If $T_R\mathbf{A}(t)=\mathbf{A}(-t)$ as it does for $\mathbf{E}(t)$, then

$$T_R\mathfrak{A}(\omega)=\int_{-\infty}^{\infty}\mathbf{A}(-t)\exp(-i\omega t)dt/2\pi=\mathfrak{A}^*(\omega). \quad \text{(A3)}$$

If $T_R\mathbf{A}(t)=-\mathbf{A}(-t)$ as it does for $\mathbf{H}(t)$, then

$$T_R\mathfrak{A}(\omega)=-\int_{-\infty}^{\infty}\mathbf{A}(-t)\exp(-i\omega t)dt/2\pi =-\mathfrak{A}^*(\omega). \quad \text{(A4)}$$

Consider a real scalar quantity Φ:

$$\Phi=2\operatorname{Re}[\chi(\omega_a,\omega_b,\omega_c,\cdots)_{ijk\cdots} \times\mathfrak{A}(\omega_a)_i\mathfrak{B}(\omega_b)_j\mathfrak{C}(\omega_c)_k\cdots], \quad \text{(A5)}$$

where each of the quantities $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ is either a vector or a pseudovector. Under time reversal A will transform like $T_R\mathfrak{A}(\omega_a)_i=t_A\mathfrak{A}^*(\omega_a)_i$, where t_A is either $+1$ if $\mathbf{A}$ transforms like $\mathbf{E}$ or -1 if $\mathbf{A}$ transforms like $\mathbf{H}$. Similar transformations hold for $\mathbf{B}$, $\mathbf{C}$, etc. Then, under time reversal

$$T_R\Phi=\Phi=(-1)^n 2\operatorname{Re}[T_R\chi(\omega_a,\omega_b,\omega_c,\cdots)_{ijk\cdots} \times\mathfrak{A}^*(\omega_a)_i\mathfrak{B}^*(\omega_b)_j\mathfrak{C}^*(\omega_c)_k\cdots], \quad \text{(A6)}$$

where n is the number of quantities $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, etc., that transform like $\mathbf{H}$. If Φ is to be a real scalar, χ must transform as

$$T_R\chi(\omega_a,\omega_b,\omega_c,\cdots)=(-1)^n\chi^*(\omega_a,\omega_b,\omega_c,\cdots), \quad \text{(A7)}$$

where the tensor χ has n indices that transform like $\mathbf{H}$ on time reversal.

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Influence of the Spin of the Electron on the Quantum Magnetoacoustic Effect in Metals*

SERGIO RODRIGUEZ
Department of Physics, Purdue University, Lafayette, Indiana
(Received 6 December 1962)

A discussion is given of the influence of the orientation of the electron spins on the quantum oscillations of the ultrasonic attenuation in metals. This effect occurs for the situation in which a longitudinal acoustic wave propagates along the direction of an applied magnetic field in a sufficiently pure crystal and at low temperatures. It is shown that the attenuation consists of two series of spikes which occur periodically as a function of the reciprocal of the intensity of the magnetic field. The period of either series is related to an extremal cross-sectional area of the Fermi surface of the material and the shift between the two series is proportional to the cyclotron effective mass of the electrons.

WITHIN the framework of the semiclassical theory of ultrasonic absorption by metals,[1] the attenuation of a longitudinal acoustic wave propagating parallel to the direction of an applied dc magnetic field $\mathbf{B}_0$ is independent of the magnitude B_0 of $\mathbf{B}_0$. However, Gurevich *et al.*[2] have shown that, if quantum effects are taken into account, the coefficient of ultrasonic attenuation γ experiences large oscillations as a function of B_0. These oscillations have been observed by Korolyuk and Prushack[3] in Zn at liquid-helium temperatures.

The mechanism responsible for ultrasonic attenuation is absorption of phonons by the conduction electrons of the metal. In the simplest model of electrons having a spherical effective mass m^*, the stationary states of an electron in the magnetic field $\mathbf{B}_0$ are described by the wave functions[4]

$$|nk_yk_z\rangle=L_0^{-1}\exp(ik_yy+ik_zz)u_n(x+\hbar k_y/m^*\omega_c), \quad (1)$$

and their corresponding eigenvalues

$$E_n(k_z)=\hbar\omega_c(n+\tfrac{1}{2})+\hbar^2k_z^2/2m^*. \quad (2)$$

Here we have taken $\mathbf{B}_0$ parallel to the z axis of a Cartesian coordinate system (x,y,z). The length L_0 is the dimension of a cubic box of volume $V=L_0^3$ which contains the electrons, $\omega_c=eB_0/m^*c$ is the cyclotron frequency, e is the charge on a proton, and c is the speed of light in empty space. The wave numbers k_y and k_z can take any values consistent with periodic boundary conditions with the fundamental period taken as the volume V. The functions $u_n(x)$ are normalized harmonic

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[1] M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. **117**, 937 (1960).
[2] V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Soviet Phys.—JETP **13**, 552 (1961).
[3] A. P. Korolyuk and T. A. Prushack, Soviet Phys.—JETP **14**, 1201 (1962).
[4] L. D. Landau, Z. Physik **64**, 629 (1930). The energy levels defined by the different values of n in Eq. (2) are called Landau levels.

oscillator wave functions[5] ($n=0, 1, 2, \cdots$). The mass is m^* and the characteristic frequency ω_c. Now, not all electrons can absorb phonons. In fact, the restrictions imposed by the laws of conservation of energy and momentum (longitudinal phonons propagating parallel to $\mathbf{B}_0$ can be shown[6] to be able to induce transitions between electron states belonging to the same Landau level only, i.e., between states having the same value of n) show that only electrons having k_z such that

$$\hbar k_z/m^* = \omega/q - \hbar q/2m^* \tag{3}$$

can participate in the absorption. Here ω and $\mathbf{q}$ are the angular frequency and wave vector of the acoustic wave. For acoustic waves of frequency $\omega \ll 2\times 10^{11}$ sec^{-1}, the second term on the right-hand side of Eq. (3) is negligible[7] as compared with the first. This indicates that only those electrons whose velocity in the z direction is equal to the velocity of sound s are capable of absorbing phonons. However, because of the Pauli principle, at 0°K only electrons having energy between $E_0-\hbar\omega$ and E_0 can participate in the absorption. Here E_0 is the Fermi energy at 0°K but in the presence of $\mathbf{B}_0$. In this paper we shall make the assumptions that $\hbar\omega_c \ll E_0$ and $\omega \gg \tau^{-1}$, where τ is the electron relaxation time. Because $\hbar\omega_c \ll E_0$, the quantity E_0 differs from the Fermi energy ζ_0 in the absence of a magnetic field by a negligible amount. A simple calculation shows that absorption of a phonon is possible whenever there is an integer between the numbers Λ_+ and Λ_- defined by

$$\Lambda_\pm = \frac{E_0}{\hbar\omega_c} - \frac{1}{2} - \frac{m^*\omega^2}{2\hbar\omega_c q^2}\left(1 \pm \frac{\hbar q^2}{2m^*\omega}\right)^2. \tag{4}$$

The difference $\Lambda_- - \Lambda_+ = \omega/\omega_c$ so that, whenever $\omega < \omega_c$, the attenuation coefficient is zero if there is no integer within the interval (Λ_+,Λ_-) and $\gamma = \gamma_0(\omega_c/\omega)$ otherwise. The quantity $\gamma_0 = \pi\omega/2v_F$ (where v_F is the Fermi velocity) is the coefficient of ultrasonic attenuation in the absence of a magnetic field. The result quoted above follows from the fact that the attenuation averaged over a sufficiently large range in B_0 should yield γ_0 in agreement with the semiclassical theory. This means that the ratio γ/γ_0, when plotted as a function of $(E_0/\hbar\omega_c)-\frac{1}{2}$ has spikes of width ω/ω_c and height ω_c/ω which occur, approximately, at integral values of $(E_0/\hbar\omega_c)-\frac{1}{2}$.

The purpose of this paper is to investigate the effect of the alignment of the electron spins in the field B_0 upon the results outlined above. First, we discuss this question and how it could be used to obtain information about the electronic structure of metals. Second, we investigate the effect of a finite temperature on the consequences deduced for 0°K.

If the orientation of the electron spins in the field $\mathbf{B}_0$ is taken into account, the energy levels (2) become

$$E_n^{(\pm)}(k_z) = E_n(k_z) \pm \tfrac{1}{2} g\mu_B B_0, \tag{5}$$

where the positive and negative signs correspond to states having spin parallel to and antiparallel to B_0, respectively, $\mu_B = e\hbar/2mc$ is the Bohr magneton (m is the free electron mass), and g is the g factor. The argument given above can be repeated in exactly the same fashion. The conclusion is that each of the spikes in γ/γ_0 as a function of $(E_0/\hbar\omega_c)-\frac{1}{2}$ splits into two a distance $(g\mu_B B_0/\hbar\omega_c) = (gm^*/2m)$ apart. The conditions for resolution of these lines depend, obviously, on how m^*/m compares with unity and with the ratio ω/ω_c. Thus, the effect of the spin is to give rise to two sets of spikes that occur at regular intervals of B_0^{-1}. One set owes its origin to electrons with spin parallel to $\mathbf{B}_0$ and the other to electrons with spin antiparallel to $\mathbf{B}_0$.

We turn now our attention to a metal with an arbitrary electronic structure. The maxima and minima in the ratio γ/γ_0 occur whenever the field B_0 is such that a state exists having average velocity in the z direction equal to the velocity of sound and energy within $\hbar\omega$ of E_0. Since $s \ll v_F$ such states correspond, for practical purposes, to extremal orbits on the Fermi surface.[8] Onsager[9] has developed a method for describing the stationary states of electrons in a real crystal in the presence of a magnetic field. It is assumed that the motion in $\mathbf{k}$ space is governed by the Lorentz equation $\hbar d\mathbf{k}/dt = -(e/c)\mathbf{v}\times\mathbf{B}_0$, where $\mathbf{v}$ is the velocity of an electron characterized by wave vector $\mathbf{k}$. Using the Bohr-Sommerfeld quantization rule it is possible to show that only those states which correspond to orbits in planes characterized by a constant k_z having areas

$$S = 2\pi(n+\delta)eB_0/\hbar c \tag{6}$$

are stationary states. (Here δ is a constant phase factor that lies between zero and unity; for free electrons $\delta=\frac{1}{2}$.) This indicates that, in the absence of the Zeeman splitting of the electronic energy levels, there is a maximum whenever B_0 is such that an integer n exists for which the left-hand side of S is S_0 the area of the extremal orbit on the Fermi surface (see footnote 8). Now the effect of the spin is to give absorption arising from separate contributions by the electrons of either spin orientation. The Zeeman energy of the electrons in the field $\mathbf{B}_0$ causes them to repopulate their states so that the extremal orbits on the Fermi surface for electrons with spin up or down are $S_\pm$. A maximum of attenuation occurs when an integer n exists for which

$$S_\pm = 2\pi(n+\delta)eB_0/\hbar c. \tag{7}$$

[5] See, for example, L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1955), 2nd ed., pp. 64–65.

[6] J. J. Quinn and S. Rodriguez, Phys. Rev. **128**, 2487, 2494 (1962).

[7] For the purpose of this numerical estimate we assumed $s=\omega/q=3\times 10^5$ cm/sec and m^* of the order of the free electron mass.

[8] In what follows we shall describe these orbits as extremal. Strictly speaking they are not, since extremal orbits have zero average velocity in the direction of the magnetic field.

[9] L. Onsager, Phil. Mag. **43**, 1006 (1952).

But

$$S_{\pm}=S_0\pm(dS/dE)_0\tfrac{1}{2}g\mu_B B_0, \tag{8}$$

where $(dS/dE)_0$ is the rate of change with respect to the energy of the extremal cross sectional area of the Fermi surface by a plane perpendicular to $\mathbf{B}_0$. We know that $(dS/dE)_0$ can be related to the effective mass m^* which one obtains in cyclotron resonance experiments,[10] namely $m^*=(\hbar^2/2\pi)(dS/dE)_0$. (We use the same symbol m^* as in our previous discussion; no confusion need arise.) Thus, Eq. (7) becomes

$$S_0=[n+\delta\mp(gm^*/4m)]2\pi eB_0/\hbar c. \tag{9}$$

This result implies that if γ/γ_0 is plotted as a function of B_0^{-1}, we obtain two sets of spikes which occur at regular intervals of B_0^{-1} with period

$$\Delta(1/B_0)=2\pi e/\hbar cS_0. \tag{10}$$

Two contiguous spikes belonging to different sets are separated by[11] $g\pi em^*/\hbar cS_0 m$. This result is the same as that given for a spherical band but now m^* has a well-defined meaning in terms of the geometrical properties of the Fermi surface.

Let us discuss now the effect of a finite temperature upon the results described above. The results of the calculation which we shall now present are similar to those found by several authors[12–16] for the case of the de Haas–van Alphen effect. For this reason our description will be brief. The coefficient of ultrasonic attenuation can be obtained from the imaginary part of the sound frequency given by[17,18]

$$\omega^2=\Omega_p^2/\epsilon_{zz}(\mathbf{q},\omega). \tag{11}$$

Here Ω_p is the plasma frequency of the positive ions of the metal and

$$\epsilon_{zz}(q,\omega)=1+4\pi\int f_0(E)(d\chi/dE)dE, \tag{12}$$

where $f_0(E)$ is the Fermi-Dirac distribution function and

$$\chi(E)=\frac{m^*\omega_p^2}{4\pi Nq^2}\sum_{nk_yk_z}{}' \{[E_n(k_z+q)-E_n(k_z)-\hbar\omega+i\hbar/\tau]^{-1} +[E_n(k_z+q)-E_n(k_z)+\hbar\omega-i\hbar/\tau]^{-1}\}. \tag{13}$$

We assume we are dealing with electrons with a spherical effective mass m^*, that $\omega\tau\gg 1$, and that $\mathbf{q}=(0,0,q)$ points in the direction of $\mathbf{B}_0$ (longitudinal wave propagating along the z axis). The quantity ω_p is the electron plasma frequency and N the number of electrons in the system. The sum in Eq. (13) extends over all states having energy less than E [i.e., $E_n(k_z)<E$]. The important parameter in the determination of γ is the imaginary part of ϵ_{zz}. We find $(\omega\gg\tau^{-1})$

$$\operatorname{Im}\chi(E) =-\frac{3\omega_p^2\omega_c}{16q^3v_F^3}[\Lambda(E+\tfrac{1}{2}g\mu_B B_0)+\Lambda(E-\tfrac{1}{2}g\mu_B B_0)]. \tag{14}$$

$\Lambda(E)$ is the number of integers within the interval defined by the quantities $\Lambda_{\pm}$ given by Eq. (4) except that we replace E_0 by E. If $\omega<\omega_c$, we can perform a Fourier expansion of $\Lambda(E)$ and after use of the integral

$$\int dE\frac{df_0}{dE}\exp\left(\frac{2\pi n(E-\zeta)i}{\hbar\omega_c}\right) =-\frac{2n\pi^2kT}{\hbar\omega_c\sinh(2n\pi^2kT/\hbar\omega_c)}, \tag{15}$$

we obtain

$$\gamma/\gamma_0=1+\sum_{n=1}^{\infty}(-1)^n\frac{4\pi kT}{\hbar\omega}\sin\left(\frac{\pi n\omega}{\omega_c}\right) \times\cos\left(\frac{g\pi nm^*}{2m}\right)\frac{\cos(2\pi n\zeta_0/\hbar\omega_c)}{\sinh(2\pi^2nkT/\hbar\omega_c)}. \tag{16}$$

If only the first Fourier coefficient in Eq. (16) is observed, the effect of the electron spin is simply to introduce the factor $\cos(g\pi m^*/2m)$ which is absent if spin is not taken into account. However, if $2\pi^2kT\ll\hbar\omega_c$ several Fourier components of Eq. (16) are important and the two sets of spikes in the graph of γ/γ_0 as a function of B_0^{-1} may be detected.

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[10] See, for example, R. G. Chambers, Can. J. Phys. **34**, 1395 (1956).

[11] This is true if $gm^*<2m$. If this is not the case, the problem of interpretation is obviously more difficult. In particular, if $gm^*=2m$ it is impossible to detect the effect due to the electron spins.

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