q_α values obtained in each case were constant to within 0.1%. Such accuracy was possible because charge and number of pulses were simultaneously measured.[12]

The measured uncorrected fractional differences $(q_\alpha - q_c)/q_\alpha$ (when $P_\alpha = P_c$) are listed in column 4. The corrections for the dead time, finite collection time and grid inefficiency are given in columns 5, 6, and 7, respectively. Column 8 lists the corrected values of the fractional differences $(q_\alpha - q_c)/q_\alpha$ and represents the results of this experiment.

The uncertainties of the results are listed in the last column. These were compounded from the standard error of the mean and the estimated uncertainties of the corrections and calibrations. The uncertainty of the grid inefficiency correction was estimated to be 30% of its own value due to the uncertainty in the determination of σ. The uncertainty of the dead-time correction was taken to be 10% of its value due to uncertainties in the knowledge of the filters' time constants. An uncertainty of $\pm 0.1\%$ of the value of q_α was ascribed to the finite-collection-time correction (except in the $Ar+CH_4$ case, where the correction was negligible), because of uncertainties in the knowledge of P_α with two different time constants.

VI. DISCUSSION

As seen in Table I, none of the corrected fractional differences $(q_\alpha - q_c)/q_\alpha$ exceeds the uncertainty of the measurement. Thus, it is safe to say that, in all cases investigated, the slow and fast measurements give the same result to within somewhat better than one half of one percent. In other words, there is no slow (or delayed) component of the alpha current which is detected in a slow measurement but which remains undetected in fast methods based on electron collection.

It is not the task of the present paper to decide whether in past measurements the results of slow or fast methods should be accepted with greater confidence. The experiences of this work indicate, however, that there are more sources of error inherent in the fast method. Two of these, the calibration of pulse heights in terms of absolute charge and effects of insufficient saturation, were eliminated in the present work. It seems likely that this experiment thereby avoided the main sources of discrepancy between the two methods.

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Observations on the Energies of Single-Particle Neutron States*

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The available information on location of neutron single-particle levels is analyzed and the following conclusions are reached: (1) There is strong evidence that the depth of the shell-model potential well varies with symmetry energy, getting shallower as the neutron excess increases. (2) There is a stronger than average interaction between nucleons with the same orbital angular momentum; this causes a level to move down in energy as it fills (self-binding effect), or as a proton state of the same l fills, but the effect seems to be weaker on the $(l-\frac{1}{2})$ neutron state as the $(l+\frac{1}{2})$ neutron state fills. (3) Spin-orbit splittings are extra large when the members of the doublet are in different shells, one full and the other empty; this is attributed to the self-binding effect. (4) The rate of change of binding energy with mass number for a given level, dE/dA, is considerably smaller than calculations would indicate; this may be explained as a decrease in potential well depth with A, or as a velocity dependence giving an effective mass of nucleons in nuclei somewhat larger than the free nucleon mass. (5) The spacings between oscillator shells is somewhat smaller than in the harmonic oscillator potential, and in available calculations for a Saxon potential; this again may indicate an effective mass greater than the free nucleon mass. (6) The l dependence of the energies of shell-model levels is much smaller than given by Nilsson when the levels are empty, but the Nilsson term gives reasonable agreement when the levels are full; this indicates that the self-binding effect increases with increasing l. All of these effects are discussed and quantitative estimates of their magnitudes are given.

INTRODUCTION

THE location of neutron single-particle states by means of stripping reaction studies has recently been reviewed.[1] The excitation energies from reference 1 are listed in Table I, and the absolute binding energies of ground states from mass and reaction Q-value data are listed in Table II. By use of this data, one can readily determine the binding energy of neutrons in the various hole states to the closed-shell nucleus, and the binding energy of neutrons in the various particle states to the closed-shell-plus-one nucleus. A plot of these is given

* Sponsored by the National Science Foundation and the Office of Naval Research.
[1] B. L. Cohen, P. Mukherjee, R. H. Fulmer, and A. L. McCarthy, Rev. Mod. Phys. (to be published).

TABLE I. Excitation energies of various single-particle and single-hole states. All energies are in MeV above the ground state.

O^{15}—Reference a	O^{17}—Reference a	Ca^{39}—Reference b	Ca^{41}—Reference a, c	Ca^{49}—Reference d	Fe^{53}—Reference e
$p_{1/2}$ 0	$d_{5/2}$ 0	$d_{3/2}$ 0	$f_{7/2}$ 0	$p_{3/2}$ 0	$f_{7/2}$ ~2.0
$p_{3/2}$ 6.16	$s_{1/2}$ 0.87	$s_{1/2}$ 2.6	$p_{3/2}$ 2.1	$p_{1/2}$ 2.03	
	$d_{3/2}$ 5.08	$d_{5/2}$ ~7	$p_{1/2}$ 3.9	$f_{5/2}$ 3.6	
			$f_{5/2}$ 6.5		

Fe^{55}—Reference f	Ni^{59}—Reference f	Zr^{89}—Reference g	Zr^{91}—Reference h	Zr^{97}—Reference h	Ba^{137}—Reference i
$p_{3/2}$ 0.11[l]	$p_{1/2}$ 1.4[k]	$g_{9/2}$ 0	$d_{5/2}$ 0	$s_{1/2}$ 0	$d_{3/2}$ 0
$f_{5/2}$ 1.4	$g_{9/2}$ 3.0	$p_{1/2}$ 0.8	$s_{1/2}$ 1.55	$d_{3/2}$ 1.37	$s_{1/2}$ 0.29
$p_{1/2}$ 3.26[l]	$d_{5/2}$ ~5.2		$g_{7/2}$ 2.70	$g_{7/2}$ 1.64	$h_{11/2}$ 0.66
$g_{9/2}$ 3.86	$s_{1/2}$ ~6.2		$d_{3/2}$ 2.70	$h_{11/2}$ >4.0	$g_{7/2}$ 1.40
$d_{5/2}$ ~6.0			$h_{11/2}$ >5.1		$d_{5/2}$ 2.4
$s_{1/2}$ ~7.6					

Ba^{139}—Reference i	Ce^{139}—Reference i	Ce^{141}—Reference i	Pb^{207}—Reference j	Pb^{209}—Reference j
$f_{7/2}$ 0	$d_{3/2}$ 0	$f_{7/2}$ 0	$p_{1/2}$ 0	$g_{9/2}$ 0
$p_{3/2}$ 0.78	$s_{1/2}$ 0.25	$p_{3/2}$ 0.88	$f_{5/2}$ 0.57	$i_{11/2}$ 0.77
	$h_{11/2}$ 0.75	$f_{5/2}$ 1.88	$p_{3/2}$ 0.90	$j_{15/2}$ 1.41
	$g_{7/2}$ 1.34	$h_{9/2}$ ~1.9	$i_{13/2}$ 1.64	$d_{5/2}$ 1.56
	$d_{5/2}$ 2.2	$p_{1/2}$ 2.25	$f_{7/2}$ 2.35	$s_{1/2}$ 2.03
			$h_{9/2}$ 3.47	$g_{7/2}$ 2.47
				$d_{3/2}$ 2.52

[a] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington D. C., 1960, 1962).
[b] C. D. Kavaloski, G. Bassani, and N. Hintz, University of Minnesota Linear Accelerator Laboratory Progress Report, 1962. Also P. E. Cavanaugh, C. F. Coleman, G. A. Garol, B. W. Ridley, and J. F. Turner (private communication). We are greatly indebted to these authors for making their results available in advance of publication.
[c] K. Ramavatram, Bull. Am. Phys. Soc. **7**, 302 (1962).
[d] E. Kashy, A. Sperduto, H. A. Enge, and W. W. Buechner, Bull. Am. Phys. Soc. **7**, 315 (1962).
[e] B. Zeidman and T. H. Braid, Bull. Am. Phys. Soc. **7**, 315 (1962); R. D. Lawson and B. Zeidman, Phys. Rev. **128**, 821 (1962).
[f] R. H. Fulmer, A. L. McCarthy, and B. L. Cohen (to be published).
[g] C. D. Goodman (to be published). We are greatly indebted to Dr. Goodman for making his results available in advance of publication.
[h] B. L. Cohen (to be published); see reference 10 for preliminary results.
[i] R. H. Fulmer, A. L. McCarthy, and B. L. Cohen, Phys. Rev. **128**, 1302 (1962).
[j] P. Mukherjee and B. L. Cohen, Phys. Rev. **127**, 1284 (1962).
[k] This level is slightly filled; a 0.3-MeV correction (from pairing theory) has been applied to correct to the situations where the level is empty.
[l] See footnote b of Table V.

in reference 1. It is the purpose of this paper to analyze this plot for various systematic trends and effects.

A. SYMMETRY ENERGY EFFECT

The data for the Ca and Zr isotopes are plotted in Fig. 1 just as they appear in the figure of reference 1.

TABLE II. Neutron binding energies (MeV) in ground states of various nuclei, and corrections for symmetry energy to the line of maximum beta stability (M.B.S.), which is at the A value listed for that element.

Nucleus	Binding energy (MeV) Actual	Corrected	A for M.B.S.
O^{16}	15.65	15.25	16.5
O^{17}	4.15	4.55	16.5
Ca^{40}	15.81	a	43
Ca^{41}	8.36	a	43
Ca^{48}	9.96	a	43
Ca^{49}	5.14	a	43
Fe^{54}	13.34	12.20	56.3
Fe^{55}	9.29	8.65	56.3
Ni^{59}	9.01	8.1	61.3
Zr^{90}	11.79	a	91
Zr^{91}	7.20	a	91
Zr^{96}	7.81	a	91
Zr^{97}	5.53	a	91
Ba^{138}	8.67	9.07	136
Ba^{139}	4.65	5.25	136
Ce^{140}	9.04	9.04	140
Ce^{141}	5.44	5.64	140
Pb^{208}	7.38	7.48	207.5
Pb^{209}	3.88	4.18	207.5

[a] Corrections applied directly in Fig. 1; see discussion in text.

One is immediately struck by the fact that among the isotopes of a single element, the binding energy of levels that are not filling decreases with increasing A, contrary to the general trend through the periodic table. The only simple explanation for this behavior is that the depth of the shell-model potential depends on the symmetry energy, becoming shallower for neutrons as the neutron excess increases. A similar effect is well established in optical model calculations.[2]

Further evidence for this may be seen from the behavior of ground state Q values near closed shells. All effects usually considered, the size effect (see Sec. D), the self-binding effect (see Sec. B), and the pairing energy act to increase the binding energy as neutrons are added beyond a closed shell. Eventually, of course, the binding energy must decrease as additional neutrons are forced to go into higher single-particle levels, but this effect cannot even begin until the lowest level available is at least half full. Some of the experimental evidence on this point is shown in Table III; much additional evidence of this type could be obtained if odd proton nuclei were included. In Table III, the binding energy decreases as neutrons are added beyond closed shells in every case except in Pb where there is a slight increase. In Pb, one expects the smallest symmetry energy effect, and the largest self-binding and pairing energy effects to overbalance it (the lowest three states,

[2] See, for example, F. G. Perey, in *Proceedings of Padua Conference on Nuclear Reaction Mechanisms, 1962* (to be published).

$g_{9/2}$, $i_{11/2}$, and $j_{15/2}$, all have very large angular momenta and thus large pairing and self-binding energies). Furthermore, in Po where the same states are involved, the binding energy decreases as neutrons are added in agreement with the situation throughout the rest of the periodic table. These effects can only be explained by a symmetry energy dependence for the depth of the shell-model potential well.

A third line of evidence, and one which bears directly on our interest here, derives from a study[3] of the location of the $3s_{1/2}$ neutron state in the $A=86$–140 region. Here a symmetry energy effect of about 0.25 MeV/mass unit was found, and a reasonable behavior for the binding energy could only be obtained if the data for each element were corrected for this effect to the mass of maximum beta stability.[4]

This latter method is therefore used in this paper. A correction for the symmetry energy effect is applied to the ground-state binding energies to extrapolate them to the mass of maximum beta stability. This correction was derived by taking a weighted average of the results of reference 2, the results for Ca and Zr from Fig. 1, and the values derived from Table III and similar data on odd-Z nuclei. The corrections are shown in Table II. The binding energies of all states as derived from these corrected ground-state binding energies and the data of Table I are plotted in Fig. 1. For Ca and Zr, the correction to the mass of maximum beta stability is obtained directly by interpolation of the data for the two isotopes of each. In Fig. 1 the points corresponding to the binding energy of a given single-particle state in various nuclei are connected by straight lines. The pair of horizontal wavy lines marks the separation between full and empty levels.

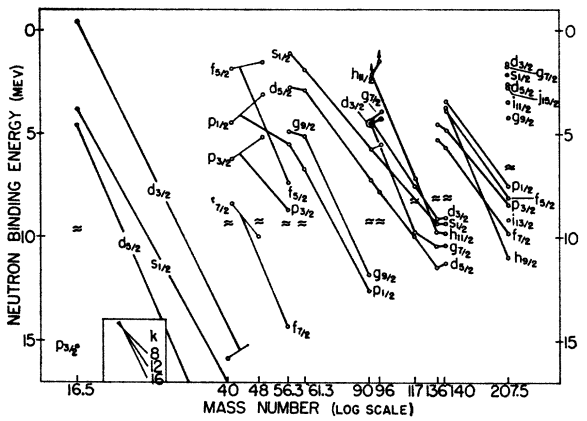

FIG. 1. Binding energy of single-particle and single-hole states corrected for symmetry energy. Data are from Tables I and II. For Sn data, see discussion in text. Double wavy horizontal lines indicate separation between particle and hole states. Note that some states in $A=16$ and $A=40$ are off-scale; their positions may be obtained from Table I. Note that abscissa scale is logarithmic. Insert gives slopes corresponding to different values of k in (1).

[3] B. L. Cohen and R. E. Price, Nucl. Phys. **17**, 129 (1960).
[4] W. H. Sullivan, *Trilinear Chart of Nuclides* (U. S. Government Printing Office, Washington, D. C., 1957).

TABLE III. Comparison of neutron binding energies in nuclei just beyond closed shells.

Nucleus	Neutron number	Bind. en. (MeV)	Nucleus	Neutron number	Bind. en. (MeV)
O^{17}	9	4.15	Mo^{94}	52	9.76
O^{19}	11	3.96	Mo^{96}	54	9.15
Ca^{41}	21	8.36	Ce^{141}	83	5.44
Ca^{43}	23	7.92	Ce^{143}	85	5.17
Ca^{42}	22	11.47	Ce^{142}	84	7.06
Ca^{44}	24	11.13	Ce^{144}	86	7.05
Cr^{53}	29	7.94	Nd^{143}	83	6.62
Cr^{55}	31	6.01	Nd^{145}	85	5.96
Fe^{55}	29	9.29	Nd^{144}	84	7.95
Fe^{57}	31	7.63	Nd^{146}	86	7.49
Fe^{56}	30	11.19	Pb^{209}	127	3.88
Fe^{58}	32	10.02	Pb^{211}	129	4.08
Zr^{91}	51	7.20	Pb^{210}	128	4.93
Zr^{93}	53	6.68	Pb^{212}	130	5.20
Zr^{92}	52	8.65	Po^{211}	127	4.56
Zr^{94}	54	8.23	Po^{213}	129	4.31
Mo^{93}	51	7.88	Po^{212}	128	6.01
Mo^{95}	53	7.30	Po^{214}	130	5.61

It may be noted that the symmetry energy corrections obtained here from the Ca and Zr data are somewhat less than those estimated from the other sources. In Ca where only the p states should be included (see Sec. B), the correction is about 0.16 MeV/mass unit; in Zr where only the s and g states should be considered, the correction is 0.12 MeV/mass unit.

There is clearly some uncertainty in the symmetry energy correction, and the corrections adopted here are not highly reliable. However, it should be kept in mind that the entire correction is relatively small, and none of the results discussed in the rest of this paper would be qualitatively altered if this correction were either doubled in magnitude or neglected completely.

It may be noted that data for Sn^{117} are included in Fig. 1, although the Sn isotopes do not have closed neutron shells and, in fact, all the states in question are partly full. However, a pairing theory analysis was rather successful here[5] except for the fact that the single-particle energies derived from it were more closely spaced than the energies for these same levels at both the beginning and end of the shell. To allow for this, the level spacings derived from the pairing theory analysis were increased by a constant factor. The absolute binding energy was determined by assuming that the $s_{1/2}$ state in Sn is on the line joining the $s_{1/2}$ states in Zr and Ba.

B. INCREASED INTERACTION BETWEEN STATES OF THE SAME ORBITAL ANGULAR MOMENTUM (l)

In spite of the symmetry energy effect, in both Ca and Zr, the subshell that is filling—the $f_{7/2}$ in Ca and the $d_{5/2}$ in Zr—moves down while the other levels move up. This indicates that a subshell moves down in energy as it is filled due to attractions among the nucleons in

[5] B. L. Cohen and R. E. Price, Phys. Rev. **121**, 1441 (1961).

TABLE IV. Magnitude of energy shift due to self-binding effect. Numbers in parentheses are explained in Sec. C of text.

Level	A	Energy shift (MeV) Direct	Energy shift (MeV) Spin-orbit splitting
$1f_{7/2}$	40–48	2.2	
	48		3.4 (1.7)
	54		0 (~2)
$2d_{5/2}$	90–96	0.8	
$1g_{9/2}$	90		1.6
$1h_{9/2}$	140		1.8
$1i_{13/2}$	208		2.3

that subshell; the overlap among their wave functions is better than between a random pair of wave functions because they all have identical radial dependence.[6] This effect has been discussed and emphasized by Baranger[7] and by Kerman[8]; it will be referred to hereafter as the "self-binding effect". It will be considered frequently in this paper.

The magnitude of this effect may easily be estimated from Fig. 1 if one assumes that in its absence, the slopes of the lines would be the same for all states. The results are listed in Table IV under "Direct"; they are 2.2 MeV for the $1f_{7/2}$ state and 0.8 MeV for the $2d_{5/2}$ state.

The self-binding effect is a special case of an extra-strong attraction between particles of the same orbital angular momentum (l) arising from the fact that the radial wave functions overlap perfectly. This perfect overlap is also achieved if the total angular momentum (j) is not the same, and/or if one of the particles is a neutron and the other a proton. One example of this was pointed out by Talmi[9] as an explanation of the shift of the $g_{7/2}$ neutron state between Zr [10] and Sn.[4] This is a case where the $(l-\frac{1}{2})$ neutron state is lowered as the $(l+\frac{1}{2})$ proton state fills. Several other examples of this type have been described.[11] and they can be seen graphically in Fig. 1. In the $A=96$–118 region the slope of the $g_{7/2}$ state is much steeper than that of the s and d states, presumably because the $g_{9/2}$ proton shell is filling in this region. In the $A=140$–208 region where the $h_{11/2}$ proton state is filling, the slope of the $h_{9/2}$ neutron state is much steeper than that of the others; in the $A=42$–54 region where the $f_{7/2}$ proton state is filling, the slope of the $f_{5/2}$ neutron state is extra steep; and in the $A=16$–40 region where the $d_{5/2}$ and $d_{3/2}$ proton states are filling, the slope of the $d_{3/2}$ is greater than that of the $s_{1/2}$. All of these cases were described in reference 11. Two additional cases are the extra steep slopes of the $f_{7/2}$ state in the $A=42$–54 region where the $f_{7/2}$ proton state is filling, and of the $d_{5/2}$ in the $A=16$–40 region where the $d_{5/2}$ and $d_{3/2}$ proton states are filling.

On the other hand, at least two difficulties for this hypothesis are also evident in Fig. 1. In the $A=118$–138 region where the $d_{5/2}$ and $g_{7/2}$ proton states are filling, the $d_{5/2}$, $d_{3/2}$, and $g_{7/2}$ neutron states do not have unusually steep slopes. Also in the $A=90$–138 region, the slope of the $h_{11/2}$ state is fully as steep as that of the $g_{7/2}$ state. This may be evidence for Silverberg's theory[12] that the strong neutron-proton interaction is between states with the same number of radial nodes in their wave functions (the $g_{7/2}$ and $h_{11/2}$ both have only one node). That theory should have many other consequences which must be investigated.

As another example of this type of effect, one would expect the $(l-\frac{1}{2})$ neutron state to be lowered when the corresponding $(l+\frac{1}{2})$ neutron state fills. The evidence for this in Fig. 1 is rather negative.[12a] The $1f_{5/2}$ state is indeed lowered relative to the p states between Ca^{40} and Ca^{48} where the $f_{7/2}$ neutron state is filling, but only by about 1 MeV whereas when the $f_{7/2}$ proton state fills, it is lowered relative to the center of gravity of the $p_{3/2}$ and $p_{1/2}$ states by about 4 MeV. Between Zr^{90} and Zr^{96} where the $d_{5/2}$ neutron state is filling, there is very little lowering of the $d_{3/2}$ state relative to the $s_{1/2}$ and $g_{7/2}$ states.

C. SPIN-ORBIT SPLITTINGS

The energy spacing between the two members of spin-orbit doublets can be readily calculated from Tables I and II, and seen graphically in Fig. 1. These have been discussed in a previous paper,[13] but the method used for applying a correction there is different here. This correction arises from the fact that when the two members of a spin-orbit doublet are in different major shells, their location is available in different isotopes. For example, from Tables I and II we can obtain the binding energy of the $i_{13/2}$ state to Pb^{208} and of the $i_{11/2}$ state to Pb^{209}. Thus a symmetry energy correction is necessary, and is applied as part of the general symmetry energy correction used in obtaining Fig. 1. In reference 13, on the other hand, the two states were located in the same nucleus, but a pairing energy correction was necessary and was taken from theory. It was pointed out by Baranger[14] that theoretical pairing energies can have large errors, and that the method used here is essentially equivalent to the previous method if "empirical" rather than theoretical pairing energy corrections are used. Furthermore, the corrections applied here are only a few-tenths MeV whereas those in reference 13 are frequently more than 1 MeV.

[6] A. de-Shalit and M. Goldhaber, Phys. Rev. **92**, 1211 (1953).
[7] M. Baranger, Phys. Rev. **120**, 957 (1960).
[8] A. Kerman (private communication via N. Austern).
[9] I. Talmi (private communication).
[10] B. L. Cohen, Phys. Rev. **125**, 1358 (1962).
[11] B. L. Cohen, Phys. Rev. **127**, 597 (1962).
[12] L. Silverberg (private communication).
[12a] *Note added in proof.* E. P. Wigner (private communication) has pointed out that this might be due to the well-known repulsion between half-shells resulting from the tensor force. This repulsion is only present between identical particles in $(l+\frac{1}{2})$ and $(l-\frac{1}{2})$ shells and thus would not affect the other cases cited above.
[13] B. L. Cohen, P. Mukherjee, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. **127**, 1678 (1962).
[14] E. Baranger (private communication). The author is greatly indebted to Dr. Baranger for her very helpful suggestion.

In addition to the improved correction for cases where members of the spin-orbit doublet are in different major shells, somewhat more data is now available, and in a few cases, the old data have been refined. The present information, as obtained from Tables I and II is summarized in Table V. The $(2l+1)$ and $A^{1/3}$ dependence of the spin-orbit splitting are tested in the final two columns of Table V. All conclusions from reference 13 are still valid.[14a]

Perhaps the most striking of these conclusions was the observation that the spin-orbit splitting is abnormally large in the cases where the two members are in different major shells. This may be understood as a manifestation of the self-binding effect (discussed in Sec. B) since only in such cases is the upper member of the doublet empty and the lower full. This explanation allows us to estimate the magnitude of the self-binding effect.

To do this, we assume that the difference between the average value of $\Delta E/(2l+1)$ for spin-orbit doublets in which both members are filled or empty is the value for the doublet in question in the absence of the self-binding effect. The difference between this and the observed value, multiplied by $(2l+1)$, is then the shift due to the self-binding effect. The results are listed in Table IV under "spin-orbit splitting." The numbers in parenthesis were obtained as follows: In Fe^{54}, the spin-orbit splitting for the $l=1$ states is abnormally large (see footnote b of Table V); if it were in line with expectations from other data the value in parenthesis would be obtained. In Ca^{48}, an alternative method of calculation is to take the spin-orbit splitting in the absence of self-binding effect from Ca^{40} where the $f_{7/2}$ is empty; if this is done, the value in parenthesis is obtained.

From Table IV it seems evident that the magnitude of the self-binding effect is about 2 MeV for high-spin states, and somewhat less for low-spin states.

TABLE V. Spin-orbit splittings.

Mass	l	ΔE (MeV)	$\Delta E/(2l+1)$	$[\Delta E/(2l+1)]A^{1/3}$
208	1	0.90	0.30	1.77
	2	0.96	0.19	1.14
	3	1.78	0.26	1.51
	4	2.47	0.28	1.62
	6	5.71	0.44[a]	2.6
140	1	1.37	0.46	2.4
	2	2.2	0.44	2.3
	3	1.88	0.27	1.40
	5	6.05	0.55[a]	2.9
138	2	2.4	0.48	2.5
96	2	3.32	0.66	3.0
90	2	2.70	0.54	2.4
	4	6.74	0.75[a]	3.4
54	1	3.15	1.05[b]	4.0
	3	7.0	1.0[a]	3.8
48	1	2.03	0.68	2.5
	3	8.17	1.17[a]	4.3
40	1	1.8	0.60	2.0
	2	~7.0	1.4	4.1
	3	6.5	0.93	3.2
16	1	6.16	2.05	5.2
	2	5.08	1.02	2.6

[a] Two members of doublet are in different major shells.
[b] *Note added in proof.* Recent investigations by J. P. Schiffer and L. L. Lee at Argonne (private communication) and by G. A. Bartholemew at Chalk River (private communication) indicate that some states assigned as $P_{1/2}$ are actually $P_{3/2}$. This reduces the $l=1$ spin-orbit splitting by almost a factor of 2.

[14a] *Note added in proof.* The conclusion from reference 13 that the $A^{1/3}$ dependence does not fit the data has been explained by Donald Sprung of McMaster University who points out (private communication) that the spin-orbit force is a surface force and hence the spin-orbit splitting should be proportional to $A^{2/3}$. His analysis of this data verifies this A dependence.

TABLE VI. Rate of change of binding energy with mass. The third column lists measured values of $k=-dE/d(\ln A)$. The fourth column shows this quantity corrected for the self-binding effect, which is assumed to lower all filled states by 1.0 MeV; this probably underestimates the correction.

State	Mass range	$k=-dE/d(\ln A)$ Measured (MeV)	Corrected (MeV)
$2s_{1/2}$	16–43	14	13
$2p_{1/2}$	43–91	11	10
$2p_{3/2}$	43–56	10	10
$1g_{9/2}$	56–91	14	12
$2d_{5/2}$	56–138	10	8
$3s_{1/2}$	56–138	9	8
$2d_{3/2}$	91–138	11	9
$2f_{7/2}$	136–208	11	9
$3p_{3/2}$	136–208	9	7
$2f_{5/2}$	140–208	11	9
$3p_{1/2}$	140–208	11	8

D. THE RATE OF CHANGE OF BINDING ENERGY WITH MASS NUMBER $-dE/dA$

Aside from the special cases mentioned in Sec. B, the slopes of the lines through give single particle states are roughly constant through the periodic table. This indicates that $-dE/d(\ln A)=k$, a constant, or

$$dE/dA=-k/A. \quad (1)$$

The slopes corresponding to various values of k are shown in an insert in Fig. 1, and a list of values of k obtained from the data of that figure is given in Table VI. All levels are included which are not affected by the special considerations of Sec. B. The final column of Table VI shows the values of k obtained if one assumes the self-binding effect lowers all levels by 1 MeV when they become full. Judging from the data of Table IV, this is an underestimate of that correction. One may thus conclude from Table VI that the experimental value of k is almost certainly not larger than 11 MeV, and is more probably about 9 MeV.

Many calculations of positions of energy levels are available from the literature and can be fitted to (1) over the range of interest to give values of k. Perhaps the most realistic calculation is that by Ross, Mark, and Lawson[15] for a Saxon well. Their results, fitted to (1),

[15] A. A. Ross, H. Mark, and R. D. Lawson, Phys. Rev. **102**, 1613 (1956).

give $k\simeq 15$ MeV. These same authors report calculations for a square well[15] and a Saxon well with velocity dependence[16] (effective mass $-\frac{1}{2}$); these results give $k\simeq 18$ MeV and $k\simeq 22$ MeV respectively. Schröder has reported[17] calculations for a well with linearly sloping sides, and his results give $k\simeq 17$ MeV. Elementary calculations may be made for a harmonic oscillator potential and an infinite square well; these give $k\simeq 16$ MeV and $k\simeq 30$ MeV, respectively.

It is notable that all theoretical calculations give values of k considerably larger than the observed values. Two possible explanations for this may be suggested:

(1) The depth of the shell-model potential well may be decreasing with increasing A. Actually, the quantity calculated here is not dE/dA, but rather $d(E-V_0)/dA$. To explain our discrepancy would therefore require $dV_0/dA\simeq -5$ MeV/A, at least for $A>40$. This corresponds to a decrease of well depth by about 8 MeV between Ca and Pb.

(2) Presumably due to a velocity dependence in the potential, the neutrons in the nucleus may have an effective mass, m^*, greater than the free nucleon mass, m_0. All theoretical calculations for k would thus give results m_0/m^* times the previous results. The discrepancy discussed above could thus be explained by $m^*/m_0\simeq 1.5$. Brown[18] has proposed a similar effective mass for a closely related reason (see Sec. G below), and other evidence for an increased effective mass is discussed in Sec. E.

One interesting application of (1) is to determine the location of the neutron giant resonances. These occur where the $s_{1/2}$ states cross zero binding energy. Extrapolations in Fig. 1 locate these at $A\simeq 155$ and $A\simeq 50$. Both of these results are in good agreement with the neutron data. It is interesting to note that if the symmetry energy correction is not applied, this agreement is destroyed. It would be interesting to look for symmetry energy shifts of the giant resonance in the neutron data.

TABLE VII. Spacings between major shells.

A	N	States included	ΔE (MeV)	$\Delta E\,A^{1/3}$ (MeV)	ΔE— Reference 13
208	6	s, d, g	6.01	35	7.0
	5	p, f			
140	5	p	5.8	30	8.0
	4	s, d			
90	4	s, d	7.3	33	9.0
	3	p			
40	3	p	12.5	43	
	2	s, d			
16	2	s, d	15.8	40	
	1	p			

[16] A. A. Ross, H. Mark, and R. D. Lawson, Phys. Rev. **104**, 401 (1956).
[17] A. Schröder, Nuovo Cimento **7**, 461 (1958).
[18] G. E. Brown (private communication).

E. SPACINGS BETWEEN OSCILLATOR SHELLS

From Tables I and II and Fig. 1, the spacings between "oscillator" shells can be determined. These are shown in Table VII. States belonging to spin-orbit doublets whose members are in different major shells are not included. In addition the f states in Ca^{41} and Ce^{141} are not included so as to equalize the average l value of the states being compared. This eliminates systematic errors due to dependence of energy on l. Actually, inclusion of these f states would not have changed the results significantly. In a harmonic oscillator potential, the spacing between adjacent shells is

$$\Delta E=41\text{ MeV}/A^{1/3}.$$

The fifth column of Table VII shows the experimental spacings multiplied by $A^{1/3}$, so that if the harmonic oscillator potential were realistic, all figures in that column should be 41 MeV. It is apparent that they are somewhat smaller. Actually the data of Table VII should be corrected for the self-binding effect, since the lower of the two shells under consideration is always full while the upper is always empty. This correction would reduce ΔE (by about 1 MeV according to Table IV) and thus further increase the discrepancy with theory.

Some estimate of the gaps predicted by the Saxon potential may be obtained from the calculations of reference 15. They give approximately the values listed in the last column of Table VII. These again are larger than the experimental values of ΔE, and a correction for the self-binding effect would increase the discrepancy. These discrepancies could also be removed by introducing an effective mass somewhat larger than unity.

F. THE l DEPENDENCE OF SHELL-MODEL ENERGIES

It is commonly assumed that within a major shell, the higher orbital angular momentum states lie lower than those of lower l. This seems most logical since these states are degenerate in an oscillator well, and a realistic well differs from an oscillator well principally in supplying added well depth at large radii which should lower the high l states. To take this into account, Nilsson[19] adds to the shell-model potential a term $-Dl^2$, where $D=0$ for $N=0$, 1, 2, $D=0.74A^{-1/3}$ for $N=3$, and $D=0.95A^{-1/3}$ for $N=4$, 5, 6.

The experimental information on this effect is summarized in Table VIII. Here, the energy of the $s_{1/2}$ state is compared with the centers of the d, g, and i states, and the energy of the centers of the p, f, and h states are compared. By "center" is meant the location of the doublet in the absence of a spin-orbit force; since the spin-orbit force lowers the $(l+\frac{1}{2})$ state and raises the $(l-\frac{1}{2})$ state by energies in the ratio $l/(l+1)$, the center is taken as $l/(2l+1)$ of the distance from the $(l+\frac{1}{2})$ state to the $(l-\frac{1}{2})$ state.

The results are compared in Table VIII with the pre-

[19] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **29**, No. 16 (1960).

TABLE VIII. Energy spacings between centers of l substates in same oscillator shell. Entries above and below line are for cases in which all levels involved are empty and full, respectively. For the h and i states, one level is full and the other empty. The theoretical values are calculated according to reference 19.

	s–d ΔE(MeV)			p–f ΔE(MeV)			s–g ΔE(MeV)			p–h ΔE(MeV)			s–i ΔE(MeV)	
A	Exp.	Theor.	A	Exp.	Theor.	A	Exp.	Theor.	A	Exp.	Theor.	A	Exp.	Theor.
16	−1.16	0	40	−0.06	2.2	90	1.8	4.0	140	2.8	5.1	208	4.4	6.7
90	0.32	1.2	140	0.53	1.8									
208	0.09	0.96				208	0.93	3.2						
40	2	0	208	0.99	1.6	138	4.5	3.7						
138	1.15	1.08				140								
140	1.07	1.07												

dictions of Nilsson's potential, $\Delta E = Dl(l+1)$. Cases where the shells are full and empty are considered separately. In the comparisons between the s and g states, an extrapolation of the $g_{9/2}$ state from Fig. 1 is used to obtain the $A = 138$–140 data, and a self-binding correction from Table III is applied to it to obtain the $A = 90$ data.

For cases where the shells are empty (above the line in Table VIII), there seems to be almost no lowering of the higher l states for $l < 4$, and the lowering for $l = 4$ is much less than that given by the Nilsson potential. For cases where the shells are full, on the other hand, the Nilsson potential gives rather good results. This may indicate that the self-binding lowering is larger for high l states; this seems most reasonable as there are more particles involved in the process for higher l.

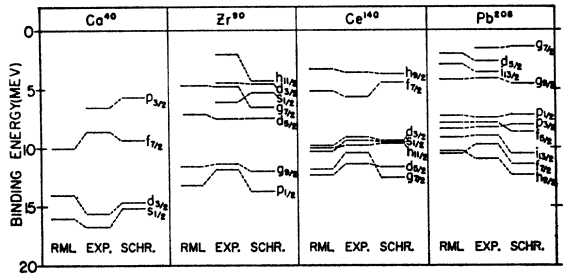

FIG. 2. Level schemes in some closed-shell nuclei. Center spectra are from Fig. 1 and left and right spectra are from theoretical calculations by Ross, Mark, and Lawson (reference 15) and by Schröder (reference 17), respectively. The absolute binding energies in the theoretical spectra have been shifted arbitrarily to obtain best agreement with experiment.

For the h and i states in Table VIII, one member is full and the other empty. The energy spacings as compared with the predictions of the Nilsson term are intermediate between the situation where both levels are full and where both levels are empty.

G. COMPARISON OF LEVEL STRUCTURE WITH CALCULATIONS

The level structure for Ca^{40}, Zr^{90}, Ce^{140} and Pb^{208} are compared in Fig. 2 with the calculations of Ross, Mark, and Lawson[15] and of Schröder.[17] The calculated spectra have been shifted (as a whole) so as to eliminate absolute binding energies as a factor in comparing them with the experimental results. In general, the agreement is quite good considering the simplicity of the models used in the calculations.

Brown[18] has pointed out that the calculated spectra are somewhat more spread-out in energy than the experimental ones. This is an indication of an effective mass greater than unity. A correction to the experimental results for self-binding would increase this spreading somewhat. Furthermore the potential well depth used in reference 15 is probably too shallow, and deepening it would spread the states further apart.

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