3π Decay Modes of $K^{\pm}$ and η

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Present experimental results on τ, τ', and $\eta\rightarrow 3\pi$ decay are shown to imply either that these decays all pass through an intermediate virtual one-pion state; or that there exists an S-wave 2π resonance of low mass, such as the recently reported ϕ, which, however, has yet to be confirmed. It is not possible simply to invoke a pure $I=1$ final state in each of the three decays, since other evidence shows that, in the absence of such a resonance, the $K^{\pm}\rightarrow 3\pi$ effective weak Lagrangian density contains both nonderivative and derivative coupling. The ηx spectrum, while presently unreliable, shows evidence against such a resonance, but further experiments are needed. Here x is the usual Dalitz variable, related to the $\pi^+\pi^-$ energy difference.

WE make some comments on the experimental similarity of the Dalitz plots for τ' and η decay in the light of various theoretical models and calculations, in particular the pion pole model (ppm) of Bég and DeCelles.[1] We also mention a possible difficulty in reconciling the η spectra with a model based on the possible ϕ resonance.[2]

Barton and Rosen[3] suggested that both the η decay mechanism proposed by Gell-Mann[4] and the ppm for $K^{\pm}\rightarrow 3\pi$ (cf. Fig. 1) would simultaneously be verified if the τ' and η Dalitz plots agreed experimentally, since the η Dalitz plot then indicates that this passes primarily through the one-pion intermediate state. The accumulating experimental results on η have borne out this agreement, as is most strikingly demonstrated in a compilation by Berley, Colley, and Schultz.[5] Recently Bég[6] and, independently, Wali[7] have pointed out that such agreement is not *necessarily* due to the predominant ppm in $K^{\pm}$ decay, but is already a consequence of the assigned quantum numbers (0^{-+}) for η together with a pure $I=1$ final state for $K^{\pm}$ decay, provided that the *whole* structure of the $K^{\pm}$ and η decay amplitudes is determined by final-state interactions (fsi). We wish to point out that this proviso requires the existence of a rather low energy S-wave $\pi\pi$ resonance. Such a resonance called ϕ has been reported by Samios *et al.*[2] at 395 ± 10 MeV, with $I=0$, but has yet to be confirmed by other workers.[8] If such a resonance is not confirmed, we show that the ppm is actually correct; if the ϕ is confirmed certain consequences follow for the spectrum.

Assume first that no such resonance exists, so that the low-energy $\pi\pi$ scattering is well characterized by scattering lengths a_I. Further, as various analyses of the πN system have indicated,[9] suppose that a_0 is fairly large and positive, i.e., attractive, and a_1 and a_2 are very small. It is then impossible to explain why in $\tau^+\rightarrow\pi^+ +\pi^+ +\pi^-$ decay the π^- carries away more than its fair share of energy, if one assumes that all the observed structure is due to fsi. This is clear if one only includes two-body fsi effects, since one then requires either a $\pi^+\pi^-$ repulsion and/or a $\pi^+\pi^+$ attraction (cf. Thomas and Holladay,[10] also other treatments quoted by Smith *et al.*[9]). Khuri and Treiman[11] included true three-body effects using a dispersion relation treatment. They worked to lowest order in the scattering lengths a_I, and were led to the requirement $a_2-a_0\approx 0.7$; which is inconsistent with the πN data. Working to all orders in the a_I (and with more recent data) does not alter these results qualitatively,[12] and we would argue very strongly that any valid theoretical treatment must reach this result. The conclusion under the above assumptions is that the *effective* $K^{\pm}\rightarrow 3\pi$ Lagrangian density $\mathcal{L}_{\text{eff}}$ must contain P-wave effects, i.e., $g_P(K\partial_\mu\pi)(\pi\partial_\mu\pi)$ as well as $g_S K\pi^3$. (These are symbolic.)

Here we distinguish $\mathcal{L}_{\text{eff}}$ which specifically causes $K^{\pm}\rightarrow 3\pi$, from the intrinsic Lagrangian density $\mathcal{L}_{\text{int}}$. The difference is that $\mathcal{L}_{\text{eff}}$ corresponds to $\mathcal{L}_{\text{int}}$ after renormalization due to the strong $\pi\pi$ interaction. In the ppm, for which $\mathcal{L}_{\text{int}}=gK\pi$, $\mathcal{L}_{\text{eff}}$ does contain a P-wave

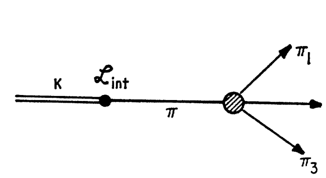

FIG. 1. Diagram for the pion-pole-model decay of K into 3π.

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[1] M. A. Baqi Bég and P. C. DeCelles, Phys. Rev. Letters **8**, 46 (1962).

[2] N. P. Samios, A. H. Bachman, R. M. Lea, T. E. Kalogeropoulos, and W. D. Shephard, Phys. Rev. Letters **9**, 139 (1962).

[3] G. Barton and S. P. Rosen, Phys. Rev. Letters **8**, 414 (1962).

[4] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); also M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

[5] D. Berley, D. Colley, and J. Schultz, Phys. Rev. Letters **10**, 114 (1963).

[6] M. A. Baqi Bég, Phys. Rev. Letters **9**, 67 (1962).

[7] K. C. Wali, Phys. Rev. Letters **9**, 120 (1962).

[8] See, in particular, C. Alff, D. Berley, D. Colley *et al.*, Phys. Rev. Letters **9**, 322 and 325 (1962).

[9] See the survey and references in H. J. Schnitzer, Phys. Rev. **125**, 1059 (1962); also J. Kirz, J. Schwartz, and R. D. Tripp, *ibid.* **126**, 763 (1962); M. Jacob, G. Mahoux, and R. Omnès, Nuovo Cimento **23**, 838 (1962); J. D. Jackson and G. L. Kane, *ibid.* **23**, 444 (1962); also the introductory remarks and references in L. T. Smith, D. J. Prowse, and D. H. Stork, Phys. Letters **2**, 204 (1962) (note the misprint $I=0$ for $I=2$ in line 22).

[10] B. S. Thomas and W. G. Holladay, Phys. Rev. **115**, 1329 (1959).

[11] N. N. Khuri and S. B. Treiman, Phys. Rev. **119**, 1115 (1960).

[12] C. Kacser and G. Barton, Clarendon Laboratory, Oxford, England Report 95/62 (to be published); an initial account of this work (to lowest order in the a_I) appears in Phys. Rev. Letters **8**, 226 and 353(E) (1962).

part arising from $\pi \to 3\pi$, with the virtual pion having a mass corresponding to that of the decaying particle. The crucial feature of this model is that g_P/g_S depends only on the $\pi\pi$ interaction, and *not* on $\mathcal{L}_{\text{int}}$, in distinction to most other P-wave models.

Let us now compare $\tau'^{+} \to \pi^0+\pi^0+\pi^+$ and $\eta^0 \to \pi^+ +\pi^- +\pi^0$, assuming that both final states are pure $I=1$. If all the structure were due to fsi in each case, the two spectra would be identical except for the (hopefully) minor effects due to the different decaying mass, etc. This is not the case once $K^{\pm} \to 3\pi$ has an effective P-wave part, unless the ratio g_P/g_S is the same for $K^{\pm}$ and η; which will only be so if both $K^{\pm}$ and η go through the ppm, and not otherwise (except if there is a remarkable coincidence). The final remarks on this point in the letter of Bég[6] are, in fact, incorrect if taken literally, and there will be no direct correspondence between any terms in the two spectra. Thus if there is no low-energy S-wave $\pi\pi$ resonance the very close agreement between the τ' and η spectra is a very strong indication of the validity of the ppm for *both* $K^{\pm}$ and η decay.

On the other hand, suppose that the S-wave ϕ resonance is confirmed. It is then consistent to assume $\mathcal{L}_{\text{eff}}$ has no structure and the fsi is predominantly through ϕ. The simplest possible calculation is then reasonably consistent with the present τ, τ', and η data, provided the full width of the ϕ (experimentally 50 ± 20 MeV) is taken to be at least 70 MeV, preferably more. In fact such a resonance was originally postulated by Brown and Singer[13] to explain both the $\eta \to 3\pi$ spectrum and also various η branching ratios. There is, however, one trend in the present data which throws doubt on the ϕ resonance model, as discussed below.

The quantity measured experimentally is $|M(x,y)|^2$, where x and y are the usual Dalitz variables and M is the invariant matrix element. To improve statistics one averages over x or y, so obtaining $Y(y)$ and $X(x)$, respectively. These are then analyzed in the form $Y(y) = Y(0)[1+\alpha y+\beta y^2]$ and $X(x)=X(0)[1+\gamma x^2]$, there being no odd powers of x provided Bose-Einstein statistics and charge symmetry are valid.

Since there are only 511 η events at the moment β_η is very uncertain. Berley *et al.*,[5] therefore, analyzed the η spectrum in the form $Y^{1/2}=Y^{1/2}(0)[1+\alpha' y]$ and found a very good straight line fit between the η and τ' spectra when expressed in invariant variables rather than y. Note that in this linear form one many also make use of τ data, by invoking *only* an $I=1$ final state; thus τ, τ', and η are actually compared.

One should also compare the X spectra for η and τ' (or, invoking $\Delta I=\frac{1}{2}$, also $K_2{}^0 \to \pi^+\pi^-\pi^0$ decay). No K data are available, since τ is *not* directly related. The η X spectrum shows some asymmetry between $-x$ and $+x$ which indicates that some background bias is actually present (which, however, is unlikely to cause serious error in the Y spectrum). Disregarding this bias one finds $X_\eta \propto 1-(0.3\pm0.2x^2)$. This should not be taken too seriously; but if future data confirm such a large x^2 dependence, and also demonstrate that this is not a "spurious" effect due to averaging over y [for note $\langle y^2\rangle_{\text{av}} \approx \frac{1}{3}(1-x^2)$, $\langle x^2\rangle_{\text{av}} \approx \frac{1}{3}(1-y^2)$ so that x^2 and y^2 dependences in $|M|^2$ becomes interrelated after averaging, both here and in $Y^{1/2}$], this will throw serious doubt on the predominant $I=0$ ϕ resonance model, which has *no* x^2 dependence other than other *small* fsi effects.

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[13] L. M. Brown and P. Singer, Phys. Rev. Letters **8**, 460 (1962).