tions the curves (not shown here) are slightly different; the most striking feature is that in the combinations He+, Ar and He+, Kr the emission cross sections are still rising with decreasing energy at 5 keV, and will reach maxima below this energy. However, in case of $^2D$ excitation the change of internal energy $\Delta E$ is larger than with $^2S$ excitation, so that the emission maxima of the $^2D-^2P$ transitions should be expected at higher energies, in contradiction to our results.

The results for H+ and He+ shot into neon gas are given in Fig. 3, for the excitation of the Ne I $2p^6\,{}^1S$ level giving rise to the emission of the Ne I lines $2p^6\,{}^1S-3s[1\frac{1}{2}]^0$ and $2p^6\,{}^1S-3s'[\frac{1}{2}]^0$, which could not be separated. Ar I and Kr I lines were observed only at high pressures (15–50 $\mu$), partly because the sensitivity of the particle multiplier is lower in this spectral region

(1000–1250 Å). The cross section for the He+, Ne combination rises very sharply at lower energy. However, the adiabatic criterion predicts a maximum around 60 keV.

A theory explaining the rise in the emission cross sections at very low energy cannot be given at present. An approach towards the problem is under study, in which we assume that for low velocities the colliding particles may form intermediate molecular states. Such intermediate states may show lower changes of internal energy than the atomic states giving rise to maxima in the emission cross sections at correspondingly lower energies.[5]

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[5] See also E. S. Solov'ev, R. N. Ill'in, V. A. Oparin, and N. V. Fedorenko, Soviet Phys.—JETP **15**, 459 (1962).

# Model for the Scattering of Slow Electrons by Cesium Atoms*

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The computation of the total cross section for the elastic scattering of slow electrons from cesium atoms was investigated. The model adopted consisted of a combination of the Biermann and Harting polarization potential with the Hartree potential of a cesium atom. The cross section was computed by the phase shift method. Using the reported values of the polarizability of cesium and a value of 6.13 $a_0$ for the cutoff parameter, $fr_0$, in the polarization potential, it was found that this model yields values of the cross section in agreement with current experimental results. A method is proposed for the choice of the cutoff parameter.

## I. INTRODUCTION

EXACT calculations of electron scattering by alkali atoms are not possible at present because of the computational difficulties involved. Calculations using the Hartree or Hartree-Fock atomic potential functions give unsatisfactory results at low incident energies because of large contributions to the interaction potential due to the polarization of the target atom. Some method for calculating scattering cross sections for the alkali atoms is desirable which includes this polarization effect. Robinson[1] has recently given results of calculations made for electron-cesium atom cross sections where the potential function used was constructed from one-electron Slater type wave functions plus a polarization term of the form:

$$V_p(x) = -\frac{\epsilon^2}{2a_0}\frac{\alpha}{(x_p^2+x^2)^2}. \tag{1}$$

Here $\alpha$ is the polarizability, $\epsilon$ the energy, and $x_p$ the "screening constant." His results are not completely in accord with the experimental data.

We attempt to build a model of the alkali atom which includes the rather large polarization contribution to the interaction potential in such a way that calculated cross sections can be made to fit experimental data in a satisfactory manner. In the present calculations the Hartree potential for the neutral cesium atom was obtained. To this was added a polarization term of the Biermann-Harting[2] type which has been used previously in calculations of excited state wave functions of alkali atoms.[3,4] It might be argued that such a representation of the potential function is not well founded, especially if the velocity of the incident particle is comparable with the orbital velocities of the bound electrons. Here we consider incident velocities much less than those of the orbital electrons.

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[1] L. B. Robinson, Phys. Rev. **127**, 2076 (1962).

[2] L. Biermann and H. Harting, Z. Astrophys. **22**, 87 (1942).
[3] H. J. Brudner and S. Borowitz, Phys. Rev. **120**, 2053 (1960).
[4] D. S. Villars, J. Opt. Soc. Am. **42**, 552 (1952).

## II. PROCEDURE

The Hartree potential function for the cesium ion has been previously calculated,[5] but that for the neutral atom has not, to our knowledge, been published. The calculation of the neutral cesium atom potential function was made by using Hartree's previous result for the ion as a potential in which the 6s electron moves. The wave function for the 6s electron of cesium was then calculated by the standard Hartree procedure. With the 6s wave function thus obtained, a potential function for the total atom was calculated by adding the contribution due to this electron to the potential of the ion. This procedure yields a Hartree potential for the neutral cesium atom which is approximate in that complete self-consistency, which is difficult to obtain in the heavier alkali atoms, was not achieved. Since we are using a polarization potential, slight inaccuracy due to this procedure should not be important. The polarization term is very much larger than the contribution due to the outermost electron in the region where the Hartree potential function thus obtained lacks self-consistency.

The polarization term which was added to the Hartree potential function was of the form given by Biermann and Harting[2]:

$$V_p(r) = (\alpha/2r^4)\{1 - \exp[-(r/fr_0)^8]\}. \qquad (2)$$

In this equation, $\alpha$ is the polarizability of the atom, $r$ the radial distance, and the quantity $fr_0$ is the distance at which the polarization induced force disappears.

The complete potential function then has the form

$$V(r) = [V_H(r)/r] + (\alpha/2r^4)(1 - \exp[-(r/fr_0)^8]), \qquad (3)$$

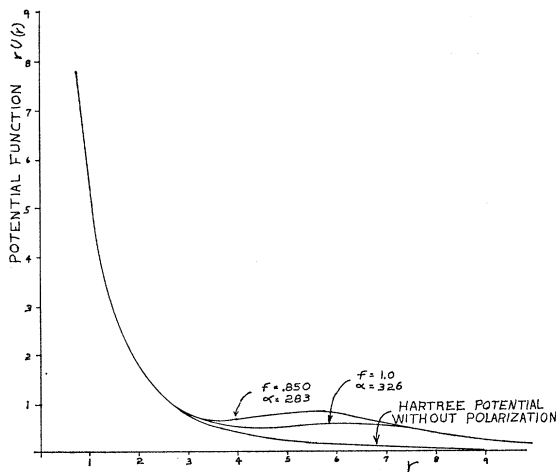where $V_H(r)/r$ is the Hartree potential function (Fig. 1).



FIG. 1. Potential function for the neutral cesium atom with and without the polarization term. (Atomic units.)

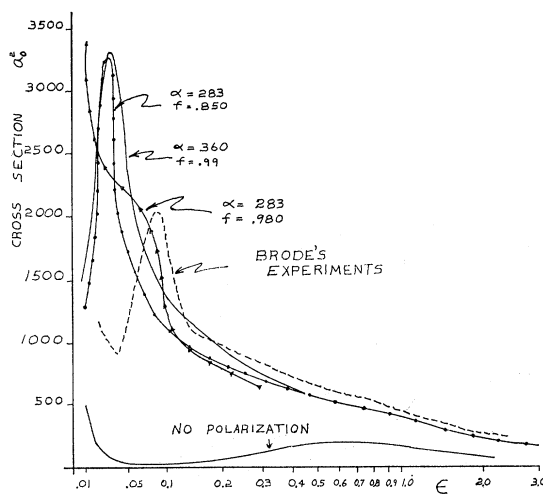[5] D. R. Hartree, Proc. Roy. Soc. (London) A143, 506 (1934).



FIG. 2. Calculated total elastic cross sections for electrons incident on cesium atoms showing the sensitivity to the parameters in the polarization potential. (Atomic units, where the unit of energy $\epsilon$ is approximately 27.2 eV.)

## III. METHOD OF SOLUTION

In the polarization term, Eq. (2), the cutoff parameter $fr_0$ has been taken to be the radial distance of the last inflection point of the outermost electron wave function.[2,4] Since this is a somewhat arbitrary choice,[2] it seems reasonable to choose $r_0$ to be the distance of the last maximum of the valence electron wave function (here $r_0 = 6.13a_0$) and to vary the parameter $f$ over a small range to determine its effect on the scattering cross section. In this way the polarization contribution was varied somewhat in order to give a best fit to the experimental data. The polarizability $\alpha$, in Eq. (2), is also somewhat uncertain since it has not been measured with high accuracy. Three experimentally determined values were taken from the literature.[6-8] The values used were $\alpha = 42$, 48, and 53 Å$^3$, or $\alpha = 283$, 326, and $360a_0^3$, respectively, in atomic units. Since the two latter values are more recent we are inclined to accept these. By using all three values above, one may determine how inaccuracies in the value of the polarizability effect the resulting computed cross section.

Having made a choice for the parameter $fr_0$ in Eq. (3), the radial wave equation was solved numerically by standard procedures. For a given value of the incident energy the radial equation was solved for each value of $l$ from $l = 0$ to $l = l_{max}$, where $l_{max}$ is determined by the incident energy (values of $l$ through $l = 15$ were used at the highest energies considered). A phase shift $\delta_l$ was then obtained directly for each radial function by comparing the solution to that of a free particle at large values of $r$ where the potential is essentially zero[8]

[6] H. Scheffers and J. Stark, Physik. Z. 35, 625 (1934).
[7] G. E. Chamberlin and J. C. Zorn, Bull. Am. Phys. Soc. 7, 70 (1962).
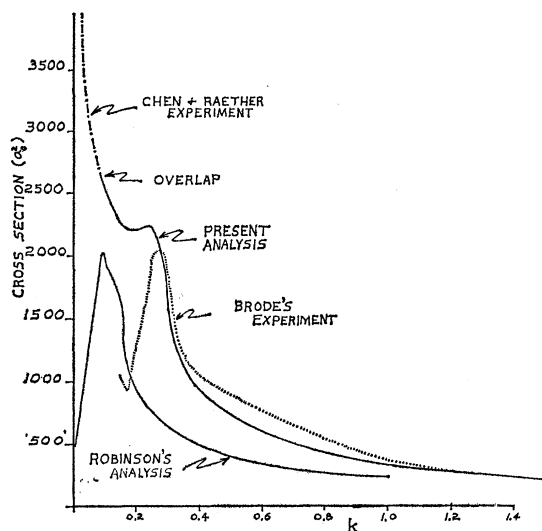[8] A. Salop, E. Pollack, and B. Bederson, Phys. Rev. 124, 1431 (1961).

FIG. 3. Comparison of present analysis with that of Robinson and with the experimental results of Brode and of Chen and Raether. (Atomic units, $k = \sqrt{\epsilon}$.)

($<0.1\%$ of the incident energy). The total cross sections were then obtained using the expression

$$\sigma = (4\pi/k^2)\sum_l (2l+1)\sin^2\delta_l. \tag{4}$$

## IV. RESULTS

In Fig. 1 the Hartree potential function is plotted and compared with the total potential including polarization. From this figure we see that the polarization contribu-

tion is very much larger than the Hartree potential in the region $r>3$, and hence the statement that small errors in $V_H(r)$ should be negligible as compared with $V_p$ is justified. The change in the potential function due to a small change in the parameter $fr_0$ is also shown. It was found that in the low energy region the cross section varied over a wide range of values with small changes in the cutoff parameter. This indicates a strong dependence on the polarization contribution to the potential for values of $r$ comparable to the atomic radius. In Fig. 2 the cross section for various values of $\alpha$ and $fr_0$ are plotted. We note that in the energy region $\epsilon > 0.4$ ($E>11$ eV) little change is produced in the total cross section by small variations in the cutoff parameter. In Fig. 3 the theoretical cross section which best fits the experimental values given by Brode[9] as well as more recent values given by Chen and Raether[10] is given. We see that a good fit to Brode's values is achieved in all but a very small region in the low energy range, and an almost perfect fit to the values of Chen and Raether in the thermal region is obtained ($E \sim 0.06$–0.075 eV).

## V. CONCLUSIONS

From the results obtained it appears that the model discussed describes the collision process. Clearly, if the method used for selecting the value of $fr_0$ is valid for different atoms, then a simple model may be used to describe low energy electron scattering as only the experimental polarizability is required.

[9] R. B. Brode, Phys. Rev. 34, 673 (1929).
[10] C. L. Chen and M. Raether, Phys. Rev. 128, 2679 (1962).

# Low-Energy Spectrum of Electrons Scattered on Bound Electrons*

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The spectrum of electrons at low energies and large scattering angles resulting from the scattering of high-energy electrons on bound atomic electrons is calculated to lowest order in $\alpha Z$ and highest order in the incident energy. Relativistic Coulomb field effects are included by the use of wave functions correct to two orders in $\alpha Z$. Inclusion of these relativistic Coulomb effects leads to a cross section significantly different from that obtained previously by the use of the plane wave approximation. The results show that the low-energy spectrum of electrons scattered on bound atomic electrons completely dominates the peak predicted by Parzen and co-workers in the low-energy spectrum of bremsstrahlung-producing electrons.

## I. INTRODUCTION

IT has been pointed out by Parzen and co-workers[1] that the measurement of the energy spectrum of the electrons which have lost energy in bremsstrahlung pro-

[1] D. G. Keiffer and G. Parzen, Phys. Rev. 101, 1244 (1956); P. T. McCormick, D. G. Keiffer, and G. Parzen, *ibid.* 103, 29 (1956).

duction would complement the direct measurement of the bremsstrahlung energy spectrum and thus provide a check on the Bethe-Heitler formula. Of particular interest is the form of the bremsstrahlung spectrum near the high-energy limit. This portion of the bremsstrahlung spectrum corresponds to the low-energy spectrum of the scattered electrons. Using the Bethe-Heitler formula, Parzen *et al.* have shown that there is a peak in the spectrum of electrons for very low energies and large scattering angles.