K^--Mesonic Atoms

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Computational techniques previously used by the authors for the treatment of μ^-- and π^--mesonic atoms are extended to K^--mesonic atoms and x-ray yields for the most important transitions are calculated as a function of atomic number. It is shown that the experimental measurement of these yields will provide a sensitive determination of the imaginary part of the K^--nucleus optical potential. The level shifts due to the meson scattering interaction are also discussed. In principle, one can relate the level shifts to the real part of the zero-energy scattering lengths with the help of the theory of Deser *et al.*; however, the calculations show that, for the ground state, such measurements will be more difficult to perform than in π^--mesonic atoms because of the low yields of the K lines. Therefore, an attempt was made to predict the shifts of the $2p$ state. It turns out that these shifts are expected to be measurable, both from the point of view of magnitude and yield, for the elements from carbon to fluorine. The Auger electron spectra from stopping K^- mesons in nuclear emulsion are derived from the cascade calculations. In particular, it is shown that the expected fraction of K^- captures associated with at least one Auger electron of more than 15-keV energy is 4% in the light (C,N,O) and 88% in the heavy (AgBr) emulsion elements. An experiment on Auger electrons in nuclear emulsion is described. Whereas the theoretical and experimental numbers of K^- stoppings associated with electrons are in good agreement, there are systematic discrepancies in the details of the energy and multiplicity spectra which are, most probably, due to experimental uncertainties. The level shifts in π-mesonic atoms are briefly discussed in an Appendix.

1. INTRODUCTION

In two previous papers[1,2] we have considered the problem of μ^-- and π^--mesonic atoms. It was shown there that the observed intensities of K and L lines can be fitted by simple cascade calculations using the conventional radiative and Auger transition probabilities. Such a theory involves two additional assumptions, namely, the initial distribution of the mesons in the state from which the cascade is started, and, in the case of π^- mesons, a parameter characterizing the strength of absorption of the meson by the nucleus.

The initial population of the $n=14$ level which was chosen as the starting point of the cascades could be inferred from a comparison of the predicted *relative* intensities, i.e., the ratios of the basic K_α (or L_α) lines to the total intensity of all K (or L) lines, with the experimental values. Similar information was obtained from the *absolute* intensities of the π-mesonic L x rays which were shown to be insensitive to the strength of absorption, but to depend on the initial population. From all these data it was concluded that the initial distribution in the $n=14$ level is more peaked towards higher l's than the statistical $(2l+1)$ distribution. If a "modified statistical" distribution of the type $(2l+1)\exp(al)$ is assumed, the best fit is obtained for $a\sim0.2$ for both μ^-- and π^--mesonic atoms. This peaking towards higher angular momentum states can be qualitatively understood by the gradual enrichment of states of high l when the meson cascades down.[3]

The parameter describing the strength of nuclear absorption of π^- mesons was obtained from a fit of the calculated K yields as a function of Z to the experimental values. We obtained for the mean lifetime of π^- in nuclear matter of normal density the value $\tau_\pi=2.75\times10^{-23}$ sec. According to the relation $\tau=\hbar/2W$, the imaginary part of the optical potential $(V+iW)$ is then $W=-12$ MeV, in good agreement with the values derived from low-energy scattering of π^- mesons in complex nuclei.[4–6]

The only problem which is still not understood is that of the "missing x rays"[1,2,7–17]: the lightest elements (Li for K lines and C, N, and O for L lines) show abnormally

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[1] Y. Eisenberg and D. Kessler, Nuovo Cimento **19**, 1195 (1961), hereafter referred to as I.

[2] Y. Eisenberg and D. Kessler, Phys. Rev. **123**, 1472 (1961), hereafter referred to as II.

[3] Note that the circular orbits, $l=n-1$, act as a "trap" for mesons: Once a meson has arrived in a circular orbit, it stays in circular orbits all along the cascade.

[4] A. Pevsner, J. Rainwater, R. E. Williams, and S. J. Lindenbaum, Phys. Rev. **100**, 1419 (1955).

[5] R. E. Williams, W. F. Baker, and J. Rainwater, Phys. Rev. **104**, 1695 (1956).

[6] W. F. Baker, H. Byfield, and J. Rainwater, Phys. Rev. **112**, 1773 (1958).

[7] M. B. Stearns and M. Stearns, Phys. Rev. **105**, 1573 (1957).

[8] M. Stearns and M. B. Stearns, Phys. Rev. **107**, 1709 (1957).

[9] M. B. Stearns, M. Stearns, and L. Leipuner, Phys. Rev. **108**, 445 (1957).

[10] D. West, in *Reports on Progress in Physics* (The Physical Society, London, 1958), Vol. 21, p. 271.

[11] T. B. Day and P. Morrison, Phys. Rev. **107**, 912 (1957).

[12] J. Bernstein and T. Y. Wu, Phys. Rev. Letters **2**, 404 (1959).

[13] T. B. Day and J. Sucher, Air Force Office of Scientific Research Report TN-59-771, 1959 (unpublished).

[14] N. A. Krall and E. Gerjuoy, Phys. Rev. Letters **3**, 142 (1959).

[15] R. A. Ferrell, Phys. Rev. Letters **4**, 425 (1960).

[16] M. A. Ruderman, Phys. Rev. **118**, 1632 (1960).

[17] J. L. Lathrop, R. A. Lundby, V. L. Telegdi, and R. Winston, Phys. Rev. Letters **7**, 147 (1961).

low x-ray yields, which cannot be explained by theory. The present status of this problem is discussed in a separate paper.[18] The discrepancy is limited to x rays of low energy (less than ~80 keV) and a slight possibility still exists that it might be experimental.

In view of the good general agreement of the previous calculations with the measurements,[19] it seemed to us worthwhile to extend this work to K-mesonic atoms. To our knowledge, no experiments on K-mesonic atoms have been reported so far, but it seems that the recent development of intense K^- beams will allow such experiments to be performed in the near future. It is hoped that the present work will be of some help to experimenters in this field in planning the experiments.

The methods of calculation are reviewed briefly in the next section. In Sec. 3, the yields of L, M, N, etc., x rays are given for all elements. For the purpose of the calculation, three different values for the absorption

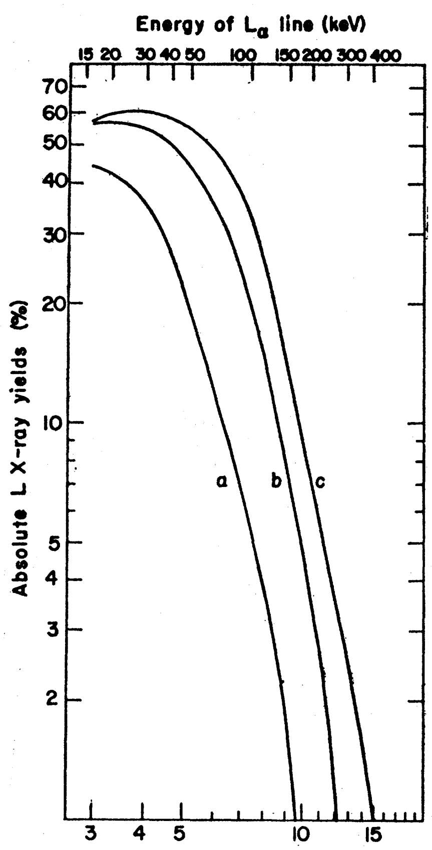

FIG. 1. Calculated total yield of L x rays in K-mesonic atoms as a function of Z. The corresponding L_α energies are plotted at the top of the figure. In Figs. 1 to 5 and 7, the curves a, b, and c were derived from 3 values of τ_K: 0.6, 3.0 and 6.0×10^{-23} sec. The actual values will probably lie slightly above curve a.

[18] Y. Eisenberg and D. Kessler, preceding paper, Phys. Rev. **130**, 2349 (1963).

[19] The Auger-electron spectra from π^- capture in nuclear emulsion predicted in II were recently confirmed experimentally by J. E. Cuevas and A. G. Barkow, Nuovo Cimento **26**, 855 (1962).

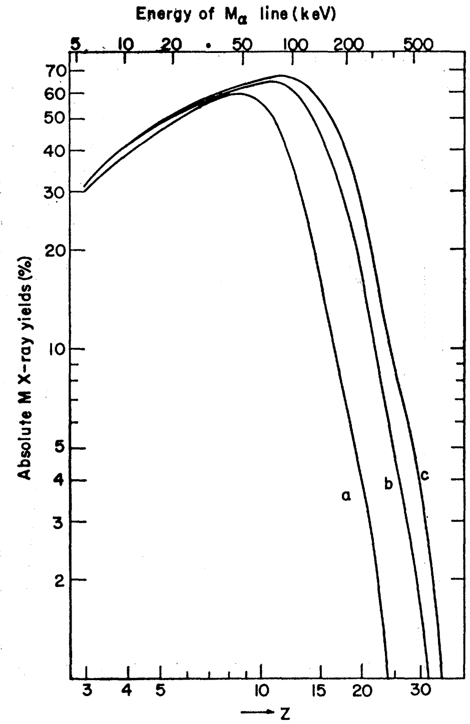

FIG. 2. Calculated total yield of M x rays.

parameter τ_K were assumed. It is shown that in certain regions of Z the x-ray yields are very sensitive to this parameter and will permit its experimental determination. As mentioned above, τ_K is simply related to the imaginary part of the K^- meson-nucleus optical potential.

In Sec. 4, we present the calculated Auger-electron spectrum for K^- absorption at rest in nuclear emulsion. The practical importance of this result lays in the fact that the observation of Auger-electrons associated with capture-stars are generally considered as an indication of capture by a *heavy* emulsion element.[20–23] We also describe in this section an emulsion experiment which was performed in order to test the theory. As expected, due to the considerable experimental difficulties, the agreement between the measurements and calculations is only rough, but, on the other hand, no striking discrepancy is found.

In Sec. 5, we discuss the expected level shifts of K^--mesonic atoms. These calculations are based on the treatment of Deser *et al.*[24] (slightly modified, see Appendix) which relates the level shifts to the real part of the meson-nucleon scattering lengths.

[20] M. Nicolic, Y. Eisenberg, W. Koch, M. Schneeberger, and H. Winzeler, Helv. Phys. Acta **33**, 221, 237 (1960).

[21] C. Grote, I. Hauser, U. Kundt, U. Krecker, K. Lanius, K. Lewin, and H. W. Meier, Nuovo Cimento **14**, 532 (1959).

[22] E. B. Chesik and J. Schneps, Phys. Rev. **112**, 1810 (1958).

[23] Y. Eisenberg, M. Friedmann, G. Alexander, and D. Kessler, Nuovo Cimento **22**, 1 (1961).

[24] S. Deser, M. L. Goldberger, K. Baumann, and W. Thirring, Phys. Rev. **96**, 774 (1954).

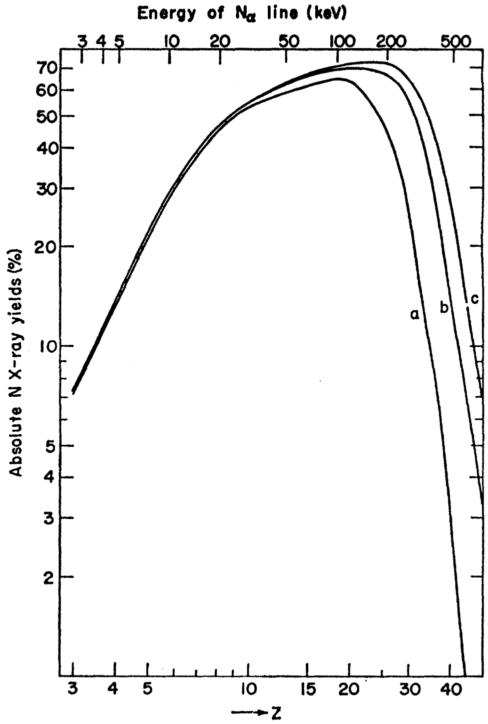

FIG. 3. Calculated total yield of N x rays.

2. CALCULATIONS

The calculations were conducted along similar lines as for μ- and π-mesonic atoms (I and II).

For rather historical reasons the $n=14$ level was chosen as the starting point of the cascades: In μ-mesonic atoms this is the highest level which is still inside the K-electron shell so that unperturbed hydrogenic wave functions could be used for all mesonic states. For the same reason, the K^- cascade could, in principle, start as high as at about $n=30$. This would, however, involve prohibitively lengthy calculations without much effect upon the final results. Indeed, our previous experience with mesonic atoms has shown that the results are quite insensitive to the starting point of the cascades, as long as the latter is chosen high enough (in the region where the Auger effect predominates). The reason for this is that the Auger transitions occur mostly in small steps ($n\rightarrow n-1$) so that the meson distribution among angular momentum states changes relatively slowly in cascading down. In the high Z elements ($Z>40$) the radiative transitions dominate in the $n=14$ level, since there is not enough energy for the emission of an Auger electron from the K shell. However, for these elements the radiation also occurs mostly in small steps and thus it is still true that the angular momentum distribution changes slowly. From a different point of view, the above choice of initial level ensures that all Auger electrons of more than 15 keV are included in the calculations of the elements of nuclear emulsion (see Sec. 4).

For μ- and π-mesonic atoms, we have shown previously (I and II) that the initial distribution in the $n=14$ level must be assumed to be more peaked towards the high l values than a statistical $(2l+1)$ distribution. All experimental data which are sensitive to the initial distribution, viz., the ratios of higher to basic x-ray yields of μ- and π-mesonic atoms and the π-mesonic absolute L x-ray yields for the light elements ($Z<20$), could be fitted with the help of a "modified statistical" distribution of the type $(2l+1)\exp(0.2l)$. Recent experimental results agree with our predictions.[25]

In the present calculation we have again assumed the above initial distribution. There seems to be no obvious reason to suspect the K^- initial distribution to be drastically different from that of both π^- and μ^- mesons. So far, there are no experimental measurements of K^--mesonic atoms available in order to test this hypothesis, the Auger-electron data from emulsions (see Sec. 4) not being sufficiently accurate for this purpose. On the other hand, the task of predicting the initial distribution theoretically seems quite hopeless, as the problem of the penetration of the meson through the atomic electron cloud is extremely complicated. The correct initial distribution will, therefore, have to be determined by experiment: Some of our predictions, such as the M yields for $Z<10$, the N yields for $Z<20$ and the ratios of higher to basic x ray yields are moderately sensitive to the initial distribution and can serve the purpose of testing our assumption. However, the L x-ray yields and the M, N, etc., yields for higher Z are insensitive to the assumed initial distribution and relatively free from any ambiguity on this account.

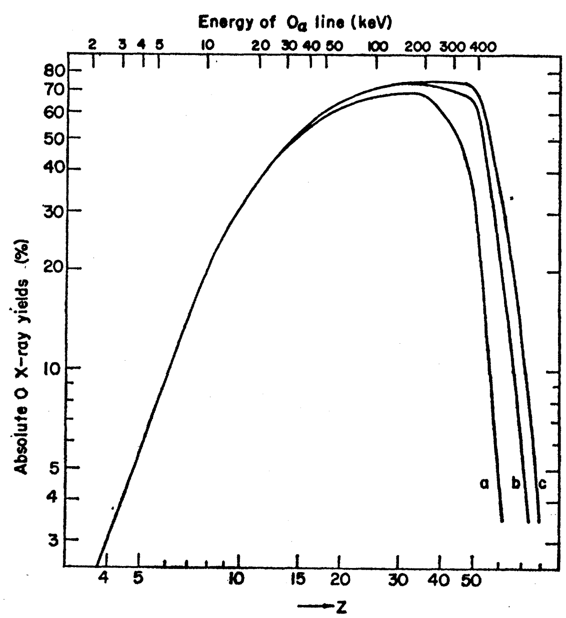

FIG. 4. Calculated total yield of O x rays.

[25] C. Scott Johnson, E. P. Hincks, and H. L. Anderson, Phys. Rev. **125**, 2102 (1962).

The Auger and radiative transitions were calculated with the help of formulas of I.

The capture probability of the K^- meson by the nucleus was taken as

$$P_c = J/\tau_K, \quad (1)$$

where τ_K is the mean lifetime of the meson in nuclear matter of normal density and infinite extension. It is implicitly assumed that τ_K can be obtained from an effective nuclear potential for the K^- meson and does not depend upon the relative meson-nucleus angular momentum (velocity independent potential). As in the case of π-mesonic atoms (II), we have here a one-parameter theory. This assumption will be checked by comparison with the experimental x-ray yields. It will be shown in the next section, however, that our calculations will lend themselves to a reinterpretation in terms of τ_K as a function of angular momentum, if necessary. The calculations were performed for 3 different values of τ_K ($\tau_K = 0.6$, 3.0, and 6.0×10^{-23} sec) in order to show how sensitive the results are to the choice of this parameter. It is hoped that the correct value of τ_K will be found in this range. (For comparison, in II we found for negative pions $\tau_\pi \approx 2.75 \times 10^{-23}$ sec.)

J in formula (1) is the "overlap integral"

$$J = 4\pi \int \rho(r) |\psi(r)|^2 r^2 dr, \quad (2)$$

where $\rho(r)$ is the density distribution of the nucleus and $\psi(r)$ the mesonic wave function. Since absorption takes place mainly from high angular momentum states, as previously argued by Jones[26] and confirmed in this work, the tail of the distribution of $\rho(r)$ plays an important role. Therefore, a Fermi distribution of the following form was chosen:

$$\rho(r) = \rho(0)(1 + e^{-c/a})/(1 + e^{(r-c)/a}), \quad (3)$$

where $\rho(0)$ is the density of matter at the center of the nucleus; the mean lifetime τ_K refers to this "normal" density. c and a are the parameters determined by electron scattering and given by Elton.[27]

For reasons of simplicity, we have again taken hydrogenic wave functions for $\psi(r)$. This is permissible as a first approximation since the relevant states are dominated by a high centrifugal barrier compared to which the nuclear potential and the deviation from a pure Coulomb field are small, at least on the fringe of the nucleus where the absorption takes place. It is hoped that by comparing the present calculations with experiment, we will be able to learn enough about the K-meson nuclear potential, so that in a second approximation the modification of the meson wave function by the nuclear potential and deviation from Coulomb field could be taken into account.

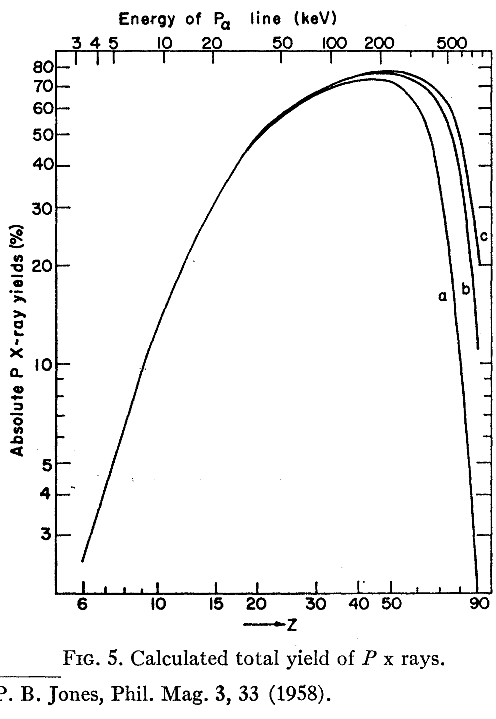

FIG. 5. Calculated total yield of P x rays.

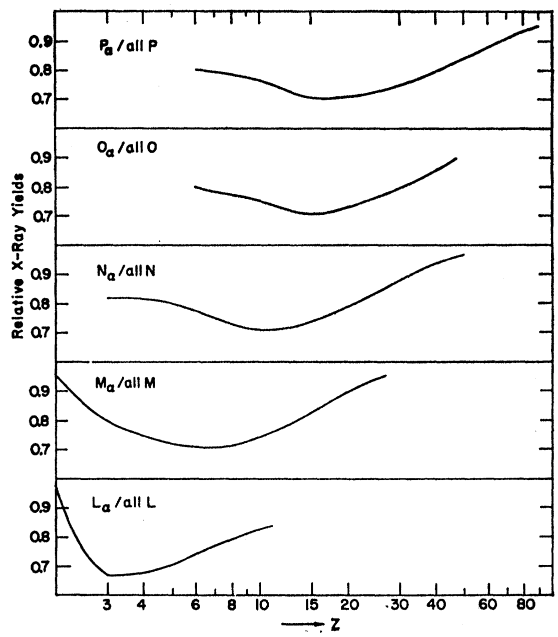

FIG. 6. Predicted relative yields of the basic α lines to the total intensity of given order for $\tau_K = 3.0 \times 10^{-23}$ sec.

3. RESULTS

The yields of L, M, N, O, and P x rays per stopped meson as a function of Z are presented in Figs. 1 to 5. For orientation, the energies of the corresponding α lines are also plotted in the figures. Three different values of τ_K were assumed.

The general shape of all curves, is very similar to that of π-mesonic L rays which is already familiar to us: At

[26] P. B. Jones, Phil. Mag. 3, 33 (1958).

[27] L. R. B. Elton, *Nuclear Sizes* (Oxford University Press, London, 1961).

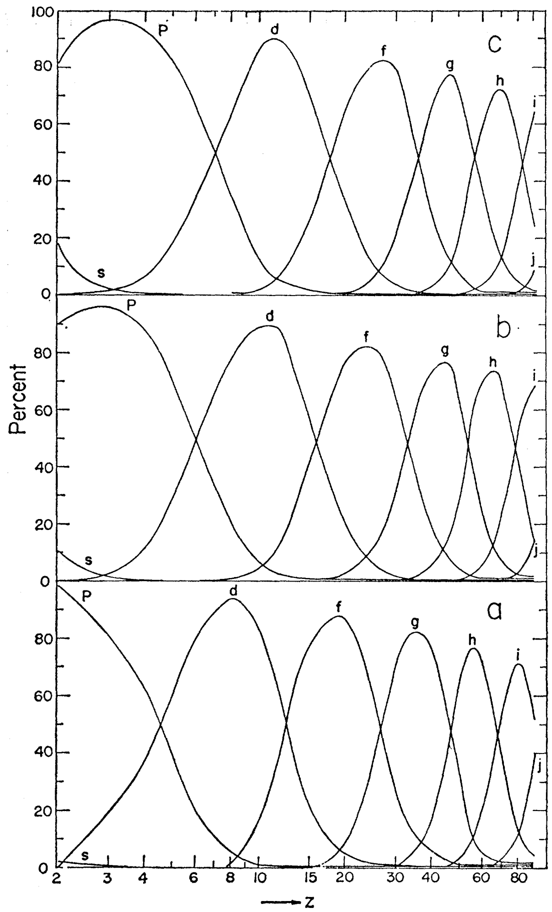

FIG. 7. Percentage contribution of different angular momentum states to K^- absorption as a function of Z for 3 assumed values of τ_K (see Fig. 1).

first, the curves rise due to the competition between radiation (proportional to Z^4) and Auger effect (roughly Z-independent). Thereafter, there is a steep decrease when absorption from the upper levels sets in.

It is clearly seen from the figures that the increasing part of the yield curves is almost unaffected by the value of τ_K. This part of the curve depends, however, upon the initial meson distribution in a way similar to that of the π-mesonic L rays treated in II. In order not to overload the figures, only the results derived from the "modified statistical" distribution are shown. Qualitatively speaking, different initial populations lead to curves of roughly the same slope, but displaced towards lower or higher x-ray yields according to whether the initial population is chosen more uniform or more peaked towards high angular momenta. This part of the curves will, therefore, be used to test the initial population.

The decreasing part of the yield curves, on the contrary, is dominated by the absorption parameter and depends only weakly on the initial distribution. This part will be used to determine τ_K. As pointed out in Sec. 2, it might be that τ_K is not a constant, but depends upon the relative angular momentum of the meson-nucleus system. This will be checked by experiment; the L x rays arise almost exclusively from transitions from d states to the $2p$ state, so that the decrease in the yield curve is produced by d-state absorption and determines $\tau_K(d)$. Similarly, $\tau_K(f)$, $\tau_K(g)$, etc., could be determined from the M, N, etc., yield curves, respectively.

Figure 6 shows the predicted ratios of the basic α lines to the total intensity of given order. These ratios are insensitive to the absorption parameter but depend upon the initial population. Different initial populations give rise to curves of similar shape, but higher ratios are obtained for distributions which are more peaked than for those which are more uniform. The relative yields plotted in Fig. 6 were again obtained from the "modified statistical" distribution.

Figure 7 shows the percentage contribution of different angular momentum states to K^- absorption as a function of Z and for 3 assumed values of τ_K. It is seen that for a given element only three l states at most contribute to absorption and for certain elements one has almost pure p, d, or f absorption. This fact may find important experimental applications.

It should be pointed out here that all the results presented refer to isolated atoms. Thus, in Fig. 7 no account was taken of the fact that in *liquid* helium, for instance, the p-absorption will be quenched by Stark mixture of levels with high n.[28–30] A measurement of the K_α yield in liquid helium will be of particular interest, as the amount of Stark quenching can be deduced from a comparison of the measured and predicted yields.

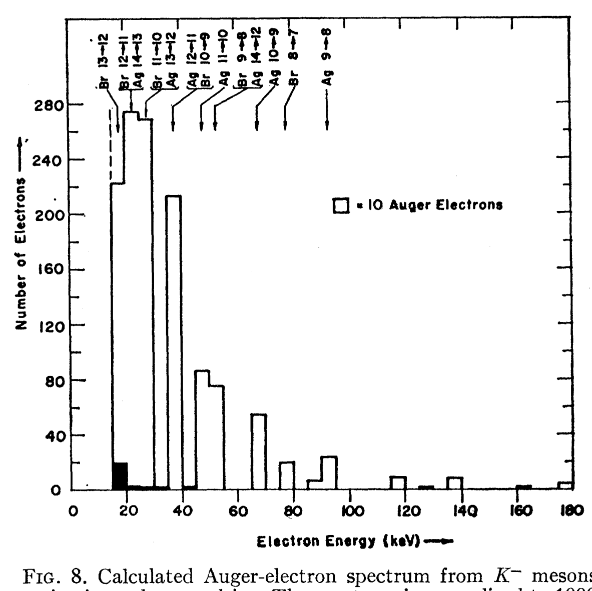

FIG. 8. Calculated Auger-electron spectrum from K^- mesons stopping in nuclear emulsion. The spectrum is normalized to 1000 stopping mesons, and all electrons in events where more than one Auger electron is emitted, are included separately. (Light elements are shown in black.)

[28] T. B. Day, G. A. Snow, and T. Sucher, Phys. Rev. Letters **3**, 61 (1959); Phys. Rev. **118**, 864 (1960).
[29] T. B. Day and G. A. Snow, Phys. Rev. Letters **5**, 112 (1960).
[30] M. Leon and H. A. Bethe, Phys. Rev. **127**, 636 (1962).

TABLE I. Auger electron multiplicity distribution (percent) for K^- absorption in nuclear emulsion [for an assumed distribution of 43% (C, N, O) and 57% AgBr].

Numbers of electrons (≧15 keV) per capture		0	1	2	3	4	5	6	Mean multiplicity per atom
Calculated	AgBr	7.1	12.8	14.3	12.8	7.4	2.3	0.3	2.31
	C, N, O	41.0	2.0	0	0	0	0	0	0.05
	Total	48.1	14.8	14.3	12.8	7.4	2.3	0.3	1.25
Experiment		47.1	36.0	11.7	4.5		0.7		0.76

4. AUGER ELECTRONS ASSOCIATED WITH K^- CAPTURE IN EMULSIONS

From the cascade calculations we can derive the energy spectrum and total yield of Auger electrons associated with K^- capture at rest in nuclear emulsions. For comparison with the experimental data one needs to know, in addition, the branching ratio of K^- captures between the light elements of the emulsion (C, N, O) and the heavy elements (AgBr). In the following we have used the recent value obtained by Pevsner *et al.*[31] for μ^- captures at rest: 43% in C, N, O and 57% in AgBr. We shall later show that this proportion agrees with our experimental results for K^- mesons as well.

The calculated Auger-electron energy spectrum is shown in Fig. 8, where cascade C^{IV} and $\tau_K=3.0\times10^{-23}$ sec have been used; however, the results are insensitive to the value of τ_K. We have plotted the energy spectrum of all the electrons having an energy ≧15 keV. Figure 8 represents, therefore, the total yield of electrons and if, for example, two electrons are associated with the capture of a single K^- meson, both are counted separately.

Another quantity which could be, in principle, directly compared with experiment is the electron multiplicity distribution—namely, the calculated number of Auger electrons (over 15 keV) per K^- capture star. This was obtained by performing a Monte Carlo calculation (200 trials) using our C^{IV} cascades. The results of this calculation are presented in Table I.

Quantitative comparison between the calculations and the experimental results is not straightforward because of the following difficulties: (a) The relation between the number of grains (or range) of a given δ ray and its energy is rather poorly known; (b) if several electrons are emitted from the same K^- star, one is not always able to differentiate between them; and (c) the nuclear prongs quite often obscure the low-energy electrons.

The experiment consisted in examining 278 randomly chosen K^- stoppings for Auger electrons and blobs. All clearly defined Auger electrons or blobs of 3 or more grains were recorded by two observers. In cases of disagreements the events were critically re-examined by three observers. About 10% of all cases were classified as "ambiguous" when there remained some doubt about the correct classification. The main ambiguities were between "blobs" and recoils, Auger electrons and very short prongs, or Auger electrons and background electrons. In such cases each alternative was given the weight of $\frac{1}{2}$ event.

[31] A. Pevsner, R. Strand, L. Madansky, and T. Toohig, Nuovo Cimento **19**, 409 (1961).

For estimating the electron energies, we have assumed that each grain is equivalent to 5-keV kinetic energy. This is possibly an overestimate, but it is somewhat compensated by a probable underestimation in the grain counting, especially of blobs. This method differs from the one of Chesick and Schneps,[22] who used the electron-range criterion. The main reason for the choice of our method was our desire to use a unique criterion for *all* electrons, whether they appear as clearly resolved electrons or as blobs.

In order to compare our results with those of Chesick and Schneps[22] and Grote *et al.*[21] we have divided our events into four classes: 47% of all capture stars showed no electron nor blob, 29% were accompanied by one or more electrons of 3 or more grains but no blob, 14% showed a blob of 3 or more grains but no electron, and 10% showed both a blob and one or more Auger electrons. Thus, there were 39% "electron events" and 24% "blob events," which compare with 32 and 29%, respectively, in the work of Chesick and Schneps[22] and 25.5% and 37.7% in the work of Grote *et al.*[21] From this comparison it would seem that there are quite important differences in the criteria defining blobs and electrons used by the three groups; it is, however, noteworthy that the sum of "electron" and "blob" events is almost identical (about 62%).

In the following, we shall drop the descriptive term "blob" and count each blob as a single electron.

The experimental Auger electron spectrum is shown in Fig. 9 and the observed multiplicity distribution in Table I. The latter is not notably different from the one observed by Chesick and Schneps.[22] Comparison with the theoretical predictions shows a considerable loss of slow Auger electrons (below 30 keV) and of higher multiplicities. Part of these discrepancies can be attributed to the unavoidable assignment of each blob to a *single* electron. In addition, there seems to exist a systematic observational loss of electrons of low energy especially when other electrons are present. Experimentally, one would expect the number of stars without electrons to be rather reliable; in fact, Table I shows a good agreement between the calculated and measured

TABLE II. The calculated shift of the K_α line of K-mesonic He^4, Li^6, and Li^7.

Isotope	Klein-Gordon energy $E(K_\alpha)$ (keV)	Finite size correction (keV)	Vacuum polarization (keV)	$E(K_\alpha)$ corrected (keV)	$\Delta E(K_\alpha)$ Calculated line shift (keV) Sol. I[a]	Sol. II[b]
He^4	34.84	−0.17	+0.27	34.94	−1.23	(+23.2)
Li^6	81.58	−2.57	+0.74	79.75	−6.26	(+117.8)
Li^7	82.53	−2.02	+0.74	81.25	−5.94	(+154.4)

[a] $\text{Re}a_0=-0.22\pm1.07$ F; $\text{Re}a_1=+0.02\pm0.33$ F.
[b] $\text{Re}a_0=-0.59\pm0.46$ F; $\text{Re}a_1=+1.20\pm0.06$ F (reference 33).

number of zero-electron stars. Since about 88% of the K^- captures in AgBr and only 4% of the captures in C, N, O are expected to give rise to one or more electrons of more than 15 keV, the number of zero-electron stars must be very sensitive to the assumed ratio of K^- captures in the heavy and light elements. One can, therefore, consider the above agreement as an indication that the capture law is not significantly different for μ^- and K^- mesons, as mentioned at the beginning of this section.

In view of the good agreement of the predicted energy spectrum of Auger-electrons of more than 30 keV, with the experiment, we conclude that the discrepancy in the number of low-energy electrons is, very probably, experimental.

5. LEVEL SHIFTS

The present section will be devoted to a discussion of the possibility of measuring the level shifts in K^--mesonic atoms. The importance of such measurements lays in the fact that they can provide, in principle, an unambiguous determination of the real part of the elementary kaon-nucleon phase shifts.

Four possible sets of scattering lengths, solutions $(a\pm)$ and $(b\pm)$, were derived from low-energy K^- cross sections by Dalitz and Tuan.[32] Two different solutions, based upon a more complete set of low-energy data, were recently published by Humphrey and Ross.[33] In addition to the ambiguity as to which set is the correct one, the errors on some of the phase shifts are so large that even the sign is not certain. It, therefore, seems very desirable to supplement the low-energy scattering data with measurements on mesonic atoms.

The energy shifts of the K lines of π-mesonic atoms were calculated by Deser *et al.*[24] and related to the pion-nucleon phase shifts. Reasonable agreement was obtained with the experimental data[34] and it can be improved by a trivial modification of Deser's formula, as shown in the Appendix.

Brueckner[35] calculated an additional contribution to the phase shift due to the virtual absorption of pions by nucleon pairs. However, his results show much less agreement with experiment than the Deser term alone.[34] A possible explanation for this discrepancy was offered by Wolfenstein.[36]

For K mesons, it is expected that the modified Deser formula alone (see Appendix) will give reasonable results, especially when it is remembered that kaons are mostly absorbed by *one*-nucleon interactions and that the effect of absorption by single nucleons is already included in the phase shifts.

Unfortunately, it appears from Fig. 7 that K^- mesonic atoms will produce K lines only in the lightest elements up to Li. Moreover, in liquid hydrogen, the K lines will be quenched by the Stark effect mentioned at the end of Sec. 3. In helium, the calculated yield of K_α is about 3–10% for isolated atoms. In liquid helium, the K_α yield is again expected to be reduced by an hitherto unknown amount,[29] so that it is not clear if this line will be measurable at all. We have already pointed out the interest of a measurement of the K_α yield in liquid helium, which will permit a determination of the amount of Stark quenching. If this effect is not too strong, one can hope to measure the shift $\Delta E(K_\alpha)$ in helium. It will probably also be possible to observe the shift of the ground state in lithium, where the expected yield is as low as 1–4%, depending on the value of τ_K. In Table II we present the calculated energy of the K_α line of K-mesonic He^4, Li^6, and Li^7, corrected for finite size and vacuum polarization. The finite size correction was calculated according to Cooper and Henley[37] and the nuclear radii were taken from Elton,[27] $R=2.07$ F for He^4, $R=3.41$ F for Li^6 and $R=2.97$ F for Li^7. For the vacuum polarization we used the expression given by Mickelwait

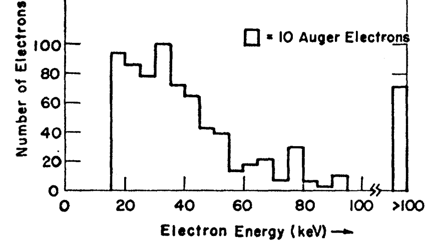

FIG. 9. Experimental Auger-electron spectrum from K^- mesons in nuclear emulsion, normalized to 1000 stoppings.

[32] R. H. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) **10**, 307 (1960).
[33] W. E. Humphrey and R. Ross, Phys. Rev. **127**, 1305 (1962).
[34] M. B. Stearns, *Progress of Nuclear Physics* (Pergamon Press, Inc., New York, 1957), Vol. 6, p. 108.
[35] K. A. Brueckner, Phys. Rev. **98**, 769 (1955).
[36] L. Wolfenstein, Bull. Am. Phys. Soc. **2**, 39 (1957).
[37] L. N. Cooper and E. M. Henley, Phys. Rev. **92**, 80 (1953).

and Corben.[38] The line shifts were derived from the solutions I and II of Humphrey and Ross[33] with the help of formula (A2) of the Appendix. Because of the large uncertainties in the experimental scattering lengths, the calculated shifts ought to be considered only as indicative of the order of magnitude of the expected effect. In addition, the Born approximation is evidently no longer valid in the case of the large attractive optical potential corresponding to solution II, where more refined methods of calculation will have to be used. On the other hand, it is clear that experimental values of the line shifts will help to reduce considerably the present uncertainties of the scattering lengths. In view of the importance of the problem, it would seem that the K_α yield of the lithium isotopes is not so prohibitively low that the K_α line could not be measured.

Considering the limited possibilities of measuring the shift of the ground state, it is interesting to inquire into the feasibility of a measurement of the shift of $2p$ states in some elements. In the case of π-mesonic atoms, the shifts of the L lines are so small that they are at the limit of present experimental techniques. There has been a discrepancy between the measured value of the shifts and theoretical predictions, as discussed in the Appendix. Before we attempt to predict the expected shifts of the K-mesonic L lines we must try to get a better insight into this problem.

In analogy with the formula

$$\Delta E_s = -(2\pi\hbar^2/\bar{\mu})(\mathrm{Re}\delta_s/k)|\psi_s(0)|^2, \qquad (4)$$

relating the level shift of the s state to the real part of the s-wave phase shift δ_s of the meson-proton system,[24] one can write the level shift of the p state

$$\Delta E_p = -(6\pi\hbar^2/\bar{\mu})(\mathrm{Re}\delta_p/k^3)|\nabla\psi_p(0)|^2. \qquad (5)$$

In (4) and (5), $\bar{\mu}$ is the reduced mass of the $K-p$ system, k the wave number, and $\psi(0)$ the wave function at the origin. Introducing the derivative of the wave function of the $2p$ state, $|\nabla\psi(0)|^2=(1/32\pi)(Z/a_0)^5$, a_0 being the mesonic Bohr radius, one obtains

$$\Delta E_{2p}/E_{2p} = -\tfrac{3}{2}(Z\alpha)^3(\mathrm{Re}\delta_p/\eta^3), \qquad (6)$$

where η is the center-of-mass momentum in units of μc.

For complex nuclei, one is tempted, in analogy with the case of the s states, to assume that the effects of the nucleons are additive and to replace δ_p in (6) by $Z(\delta_{p0}/2)+(Z+2N)(\delta_{p1}/2)$, where δ_{p0} is the kaon-nucleon p phase shift in the state of isotopic spin 0 and δ_{p1} the p phase shift in the state of isotopic spin 1. This leads to an expression similar, apart from the factor $\frac{3}{2}$, to the formula of Wolfenstein[39] for the case of π-mesonic atoms. Wolfenstein's expression, however, does not agree with the experiment (see Appendix), the reason being the following: If one assumes that the scattering of the meson by the nucleus can be described by an optical potential $V(r)$, one obtains in the Born approximation the following expression for the phase shift δ_l[40]:

$$\delta_l = \frac{2\mu}{\hbar^2}\int_0^\infty kr^2 j_l^2(kr)V(r)dr, \qquad (7)$$

where $j_l(r)$ is the spherical Bessel function of order l. In the case of s waves, one obtains for $kR \ll 1$, where R is the "radius" of the potential

$$\delta_s = -\frac{2\mu k}{\hbar^2}\int V(r)r^2dr = -\frac{\mu k}{2\pi\hbar^2}\int V(r)d\tau. \qquad (8)$$

Therefore, assuming a constant average optical potential, the s phase shift turns out to be proportional to the volume of the nucleus or to the number of nucleons present. This justifies the usual assumption of simple additivity of the contribution of all nucleons to the total phase shift. However, in the case of p waves, (7) leads to

$$\delta_p = -\frac{2\mu k^3}{9\hbar^2}\int V(r)r^4dr. \qquad (9)$$

This shows that the assumption of additivity is inconsistent in the case of p waves.

In order to calculate the order of magnitude of the expected level shifts we therefore adopt the following procedure: We use the measured s wave scattering lengths in order to derive $V(r)$ with the help of (8) and then use the same $V(r)$ in (9) in order to calculate δ_p. The line shift is then obtained by (5) applied to the kaon-nucleus system. We use a square well potential, $V(r)=\text{const}$ for $r\leqq R$ and $V(r)=0$ for $r>R$, where R is the nuclear radius taken as $r_0A^{1/3}$ with $r_0=1.4$ F. Also, the zero-range approximation $a_p=\delta_p/k^3$ is sufficient for this problem. For π mesons, this procedure leads to results which are not in disagreement with the experiment, as shown in the Appendix. From (8) one obtains, for nuclei with equal number of protons and neutrons,

$$V(r\leqq R) = -\frac{3\hbar^2}{8\mu}\frac{a_0+3a_1}{r_0^3}. \qquad (10)$$

Using the scattering lengths a_0 and a_1 of solution II of Humphrey and Ross[33] (see Table II), one obtains an attractive potential $V=-32$ MeV, in good agreement with the work of Alles *et al.*[41] and Jones.[42] As it stands, solution I of Humphrey and Ross, although favored by a lower χ^2, would lead to a slightly repulsive potential, in contradiction with the above-mentioned experiments. The errors in the scattering lengths of solution I are, however, so large that no conclusion can be derived from this solution as to the sign and magnitude of the optical

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[39] L. Wolfenstein, quoted by M. Stearns and M. B. Stearns, Phys. Rev. **103**, 1534 (1956); see also the review article, reference 10.

[40] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), Sec. 26.

[41] W. Alles, N. N. Biswas, M. Ceccarelli, and J. Crussard, Nuovo Cimento **6**, 571 (1957).

[42] P. B. Jones, Phys. Rev. Letters **4**, 35 (1960).

TABLE III. The level shifts for $V=-32$ MeV together with the Klein-Gordon energy and the expected yields of the L_α lines.

Element	$E(L_\alpha)$ (keV) (Klein-Gordon energy)	$\Delta E(L_\alpha)$ (keV) (for $V=-32$ MeV)	L_α-yield (%) (for $W=-55$ MeV)
Be^9	27.6	+0.09	35
B^{10}	43.4	0.33	21
C^{12}	63.0	1.15	11
N^{14}	86.3	3.3	7
O^{16}	113.1	8.2	4
F^{19}	144.1	20.4	2
Ne^{20}	177.9	37.7	0.7

potential. From (9) and (5) one obtains

$$\Delta E_{2p}=\frac{1}{120}V(ZR/a_0)^5. \tag{11}$$

The level shifts are presented in Table III, for $V=-32$ MeV together with the Klein-Gordon energy and the expected yields of the L_α lines. The latter are derived from $\tau_K\approx0.6\times10^{-23}$ sec or an imaginary potential $W=-55$ MeV. Note that solution I of Humphrey and Ross[33] corresponds to $W=-64$ MeV and solution II, to $W=-43$ MeV.

From the point of view of the expected yield and magnitude of the shift, it would seem that the measurement is feasible for the elements from carbon to fluorine. One notes that the p level shift is a very sensitive function of the nuclear "radius," so that this type of measurement will give additional information about the nuclear size and shape of the elements considered above. The interpretation of such experiments will, however, require a more sophisticated theory than the crude model used here for evaluation of orders of magnitude only.

We would like to conclude with the following remark: An experiment set up for the measurement of K-mesonic x rays will also yield Σ^--hyperonic x rays as a by-product. The information obtained from such an experiment about the Σ^--nucleon interaction will be particularly interesting in view of the difficulty of observing the absorption and scattering of the Σ's directly. Calculations on Σ-hyperonic atoms are now in progress.

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APPENDIX

Remarks on the Energy Level Shifts in π-Mesonic Atoms

According to Deser *et al.*,[24] the level shift of a meson in an s state can be related to the zero-energy scattering lengths by the formula

$$\Delta E_s=-|\psi_0(0)|^2(2\pi\hbar^2/\bar{\mu})(Za_P+Na_N), \tag{A1}$$

TABLE IV. Percentage shift $\Delta E/E(K_\alpha)$ in π-mesonic atoms.

Element	Experimental shift[a] (%)	Calculated according to Deser *et al.*[b]	Calculated according to formula (A2)
Li^7	3.35±0.3	3.09	2.90
Be^9	4.25±0.2	4.75	4.46
Be^{10}	5.00±0.3	4.42	4.02
B^{11}	7.8 ±0.3	6.87	6.23
C^{12}	6.5 ±0.4	6.37	5.64
N^{14}	6.55±0.2	8.97	7.49
O^{16}	11.7 ±0.5	11.37	9.49
F^{19}	11.4 ±0.5	18.98	14.81

[a] Reference 34.
[b] Reference 24.

where $\psi_0(0)$ is the wave function at the origin, $\bar{\mu}$ is the meson-nucleon reduced mass, a_P and a_N are the zero-energy scattering lengths for protons and neutrons, respectively, and Z and N are the numbers of protons and neutrons of the complex nucleus, respectively. For the ground state in π-mesonic atoms, this formula leads to qualitative agreement with the experimental results, as shown by Stearns.[34] For heavy mesons, or heavy elements, or higher angular momentum states, the approximation consisting of replacing the wave function by its value at the origin becomes rather poor. We, therefore, use the expression

$$\Delta E_s=-(2\pi\hbar^2/\bar{\mu})(Za_P+Na_N)\int|\psi_0(r)|^2\rho(r)d\tau, \tag{A2}$$

where $\rho(r)$ is the nuclear density distribution normalized so that $\int\rho(r)d\tau=1$. Even for the ground state of π-mesonic atoms, the use of (A2) leads to some improvement, as shown in Table IV, especially for F^{19}.

In this table the Orear scattering lengths[43] $a_1=0.16\lambda$ and $a_3=-0.11\lambda$ and a Fermi distribution $\rho(r)$ with the parameters given by Elton[27] were used.

The discrepancy between the $2p$ level shift in π-mesonic atoms and Wolfenstein's formula[39] was discussed in the review articles of West.[10] If we assume that the shifts contributed by the individual nucleons are additive, we obtain a formula similar to that of Wolfenstein (see Sec. 5), but multiplied by a factor $\frac{3}{2}$. This would make the discrepancy even worse: the predicted value for the relative shift $\Delta E/E(L_\alpha)$ of calcium would be -6% against a measured value[44] of $-(1\pm1)\%$. In Sec. 5, we have, however, given reasons why, in our opinion, the assumption of additivity is not correct in the case of p states. Let us now apply the procedure outlined there, namely, to assume that the same average optical potential is responsible for the shifts of s and p states in complex nuclei. From the s-wave scattering lengths of Orear[43] we calculate that the average real optical potential is repulsive, $V\approx+7$ MeV. Formula (11) then leads to $\Delta E/E(L_\alpha)\approx-0.6\%$, which is not in disagreement with the experimental value. To our knowledge, there have, unfortunately, been no measurements on atoms heavier than Ca for a better test of our method.

[43] J. Orear, Phys. Rev. **96**, 176 (1954).
[44] M. Stearns and M. B. Stearns, Phys. Rev. **103**, 1534 (1956).