Anisotropy of the Conductivity of *n*-Type Germanium at High Electric Fields*

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Current density vs electric field has been measured up to 10 000 V/cm in *n*-type germanium at 77° and 297°K for the field in the [100], [110], and [111] crystallographic directions. At 77°K for 1000 V/cm the drift velocity in the [100] direction is 60% greater than in the [111] direction. By 10 000 V/cm, this difference (the longitudinal anisotropy) has decreased from its maximum value at 1000 V/cm to 15%. The [110] curve lies between the other two. This anisotropy can be explained in terms of the effective mass anisotropy and population changes of the conduction band valleys oriented differently with respect to the field. The drift velocity of a single valley and the change of intervalley scattering rate as a function of field are deduced. The latter increases by a factor of 30 over the range of applied field. At 297°K negligible anisotropy is found.

INTRODUCTION

In a cubic semiconductor the conductivity at high electric fields (when Ohm's law fails) is, in general, anisotropic. Shibuya[1] has calculated the anisotropy for *n*-type germanium. His calculation takes into account the multiellipsoidal nature of the energy surfaces, but allows only for acoustic mode scattering. He finds that at high fields the conductivity is no longer scalar and that, in general, the field **E** and the current density **J** are not parallel. The angle, ϵ, between **J** and **E** has been measured as a function of crystallographic orientation and field by Sasaki *et al.*[2] The results agree qualitatively with the Shibuya theory. However, at 90°K, $\tan\epsilon$ is a strong function of E which has a maximum at about 500 V/cm that is about seven times that predicted by Shibuya.

This discrepancy was explained by Sasaki *et al.*[2] to be caused by a net transfer of electrons among ellipsoids oriented differently with respect to the electric field. This transfer is caused by an unequal intervalley scattering probability among the valleys because of a difference of electron temperature among the valleys.

It has been pointed out by Gunn (private communication) that the anisotropy measurements of Sasaki *et al.*[2] and, to a lesser extent, those of Koenig[3] are subject to geometric errors in the region of high fields. These geometric effects have been minimized in the experiments of Koenig *et al.*,[4] and Schmidt-Tiedemann.[5] Reik and Risken[6] have calculated ϵ as a function of E, taking into account optical mode and intervalley scattering and have obtained nearly quantitative agreement with experiment.

Because of symmetry conditions, $\epsilon \neq 0$ only if the field is not along a rotation axis, so that there are at least three nonequivalent ellipsoid orientations with respect to the field. It is, therefore, cumbersome to make a quantitative comparison of ϵ with theoretical predictions. A quantity which is easier to analyze is the longitudinal current. It is possible to make measurements with the field along rotation axes of the crystal, so that there are at most two nonequivalent sets of ellipsoids.

In this paper measurements at 77 and 297°K of longitudinal current density, J_l, vs longitudinal field, E_l, for three crystallographic orientations [100], [111], and [110] are reported. The relative intervalley scattering rate is calculated from the results on [100] and [111] oriented samples. The **J** vs **E** curve can then be calculated for any orientation. Comparison with experiment is given for [110] orientation. Good agreement is found.

EXPERIMENTAL

Measurements of current density as a function of applied field were made using pulsed fields. The results showed no dependence on pulse repetition rate or duration time ($\cong 0.5\ \mu$sec) for the range of these parameters in the experiment. Samples were cut with an ultrasonic cutter into shapes shown in Fig. 1. The length of the cross bar was 8 or 2.5 mm. They were etched in CP_4 and tin contacts were soldered to the large ends.

The results of measurements of J_l vs E_l for three

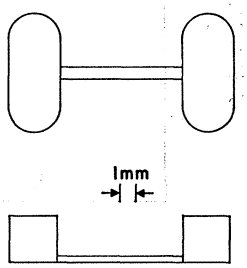

FIG. 1. Shape of samples used for current density vs field measurements. The length of the cross bar was varied between 2.5 and 8 mm.

* A preliminary account of this work was presented at the 1960 APS Meeting, [Bull. Am. Phys. Soc. **5**, 194 (1960)].

[1] M. Shibuya, Phys. Rev. **99**, 1189 (1955).

[2] W. Sasaki, M. Shibuya, and K. Mizuguchi, J. Phys. Soc. Japan, **13**, 456 (1958).

[3] S. H. Koenig, Proc. Phys. Soc., (London) **B73**, 959 (1960).

[4] S. H. Koenig, M. I. Nathan, W. Paul, and A. C. Smith, Phys. Rev. **118**, 1217 (1960).

[5] K. J. Schmidt-Tiedemann (unpublished), referred to by Reik and Risken, reference 6.

[6] H. G. Reik and H. Risken, Phys. Rev. **126**, 1737 (1962).

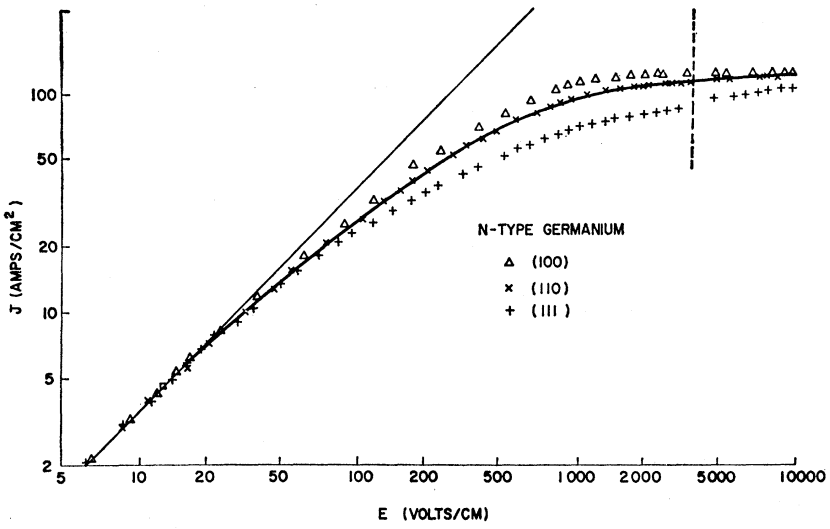

FIG. 2. Current density for three orientations vs electric field at 77°K $\rho=18$ Ω-cm at 297°K. For $E<3300$ V/cm sample length=8 mm; for $E>3300$ V/cm sample length=2.5 mm. The curve is calculated for [110] orientation—see text.

orientations ([100], [110], and [111]) at 77 and 297°K are shown in Figs. 2 and 3, respectively, for typical sets of samples. For measurements at 77°K several sets of samples cut from crystals with resistivities at room temperature between 15 and 40 Ω-cm were used. At 297°K crystals with resistivities between 1.5 and 4 Ω-cm were used. The results did not depend on sample resistivity within these ranges. A large anisotropy is found at 77°K while at 297°K the current density is almost isotropic. We shall discuss the interpretation for this in the next section.

In the data of Figs. 2 and 3 the field is taken to be the applied voltage divided by the length of the cross bar. Corrections for the voltage drop in the ends have been ignored. This causes the estimated field to be too high, especially at low fields. At 77°K for $E<3300$ V/cm samples 8 mm long were used and the error in the field is estimated to be less than 10%. (The effect on the anisotropy between different orientations is much less because similar corrections should be made to the data for all samples.) For $E>3300$ V/cm the sample length was 2.5 mm. The error due to end effects is about 30% at low fields. However, it has been corrected for by ignoring the low-field data for this sample length, where the correction is largest, and translating current density data vertically so that it joins continuously onto the data for the longer samples. It can be seen that all three curves join continuously indicating that error in the anisotropy is small.

The room-temperature data were all taken on samples 2.5 mm long. This does not affect significantly the anisotropy since it is very negligible.

DISCUSSION

The major problem in understanding hot electron effects is the determination of the distribution function. We shall avoid this problem by taking a phenomeno-

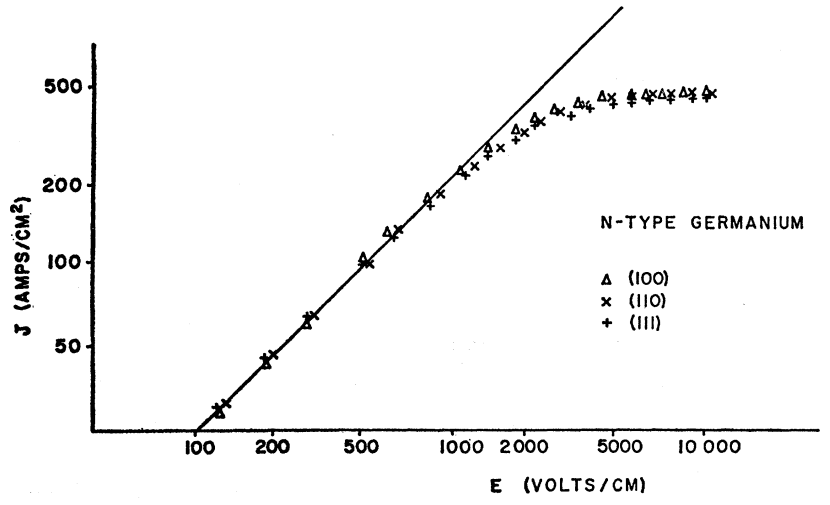

FIG. 3. Current density vs electric field at 297°K. $\rho=4$ Ω-cm.

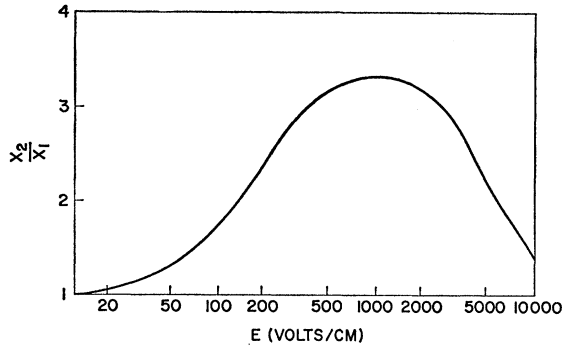

FIG. 4. Ratio of the number of carriers in the cool valleys to the number in a hot valley vs field. $T=77°K$.

logical approach. Price (unpublished) has shown for a single ellipsoidal energy surface in reduced momentum or $\mathbf{k}$ space that if the scattering is isotropic

$$\mathbf{J}=\gamma\cdot\mathbf{E}g(\mathbf{E}\cdot\gamma\cdot\mathbf{E}), \tag{1}$$

where γ is the reciprocal effective mass tensor of the ellipsoid[7] and g is a function that does not depend on field direction except through its argument.

Price also showed that the energy distribution of electrons in the ith valley will depend on $(\mathbf{E}\cdot\gamma\cdot\mathbf{E})$. Thus, the valleys can have different energy distributions if they are oriented differently with respect to the field. Because of the energy dependence of the intervalley scattering, there will be redistribution of carriers among the valleys.

Equation (1) will be true for each valley of a multivalley semiconductor provided that the intervalley scattering time is long compared to the intravalley scattering time so that the distribution function in each valley is determined only by intravalley scattering.

In n-type germanium, where the conduction band valleys are along [111] directions in $\mathbf{k}$ space, the quantities $(\mathbf{E}\cdot\gamma_i\cdot\mathbf{E})$ are all equal for a field in a [100] direction. The intervalley scattering probabilities are all equal and there will be no carrier redistribution. Thus, from the [100] data we can determine the function $g(\mathbf{E}\cdot\gamma\cdot\mathbf{E})$.

For the field in the [111] direction there are two sets of nonequivalent valleys: One "cool" valley with its high mass parallel to the field and three "hot" valleys with their low mass almost parallel to the field. Using the function g determined from the [100] data, we can compute J_l vs E_l for each set. For 297°K if the densities of electrons in all the valleys are assumed to be equal, independent of field, the J_l vs E_l curve obtained is in reasonably good agreement with the data. Thus, there is no appreciable carrier redistribution.

At 77°K, however, the agreement is poor. It is necessary to take account of electron redistribution between the two sets of valleys. It is not unreasonable that electron redistribution should be important at 77°K but not at 297°K. Intervalley scattering occurs with the interaction of phonons of energy 300°K or greater since the transverse acoustic mode transition is forbidden.[8] At 297°K the energy dependence of intervalley scattering is relatively weak since both the absorption and emission of phonons are important. At 77°K, since the high-energy phonons are "frozen out" there can only be emission of phonons. The dependence on energy is stronger since there will only be a small amount of scattering at energies less than the interacting phonon energy. This accounts for the fact that the transverse anisotropy is small at room temperature but large at 77°K.[2,4,5]

If we allow the density of electrons in each valley to change and require the calculation to fit the [111] data at 77°K, we can determine the fraction x_i of electrons in each valley as function of field. The ratio of the number of electrons in the cool valley to that in the hot valley as a function of effective field in the cool valley is shown in Fig. 4. This ratio has a maximum value of 3.4 at about 1000 V/cm. If we assume that the intervalley scattering rate is isotropic, it follows that

$$x_i(E)=(1/\nu_{iv})/\textstyle\sum_i(1/\nu_{iv}), \tag{2}$$

where x_i is the fraction of carriers in the ith valley, ν_{iv} is the rate of intervalley scattering out of the ith valley. From this, ν_{iv} can be determined as a function of effective field. The result is shown in Fig. 5.

We can now calculate the current density for any direction of field. The results for a [110] direction are given by the solid line in Fig. 2. The agreement with the experimental data is seen to be quite good.

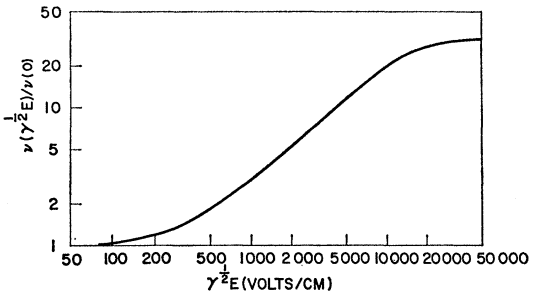

FIG. 5. Normalized intervalley scattering rate as a function of effective field. $T=77°K$.

[7] γ is a diagonal tensor in a coordinate system along the axes of the valley whose elements are $(1/m_l, 1/m_t, 1/m_t)$. $m_l=1.64\ m_0$, $m_t=0.0819\ m_0$, m_0=free electron mass. The numerical values are from R. N. Dexter, H. J. Zeiger, and B. Lax, Phys. Rev. **104**, 637 (1956).

[8] C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).

In Fig. 5 for low fields $\nu_{iv}-\nu_{iv0} \propto E^2$ holds up to about 100 V/cm, where ν_{iv0} is the zero-field intervalley scattering frequency. At intermediate fields the $\nu_{iv} \propto E^{0.87}$. For a Maxwellian electron distribution $\nu_{iv} \propto E$ is expected for electron energies large compared to the phonon involved in intervalley scattering. The tendency toward saturation at high fields is probably associated with the breakdown of the assumption that the effect of intervalley scattering on the distribution function in a single valley can be neglected.

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Optical Absorption in the Presence of a Uniform Field*

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Expressions for the absorption coefficient in the presence of a uniform field, of semiconductors and insulators for both allowed and forbidden transitions are derived. The results are expressed in terms of Airy functions, and the limiting cases examined indicate the tunneling mechanism as a possible explanation for the experimentally observed long-wavelength tails.

1. INTRODUCTION

INSULATING and semiconducting crystals, in the absence of any perturbations, are transparent below a certain photon energy above which the absorption rises rapidly. In practice, however, this edge exhibits a complicated structure often giving an exponential type tail in the long-wavelength region.[1] It has been suggested unsuccessfully that (a) the absence of perfect periodicity, and (b) the presence of impurity states in the forbidden band leads to absorption tails in the long-wavelength region.[2,3]

An alternative explanation of the above effect rests on the tunneling of electron states into the forbidden band due to band bending; such band bending in practice can occur at the surface of the crystal, due to the termination of periodicity, or in the bulk of the crystal.[4] Franz[5] considered the effect of a uniform field, on the absorption coefficient, by using the Houston[6] wave functions to calculate the matrix elements and his results were expressed partly in terms of an infinite series.

We have, in this paper, treated the states as stationary and thus adopted the standard procedure to calculate the matrix elements. The results, expressed in terms of the well-known Airy functions in Sec. 3, are, in fact, identical to those obtained by Franz for the case of allowed transitions. Our results, however, can be extended to the case of forbidden transitions giving the expected exponential type tail in the long-wavelength region. A brief discussion of the results, including the effect of binding between electron-hole pairs, is in Sec. 4, and the definition of absorption coefficient in Sec. 2.

2. THE ABSORPTION COEFFICIENT

The absorption coefficient α is defined by[7]

$$\alpha=\frac{4\pi^2e^2}{ncm^2\omega}\sum_i |P_{if}|^2\delta(E_f-E_i-\hbar\omega), \qquad (2.1)$$

where E_i, E_f are the initial and final energies of the system interacting with photons of energy $\hbar\omega$. n is the refractive index; $\hbar$ is the Planck's constant; c is the velocity of light; and e, m are charge and mass of an electron, respectively. The matrix element P_{if} for electrons going from the initial state i to the final state f can be written as[8] (a) for allowed transitions

$$P_{if}=\Phi(0)C_0\delta_{k_i,k_f}, \qquad (2.2)$$

and (b) for forbidden transitions

$$P_{if}=\hbar|\nabla_{r_q}\Phi(0)|C_1\delta_{k_i,k_f}. \qquad (2.3)$$

In (2.2) and (2.3), C_0, C_1 involving the matrix elements between the periodic parts of the Bloch states at the band edges, are independent of the electron wave vector $\mathbf{k}$ with C_0 having the dimensions of momentum and C_1

* Supported in part by the U. S. Atomic Energy Commission.
[1] J. R. Dixon and J. M. Ellis, Phys. Rev. **123**, 1560 (1961).
[2] D. L. Dexter, in *Photoconductivity Conference* (John Wiley & Sons, Inc., New York, 1956), p. 155.
[3] D. M. Eagles, J. Phys. Chem. Solids **16**, 76 (1960).
[4] G. W. Mahlman, W. B. Nottingham, and J. C. Slater, in *Photoconductivity Conference* (John Wiley & Sons, Inc., New York, 1956), p. 489.
[5] W. Franz, Z. Naturforsch **13a**, 484 (1958).
[6] W. V. Houston, Phys. Rev. **57**, 184 (1940).
[7] J. Bardeen, F. J. Blatt, and L. H. Hall, in *Photoconductivity Conference* (John Wiley & Sons, Inc., New York, 1956), p. 146.
[8] R. J. Elliott, Phys. Rev. **108**, 1384 (1957).