξ and Imξ lie between two concentric circles centered at Re$\xi=-3.4$, Im$\xi=0$, of inner radius 4.5 and outer radius 6.0. The two radii correspond to the 15% uncertainty in B_{exp}.

We proceed to discuss the nature of the muon energy spectrum in $K^+\to\pi^0+\mu^++\nu_\mu$ if Im$\xi\neq0$. On the basis of Eqs. (1)–(3), and (8), and with the approximation $f_\pm(q^2)\cong f_\pm(0)$ (i.e., $\lambda_\pm\cong0$), this spectrum is calculated to be

$$\frac{d\Gamma(E_\mu)}{dE_\mu}=\frac{(G^2/2)|f_+(0)|^2g(E_\mu)}{2m_K(2\pi)^3}\{a(E_\mu)+2b(E_\mu)\,\text{Re}\xi+c(E_\mu)[(\text{Re}\xi)^2+(\text{Im}\xi)^2]\}, \tag{54}$$

where

$$g(E_\mu)=\frac{m_\mu{}^2}{4}(E_\mu{}^2-m_\mu{}^2)^{1/2}\left(\frac{[E_\mu]_{\max}-E_\mu}{[E_\mu]_{\max}-E_\mu+m_\pi{}^2/2m_K}\right)^2,$$
$$[E_\mu]_{\max}=(m_K{}^2+m_\mu{}^2-m_\pi{}^2)/2m_K,$$
$$a(E_\mu)=m_KE_\mu-m_\mu{}^2+\frac{8m_K{}^2}{m_\mu{}^2}E_\mu\left([E_\mu]_{\max}-E_\mu+\frac{m_\pi{}^2}{2m_K}\right), \tag{55}$$
$$b(E_\mu)=2m_K([E_\mu]_{\max}-E_\mu)+m_K{}^2-m_KE_\mu+m_\pi{}^2,$$
$$c(E_\mu)=m_KE_\mu-m_\mu{}^2,$$

and reduces to that given by Brene *et al.*[16] for Im$\xi=0$. Of course, a χ^2 fit to Eqs. (54) and (55) of any experimental muon energy spectrum will have to be carried out in order to extract the "best" values of Reξ and Imξ but the following qualitative remarks can already be made.

The functions $c(E_\mu)$ and $b(E_\mu)$ increase and decrease, respectively, with increasing E_μ while $a(E_\mu)$ has a maximum at $E_\mu=\frac{1}{2}([E_\mu]_{\max}+m_\pi{}^2/2m_K)=\frac{1}{4}[(m_K{}^2+m_\mu{}^2)/m_K]=129$ MeV. The function $g(E_\mu)$ vanishes at $E_\mu=m_\mu$, $E_\mu=[E_\mu]_{\max}$, and has a peak at an intermediate value of E_μ. The function $g(E_\mu)c(E_\mu)$ has its maximum for $E_\mu=200$ MeV, close to the value of E_μ for which the experimental muon energy spectrum of Dobbs *et al.*[8] (well described by Im$\xi=0$, Re$\xi\cong-9$) has its peak. Thus, a nonvanishing Imξ tends to enhance the high-energy portion of the muon spectrum relative to the low-energy portion. As a particular example let us take $B=1$, Im$\xi=5$ so that Re$\xi=-1.2$ or -5.7 [Eq. (53)]. Then, with Re$\xi=-1.2$, Im$\xi=5$,

$$\{d\Gamma(E_\mu)/dE_\mu\}_{E_\mu=200\text{ MeV}}/\{d\Gamma(E_\mu)/dE_\mu\}_{E_\mu=140\text{ MeV}}=1.3.$$

[14] Such a possibility has been considered in connection with the interpretation of $K^0,\bar{K}^0$ interference phenomena by R. G. Sachs and S. B. Treiman, Phys. Rev. Letters **8**, 137 (1962).
[15] For simplicity we keep in the present discussion only those terms in B which are independent of the $\lambda_\pm$.
[16] N. Brene, L. Egardt, and B. Qvist, Nucl. Phys. **22**, 553 (1961).

On the other hand, for $B=1$, $\mathrm{Im}\xi=0$, we have $\mathrm{Re}\xi=\xi=2$ or -9,

$$\{d\Gamma(E_\mu)/dE_\mu\}_{E_\mu=200\ \mathrm{MeV}}/\{d\Gamma(E_\mu)/dE_\mu\}_{E_\mu=140\ \mathrm{MeV}}=0.90, \quad \text{for} \quad \xi=2$$
$$\{d\Gamma(E_\mu)/dE_\mu\}_{E_\mu=200\ \mathrm{MeV}}/\{d\Gamma(E_\mu)/dE_\mu\}_{E_\mu=140\ \mathrm{MeV}}=2.9, \quad \text{for} \quad \xi=-9$$

which indicates a moderate enhancement. It is, therefore, not out of the question that the apparent contradiction discussed above between the data of Brown *et al.*[7] and Dobbs *et al.*[8] can be removed by use of Eqs. (53)–(55) with $\mathrm{Im}\xi\neq0$; more precise experiments on the muon energy spectrum than are now available would however be required for a detailed numerical analysis. It is also worth emphasizing that with Eq. (52) for ξ and with $\alpha_\pm=\alpha_\pm^*$ one may use Eq. (23) [assumption (I)] or Eq. (31) [assumption (I′)] to give $\xi_{1/2}(=\xi_{1/2}^*)\lesssim1$. In fact, even the result of Eqs. (19)–(23), and (27) for $\xi_{1/2}$, viz.: $\xi_{1/2}=-(m_K^2-m_\pi^2)/m_*^2=-0.29$ [which, it will be recalled, follows from the assumption that the K^* pole term alone dominates $f_{\pm;1/2,1/2}(q^2)$] becomes acceptable since the four parameters $\{\alpha_+\}_{\mathrm{Re}}$, $\{\alpha_+\}_{\mathrm{Im}}$, $\{\alpha_-\}_{\mathrm{Re}}$, $\{\alpha_-\}_{\mathrm{Im}}$ are now available to fit any observed values of B, R, $d\Gamma(E_\mu)/dE_\mu$, and $x_\pm$ [Eqs. (52)–(55)].

The most direct method of determination of any nonvanishing $\mathrm{Im}\xi$, and so the most direct test of any violation of time-reversal invariance by our $H_{\mathrm{weak}}=H_{\mathrm{weak};1/2}+H_{\mathrm{weak};3/2}$, involves a measurement of the transverse polarization of the muon in $K^+\to\pi^0+\mu^++\nu_\mu$ in a direction perpendicular to the plane determined by the momenta of the π^0 and μ^+. The average value of this muon transverse polarization for K^+ decay at rest is given by $(f_\pm(q^2)\cong f_\pm(0))$

$$\langle\gamma_4\boldsymbol{\sigma}\cdot\hat{n}\rangle=\mathrm{Tr}(\rho\gamma_4\boldsymbol{\sigma}\cdot\hat{n})/\mathrm{Tr}(\rho)\,, \tag{56}$$

with

$$\begin{aligned}(4|f_+(0)|^2)^{-1}\,\mathrm{Tr}(\rho\gamma_4\boldsymbol{\sigma}\cdot\hat{n})&=2m_\mu m_K(|\mathbf{p}_\pi\times\mathbf{p}_\mu|/E_\mu p_\nu)(\mathrm{Im}\xi)\\ (4|f_+(0)|^2)^{-1}\,\mathrm{Tr}(\rho)&=2m_K^2[1+(p_\mu/E_\mu)\hat{p}_\mu\cdot\hat{p}_\nu]-(2m_\mu^2m_K/E_\mu)(1-\mathrm{Re}\xi)\\ &\quad+\tfrac{1}{2}m_\mu^2[1-(p_\mu/E_\mu)\hat{p}_\mu\cdot\hat{p}_\nu][1-2\,\mathrm{Re}\xi+(\mathrm{Re}\xi)^2+(\mathrm{Im}\xi)^2],\end{aligned} \tag{57}$$

where

$$\boldsymbol{\sigma}\equiv(2i)^{-1}(\boldsymbol{\gamma}\times\boldsymbol{\gamma})\,;\quad \hat{n}\equiv\frac{\mathbf{p}_\pi\times\mathbf{p}_\mu}{|\mathbf{p}_\pi\times\mathbf{p}_\mu|}\,;\quad \mathbf{p}_\nu=-(\mathbf{p}_\pi+\mathbf{p}_\mu)\,,$$

$$\rho=\tilde{M}(1+\tilde{\gamma}_5)\left(\frac{-\tilde{\gamma}_5\tilde{\boldsymbol{\sigma}}\cdot\hat{p}_\nu+1}{2}\right)(1+\tilde{\gamma}_5)\tilde{M}^\dagger\left(\frac{-\gamma_5\boldsymbol{\sigma}\cdot\mathbf{p}_\mu/E_\mu+\gamma_4m_\mu/E_\mu+1}{2}\right), \tag{58}$$

$$M\equiv m_\mu[f_+(0)-f_-(0)]\gamma_4+2m_Kf_+(0)\,,$$

so that a measurement of the three quantities: $\langle\gamma_4\boldsymbol{\sigma}\cdot\hat{n}\rangle$, B [Eq. (53)], and $d\Gamma(E_\mu)/dE_\mu$ [Eqs. (54) and (55)] overdetermines the two parameters $\mathrm{Re}\xi$ and $\mathrm{Im}\xi$.

Finally, we may remark that Boyarski *et al.*[8] have reported an experimental muon energy spectrum $d\Gamma(E_\mu)/dE_\mu$ which is consistent with $-27\leqq\mathrm{Re}\xi=\xi\leqq-7.6$, but which is associated with an experimental muon longitudinal polarization that corresponds (within a rather large uncertainty) to $|\mathrm{Re}\xi|=|\xi|\lesssim2$. This apparent inconsistency may conceivably be another manifestation of the fact that $\mathrm{Im}\xi\neq0$.[17]

VII. CONCLUSIONS

It is hoped that the above discussion makes clear the extent and depth of the physical information which could be extracted regarding the strangeness-changing weak interactions if precise data were available on K meson decays. Accurate measurements of muon and electron transition rates, energy spectra, and longitudinal and transverse polarizations are urgently needed.

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[17] We record the expression for the average value of the muon longitudinal polarization for K^+ decay at rest: $\langle\boldsymbol{\sigma}\cdot\hat{p}_\mu\rangle=\mathrm{Tr}(\rho\boldsymbol{\sigma}\cdot\hat{p}_\mu)/\mathrm{Tr}(\rho)$ with ρ, $\mathrm{Tr}(\rho)$ given in Eqs. (58), (57), and with $(4|f_+(0)|^2)^{-1}\,\mathrm{Tr}(\rho\boldsymbol{\sigma}\cdot\hat{p}_\mu)=2m_K^2[(p_\mu/E_\mu)+\hat{p}_\mu\cdot\hat{p}_\nu)-(2m_\mu^2m_K/E_\mu)\hat{p}_\mu\cdot\hat{p}_\nu(1-\mathrm{Re}\xi)-\frac{1}{2}m_\mu^2\times[(p_\mu/E_\mu)-\hat{p}_\mu\cdot\hat{p}_\nu][1-2\,\mathrm{Re}\xi+(\mathrm{Re}\xi)^2+(\mathrm{Im}\xi)^2]$.