Angular Distributions from Elastic Scattering of 15-MeV Deuterons*

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Angular distributions of 15-MeV deuterons elastically scattered by 23 elements between Al and Pb are presented. A high-resolution detection system provides complete separation from all inelastically scattered deuterons. In light and medium weight elements, there is a sharp diffraction pattern, but it damps out with increasing mass. In a few cases, there are marked differences between angular distributions from nuclei of nearly equal masses.

INTRODUCTION

N recent years, distorted-wave Born approximation calculations have been very successful in analyzing data on various types of direct nuclear reactions. The basic input data for these calculations are the optical model parameters, which are obtained from analyses of elastic scattering angular distributions. Since many of the most useful direct interactions involve deuterons [e.g., (d,p), (d,n), (d,t), (d,α) , (d,Li^6) , (p,d), (α,d) , etc.], it is most important to have good data on elastic deuteron scattering over a range of energies.

A rather complete series of measurements of elastic deuteron scattering angular distributions at 11.8 MeV has been reported by Igo et al., and a few results have been reported at 13.0, 13.5, and 15.0 MeV by Cindro and Wall, by the Cracow group, and by Gofman et al.4 at the Institute of Physics of the Academy of Sciences of the Ukrainian S.S.R. We here report a rather extensive study of this type with 15 MeV deuterons. In addition to being more extensive than previous studies, the present work utilizes better energy resolution than the previous work, which is important in some cases.

EXPERIMENTAL PROCEDURE AND ANALYSIS

The scattering facility used in conjunction with the University of Pittsburgh cyclotron has been described in detail previously⁵; it provides focusing and magnetic analysis of the incident deuteron beam, and magnetic analysis of the scattered particles. With its most recent modifications, a continuous range of angles from -30° to +140° may be studied, although measurements

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¹ G. Igo, W. Lorenz, and U. Schmidt Rohr, Report No. 29 (Max Planck Institute for Nuclear Physics, Heidelberg, Germany,

² Nikola Cindro and N. S. Wall, Phys. Rev. 119, 1340 (1960). ⁸ A. Buezanowski and K. Grotowski, Rept. No. 201 (Institute of Nuclear Physics, Cracow, 1962); A. Strzalkowski, Rept. No. 202 (Institute of Nuclear Physics, Cracow, 1962) L. Freindl, H. Nicwodniczanski, J. Nurzynski, M. Slapa, and A. Strzalkowski, Rept. No. 203 (Institute of Nuclear Physics, Cracow, 1962). H. Niewodniczanski, J. Nurzynski, and J. Wilczynski, Rept. No. 204 (Institute of Nuclear Physics, Cracow, 1962).

4 Y. U. Gofman and O. F. Nemets, Zh. Eksperim. i Teor. Fiz. 39, 1489 (1960) [translation: Soviet Phys.—JETP 12, 1035].

(1961)].

⁵ R. S. Bender, E. M. Reilly, A. J. Allen, R. Ely, J. S. Arthur, and H. J. Hausmann, Rev. Sci. Instr. 23, 542 (1952).

beyond 90° require a time consuming change of scattering chambers.

In the present experiment, the scattered particles are detected on the focal plane of the reaction product magnetic analyzing system by a triple scintillator apparatus shown in Fig. 1. It consists of three thin CsI (Tl) scintillation crystals, two 1/8-in.-wide "side" crystals on each side of the 1/4-in.-wide "center" crystal. In measuring the count rate of elastically scattered deuterons, the magnetic field is varied until practically all counts come from the center crystal and the count rates from the side crystals are equal; this assures that essentially all elastically scattered deuterons strike the center crystal, so that their intensity can be measured with a single magnet setting. It also provides sufficient energy resolution, (~150 keV) so that there can be no contribution from inelastic scattering even to first excited states for any of the nuclei studied. The detector is covered with sufficient absorber to eliminate tritons, and neither protons nor alpha particles have sufficient magnetic rigidity to reach the detector. In a few cases at back angles, the elastic peak was too wide to be detected by the center crystal only, so that corrections were applied using the count rate in the side

The incident deuteron energy cannot be carefully

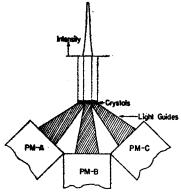


Fig. 1. Schematic diagram of the detector for the 15-MeV deuteron elastic angular distributions. Details of design and operation are explained in the text. PM-A, PM-B, and PM-C are photomultiplier tubes. The graph in the top part is the intensity distribution of elastically scattered deuterons across the crystals. By varying the analyzing magnetic field almost the entire elastic deuteron group could be made to fall on the center crystal as indicated in this figure.

Table I. $(\sigma/\sigma_R)_{\rm c.m.}$ vs $\theta_{\rm c.m.}$ for 15-MeV deuterons elastically scattered from several elements.

			-		
$\theta = (\sigma/\sigma r)$	${ m Ti} \ heta_{ m c.m.} \ \ (\sigma/\sigma_R)_{ m c.m.}$	Fe $\theta_{ ext{c.m.}}$ $(\sigma/\sigma_R)_{ ext{c.m.}}$	$ heta_{ m c.m.}$	$ m Ni^{58}$ $(\sigma/\sigma_R)_{c.m.}$	$ ext{Cu} hinspace hins$
$\theta_{\rm c.m.}$ $(\sigma/\sigma_R)_{\rm c.m.}$	$\theta_{ m c.m.}$ $(\sigma/\sigma_R)_{ m c.m.}$	$\theta_{\mathrm{c.m.}}$ $(\sigma/\sigma_R)_{\mathrm{c.m.}}$	0 _{c, m,}	(σ/σ _R) _{c, m,}	$\theta_{\rm c.m.}$ $(\sigma/\sigma_R)_{\rm c.m.}$
21° 18′ 0.920	19° 27′ 0.771	19° 17′ 0.735	20° 44′ 0.723	94° 50′ 0.138	20° 37′ 0.734
23° 57′ 0.799	22° 03′ 0.665	21° 55′ 0.686	23° 19′ 0.578	97° 20′ 0.122	23° 11′ 0.689
26° 36′ 0.560	24° 37′ 0.497	23° 29′ 0.481	25° 54′ 0.477	99° 49′ 0.0764	25° 45′ 0.523
29° 12′ 0.416	27° 21′ 0.366	27° 03′ 0.415	28° 29 0.386 31° 03 0.352	102° 19′ 0.0540	28° 19 0.482
31° 53′ 0.311 34° 31′ 0.294	29° 48′ 0.275 32° 23′ 0.283	29° 38′ 0.335 32° 22′ 0.332	31° 03 0.352 33° 37 0.397	104° 48′ 0.0498 107° 17′ 0.0456	30° 53 0.465 33° 27 0.454
37° 10′ 0.413	34° 58′ 0.377	34° 43′ 0.424	36° 11 0.479	107 17 0.0430 109° 45′ 0.0449	36° 01 0.502
39° 47′ 0.583	37° 33′ 0.427	37° 31′ 0.472	38° 46′ 0.506	112° 13 0.0747	38° 35′ 0.528
42° 25′ 0.773	40° 08′ 0.469	39° 53′ 0.503	41° 20′ 0.531	114° 40′ 0.0948	41° 8′ 0.490
45° 02′ 0.915	42° 43′ 0.497	42° 28′ 0.508	43° 53′ 0.507	117° 08′ 0.0940	43° 41′ 0.442
47° 39′ 0.852	45° 16′ 0.476	45° 01′ 0.451	46° 27′ 0.426	119° 36′ 0.0955	46° 14′ 0.345
50° 15′ 0.679	47° 51′ 0.427	47° 35′ 0.360	49° 00′ 0.349	122° 04′ 0.0948	48° 47′ 0.250
52° 51′ 0.471 55° 37′ 0.314	50° 24′ 0.367 52° 59′ 0.342	50° 07′ 0.282 52° 41′ 0.248	51° 33′ 0.269 54° 06′ 0.218	124° 31′ 0.0905 127° 0′ 0.0822	51° 20′ 0.190 53° 53′ 0.178
58° 02′ 0.193	55° 31′ 0.312	55° 13′ 0.197	56° 39′ 0.188	129° 25′ 0.0731	56° 26′ 0.184
60° 37′ 0.165	58° 05′ 0.318	57° 47′ 0.209	59° 11′ 0.195	131° 52′ 0.0556	58° 58′ 0.208
63° 12′ 0.229	60° 37′ 0.309	60° 15′ 0.213	61° 43′ 0.205	134° 19′ 0.0449	61° 30′ 0.246
65° 46′ 0.367	63° 11′ 0.295	62° 51′ 0.218	64° 16′ 0.218	136° 45′ 0.0531	63° 32′ 0.284
68° 20′ 0.521	65° 43′ 0.263	65° 23′ 0.222	66° 48′ 0.218	139° 11′ 0.0465	66° 34′ 0.260
70° 53′ 0.680	68° 15′ 0.220	67° 54′ 0.219	69° 20′ 0.225	141° 38 0.0432	68° 36′ 0.251
73° 26′ 0.803 75° 58′ 0.848	70° 47′ 0.162	70° 27′ 0.199 72° 59′ 0.196	71° 52′ 0.218 74° 24′ 0.219		71° 38′ 0.242 73° 39′ 0.221
75° 58′ 0.848 78° 31′ 0.800	73° 20′ 0.121 75° 51′ 0.124	75° 30′ 0.192	74° 24′ 0.219 76° 55′ 0.220		73° 39′ 0.221 76° 40′ 0.188
81° 02′ 0.704	78° 23′ 0.139	78° 01′ 0.195	79° 26′ 0.218		78° 41′ 0.162
83° 34′ 0.600	80° 53′ 0.176	80° 32′ 0.202	81° 57′ 0.210		81° 42′ 0.146
86° 04′ 0.467	83° 24′ 0.230	83° 03′ 0.206	84° 27′ 0.193		83° 42′ 0.135
88° 35′ 0.356	85° 54′ 0.284	85° 33′ 0.196	86° 58′ 0.170		86° 42′ 0.114
91° 05′ 0.265	88° 25′ 0.321	88° 03′ 0.180	89° 28′ 0.157		88° 43′ 0.113
93° 35′ 0.197	90° 55′ 0.331	90° 34′ 0.170	92° 00′ 0.130		91° 43′ 0.100
Zn		Zr	Y	Nb	Мо
$Z_{\rm n}$ $\theta_{ m c.m.}$ $(\sigma/\sigma_R)_{ m c.m.}$	$ heta_{ m c,m.}$	$ m Zr$ $(\sigma/\sigma_R)_{c.m.}$	$egin{array}{c} Y \ heta_{ m c.m.} & (\sigma/\sigma_R)_{ m c.m.} \end{array}$	Nb $\theta_{\mathrm{c.m.}}$ $(\sigma/\sigma_R)_{\mathrm{c.m.}}$	$egin{aligned} & ext{Mo} \ heta_{ ext{c.m.}} & (\sigma/\sigma_R)_{ ext{c.m.}} \end{aligned}$
$\theta_{ m c.m.}$ $(\sigma/\sigma_R)_{ m c.m.}$		$(\sigma/\sigma_R)_{ m c.m.}$	$\theta_{\rm c.m.}$ $(\sigma/\sigma_R)_{\rm c.m.}$	$\theta_{ m c,m.}$ $(\sigma/\sigma_R)_{ m c,m.}$	$\theta_{ m c.m.}$ $(\sigma/\sigma_R)_{ m c.m.}$
	θ _{c,m.} 20° 28′ 0.905 23° 01′ 0.840				$\theta_{\text{c.m.}}$ $(\sigma/\sigma_R)_{\text{c.m.}}$ 19° 57′ 0.926 22° 30′ 0.855
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696	(σ/σ _R) _{e.m.} 94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098	$\theta_{\text{c.m.}}$ $(\sigma/\sigma_R)_{\text{c.m.}}$ $20^{\circ} 26' 0.800$ $22^{\circ} 59' 0.787$ $25^{\circ} 32' 0.692$	$\theta_{\rm c.m.}$ $(\sigma/\sigma_R)_{\rm c.m.}$ 19° 46′ 0.851	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ $19^{\circ} 57' 0.926$ $22^{\circ} 30' 0.855$ $25^{\circ} 03' 0.731$
θ _{c,m.} (σ/σ _R) _{c,m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621	(σ/σ _R) _{e.m.} 94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070	θ _{e,m.} (σ/σ _R) _{e,m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616	θ _{e,m.} (σ/σ _R) _{e,m.} 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058	θ _{e,m.} (σ/σ _R) _{e,m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551	θ _{e,m.} (σ/σ _R) _{e,m.} 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492	$\theta_{\text{c.m.}}$ $(\sigma/\sigma_R)_{\text{c.m.}}$ $19^{\circ} 46' 0.851$ $22^{\circ} 18' 0.825$ $24^{\circ} 51' 0.740$ $27^{\circ} 24' 0.690$ $29^{\circ} 42' 0.566$ $32^{\circ} 29' 0.504$	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462
$\theta_{e.m.}$ $(\sigma/\sigma_R)_{e.m.}$ 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057	$\theta_{\text{c.m.}}$ $(\sigma/\sigma_R)_{\text{c.m.}}$ 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419
$\theta_{e.m.}$ $(\sigma/\sigma_R)_{e.m.}$ 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073	$\theta_{\text{c.m.}}$ $(\sigma/\sigma_R)_{\text{c.m.}}$ 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435
θ _{c.m.} (σ/σ _R) _{c.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419 40° 18′ 0.382 42° 51′ 0.350 45° 23′ 0.307
$\theta_{e,m}$ $(\sigma/\sigma_R)_{e,m}$. 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419 40° 18′ 0.382 42° 51′ 0.350 45° 23′ 0.307 47° 55′ 0.286
$\theta_{e,m.}$ $(\sigma/\sigma_R)_{e,m.}$ 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257	$\theta_{e,m}$. $(\sigma/\sigma_R)_{e,m}$. 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.334	$\theta_{e.m.}$ $(\sigma/\sigma_R)_{e.m.}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419 40° 18′ 0.382 42° 51′ 0.350 45° 23′ 0.307 47° 55′ 0.286 50° 26′ 0.287
θ _{c.m.} (σ/σ _R) _{c.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348 53° 32′ 0.355	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.506 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.334 52° 50′ 0.340	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419 40° 18′ 0.382 42° 51′ 0.350 45° 23′ 0.307 47° 55′ 0.286 50° 26′ 0.287 52° 58′ 0.287
θ _{c.m.} (σ/σ _R) _{c.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175 56° 26′ 0.190	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348 53° 32′ 0.355 56° 04′ 0.360	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096 128° 51′ 0.087	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306 56° 02′ 0.334	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.334 52° 50′ 0.340 55° 22′ 0.340	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419 40° 18′ 0.382 42° 51′ 0.350 45° 23′ 0.307 47° 55′ 0.286 50° 26′ 0.287 52° 58′ 0.287 55° 30′ 0.279
θ _{c.m.} (σ/σ _R) _{c.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348 53° 32′ 0.355	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.506 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.334 52° 50′ 0.340	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419 40° 18′ 0.382 42° 51′ 0.350 45° 23′ 0.307 47° 55′ 0.286 50° 26′ 0.287 52° 58′ 0.287
θ _{c.m.} (σ/σ _R) _{c.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175 56° 26′ 0.190 58° 58′ 0.217 61° 30′ 0.250 63° 32′ 0.252	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348 53° 32′ 0.355 56° 04′ 0.360 58° 35′ 0.348 61° 07′ 0.296 63° 39′ 0.244	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096 128° 51′ 0.087 131° 19′ 0.077 133° 47′ 0.067 136° 15′ 0.054	θ _{e, m.} (σ/σ _R) _{e, m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306 56° 02′ 0.334 58° 33′ 0.352 61° 05′ 0.344 63° 36′ 0.271	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.334 52° 50′ 0.340 55° 22′ 0.340 57° 54′ 0.332 60° 25′ 0.298 62° 57′ 0.259	θ _{e.m.} (σ/σ _R) _{e.m.} 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419 40° 18′ 0.382 42° 51′ 0.350 45° 23′ 0.307 47° 55′ 0.286 50° 26′ 0.287 52° 58′ 0.287 55° 30′ 0.279 58° 02′ 0.263 60° 33′ 0.242 63° 04′ 0.206
θ _{c.m.} (σ/σ _R) _{c.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175 56° 26′ 0.190 58° 58′ 0.217 61° 30′ 0.250 63° 32′ 0.252 66° 34′ 0.238	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348 53° 32′ 0.355 56° 04′ 0.360 58° 35′ 0.348 61° 07′ 0.296 63° 39′ 0.244 66° 10′ 0.179	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096 128° 51′ 0.087 131° 19′ 0.077 133° 47′ 0.067 136° 15′ 0.054 138° 43′ 0.050	θ _{e, m.} (σ/σ _R) _{e, m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306 56° 02′ 0.334 58° 33′ 0.352 61° 05′ 0.344 63° 36′ 0.271 66° 07′ 0.220	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.334 52° 50′ 0.340 55° 22′ 0.340 57° 54′ 0.332 60° 25′ 0.298 62° 57′ 0.259 65° 29′ 0.213	θ _{e.m.} (σ/σ _R) _{e.m.} 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419 40° 18′ 0.382 42° 51′ 0.350 45° 23′ 0.307 47° 55′ 0.286 50° 26′ 0.287 52° 58′ 0.287 55° 30′ 0.279 58° 02′ 0.263 60° 33′ 0.242 63° 04′ 0.206 65° 35′ 0.172
θ _{e.m.} (σ/σ _R) _{e.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175 56° 26′ 0.190 58° 58′ 0.217 61° 30′ 0.250 63° 32′ 0.252 66° 34′ 0.238 68° 36′ 0.238	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348 53° 32′ 0.355 56° 04′ 0.360 58° 35′ 0.348 61° 07′ 0.296 63° 39′ 0.244 66° 10′ 0.179 68° 42′ 0.124	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096 128° 51′ 0.087 131° 19′ 0.077 133° 47′ 0.067 136° 15′ 0.054	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306 56° 02′ 0.334 58° 33′ 0.352 61° 05′ 0.241 66° 07′ 0.220 68° 39′ 0.167	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.506 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.334 52° 50′ 0.340 55° 22′ 0.340 55° 22′ 0.340 57° 54′ 0.332 60° 25′ 0.298 62° 57′ 0.259 65° 29′ 0.213 68° 00′ 0.172	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419 40° 18′ 0.382 42° 51′ 0.350 45° 23′ 0.307 47° 55′ 0.286 50° 26′ 0.287 52° 58′ 0.287 55° 30′ 0.279 58° 02′ 0.263 60° 33′ 0.242 63° 04′ 0.206 65° 35′ 0.172 68° 07′ 0.146
θ _{c.m.} (σ/σ _R) _{c.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175 56° 26′ 0.190 58° 58′ 0.217 61° 30′ 0.250 63° 32′ 0.252 66° 34′ 0.238 68° 36′ 0.238 71° 38′ 0.206	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348 51° 0′ 0.348 51° 0′ 0.348 61° 07′ 0.296 63° 39′ 0.244 66° 10′ 0.179 68° 42′ 0.124 71° 13′ 0.103	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096 128° 51′ 0.087 131° 19′ 0.077 133° 47′ 0.067 136° 15′ 0.054 138° 43′ 0.050	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306 56° 02′ 0.334 58° 33′ 0.352 61° 05′ 0.344 63° 36′ 0.271 66° 07′ 0.220 68° 39′ 0.167 71° 11′ 0.125	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.334 52° 50′ 0.340 55° 22′ 0.340 57° 54′ 0.332 60° 25′ 0.298 62° 57′ 0.259 65° 29′ 0.213 68° 00′ 0.172 70° 31′ 0.144	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ $19^{\circ} 57'$ 0.926 $22^{\circ} 30'$ 0.855 $25^{\circ} 03'$ 0.731 $27^{\circ} 36'$ 0.630 $30^{\circ} 08'$ 0.553 $32^{\circ} 41'$ 0.462 $35^{\circ} 14'$ 0.435 $37^{\circ} 44'$ 0.419 $40^{\circ} 18'$ 0.382 $42^{\circ} 51'$ 0.350 $45^{\circ} 23'$ 0.307 $47^{\circ} 55'$ 0.286 $50^{\circ} 26'$ 0.287 $52^{\circ} 58'$ 0.287 $55^{\circ} 30'$ 0.279 $58^{\circ} 02'$ 0.263 $60^{\circ} 33'$ 0.242 $63^{\circ} 04'$ 0.206 $65^{\circ} 35'$ 0.172 $68^{\circ} 07'$ 0.146 $70^{\circ} 38'$ 0.132
θ _{e.m.} (σ/σ _R) _{e.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175 56° 26′ 0.190 58° 58′ 0.217 61° 30′ 0.250 63° 32′ 0.252 66° 34′ 0.238 8° 36′ 0.238 71° 38′ 0.206 73° 39′ 0.187	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.345 51° 0′ 0.348 51° 0′ 0.348 53° 32′ 0.355 56° 04′ 0.360 58° 35′ 0.348 61° 07′ 0.296 63° 39′ 0.244 66° 10′ 0.179 68° 42′ 0.124 71° 13′ 0.103 73° 44′ 0.105	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096 128° 51′ 0.087 131° 19′ 0.077 133° 47′ 0.067 136° 15′ 0.054 138° 43′ 0.050	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 58° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306 56° 02′ 0.334 58° 33′ 0.352 61° 05′ 0.344 63° 36′ 0.271 68° 39′ 0.167 71° 11′ 0.125 73° 41′ 0.095	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.340 55° 22′ 0.340 57° 54′ 0.332 60° 25′ 0.298 62° 57′ 0.259 65° 29′ 0.213 68° 00′ 0.172 70° 31′ 0.144 73° 02 0.136	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419 40° 18′ 0.382 42° 51′ 0.350 45° 23′ 0.307 47° 55′ 0.286 50° 26′ 0.287 52° 58′ 0.287 55° 30′ 0.279 58° 02′ 0.263 60° 33′ 0.242 63° 04′ 0.206 65° 35′ 0.172 68° 07′ 0.146 70° 38′ 0.132 73° 09′ 0.134
θ _{e.m.} (σ/σ _R) _{e.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175 56° 26′ 0.190 58° 58′ 0.217 61° 30′ 0.250 63° 32′ 0.252 66° 34′ 0.238 71° 38′ 0.206	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348 51° 0′ 0.348 51° 0′ 0.348 61° 07′ 0.296 63° 39′ 0.244 66° 10′ 0.179 68° 42′ 0.124 71° 13′ 0.103	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096 128° 51′ 0.087 131° 19′ 0.077 133° 47′ 0.067 136° 15′ 0.054 138° 43′ 0.050	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306 56° 02′ 0.334 58° 33′ 0.352 61° 05′ 0.344 63° 36′ 0.271 66° 07′ 0.220 68° 39′ 0.167 71° 11′ 0.125	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.334 52° 50′ 0.340 55° 22′ 0.340 57° 54′ 0.332 60° 25′ 0.298 62° 57′ 0.259 65° 29′ 0.213 68° 00′ 0.172 70° 31′ 0.144	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ $19^{\circ} 57'$ 0.926 $22^{\circ} 30'$ 0.855 $25^{\circ} 03'$ 0.731 $27^{\circ} 36'$ 0.630 $30^{\circ} 08'$ 0.553 $32^{\circ} 41'$ 0.462 $35^{\circ} 14'$ 0.435 $37^{\circ} 44'$ 0.419 $40^{\circ} 18'$ 0.382 $42^{\circ} 51'$ 0.350 $45^{\circ} 23'$ 0.307 $47^{\circ} 55'$ 0.286 $50^{\circ} 26'$ 0.287 $52^{\circ} 58'$ 0.287 $55^{\circ} 30'$ 0.279 $58^{\circ} 02'$ 0.263 $60^{\circ} 33'$ 0.242 $63^{\circ} 04'$ 0.206 $65^{\circ} 35'$ 0.172 $68^{\circ} 07'$ 0.146 $70^{\circ} 38'$ 0.132
θ _{e.m.} (σ/σ _R) _{e.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175 56° 26′ 0.190 58° 58′ 0.217 61° 30′ 0.250 63° 32′ 0.252 66° 34′ 0.238 88° 36′ 0.238 71° 38′ 0.206 73° 39′ 0.187 76° 40′ 0.154	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.345 51° 0′ 0.348 53° 32′ 0.355 56° 04′ 0.360 58° 35′ 0.348 61° 07′ 0.296 63° 39′ 0.244 66° 10′ 0.179 68° 42′ 0.124 71° 13′ 0.103 73° 44′ 0.105 76° 15′ 0.123	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096 128° 51′ 0.087 131° 19′ 0.077 133° 47′ 0.067 136° 15′ 0.054 138° 43′ 0.050	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306 56° 02′ 0.334 58° 33′ 0.352 61° 05′ 0.344 63° 36′ 0.271 66° 07′ 0.220 68° 39′ 0.167 71° 11′ 0.125 73° 41′ 0.095 76° 12′ 0.101	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.566 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.340 55° 22′ 0.340 55° 22′ 0.340 57° 54′ 0.332 60° 25′ 0.298 62° 57′ 0.298 62° 57′ 0.259 65° 29′ 0.213 68° 00′ 0.172 70° 31′ 0.144 73° 02 0.136 75° 33′ 0.147	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ $19^{\circ} 57' 0.926$ $22^{\circ} 30' 0.855$ $25^{\circ} 03' 0.731$ $27^{\circ} 36' 0.630$ $30^{\circ} 08' 0.553$ $32^{\circ} 41' 0.462$ $35^{\circ} 14' 0.435$ $37^{\circ} 44' 0.419$ $40^{\circ} 18' 0.382$ $42^{\circ} 51' 0.350$ $45^{\circ} 23' 0.307$ $47^{\circ} 55' 0.286$ $50^{\circ} 26' 0.287$ $52^{\circ} 58' 0.287$ $55^{\circ} 30' 0.279$ $58^{\circ} 02' 0.263$ $60^{\circ} 33' 0.242$ $63^{\circ} 04' 0.206$ $65^{\circ} 35' 0.172$ $68^{\circ} 07' 0.146$ $70^{\circ} 38' 0.132$ $73^{\circ} 09' 0.134$ $75^{\circ} 39' 0.130$
θ _{c.m.} (σ/σ _R) _{c.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175 56° 26′ 0.190 58° 58′ 0.217 61° 30′ 0.250 63° 32′ 0.252 66° 34′ 0.238 71° 38′ 0.206 73° 39′ 0.187 76° 40′ 0.154 78° 41′ 0.133 81° 42′ 0.122 83° 42′ 0.102	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348 53° 32′ 0.355 56° 04′ 0.360 58° 35′ 0.348 61° 07′ 0.296 63° 39′ 0.244 66° 10′ 0.179 68° 42′ 0.124 71° 13′ 0.103 73° 44′ 0.105 76° 15′ 0.123 78° 46′ 0.150 81° 16′ 0.191 83° 47′ 0.210	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096 128° 51′ 0.087 131° 19′ 0.077 133° 47′ 0.067 136° 15′ 0.054 138° 43′ 0.050	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306 56° 02′ 0.334 58° 33′ 0.352 61° 05′ 0.344 63° 36′ 0.271 66° 07′ 0.220 68° 39′ 0.167 71° 11′ 0.125 73° 41′ 0.095 76° 12′ 0.101 78° 43′ 0.124 81° 14′ 0.143 83° 44′ 0.198	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.506 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.334 52° 50′ 0.340 55° 22′ 0.340 57° 54′ 0.332 60° 25′ 0.298 62° 57′ 0.259 65° 29′ 0.213 68° 00′ 0.172 70° 31′ 0.144 73° 02 0.136 75° 33′ 0.147 78° 03′ 0.165 80° 34′ 0.191 83° 04′ 0.205	θ _{e.m.} (σ/σ _R) _{e.m.} 19° 57′ 0.926 22° 30′ 0.855 25° 03′ 0.731 27° 36′ 0.630 30° 08′ 0.553 32° 41′ 0.462 35° 14′ 0.435 37° 44′ 0.419 40° 18′ 0.382 42° 51′ 0.350 45° 23′ 0.307 47° 55′ 0.286 50° 26′ 0.287 52° 58′ 0.287 55° 30′ 0.279 58° 02′ 0.263 60° 33′ 0.242 63° 04′ 0.206 65° 35′ 0.172 68° 07′ 0.146 70° 38′ 0.132 73° 09′ 0.134 75° 39′ 0.130 78° 10′ 0.154 80° 40′ 0.165 83° 11′ 0.169
θ _{e.m.} (σ/σ _R) _{e.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175 56° 26′ 0.190 58° 58′ 0.217 61° 30′ 0.250 63° 32′ 0.252 66° 34′ 0.238 71° 38′ 0.206 73° 39′ 0.187 76° 40′ 0.154 78° 41′ 0.133 81° 42′ 0.102 86° 42′ 0.105	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348 53° 32′ 0.355 56° 04′ 0.360 58° 35′ 0.348 61° 07′ 0.296 63° 39′ 0.244 66° 10′ 0.179 68° 42′ 0.124 71° 13′ 0.103 73° 44′ 0.105 76° 15′ 0.123 78° 46′ 0.150 81° 16′ 0.191 83° 47′ 0.210 86° 16′ 0.221	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096 128° 51′ 0.087 131° 19′ 0.077 133° 47′ 0.067 136° 15′ 0.054 138° 43′ 0.050	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306 56° 02′ 0.334 58° 33′ 0.352 61° 05′ 0.344 63° 36′ 0.271 66° 07′ 0.220 68° 39′ 0.167 71° 11′ 0.125 73° 41′ 0.095 76° 12′ 0.101 78° 43′ 0.124 81° 14′ 0.143 83° 44′ 0.198 86° 14′ 0.216	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ $19^{\circ} 46' 0.851$ $22^{\circ} 18' 0.825$ $24^{\circ} 51' 0.740$ $27^{\circ} 24' 0.690$ $29^{\circ} 42' 0.506$ $32^{\circ} 29' 0.504$ $35^{\circ} 03' 0.452$ $37^{\circ} 36' 0.445$ $40^{\circ} 08' 0.401$ $42^{\circ} 41' 0.378$ $45^{\circ} 13' 0.347$ $47^{\circ} 46' 0.331$ $50^{\circ} 18' 0.334$ $52^{\circ} 50' 0.340$ $55^{\circ} 22' 0.340$ $57^{\circ} 54' 0.332$ $60^{\circ} 25' 0.298$ $62^{\circ} 57' 0.259$ $65^{\circ} 29' 0.213$ $68^{\circ} 00' 0.172$ $70^{\circ} 31' 0.144$ $73^{\circ} 02 0.136$ $75^{\circ} 33' 0.147$ $78^{\circ} 03' 0.165$ $80^{\circ} 34' 0.191$ $83^{\circ} 04' 0.205$ $85^{\circ} 34' 0.210$	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ $19^{\circ} 57' 0.926$ $22^{\circ} 30' 0.855$ $25^{\circ} 03' 0.731$ $27^{\circ} 36' 0.630$ $30^{\circ} 08' 0.553$ $32^{\circ} 41' 0.462$ $35^{\circ} 14' 0.435$ $37^{\circ} 44' 0.419$ $40^{\circ} 18' 0.382$ $42^{\circ} 51' 0.350$ $45^{\circ} 23' 0.307$ $47^{\circ} 55' 0.286$ $50^{\circ} 26' 0.287$ $52^{\circ} 58' 0.287$ $52^{\circ} 58' 0.287$ $55^{\circ} 30' 0.279$ $58^{\circ} 02' 0.263$ $60^{\circ} 33' 0.242$ $63^{\circ} 04' 0.206$ $65^{\circ} 35' 0.172$ $68^{\circ} 07' 0.146$ $70^{\circ} 38' 0.132$ $73^{\circ} 09' 0.134$ $75^{\circ} 39' 0.130$ $78^{\circ} 10' 0.154$ $80^{\circ} 40' 0.165$ $83^{\circ} 11' 0.169$ $85^{\circ} 41' 0.165$
θ _{c.m.} (σ/σ _R) _{c.m.} 20° 36′ 0.707 23° 11′ 0.660 25° 45′ 0.486 28° 19 0.459 30° 53 0.440 33° 27 0.423 36° 01′ 0.455 38° 35′ 0.466 41° 8′ 0.435 43° 41′ 0.360 46° 14′ 0.275 48° 47′ 0.200 51° 20′ 0.152 53° 53′ 0.175 56° 26′ 0.190 58° 58′ 0.217 61° 30′ 0.250 63° 32′ 0.252 66° 34′ 0.238 71° 38′ 0.206 73° 39′ 0.187 76° 40′ 0.154 78° 41′ 0.133 81° 42′ 0.122 83° 42′ 0.102	20° 28′ 0.905 23° 01′ 0.840 25° 34′ 0.696 28° 07′ 0.621 30° 40′ 0.562 33° 16′ 0.536 35° 46′ 0.461 38° 19′ 0.437 40° 51′ 0.404 43° 24′ 0.355 45° 56′ 0.344 48° 28′ 0.334 51° 0′ 0.348 53° 32′ 0.355 56° 04′ 0.360 58° 35′ 0.348 61° 07′ 0.296 63° 39′ 0.244 66° 10′ 0.179 68° 42′ 0.124 71° 13′ 0.103 73° 44′ 0.105 76° 15′ 0.123 78° 46′ 0.150 81° 16′ 0.191 83° 47′ 0.210	94° 7′ 0.160 96° 36′ 0.149 99° 6′ 0.098 101° 36′ 0.070 104° 06′ 0.058 106° 35′ 0.047 109° 04′ 0.057 111° 33′ 0.073 114° 02′ 0.098 116° 31′ 0.099 118° 59′ 0.115 121° 27′ 0.117 123° 55′ 0.111 126° 23′ 0.096 128° 51′ 0.087 131° 19′ 0.077 133° 47′ 0.067 136° 15′ 0.054 138° 43′ 0.050	θ _{e.m.} (σ/σ _R) _{e.m.} 20° 26′ 0.800 22° 59′ 0.787 25° 32′ 0.692 28° 05′ 0.616 30° 38′ 0.551 33° 11′ 0.492 35° 44′ 0.444 38° 17′ 0.417 40° 49′ 0.350 43° 22′ 0.321 45° 54′ 0.297 48° 26′ 0.270 50° 58′ 0.257 53° 30′ 0.306 56° 02′ 0.334 58° 33′ 0.352 61° 05′ 0.344 63° 36′ 0.271 66° 07′ 0.220 68° 39′ 0.167 71° 11′ 0.125 73° 41′ 0.095 76° 12′ 0.101 78° 43′ 0.124 81° 14′ 0.143 83° 44′ 0.198	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ 19° 46′ 0.851 22° 18′ 0.825 24° 51′ 0.740 27° 24′ 0.690 29° 42′ 0.506 32° 29′ 0.504 35° 03′ 0.452 37° 36′ 0.445 40° 08′ 0.401 42° 41′ 0.378 45° 13′ 0.347 47° 46′ 0.331 50° 18′ 0.334 52° 50′ 0.340 55° 22′ 0.340 57° 54′ 0.332 60° 25′ 0.298 62° 57′ 0.259 65° 29′ 0.213 68° 00′ 0.172 70° 31′ 0.144 73° 02 0.136 75° 33′ 0.147 78° 03′ 0.165 80° 34′ 0.191 83° 04′ 0.205	$\theta_{\text{e.m.}}$ $(\sigma/\sigma_R)_{\text{e.m.}}$ $19^{\circ} 57' 0.926$ $22^{\circ} 30' 0.855$ $25^{\circ} 03' 0.731$ $27^{\circ} 36' 0.630$ $30^{\circ} 08' 0.553$ $32^{\circ} 41' 0.462$ $35^{\circ} 14' 0.435$ $37^{\circ} 44' 0.419$ $40^{\circ} 18' 0.382$ $42^{\circ} 51' 0.350$ $45^{\circ} 23' 0.307$ $47^{\circ} 55' 0.286$ $50^{\circ} 26' 0.287$ $52^{\circ} 58' 0.287$ $52^{\circ} 58' 0.287$ $55^{\circ} 30' 0.279$ $58^{\circ} 02' 0.263$ $60^{\circ} 33' 0.242$ $63^{\circ} 04' 0.206$ $65^{\circ} 35' 0.172$ $68^{\circ} 07' 0.146$ $70^{\circ} 38' 0.132$ $73^{\circ} 09' 0.134$ $75^{\circ} 39' 0.130$ $78^{\circ} 10' 0.154$ $80^{\circ} 40' 0.165$ $83^{\circ} 11' 0.169$

Table I. $(\sigma/\sigma_R)_{\rm c.m.}$ vs $\theta_{\rm c.m.}$ for 15-MeV deuterons elastically scattered from several elements (continued).

TABLE	1. (0/0 R)c.m. VS Uc.m. 10	or 15-MeV deuterons e	lastically scattered in		
Rh $\theta_{\mathrm{c.m.}}$ $(\sigma/\sigma_R)_{\mathrm{c.m.}}$	$ ext{Pd} ext{} heta_{ ext{c. m.}} hinspace (\sigma/\sigma_R)_{ ext{c. m.}}$	$ m Ag \ heta_{c.m.} \ (\sigma/\sigma_R)_{c.m.}$	$\mathrm{Cd} \ heta_{\mathrm{c.m.}} (\sigma/\sigma_R)_{\mathrm{c.m.}}$	In $\theta_{\mathrm{c.m.}}$ $(\sigma/\sigma_R)_{\mathrm{c.m.}}$	$\mathrm{Sn^{120}}$ $ heta_{\mathrm{c.m.}}$ $(\sigma/\sigma_R)_{\mathrm{c.m.}}$
20° 15′ 0.979 22° 48′ 1.10 25° 21′ 0.952 27° 54′ 0.842 30° 26′ 0.815 32° 59′ 0.791 35° 31′ 0.745 38° 04′ 0.695 40° 36′ 0.660 43° 08′ 0.599 45° 40′ 0.547 48° 12′ 0.491 50° 44′ 0.449 53° 16′ 0.388 55° 48′ 0.365 58° 20′ 0.338 60° 51′ 0.309 63° 22′ 0.296 65° 53′ 0.299 68° 25′ 0.294 70° 56′ 0.293 73° 27′ 0.296 75° 58′ 0.290 78° 29′ 0.272 80° 59′ 0.249 83° 29′ 0.215 85° 59′ 0.183 88° 29′ 0.156 90° 59′ 0.140	20° 14′ 0.936 22° 47′ 1.03 25° 19′ 0.891 27° 51′ 0.775 30° 23′ 0.759 32° 56′ 0.716 35° 28′ 0.666 38° 00′ 0.610 40° 32′ 0.566 43° 04′ 0.519 45° 36′ 0.469 48° 08′ 0.440 50° 39′ 0.408 53° 11′ 0.373 55° 42′ 0.356 58° 14′ 0.339 60° 45′ 0.294 63° 17′ 0.270 65° 48′ 0.242 68° 19′ 0.230 70° 50′ 0.218 73° 21′ 0.220 75° 52′ 0.231 78° 23′ 0.230 80° 53′ 0.226 83° 24′ 0.162 90° 54′ 0.139	19° 40′ 1.01 22° 13′ 0.998 24° 46′ 0.835 27° 18′ 0.695 29° 50′ 0.629 32° 22′ 0.569 34° 55′ 0.530 37° 27′ 0.510 39° 59′ 0.476 42° 31′ 0.437 45° 03′ 0.393 47° 35′ 0.355 50° 07′ 0.226 55° 10′ 0.296 57° 41′ 0.279 60° 13′ 0.254 62° 45′ 0.229 65° 16′ 0.213 67° 47′ 0.200 70° 18′ 0.188 72° 49′ 0.197 75° 20′ 0.187 77° 50′ 0.182 80° 20′ 0.174 82° 50′ 0.167 85° 21′ 0.152 87° 51′ 0.140 90° 21′ 0.125	20° 22' 1.02 22° 53' 0.845 25° 26' 0.745 27° 58' 0.712 30° 30' 0.646 33° 02' 0.585 35° 35' 0.560 38° 07' 0.540 40° 39' 0.490 43° 11' 0.483 45° 43' 0.433 48° 15' 0.347 53° 19' 0.343 55° 50' 0.311 58° 31' 0.282 60° 53' 0.263 63° 25' 0.241 65° 56' 0.228 68° 27' 0.224 70° 58' 0.218 73° 29' 0.211 78° 30' 0.206 81° 00' 0.201 88° 31' 0.140 91° 01' 0.121	20° 04' 1.02 22° 35' 1.00 25° 08' 0.880 27° 40' 0.762 30° 12' 0.714 32° 44' 0.665 35° 17' 0.581 37° 49' 0.559 40° 21' 0.504 42° 53' 0.465 45° 25' 0.438 47° 57' 0.397 50° 29' 0.380 53° 01' 0.365 55° 32' 0.352 58° 03' 0.291 63° 07' 0.257 65° 48' 0.223 68° 09' 0.204 70° 40' 0.186 73° 11' 0.188 75° 42' 0.193 78° 12' 0.200 80° 42' 0.207 83° 12' 0.196 85° 43' 0.179 88° 13' 0.161 90° 43' 0.141	20° 22′ 1.08 22° 54′ 0.956 25° 27′ 0.804 27° 59′ 0.703 30° 31′ 0.644 33° 03′ 0.702 35° 35′ 0.636 38° 07′ 0.650 40° 39′ 0.608 43° 11′ 0.561 45° 43′ 0.500 48° 15′ 0.440 50° 46′ 0.387 55° 49′ 0.307 58° 21′ 0.290 60° 52′ 0.285 63° 22′ 0.277 65° 54′ 0.275 68° 25′ 0.293 70° 56′ 0.281 73° 26′ 0.272 75° 57′ 0.261 78° 27′ 0.233 80° 58′ 0.293 70° 56′ 0.128 73° 26′ 0.272 75° 57′ 0.261 78° 27′ 0.233 80° 58′ 0.135 80° 58′ 0.135 90° 59′ 0.153 88° 28′ 0.171 85° 59′ 0.153 88° 28′ 0.135 90° 59′ 0.128 93° 49′ 0.127 96° 19′ 0.139 98° 48′ 0.138 101° 18′ 0.133 103° 47′ 0.130 106° 17′ 0.108 108° 46′ 0.083 111° 16′ 0.098 113° 44′ 0.050 121° 11′ 0.050 123° 40′ 0.050 126° 09′ 0.052 128° 38′ 0.052 121° 11′ 0.050 126° 09′ 0.052 128° 38′ 0.058 131° 06′ 0.063 133° 35′ 0.062 136° 03′ 0.063 138° 31′ 0.065 140° 59′ 0.062
$ heta_{ m c.m.}$	Er $(\sigma/\sigma_R)_{c.m.}$		Υb (σ _R) _{c.m.}	Τ θ _{c.m.} (σ	$(\sigma/\sigma_R)_{c.m.}$
20° 14' 1.07 22° 46' 1.05 25° 17' 1.00 27° 49' 1.03 30° 20' 0.97 32° 51' 0.89 35° 23' 0.83 37° 54' 0.80 40° 26' 0.735 42° 57' 0.675 45° 29' 0.577 48° 00' 0.520 50° 31' 0.463 53° 02' 0.439 55° 33' 0.414 58° 04' 0.404 60° 35' 0.370 63° 06' 0.332 65° 37' 0.326 68° 08' 0.302	70° 38′ 0.278 73° 09′ 0.254 75° 39′ 0.238 78° 10′ 0.211 80° 40′ 0.199 83° 10′ 0.193 85° 40′ 0.184 88° 10′ 0.173 90° 40′ 0.167	19° 37′ 1.05 22° 09′ 1.04 24° 40′ 1.07 27° 12′ 1.07 29° 43′ 0.980 32° 14′ 0.912 34° 46′ 0.882 37° 17′ 0.838 39° 49′ 0.770 42° 20′ 0.715 44° 52′ 0.635 47° 23′ 0.568 49° 54′ 0.517 52° 25′ 0.491 54° 56′ 0.441 57° 27′ 0.438 59° 58′ 0.428 62° 29′ 0.390 65° 00′ 0.387 67° 31′ 0.332	70° 01′ 0.304 72° 32′ 0.265 75° 02′ 0.262 77° 33′ 0.238 80° 03′ 0.224 82° 33′ 0.220 85° 03′ 0.209 87° 33′ 0.189 90° 03′ 0.194	18° 57' 1.07 21° 29' 1.13 24° 00' 1.11 26° 32' 1.09 29° 03' 1.05 31° 34' 0.903 34° 06' 0.858 36° 37' 0.782 39° 08' 0.762 41° 40' 0.733 44° 11' 0.649 46° 42' 0.605 49° 13' 0.525 54° 15' 0.499 56° 46' 0.473 59° 17' 0.453 61° 48' 0.421 64° 18' 0.407 66° 49' 0.380	69° 19′ 0.349 71° 50′ 0.320 74° 20′ 0.292 76° 51′ 0.276 79° 21′ 0.261 81° 52′ 0.248 84° 22′ 0.237 86° 53′ 0.230 89° 23′ 0.221

Table I. $(\sigma/\sigma_R)_{c.m.}$ vs θ_c	m. for 15-MeV deuteron	s elastically scattered from	several elements (continued).
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W	Pt	Au	Pb
$\theta_{\mathrm{c.m.}}$ $(\sigma/\sigma_R)_{\mathrm{c.m.}}$	$\theta_{ m c.m.}$ $(\sigma/\sigma_R)_{ m c.m.}$	$ heta_{ m e.m.}$ $(\sigma/\sigma_R)_{ m e.m.}$	$ heta_{ m e.m.} (\sigma/\sigma_R)_{ m e.m.}$
20° 13′ 1.10	18° 57′ 1.00	20° 06′ 1.00	18° 55′ 1.00 91° 46′ 0.195
22° 45′ 1.07	21° 28′ 1.05	22° 37′ 1.00	21° 27′ 1.10 94° 16′ 0.259
25° 16′ 1.04	23° 59′ 1.03	25° 09′ 1.03	23° 58′ 1.10 96° 46′ 0.242
27° 48′ 1.08	26° 30′ 1.06	27° 40′ 1.02	26° 29′ 1.15 99° 16′ 0.208
30° 19′ 1.03	29° 01′ 1.04	30° 11′ 1.00	29° 00′ 1.12 101° 46′ 0.202
32° 51′ 0.951	31° 32′ 1.02	32° 42′ 0.948	31° 32′ 1.02 104° 15′ 0.160
35° 22′ 0.957	34° 03′ 0.919	35° 13′ 0.917	34° 03′ 0.970 106° 45′ 0.127
37° 53′ 0.891	36° 34′ 0.842	37° 44′ 0.862	36° 34′ 0.990 109° 14′ 0.148
40° 24′ 0.836	39° 05′ 0.828	40° 22′ 0.828	39° 05′ 0.944 111° 43′ 0.124
42° 57′ 0.801	41° 37′ 0.789	42° 53′ 0.801	41° 36′ 0.904 114° 13′ 0.094
45° 26′ 0.747	44° 08′ 0.738	45° 24′ 0.741	44° 07′ 0.869 116° 42′ 0.113
47° 58′ 0.656	46° 39′ 0.686	47° 55′ 0.705	46° 38′ 0.806 119° 12′ 0.105
50° 29′ 0.614	49° 10′ 0.658	50° 26′ 0.665	49° 09′ 0.779 121° 42′ 0.068
53° 00′ 0.554	51° 41′ 0.601	52° 57′ 0.663	51° 40′ 0.703 124° 11′ 0.068
55° 31′ 0.529	54° 12′ 0.564	55° 28′ 0.601	54° 11′ 0.671 126° 40′ 0.073
58° 02′ 0.511	56° 43′ 0.524	57° 59′ 0.505	56° 42′ 0.584 129° 9′ 0.068
60° 32′ 0.456	59° 14′ 0.474	60° 30′ 0.458	59° 12′ 0.534 131° 38′ 0.069
63° 03′ 0.414	61° 45′ 0.452	63° 00′ 0.436	61° 43′ 0.503 134° 7′ 0.068
65° 34′ 0.398	64° 15′ 0.425	65° 31′ 0.405	64° 14′ 0.493 136° 36′ 0.065
68° 04′ 0.384	66° 46′ 0.411	68° 01′ 0.381	66° 44′ 0.489 139° 5′ 0.063
70° 35′ 0.348	69° 16′ 0.384	70° 32′ 0.377	69° 15′ 0.448
73° 05′ 0.330	71° 47′ 0.363	73° 02′ 0.364	71° 45′ 0.426
75° 36′ 0.292	74° 17′ 0.340	75° 33′ 0.331	74° 15′ 0.410
78° 06′ 0.272	76° 48′ 0.321	78° 03′ 0.320	76° 46′ 0.386
80° 37′ 0.257	79° 18′ 0.301	80° 34′ 0.304	79° 16′ 0.346
83° 07′ 0.249	81° 49′ 0.286	83° 04′ 0.276	81° 46′ 0.309
85° 37′ 0.231	84° 19′ 0.256	85° 34′ 0.247	84° 16′ 0.246
88° 08′ 0.228	86° 49′ 0.237	88° 04′ 0.236	86° 46′ 0.263
90° 38′ 0.218	89° 19′ 0.229	90° 34′ 0.212	89° 16′ 0.254

controlled, so that during the course of the experiments reported here it may have varied between 14.6 and 15.0 MeV. The variations during a measurement of a single angular distribution, however, are an order of magnitude smaller than this. Tests were made to ascertain that beam misalignment or scattering in the target do not cause errors in the Faraday cup beam current measurement even for the thickest and heaviest targets. The current integrator was frequently checked with a known current from a standard electrolytic cell.

Variations in target thickness (from the average value

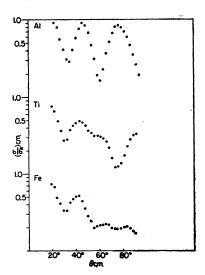


Fig. 2. Typical 15-MeV deuteron elastic scattering angular distribution data in the mass region A = 25 to 60. Data points were taken at intervals of 2.5°. Ordinates ratios of total to Rutherford cross section at the scattering angles $\theta_{\rm c.m.}$, which are the abscissas (in center-of-mass system). Each point is the average of two or three independent experimental determinations. The scatter of the adjacent points tells the size of errors in the experiment.

calculated from the total area and mass) was a particularly difficult problem in some cases; this was checked by use of several different targets of the same element.

The angle of the incident beam was checked frequently by measuring elastic scattering from a heavy element (target Au or Pt) at equal angles on each side of the nominal 0° and a zero angle correction was obtained from the intensity ratio between the two measurements by use of the Rutherford formula as follows: If the ratio of count rates at $+20^{\circ}$ and -20° (where these angles are measured from the nominal 0°) is R, the correction in degrees to the zero angle, Δ , is given by $\Delta = 2.5$ (1-R). The zero angle varied from day to day by less than $\frac{1}{2}$ deg, and was uncertain by less than $\frac{1}{4}$ deg. The acceptance angle for scattered particles was usually 2°, but in some cases it was less. Errors in setting the scattering angle and in alignment were negligible. A detailed discussion of errors in experiments of this type has been given by Low.6

The most important errors due to angular uncertainties are eliminated by the method of determining absolute cross sections. This was done by normalizing all measurements to those for Pt at 20° and 25°, and assuming the Pt cross sections to be given correctly by the Rutherford formula at these small angles as is expected from optical model calculations. This method eliminates errors not only from angular uncertainties but from uncertainties in the geometry of the system

 $^{^6}$ C. A. Low, M.S. thesis, University of Pittsburgh, 1960 (unpublished).

Fig. 3. Typical data in the mass region A = 85 to 95. Zr data from 90 to 140° is the result of a single experiment. (See also caption for Fig. 2.)

arising from vertical bending by the fringe fields of the magnet.

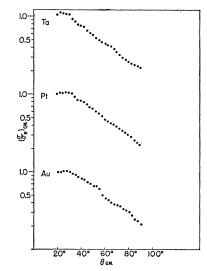
Errors from imperfect performance of electronic instrumentation were checked by simultaneously feeding the signal from the center crystal into two different amplifier-discriminator-scalar systems. Results from the two systems were recorded in all runs. The scalars had 100 kC and 10 MC maximum count rates, respectively, so that errors due to excessive count rates were immediately detected. In all cases, count rates were kept conservatively low, but the lengths of runs were adjusted to give 1% statistical accuracy. Measurements were made at 2.5° intervals between 20° and 90° for all targets, and up to 140° for Ni, Zr, Sn¹²⁰, and Pb. All angular distributions were measured at least twice on separate days (many were done three or more times) and in no case was there a significant discrepancy in the shape of the angular distributions. There were discrepancies in the determinations of absolute cross sections, so that a large number of separate runs were made to check

Fig. 4. Typical data in the mass re-

gion A = 180 to 200.

See also caption for

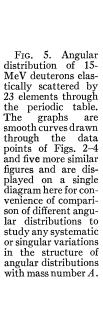
Fig. 2.)

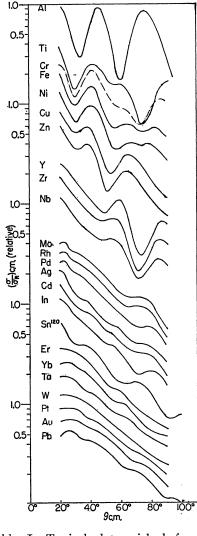


absolute cross sections at 20° and 25°. The discrepancies were found to be due to variations in thickness of the Au foils used for the early calibrations, so that Pt was used for calibrations in the later work.

RESULTS AND DISCUSSION

The values of $(\sigma/\sigma_R)_{\rm c.m.}$ at different values of $\theta_{\rm c.m.}$ for the different elements studied in the present work





are given in Table I. Typical data picked from throughout the entire range of investigation are presented in Figs. 2–4. Figure 5 presents smooth curves drawn through the data points for all the elements and put together closely for studying the systematic and singular changes in structure as one goes through the periodic table. Such features as are immediately obvious from an examination of Fig. 5, are discussed below:

(1) (σ/σ_R) vs θ shows oscillations that are quite sharp in light nuclei but are increasingly damped as the mass increases. The oscillations are the well-known diffraction effect; their damping with increasing mass is well

known from other elastic scattering angular distribution experiments and can be explained by the volume absorption model⁷ by changing the depth of the imaginary potential with nuclear mass, or by the surface absorption model⁸ where it follows from the decrease in surface to volume ratio with mass.

(2) Maxima and minima of the distribution shift to smaller angles with increasing A as one expects from Fraunhofer diffraction theory, according to which⁹

$$(d\sigma/d\Omega)_{\text{Elastic}} = (Kr^2)^2 [J_1(qr)/qr]^2, \qquad (1)$$

where J_1 is the first-order Bessel function, r is the interaction radius, K is the momentum of the incident particle, and q is the momentum transfer which, for elastic scattering, is

$$q = 2K\sin(\theta/2). \tag{2}$$

It is clear from (1) that the oscillations in the angular distributions arise from the Bessel function, so that the location of maxima and minima should depend only on the argument of the Bessel function, $q\mathbf{r}$, which is proportional to $A^{1/3}\sin(\theta/2)$. To test this, the value of $A^{1/3}\sin(\theta/2)$ for each maximum and minimum in the angular distributions is plotted vs A in Fig. 6. According to the theoretical discussion above, the points in Fig. 6 lie along horizontal lines; this expectation is at least roughly fulfilled.

(3) Perhaps the most surprising feature of Fig. 5 is the few cases where there is a sharp change of angular distribution pattern over a small mass change. The most striking situation of this type is between Nb⁹³ and Mo which has an average mass of 96. Another almost equally striking change is that between In¹¹⁵ and Sn¹²⁰. There are also striking differences between In¹¹⁵ and Cd (av A = 112), between Rh¹⁰³ and Pd (av A = 106), and between Ni and Cu (av A = 59 and 64, respectively). Such sharp changes are clearly contra-

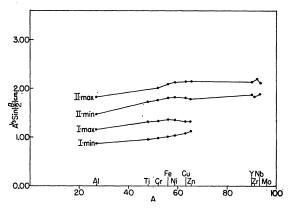


Fig. 6. Systematics of positions of maxima and minima from Fig. 5. The ordinate is $A^{1/3} \sin(\theta/2)$ and the abscissa is A, where θ is the scattering angle and A is the mass number of the scatterer.

⁹ See, for example, J. S. Blair, Phys. Rev. 115, 928 (1959).

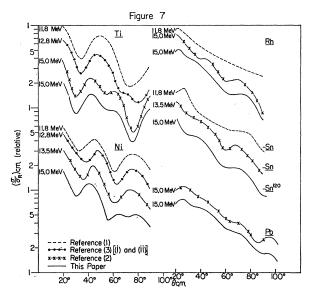


Fig. 7. The angular distributions of Ti, Ni, Rh, Sn, and Pb taken from the references indicated in the legend.

dictory to the basic assumptions of the optical model, and should be studied further.

(4) σ/σ_R averaged over the oscillations decreases with θ in all elements. The slope is mild in Al but gets large in medium and heavy elements though the change in slope is not very perceptible in the mass region A=60-200. Variations with mass in the aforesaid slope are much larger in $(p,p)^{10}$ and $(\alpha,\alpha)^{11}$ elastic scattering angular distributions.

A comparison with the 11.8 MeV data of Igo et al.¹ shows essential agreement with this data and the expected shift of peaks and valleys to smaller angles relative to their angular distributions. There are, however, discrepancies with the 15 MeV data of Cindro and Wall² in that the angular distributions for Rh and Pd, and for Cu and Fe are found to be similar in this work.

Some information on the energy dependence of elastic deuteron scattering is shown in Fig. 7, where data from references 1–3 are compared with this work. In general, the energy dependence is slowly varying, but there are a few nontrivial differences. For example, the minimum in the nickel angular distribution at $\sim 70^{\circ}$ does not appear at other energies.

An optical-model analysis of the data presented here is in progress.¹²

ACKNOWLEDGMENTS

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 ⁷ R. D. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954).
 ⁸ F. E. Bjorklund, S. Fernbach, and N. Sherman, Phys. Rev. **101**, 1832 (1956).

¹⁰ B. L. Cohen and R. V. Neidigh, Phys. Rev. 93, 282 (1954).
¹¹ I. S. Shapiro, Usp. Fiz. Nauk 75, 61 (1961) [translation: Soviet Phys.—Usp. 4, 674 (1962)].
¹² R. M. Drisko (private communication).