Approaching threshold from above,

$$\frac{d^2\sigma_{el}}{dE^2} \to \left(\frac{d\sigma_{el}}{d(\mathrm{Im}\Theta)}\right)_{E=E_{thr}} \left(\frac{d^2(\mathrm{Im}\Theta)}{dE^2}\right)_{E-E_{thr}=0^+}$$

$$= \frac{6\pi\lambda^3}{k^2} \frac{\mathrm{Im}(AB^*)}{|B-A|^2} (\mathrm{Im}A - \mathrm{Im}B)$$

$$\times (\mathrm{Re}B - \mathrm{Re}A)(E-E_{thr})^{-1/2}. \quad \text{(A16)}$$

Thus, the second derivative of the elastic cross section has an infinity at threshold when a p wave is produced.[25,26] The infinity may change sign if $(\mathrm{Re}B-\mathrm{Re}A)$ has a different sign from $(\mathrm{Im}A-\mathrm{Im}B)$. By looking at the curves in Fig. 8 or Fig. 10 we can see that this is what happened in our case. Of course, since the phase shift is a continuous function of the cross section, the same type of discontinuity will occur in a plot of α versus k as in σ versus E.

With an s wave in the outgoing channel, the usual type of cusp is found (first derivative infinite).[24] In general, if a channel with orbital angular momentum l is opened, the $(l+1)$st derivative will have a discontinuity at threshold, with no Coulomb forces present.[25,26]

It should be noticed that the above treatment is perfectly general, and can be applied to wave functions with any desired unphysical cut, insofar as an energy-dependent $\mathfrak{f}$ matrix, not singular at threshold, was permitted.

In some of our cases the produced particle is unstable. In these instances, "wooly" cusps[27] are obtained with properties which have been discussed in Ref. 27 The coupling scheme used in our formulation [Eq. (7a)] is consistent with that in the Nauenberg and Pais paper and, of course, is unitary.

[27] M. Nauenberg and A. Pais, Phys. Rev. **126**, 360 (1962).

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Modified K^* Exchange Model for ΛK^0 Production*

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A model is proposed to explain simultaneously the backward peaking of the Λ particles in the reaction $\pi^-+p\to\Lambda+K^0$, the large polarization observed, and the peak of the total cross section. The K^* exchange diagram and a resonant state in our channel are considered as the main contributions to the amplitudes. By assuming a resonant $p_{1/2}$ state, excellent fits to the angular distribution and polarization are obtained at a pion kinetic energy of 871 MeV and at an incident pion momentum of 1.01–1.05 BeV/c. A fairly good fit is obtained at a pion kinetic energy of 791 MeV. A new estimate for the $K^*\Lambda N$ coupling constant is given.

INTRODUCTION

It is well known that the model of a K^* exchange proposed by Tiomno *et al.*[1] to explain the backward peaking of the Λ's produced in the reaction

$$\pi^-+p\to\Lambda+K^0 \quad (1)$$

is incomplete, because it accounts neither for the observed large polarization of the Λ's nor for the peak in the total cross section at around an incident pion momentum of 1.03 BeV/c.

MacDowell *et al.*[2] have made fits to the angular distribution at pion kinetic energies of 960 and 1300 MeV by adding to the scheme a complex s wave. They obtained a satisfactory value for the average polarization only at the higher energy and needed two different products of coupling constants differing by a factor of 5 to obtain good fits to the angular distribution. Also, their work is incomplete in the sense that they did not attempt to fit the polarization dependence with angle.

In the present paper we propose a modification to the Tiomno scheme by adding a resonant partial wave. This model gives excellent fits to both the angular distribution and the polarization over a wide range of energy if we assume that the resonance is $p_{1/2}$. It gives also a fairly good fit to the energy dependence of the total cross section.

The idea of a $p_{1/2}$ resonance is not new. A $p_{1/2}$ or $p_{3/2}$ resonance was suggested by Kanazawa[3] in order to explain the peak in the total cross section. He ignored though the K^* exchange diagram, probably because at that time this particle was hypothetical, and considered instead the one-nucleon term and the Σ exchange term. In this paper we do exactly the opposite. We have a

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[1] J. Tiomno, A. L. L. Videira, and N. Zagury, Phys. Rev. Letters **6**, 120 (1961).

[2] S. W. MacDowell, A. L. L. Videira, and N. Zagury, Nucl. Phys. **31**, 636 (1962).

[3] Akira Kanazawa, Phys. Rev. **123**, 993 (1961).

preference for the K^* pole because it accounts for the backward peaking of the Λ's in a simple and less artificial way.

In our model there are four parameters. Three of them are the position, width, and height of the resonance. The fourth one is the product of the $K^*K\pi$ and $K^*\Lambda N$ coupling constants. From this product and the observed width of the K^* decay we have obtained a new estimate of the $K^*\Lambda N$ coupling.

KINEMATICS

Let us denote the four-momentum and the mass of the proton (Λ particle) by p (p') and m (m'), and those of the π^- (K^0) by k (k') and μ (μ'). The invariant Feynman amplitude for the reaction (1) may be written, in general,

$$F_{sr}=\bar{u}_{\Lambda s}(p')[A-\tfrac{1}{2}i\gamma_\mu(k+k')_\mu B]u_{pr}(p)\,, \qquad (2)$$

where u_p and u_Λ are the Dirac spinors of the proton and the Λ particle, respectively, and A and B are functions of the total energy W and the cosine of the angle θ between $\mathbf{k}$ and $\mathbf{k}'$ in the center-of-mass system.

The production amplitude T_{sr} is related to F_{sr} by

$$T_{sr}=\frac{(mm')^{1/2}}{4\pi}\frac{1}{W}\left(\frac{|\mathbf{k}'|}{|\mathbf{k}|}\right)^{1/2}F_{sr}. \qquad (3)$$

In terms of T_{sr} the differential cross section is given simply by

$$(d\sigma/d\Omega)_{sr}=|T_{sr}|^2. \qquad (4)$$

We can express T in terms of Pauli spinors and matrices

$$T_{sr}=\chi_s^\dagger[g+h(\boldsymbol{\sigma}\cdot\hat{p}')(\boldsymbol{\sigma}\cdot\hat{p})]\chi_r\,, \qquad (5)$$

where

$$g=\frac{1}{8\pi}\left(\frac{|\mathbf{k}'|}{|\mathbf{k}|}\right)^{1/2}\frac{[(E+m)(E'+m')]^{1/2}}{W}\times[A+(W-\tfrac{1}{2}(m+m'))B]\,,$$

$$h=\frac{1}{8\pi}\left(\frac{|\mathbf{k}'|}{|\mathbf{k}|}\right)^{1/2}\frac{[(E-m)(E'-m')]^{1/2}}{W}\times[-A+(W+\tfrac{1}{2}(m+m'))B].$$

Here E (E') denotes the energy of the proton (Λ particle).

The amplitudes g and h have the following expansions:

$$g=\sum_{l=1}^{\infty}(f^+{}_{(l-1)}-f^-{}_{(l+1)})P_l'(x)$$
$$h=\sum_{l=1}^{\infty}(f_l^- - f_l^+)P_l'(x)\,, \qquad (6)$$

where $f_l^\pm$ is the partial wave amplitude for orbital angular momentum l and total angular momentum $j=l\pm\frac{1}{2}$, $x=\cos\theta$, and $P_l'(x)$ is the derivative of the Legendre polynomial $P_l(x)$.

The M matrix defined by

$$T_{sr}=\chi_s^\dagger M\chi_r \qquad (7)$$

can then be written

$$M=a+i(\boldsymbol{\sigma}\cdot\hat{n})b\sin\theta\,, \qquad (8)$$

where

$$a=g+h\cos\theta\,,$$

and

$$b=h\,,$$
$$\hat{n}=\hat{p}'\times\hat{p}/|\hat{p}'\times\hat{p}|\,.$$

If the initial proton is unpolarized the differential cross section and the polarization of the Λ's in the $\hat{n}$ direction are given in terms of a and b by

$$d\sigma/d\Omega=\tfrac{1}{2}\mathrm{Tr}(M^\dagger M)=|a|^2+|b|^2\sin^2\theta \qquad (9)$$

$$P(d\sigma/d\Omega)=\tfrac{1}{2}\mathrm{Tr}(M^\dagger\boldsymbol{\sigma}\cdot\hat{n}M)=2\mathrm{Im}(ab^*)\sin\theta. \qquad (10)$$

The expressions for a and b for the lowest partial wave amplitudes are given in Table I.

DYNAMICS

We assume that the main contributions to a and b come from the pole term corresponding to the K^* exchange (Fig. 1), and a resonant state in the ΛK^0 system. The bases for these assumptions are:

(1) The K^* exchange term accounts in a very natural way for the backward peaking of the Λ's observed in all the energy region covered up to now by experiments.
(2) In order to obtain a large polarization we need an amplitude with an imaginary part to interfere with the K^* pole term.
(3) In order to obtain a peak in the total cross section we need to add a term with this kind of behavior to the K^* exchange term.

We proceed now to calculate these contributions.

Let us denote the mass of the K^* by M and its four-momentum by q. We calculate the contribution of the graph of Fig. 1 to the Feynman amplitude using the propagator of a stable vector particle

$$\frac{\delta_{\mu\nu}+(1/M^2)q_\mu q_\nu}{q^2+M^2}\,,$$

since the spread of mass of the K^* is not too big.

The amplitude of the ($K^0K^{*+}\pi^-$) vertex is taken as

$$\sqrt{2}f(k+k')_\mu\epsilon_\mu\,,$$

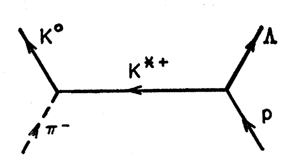

FIG. 1. K^* exchange diagram.

TABLE I. Amplitudes and angular distributions corresponding to the lower angular momentum partial waves ($x \equiv \cos\theta$).

Partial wave	a	b	$\|a\|^2+\|b\|^2\sin^2\theta$
$s_{1/2}$	f_0^+	0	$\|f_0^+\|^2$
$p_{1/2}$	$f_1^- x$	f_1^-	$\|f_1^-\|^2$
$p_{3/2}$	$2f_1^+ x$	$-f_1^+$	$\|f_1^+\|^2(3x^2+1)$
$d_{3/2}$	$f_2^-(3x^2-1)$	$3f_2^- x$	$\|f_2^-\|^2(3x^2+1)$
$d_{5/2}$	$\frac{3}{2}f_2^+(3x^2-1)$	$-3f_2^+ x$	$(9/4)\|f_2^+\|(5x^4-2x^2+1)$
$f_{5/2}$	$\frac{3}{2}f_3^-(5x^3-3x)$	$\frac{3}{2}f_3^-(5x^2-1)$	$(9/4)\|f_3^-\|^2(5x^4-2x^2+1)$
$f_{7/2}$	$2f_3^+(5x^3-3x)$	$-\frac{3}{2}f_3^+(5x^2-1)$	$\frac{1}{4}\|f_3^+\|^2(175x^6-165x^4+45x^2+9)$

where $\sqrt{2}$ is the isospin factor corresponding to a charged pion, and that of the $(K^*\Lambda N)$ vertex as

$$\bar{u}_\Lambda(ig\gamma_\nu\epsilon_\nu')u_p.$$

Terms proportional to $(k'-k)_\mu\epsilon_\mu$ and $(p'-p)_\nu\epsilon'_\nu$ in the first and second vertex, respectively, and the anomalous moment term in the second one have been neglected. This approximation seems to be justified later by the results we obtain.[4]

We obtain for the Feynman amplitude

$$F=\sqrt{2}fg\frac{1}{q^2+M^2}\left[i(k+k')_\mu\gamma_\mu-\frac{(\mu'^2-\mu^2)(m'-m)}{M^2}\right]. \quad (11)$$

This gives for the K^*-pole contribution to g and k

$$\begin{aligned} g_p&=-(E+m)^{1/2}(E'+m')^{1/2} \\ &\quad\times\left[2W-(m+m')+\frac{(\mu'^2-\mu^2)(m'-m)}{M^2}\right]C, \\ h_p&=-(E-m)^{1/2}(E'-m')^{1/2} \qquad (12) \\ &\quad\times\left[2W+(m+m')-\frac{(\mu'^2-\mu^2)(m'-m)}{M^2}\right]C, \end{aligned}$$

where

$$C=\frac{\sqrt{2}}{4}\left(\frac{fg}{4\pi}\right)\frac{1}{W|\mathbf{k}|}\frac{1}{(\beta-\cos\theta)(|\mathbf{k}||\mathbf{k}'|)^{1/2}},$$

$$\beta=\frac{2k_0k_0'+M^2-\mu^2-\mu'^2}{2|\mathbf{k}||\mathbf{k}'|}.$$

The contributions to a and b (a_p,b_p) can be obtained immediately by means of the second and third of formulas (8).

We approximate the contribution from the resonance by expressing the corresponding partial wave amplitude by means of the Breit-Wigner formula

$$f_l^{\pm}=\frac{1}{2|\mathbf{k}|}\frac{(\Gamma_1\Gamma_2)^{1/2}}{W_r-W-i\Gamma/2}, \quad (13)$$

where Γ is the full width of the resonance and Γ_1 and Γ_2 are the partial widths corresponding to the decays into the entrance and final channels.

The expressions for a and b (a_r,b_r) for different assumed angular momentum and parity can then be written with the aid of Table I or formulas (6).

DISCUSSION AND NUMERICAL RESULTS

The angular distribution and polarization are given in our model by

$$\begin{aligned} d\sigma/d\Omega=&\,g_p^2+h_p^2+2g_ph_p\cos\theta+|a_r|^2+|b_r|^2\sin^2\theta \\ &+2(g_p+h_p\cos\theta)\mathrm{Re}(a_r) \\ &+2h_p\mathrm{Re}(b_r)\sin^2\theta, \quad (14) \end{aligned}$$

$$\begin{aligned} P(d\sigma/d\Omega)=&-2(g_p+h_p\cos\theta)\mathrm{Im}(b_r)\sin\theta \\ &+2h_p\mathrm{Im}(a_r)\sin\theta. \quad (15) \end{aligned}$$

In the region of interest g_p/h_p is of the order of 10. This allows us to simplify our formulas by neglecting h_p, which will make the discussion easier.

$$d\sigma/d\Omega\approx g_p^2+|a_r|^2+|b_r|^2\sin^2\theta+2g_p\mathrm{Re}(a_r), \quad (16)$$

$$P(d\sigma/d\Omega)\approx-2g_p\mathrm{Im}(b_r)\sin\theta. \quad (17)$$

Later, for the numerical calculations we will use the more exact formulas (14) and (15).

In the region of the peak of the total cross section the polarization is large and negative and its angular dependence can be reproduced very well by assuming that

$$P=-\frac{C\sin\theta/(\beta-\cos\theta)}{d\sigma/d\Omega},$$

where C is a positive constant.

This suggests that our assumed resonance should be either $p_{1/2}$ or $p_{3/2}$ since higher partial waves will give a C depending on the angle and $s_{1/2}$ will give $P\approx 0$.

By adjusting properly the position of the resonance W_r, the product of the partial widths $\Gamma_1\Gamma_2$, the full width Γ, and the product of coupling constants $fg/4\pi$

[4] The terms proportional to $(k'-k)_\mu\epsilon_\mu$ and $(p'-p)_\nu\epsilon_\nu'$ have been included by some authors in order to have a "conserved current." See, for example, Ref. 14. The hypothesis of "exact conservation of current" leads, however, to a theoretical difficulty as pointed out by Capps who uses an approach similar to ours. [Richard H. Capps (to be published).] More recently Nambu and Sakurai [Yoichiro Nambu and J. J. Sakurai (to be published)] have made an interesting speculation in relation to this problem.

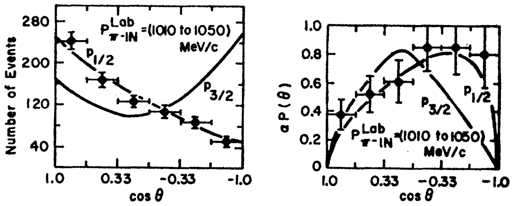

FIG. 2. (a) Angular distribution in the interval 1.01–1.05 BeV/c of incident pion momentum (Ref. 5), (b) $\alpha P(\theta)$ in the same interval (Ref. 5). The curves are the fits at $P_\pi=1.03$ BeV/c obtained by assuming $p_{1/2}$ and $p_{3/2}$ resonances in our model and assuming an asymmetry parameter $\alpha=-0.85$. [The most recent experimental value is $\alpha=-0.62\pm0.07$. See James W. Cronin and Oliver E. Overseth, Phys. Rev. **129**, 1795 (1963).] The fits to the angular distribution have been normalized to the experimental area. (These data have changed somewhat recently. See Ref. 11).

we have been able to obtain good fits to the angular distribution and polarization at incident pion momenta of[5] 1.01–1.05 and[6] 1.05 BeV/c, and incident pion kinetic energies of[7] 871 and 791 MeV (Figs. 2–5) in the $p_{1/2}$ case. We have also obtained a fairly good fit to the total cross section (Fig. 6). We have assumed an asymmetry parameter $\alpha=-0.85$ which is more consistent with our experimental data than the value $\alpha=-0.67_{-0.24}^{+0.18}$ obtained by Beall *et al.*[8]

Assuming a $p_{3/2}$ resonant partial wave and fixing the parameters so as to obtain an acceptable shape for the total cross section (Fig. 6) we have obtained the results

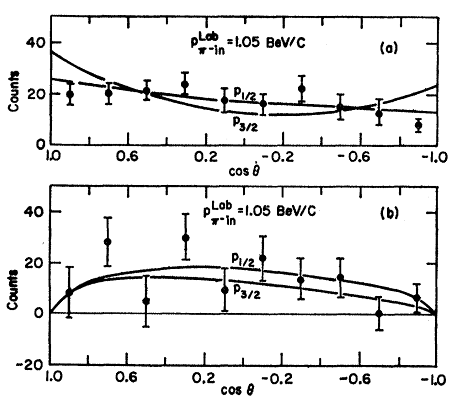

FIG. 3. (a) Angular distribution at an incident pion momentum of 1.05 BeV/c (Ref. 6), (b) $\alpha P(\theta)(d\sigma/d\Omega)$ at the same energy (Ref. 6). The curves are the fits obtained at $P_\pi=1.06$ BeV/c (assumed position of the resonance) by assuming $p_{1/2}$ and $p_{3/2}$ resonant states in our model.

[5] Experimental data from M. H. Alston, J. A. Anderson, P. G. Burke, D. D. Carmony, F. S. Crawford, N. Schmitz, and S. E. Wolf, in *Proceedings of the Tenth Annual International Conference on High-Energy Physics at Rochester, 1960*, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 378.

[6] Experimental data from Sanford E. Wolf, Norbert Schmitz, Lester J. Lloyd, William Laskar, Frank S. Crawford, Jr., Janice Button, Jared A. Anderson, and Gideon Alexander, Rev. Mod. Phys. **33**, 439 (1961).

[7] L. Bertanza, P. L. Connolly, B. B. Culwick, F. R. Eisler, T. Morris, R. Palmer, A. Prodell, and N. P. Samios, Phys. Rev. Letters **8**, 332 (1962).

[8] E. F. Beall, Bruce Cook, D. Keife, P. G. Murphy, and W. A. Wenzel, Phys. Rev. Letters **8**, 75 (1962).

shown in Figs. 2–4. It can be seen that the fit at resonance is poorer than that of the $p_{1/2}$ (due to the presence of the $1+3\cos^2\theta$ term) and below resonance much worse.

The assumed values of the parameters in both cases are given in Table II.

TABLE II. Values of the parameters assumed in our fits.

Resonant state	W_r (MeV)	Γ (MeV)	$(\Gamma_1\Gamma_2)^{1/2}$	$fg/4\pi$
$p_{1/2}$	1704	64	$\pm0.164\Gamma$	∓0.34
$p_{3/2}$	1704	64	$\pm0.116\Gamma$	±0.34

The experimental data at higher energies (above resonance) are older and, in general, less accurate. The most reliable one is the one obtained by Crawford *et al.*[9,10] at a pion momentum of 1.12 BeV/c (Fig. 7).

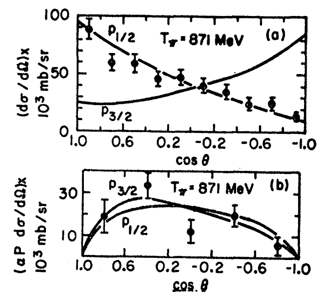

FIG. 4. (a) Angular distribution at a pion kinetic energy of 871 MeV (Ref. 7), (b) $\alpha P(d\sigma/d\Omega)$ at the same energy (Ref. 7). The curves are the results obtained at this energy by assuming $p_{1/2}$ and $p_{3/2}$ resonant states in our model.

Here the situation changes, the $p_{3/2}$ giving a much better fit to the angular distribution than the $p_{1/2}$ but still giving a poorer fit to the polarization. It should be remarked though that with the $p_{1/2}$ the lowest forward to backward ratio is obtained practically at $W=W_r+\Gamma/2$ which falls precisely at $P_\pi=1.12$ BeV/c for the assumed values of our parameters. Any change in the values of W_r or Γ will bring an increase of this ratio at this particular energy our fit being then somewhat improved. It could also happen that some other effect is present at this energy (maybe hinted by the fact that there is a change of sign of the polarization for backward angles). More experimental data are needed before a better study can be made of this region above the peak.

The reason why the two partial waves behave in an opposite way above and below resonance is the following. In order to obtain a negative polarization, $g_p\text{Im}(b_r)$ should be a positive quantity. This implies that $g_p\text{Im}(f_1^-)$ should be positive and $g_p\text{Im}f_1^+$ negative. The interference term in the angular distribution

[9] F. S. Crawford, Jr., M. Cresti, M. L. Good, K. Gottstein, E. M. Lyman, F. T. Solmitz, M. L. Stevenson, and H. K. Ticho, in *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 323.

[10] J. Steinberger, in *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 147.

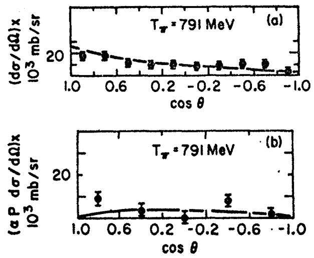

FIG. 5. (a) Angular distribution at a pion kinetic energy of 791 MeV (Ref. 7), (b) $\alpha P(d\sigma/d\Omega)$ at the same energy (Ref. 7). The curves are the results obtained by assuming a $p_{1/2}$ resonant state in our model.

$2g_p\mathrm{Re}(a_r)$ can then be written $2g_p\mathrm{Re}(f_1^-)\cos\theta$ with a positive (negative) product $g_p\mathrm{Re}(f_1^-)$ below (above) resonance for a $p_{1/2}$ partial wave, and $4g_p\mathrm{Re}(f_1^+)\cos\theta$ with a negative (positive) $g_p\mathrm{Re}(f_1^+)$ for a $p_{3/2}$. In the $p_{1/2}$ case we obtain an increase of the forward to backward ratio below resonance and a decrease above, and just the opposite behavior in the $p_{3/2}$ case. The total decreasing effect is more pronounced though in the case of the $p_{3/2}$.

This behavior of the differential cross section produces an effect in $\alpha P(\theta)$ which has a peaking that goes from backward to forward angles when we move from below to above resonance in the $p_{1/2}$ case, and the opposite behavior in the $p_{3/2}$.

We could sum up our numerical results by saying that neither with the $p_{1/2}$ choice nor with the $p_{3/2}$ do we obtain a perfect fit for all energies $|W-W_r|\leq{}^{1/2}\Gamma$, which shows that our assumptions are oversimplified, but that the experimental data available now favor the $p_{1/2}$.

In a more complete analysis we should take into account the contributions of the two other Born terms (one-nucleon and Σ exchange). At the present time, when the products of coupling constants appearing in them are not well known and there are not too many events available, this would increase the number of unknown parameters unduly. Maybe with the inclusion of these terms we could obtain a better fit for the angular distribution at $P_\pi=1.12$ BeV/c in the $p_{1/2}$ case as Kanazawa did. We do not believe that our result, that the experimental data favor a $p_{1/2}$ resonance, would be altered if we include these terms. In Kanazawa's article there is a mistake in sign in the expression for the $p_{3/2}$ resonant contribution to b (f_2 in his notation) that changes the sign of the interference term above and below resonance in the $p_{3/2}$ case, altering the backward

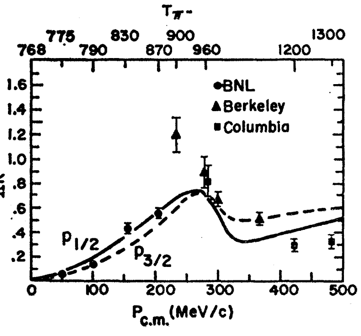

FIG. 6. Energy dependence of the total cross section. The curves are the theoretical fits obtained by assuming $p_{1/2}$ and $p_{3/2}$ resonances in our model. (The experimental point whose value is 1.19 mb has become more recently 0.73±0.028 mb which gives a much better agreement with our results. See Ref. 11).

to forward ratios. The correct results would be those that he obtained assuming a positive polarization which he qualified as a poor over-all fit.

Before ending we would like to add that at a pion kinetic energy of 829 MeV ($W=1650$ MeV) our model (with either $p_{1/2}$ or $p_{3/2}$ resonance) does not give the observed polarization dependence on angle, and to remark that at a center-of-mass energy of 1690 MeV ($P_\pi=1.03$ BeV/c) the predicted total cross section is smaller by three standard deviations than the one observed experimentally. Evidently our model has to be modified at those energies in order to take into account those anomalies.[11]

The last energy is that of the threshold of ΣK production. Baz *et al.*[12] have claimed the existence of a

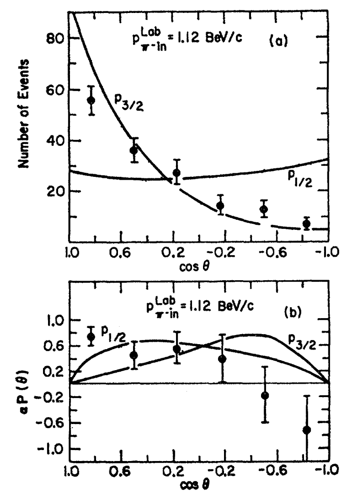

FIG. 7. (a) Angular distribution at an incident pion momentum of 1.12 BeV/c (Ref. 9), (b) $\alpha P(\theta)$ at the same energy (Ref. 10) The curves are the results obtained by assuming $p_{1/2}$ and $p_{3/2}$ resonances in our model.

bound state of the ΣK system with a binding energy of about 30 MeV that would produce a narrow peak at about $W=1660$ MeV. This could account for the high total cross section at $W=1690$ MeV observed experimentally. On the other hand, this resonance should be $s_{1/2}$ ($p_{1/2}$) for even (odd) $\Sigma\Lambda$ parity and from the phenomenological analysis of Bertanza *et al.*[7] it seems that the anomalous partial wave appearing at $W\approx1650$ MeV is of orbital angular momentum 2. Also, the total cross section should have a maximum at around this energy and this has not been observed.

From the value for the product of coupling constants $f^2g^2/(4\pi)^2=0.115$ and the now accepted full width of the $K^*(\Gamma=45$ MeV) we can obtain an estimate of the $K^*\Lambda N$ coupling constant.

[11] The total cross section at 1.03 BeV/c has changed recently and now agrees with our theoretical results. See F. S. Crawford, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki, (CERN Scientific Information Service, Geneva, 1962) p. 270.

[12] A. I. Baz, V. G. Vaks, and A. I. Larkin, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Scientific Information Service, Geneva, 1962) p. 404; A. I. Baz, V. G. Vaks and A. I. Larkin, Zh. Eksperim. i Teor. Fiz. **43**, 166 (1962) [translation: Soviet Phys.—JETP **16**, 118 (1963)].

The width of the K^* is given by

$$\Gamma=\Gamma(K^{*+}\to K^0+\pi^+)+\Gamma(K^{*+}\to K^++\pi^0)$$
$$=2\left(\frac{f^2}{4\pi}\right)\frac{p^3}{M^2}, \quad (18)$$

where p is the center-of-mass momentum of the decay pion and M the mass of the K^*.

We obtain $f^2/4\pi=0.80$ which gives $g^2/4\pi=0.144$. This value is smaller than the estimate of Chan[13] obtained under the assumption that the K^* exchange term gives the total cross section at an incident pion kinetic energy of 960 MeV and with a width for the K^* decay of 23 MeV. Our product of coupling constants falls close to the value obtained by MacDowell *et al.*[2] from the experimental data at $T_\pi=1300$ MeV.

After this work was completed we learned of a related work by Feld and Layson[14] who analyzed the experimental data on the total $\pi^\pm p$ cross sections and the differential elastic π^-p scattering cross section for energies between 0.3 and 1.3 BeV. They found that the best fitting of the angular distribution requires a $T=1/2, p_{1/2}$ resonance near 950 MeV($W=1716$ MeV) in agreement with out results. Also Kuo[15] has fitted the low energy $\gamma+p\to\Lambda+K^+$ data (excitation function, angular distribution, and one experimental point in the polarization) using a model similar to ours which included a Kanazawa resonance at $W=1700$ MeV and obtained a slightly better fit in the $p_{1/2}$ case.

We should add a comment on a work by Gourdin and Rimpault[16] in which a model somewhat similar to ours was proposed. These authors added to the K^* exchange the contributions from the Σ and Y_1^* exchanges, the nucleon pole, and the resonances $N_{1/2}^*$ and $N_{1/2}^{**}$, but an agreement with experiment for total and differential cross sections was found only in the cases of odd $\Sigma\Lambda$ parity, spin of K^* equal to 1, and even $\Sigma\Lambda$ parity, spin of K^* equal to 0. It is well known at the present time that the spin of the K^* is one[17] and the $\Sigma\Lambda$ parity even,[18] so this model is no longer valid. Their value for the $K^*\Lambda N$ coupling constant $g^2/4\pi=1.8$ should, therefore, not be considered reliable.

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[13] C. H. Chan, Phys. Rev. Letters **6**, 383 (1961).
[14] B. T. Feld and W. M. Layson, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Scientific Information Service, Geneva, 1962), p. 147. See also W. M. Layson, Nuovo Cimento **27**, 718 (1963).
[15] T. K. Kuo, Phys. Rev. **129**, 2264 (1963).
[16] M. Gourdin and M. Rimpault, Nuovo Cimento **24**, 414, (1962).
[17] See W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Rev. Letters **9**, 330 (1962).
[18] See Robert D. Tripp, Mason W. Watson, and Massimiliano Ferro-Luzzi, Phys. Rev. Letters **8**, 175 (1962). More recently, see H. Courant, H. Filthouth, P. Franzini, R. G. Glasser, A. Minquzzi-Ranzi, A. Segar, W. Willis, R. A. Burnstein, T. B. Day, B. Kehoe, A. J. Herz, M. Sakitt, B. Sechi-Zorn, N. Seeman, and G. A. Snow, *ibid.* **10**, 409 (1963).

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Dynamical Model of the K^* Resonance†

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The K^* vector meson is regarded as a P-wave resonance in the coupled, isotopic spin $\frac{1}{2}$, $\pi+K$, and $\eta+K$ states. All forces other than the K^* and ρ exchange forces are neglected, and a modification of the self-consistency technique of Zachariasen and Zemach is used to calculate the K^* mass and two relations among the three coupling-constant products $\gamma_{K^*\pi K}{}^2$, $\gamma_{K^*\eta K}{}^2$, and $\gamma_{\rho\pi\pi}\gamma_{\rho KK}$. The calculated K^* mass agrees with experiment. The factors in the self-consistency equations that depend on the $\pi-\eta-K$ and $K^*-\rho$ mass differences are isolated, and the effects of these mass differences on the results are discussed. The relationship of the results to the predictions of unitary symmetry is discussed.

I. INTRODUCTION

MANY authors have speculated that the strong-interaction coupling constants and the relative masses of the strongly interacting particles may be determinable from some form of dispersion relations.[1] Recently, several different attempts have been made to determine the ρ-meson mass and width from dispersion relations for the pion-pion scattering amplitude.[2–4]

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[1] See, for example, G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **8**, 41 (1962); R. H. Capps, Phys. Rev. **128**, 2842 (1962).
[2] F. Zachariasen, Phys. Rev. Letters **7**, 112, 268 (1961).
[3] Louis A. P. Balazs, Phys. Rev. **128**, 1939 (1962).
[4] F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962).