scattering experiment at Berkeley[21] which indicate that neither of these peaks are simple resonances.

The Fermi-Watson theorem[23] relating the phase shifts of the $S$ matrix to one another can be used in the calculation of the effect of particle exchange terms on the positions of the $\pi^0$ photoproduction peaks relative to the $\pi$-nucleon scattering peaks. The results of such a calculation are shown in Table III for the pion exchange terms; it was again assumed for the calculation that the peaks are resonances with Peierls' quantum numbers. Experimentally the $\pi^0$ photoproduction peaks are consistent with having the same positions as the scattering peaks, and except for the first resonance the positions are not consistent with the predictions based on the theorem. Since the theorem is valid only at low energies where multiple-pion production is negligible ($k \leq 0.4$ BeV), not much importance can be attached to this discrepancy.

**ACKNOWLEDGMENTS**

The author is deeply indebted to Professor Robert L. Walker for his suggestions, interest, and encouragement at all stages of the experiment. The 60° data were obtained in collaboration with Professor Walker, Dr. R. Gomez, and Dr. R. M. Talman.[9] F. Wolverton was of great assistance in obtaining the data at 90° and 120°. The author wishes to thank the various members of the Synchrotron Laboratory for their helpful comments and suggestions and for keeping the machine and peripheral equipment operating efficiently.

[23] K. M. Watson, Phys. Rev. **95**, 228 (1954).

PHYSICAL REVIEW VOLUME 130, NUMBER 5 1 JUNE 1963

# Nonleptonic Hyperon Decays in the Pole Approximation and the Strong-Coupling Constants

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We have considered the nonleptonic hyperon decays $Y \to N+\pi$ in the baryon pole approximation, assuming the validity of the $|\Delta T|=\frac{1}{2}$ rule for the weak two-fermion ($Y \to N$) vertices. The ($\Sigma\Lambda$) relative parity is assumed to be even. We have tried to solve for the two strong-coupling constants $g_\Sigma$ and $g_\Lambda$ and the four weak-vertex parameters involved in ($\Sigma \to N$) and ($\Lambda \to n$) transitions from the known (experimental) hyperon-decay parameters. We find that there exist three solutions if $\Sigma^+ \to n+\pi^+$ proceeds via $S$ wave (Case A): Solution (i)A, $g_\Lambda^2 \simeq 4g_\Sigma^2$, $g_\Sigma/g_N \simeq 0.6$; Solution (ii)A, $g_\Lambda^2 \simeq g_\Sigma^2$, $g_\Sigma/g_N \simeq -1.2$; and Solution (iii)A, $g_\Lambda^2 \simeq g_\Sigma^2/4$, $g_\Sigma/g_N \simeq 1.3$; and only one solution if $\Sigma^+ \to n+\pi^+$ proceeds via $P$ wave (Case B): Solution B, $g_\Lambda^2 \simeq g_\Sigma^2$, $g_\Sigma/g_N \simeq 0.94$. These solutions are quite different from those given previously by Singh and Udgaonkar; the main reason for the difference is that they use a value of unity for the $|P/S|$ ratio in $\Lambda \to p+\pi^-$ decay, while we use a value of nearly 0.36 for the same ratio, as found experimentally. We apply all the four solutions to radiative $Y \to N+\gamma$ decays and point out experiments which could distinguish between these solutions. Given the results of a few experiments, one may be in a position to choose the most favored solution and predict the results of other experiments, thus, subjecting the model to a direct test.

## I. INTRODUCTION

SEVERAL authors[1] have considered the nonleptonic hyperon decays in the pole approximation and Singh and Udgaonkar[2] have demonstrated that one could get an insight into the signs and magnitudes of the $\Sigma\Sigma\pi$ and $\Sigma\Lambda\pi$ coupling constants, as well as which one of the $\Sigma^\pm \to n+\pi^\pm$ decays proceeds via $S$ wave and which one via $P$ wave, from the hyperon-decay parameters, if one considers that only the $\Sigma$, $\Lambda$, and $N$ pole terms dominate the decay amplitudes. In fact, they demonstrated that if one accepts the criterion for choosing between different possible solutions that all the strong pion-baryon coupling constants should be comparable with each other, then $\Sigma^+ \to n+\pi^+$ is predicted to proceed via $S$ wave and that $g_\Sigma \approx -\frac{2}{3}g_N$ and $g_\Lambda^2 \approx g_\Sigma^2$. [This is in the framework of even ($\Sigma\Lambda$) parity.]

The present work is partly identical in spirit to that of SU. Its purpose, however, is:

(i) Firstly, to point out that one has to radically alter the conclusions reached by SU, if one uses the correct values for the experimental quantities, specially the $(P/S)$ ratio in $\Lambda$ decay,[3] to which the analysis is quite sensitive.

(ii) Secondly, to consider the radiative hyperon decays $Y \to N+\gamma$ also in the baryon-pole approximation and to apply the solutions for the weak vertex

[1] G. Feldman, P. T. Matthews, and A. Salam, Phys. Rev. **121**, 302 (1961); J. Nuyts, thesis, University Libre de Bruxelles, 1961 (unpublished); S. K. Bose and R. Marshak, Nuovo Cimento **23**, 556 (1962); A. Fujii, Phys. Letters **1**, 75 (1962).

[2] V. Singh and B. M. Udgaonkar, Phys. Rev. **126**, 2248 (1962). This is referred to in this paper as SU.

[3] Singh and Udgaonkar (reference 2) have used the value unity for the $|(P/S)|$ ratio in $\Lambda$ decay. The present value is: $|P/S|_\Lambda = 0.36_{-0.06}^{+0.05}$ (see reference 21).

parameters, obtained from nonradiative hyperon decays to the radiative ones.

In Sec. II we give the basis of certain assumptions and write down the expressions for the $Y \rightarrow N+\pi$ decay amplitudes in the baryon-pole approximation. In Sec. III we present the experimental values of the various hyperon-decay parameters and the corresponding solutions for our unknown strong and weak-vertex parameters. By using the observed (see reference 21) $(P/S)$ ratio in $\Lambda \rightarrow p+\pi^-$ decay, it is found that even with the above-mentioned criterion, used by SU, all the strong pion-baryon coupling constants should be comparable with each other; there exist too many possible different solutions and one is no longer in a comfortable position to single out one solution as decidely the most favored one. In fact, it is found that for the case $\Sigma^+ \rightarrow n+\pi^+$ proceeding via $S$ wave and $\Sigma^- - n+\pi^-$ [(4)] via $P$ wave (we denote this possibility as Case A) there exist three different sets of solutions: (i) $g_\Lambda^2 \approx 4g_\Sigma^2$, $g_\Sigma/g_N \approx +0.6$; (ii) $g_\Lambda^2 \approx g_\Sigma^2$, $g_\Sigma/g_N \approx -1.2$; and (iii) $g_\Lambda^2 \approx g_\Sigma^2/4$, $g_\Sigma/g_N \approx +1.34$; while for the case $\Sigma^+ \rightarrow n+\pi^+$ proceeding via $P$ wave and $\Sigma^- \rightarrow n+\pi^-$ [4] via $S$ wave (we denote this possibility as Case B) there exists only one set of solutions: $g_\Lambda^2 \approx g_\Sigma^2$, $g_\Sigma/g_N \approx +0.94$.

*A priori*, we can choose neither between the three solutions in Case A, nor between Cases A and B. However, there is one interesting feature to be noted, i.e., all the three strong coupling constants[5] turn out to be comparable with each other in all the four solutions, and that two of the solutions [solution (ii) of Case A and solution B] roughly correspond[6] to the so-called $G^{(-)}$ and $G^{(+)}$ global symmetries, respectively.

In Sec. IV, we apply each one of the above solutions to radiative $Y \rightarrow N+\gamma$ decays, which is also treated in the baryon-pole approximation. It is found that the observed rate of $\Sigma^+ \rightarrow p+\gamma$ decay could serve to distinguish between the various solutions, if one knew the anomalous magnetic moment of $\Sigma^+$.

In Sec. V we discuss a list of experiments which will be of great interest in the light of the present analysis. We point out that the results of a certain set of experiments could decide which is the most favored solution and which, in turn, will enable us to predict the results of other experiments.

## II. THE DECAY AMPLITUDES IN THE BARYON-POLE APPROXIMATION

We will consider the following hyperon decay modes,

$$\begin{aligned} \Lambda &\rightarrow p+\pi^-, \\ \Sigma^+ &\rightarrow n+\pi^+, \\ \Sigma^+ &\rightarrow p+\pi^0, \\ \Sigma^- &\rightarrow n+\pi^-. \end{aligned} \tag{1}$$

In order to be able to write down the amplitudes for the above decay modes, we make the following assumptions: (a) $(\Sigma\Lambda)$ relative parity is even.[7] (b) The decay amplitudes are dominated by the $\Sigma$, $\Lambda$, and $N$ pole-term contributions. (c) The two-fermion weak vertices $(Y \rightarrow N)$, satisfy the $|\Delta T| = \frac{1}{2}$ rule and can be represented by the following effective interactions:

$$H(Y \rightarrow N) = \rho m_p \bar{N}(a_Y + ib_Y\gamma_5)Y, \tag{2}$$

which involves no derivative[8] terms. $\rho$ is a dimensionless constant, typical of weak interactions. We shall take[9] $\rho \equiv 10^{-7}$, and treat the $a_Y$'s and $b_Y$'s as unknown parameters. We shall further use the same $a_Y$'s and $b_Y$'s for all the decay modes, treating them as constants.[10]

*Remark.* We leave the question of the true origin of the $|\Delta T| = \frac{1}{2}$ rule open. It could be due to either of the following two suggestions: (i)[11] The primary weak interactions satisfy the $|\Delta T| = \frac{1}{2}$ rule apart from electromagnetism. (ii)[12] The $|\Delta T| = \frac{1}{2}$ rule is mostly a dynamical effect; i.e., even though the primary weak interactions do not satisfy the $|\Delta T| = \frac{1}{2}$ rule, the dominant mechanism for the nonleptonic strange particle decays (in our case the two-fermion $Y \rightarrow N$ vertices, which give rise to the baryon-pole terms) do. In the first scheme the $Y \rightarrow N$ vertices automatically satisfy the $|\Delta T| = \frac{1}{2}$ rule. In the second, one has to either arrange the primary weak interactions, together with certain demands on strong interaction symmetries such that at least the $Y \rightarrow N$ vertices satisfy the $|\Delta T| = \frac{1}{2}$ rule, or else assume that such is the case. The second alternative has, however, the advantage that it can account for the observed

[4] By the experimental fact that the asymmetry parameters for $\Sigma^\pm \rightarrow n+\pi^\pm$ are nearly zero, that of $\Sigma^+ \rightarrow p+\pi^0$ is large ($\approx +0.8$), and the $|\Delta T| = \frac{1}{2}$ rule, which we assume, it follows that the two decay modes $\Sigma^\pm \rightarrow n+\pi^\pm$ must predominantly proceed through different, but pure, angular momentum ($S$ or $P$) channels.

[5] In our analysis, we refer to the renormalized coupling constants only.

[6] This is provided we choose the same sign for $g_\Lambda$ and $g_\Sigma$, although the sign of $g_\Lambda$ is immaterial in our analysis.

[7] At the moment there seems to be good evidence in favor of even $(\Sigma\Lambda)$ parity. See R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi, Phys. Rev. Letters **8**, 175 (1962).

[8] It should be emphasized that the existence of derivative terms cannot be ruled out. They, however, will bring too many parameters to handle. The absence of such terms have been assumed by all the previous authors, even though, it may not be mentioned explicitly in their papers. A straightforward perturbation-theoretic calculation of the $Y \rightarrow N$ vertex shows that the derivative terms do not exist if the four-fermion interactions are local, but they do if they are mediated by intermediate bosons. [See J. C. Pati, thesis, University of Maryland, 1960 (unpublished).]

[9] It may be noted that plausible perturbation-theoretic calculations [see J. C. Pait, S. Oneda, and B. Sakita, Nucl. Phys. **18**, 318 (1960)] yield $\rho \approx 10^{-7}$ with $|a_Y|$ and $|b_Y|$ of the order of unity, and these values yield the correct order of magnitude for the hyperon decay rates.

[10] In general, they are functions of square of the four-momentum of either of the two fermions associated with the vertex $(Y \rightarrow N)$. See, however, reference 26.

[11] For an illustration of such a viewpoint, see S. B. Trieman, Nuovo Cimento **15**, 916 (1960); A. Pais, *ibid.* **18**, 1003 (1960); Phys. Rev. **122**, 317 (1961); T. D. Lee and C. N. Yang, *ibid.* **119** 1410 (1960); T. D. Lee. Phys. Rev. Letters **9**, 319 (1962).

[12] For an illustration of such a viewpoint, see S. Oneda, J. C. Pati, and B. Sakita, Phys. Rev. **119**, 482 (1960); Phys. Rev. Letters **6**, 24 (1961); J. C. Pati, S. Oneda, and B. Sakita, Nucl. Phys. **18**, 318 (1960).

violations of the $|\Delta T|=\frac{1}{2}$ rule (stronger than electromagnetism) in a natural way.[12]

We will mention an argument to defend assumption (b). First of all, it is easy to see that the newly observed pion resonances $\rho$, $\omega$, $\eta$, and $\zeta$ (if it exists), etc. cannot contribute to first order in electromagnetic interactions to the decay modes $Y\to N+\pi$, due to the known conservation laws of strong interactions. The only possible mesonic pole contributions are from $K$ and $K^*$ poles. However, with odd $(YNK)$ parity the contribution of the $K$-pole term will be very small,[13] mainly because the $K$ meson has to be emitted in $P$ state at the strong vertex. (In addition the $K$ meson coupling constants seem to be weaker than the $\pi$-meson coupling constants.) The $K^*$-pole contribution may not be negligible. Plausible order-of-magnitude calculations[14] with a vector[15] $K^*$, however, indicate that its contribution is smaller by more than an order of magnitude than the baryon-pole contributions. We, therefore, *assume* that the $K^*$-pole contributions could also be neglected.

Regarding the choice of the baryon poles, it is clear that only the $J=\frac{1}{2}$ baryons are relevant, so that $Y_1^*$ (assuming its spin assignment of $J=\frac{3}{2}$ is correct) or other higher mass pion-hyperon resonances (except possibly $Y_0^*$) of known spin do not contribute; neither do any of the $\pi N$ resonances. Regarding the $Y_0^*$ pole, it can contribute only to $\Sigma^\pm\to n+\pi^\pm$ decays and not to $\Lambda\to p+\pi^-$ or $\Sigma^+\to p+\pi^0$ decay. If we accept the Berkeley assignment[16] that it has $J=\frac{1}{2}$ and that it decays to $(\Sigma+\pi)$ in $S_{1/2}$ state, then, from the observed $Y_0^*$ width of nearly 50 MeV, the $(\Sigma Y_0^*\pi)$ coupling constant turns out to be 0.4, which is more than 30 times smaller than the pion-nucleon coupling constant. This, together with the fact that $Y_0^*$ is heavier than $\Sigma$ or $\Lambda$ hyperons indicates that the $Y_0^*$ contribution is expected

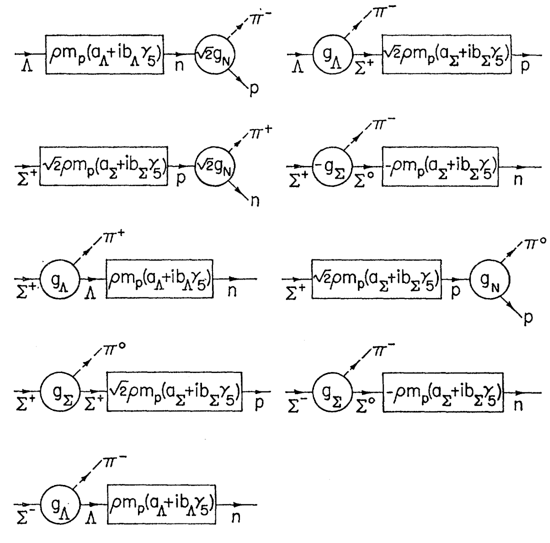


FIG. 1. Diagrams for $Y\to N+\pi$ decays in the $(\Sigma,\Lambda,N)$ pole approximation.

to be small. We will, therefore, drop its contribution also.

Under the above assumptions, the relevant diagrams for the four decay modes [Eq. (1)] are given by Fig. 1. The strong and weak vertices are denoted as shown in the figure, from which it is clear that the analysis will involve, in general, four weak vertex parameters $a_\Lambda$, $b_\Lambda$, $a_\Sigma$, and $b_\Sigma$ and two strong vertex parameters $g_\Lambda$ and $g_\Sigma$. If we write the amplitude for $\Lambda\to p+\pi^-$ decay as

$$T_\Lambda{}^-\equiv(2\pi)^4\delta^4(p_\Lambda-p_p-p_\pi{}^-)\bar{u}_p(A_\Lambda{}^-+B_\Lambda{}^-i\gamma_5)u_\Lambda,\quad(3)$$

and those for the three $\Sigma$-decay modes as

$$T^{\pm,0}\equiv(2\pi)^4\delta^4(p_\Sigma-p_N-p_\pi)\bar{u}_N(A^{\pm,0}+B^{\pm,0}i\gamma_5)u_\Sigma,\quad(4)$$

where the superscript denotes the charge of the emitted pion, then the various $A$'s and $B$'s derived from Fig. 1 are given by

$$\begin{aligned}A_\Lambda{}^-&=(-\sqrt{2}\rho m_p)[B_\Lambda g_N+B_\Sigma g_\Lambda],\\ B_\Lambda{}^-&=(-\sqrt{2}\rho m_p)[A_\Sigma g_\Lambda-A_\Lambda g_N],\\ A^+&=(-\rho m_p)[B_\Sigma(g_\Sigma+2g_N)+B_\Lambda g_\Lambda],\\ B^+&=(-\rho m_p)[A_\Sigma(g_\Sigma-2g_N)+A_\Lambda g_\Lambda],\\ A^-&=(-\rho m_p)[B_\Lambda g_\Lambda-B_\Sigma g_\Sigma],\\ B^-&=(-\rho m_p)[A_\Lambda g_\Lambda-A_\Sigma g_\Sigma],\end{aligned}\quad(5)$$

where

$$\begin{aligned}A_Y&\equiv a_Y/\Delta_Y, & B_Y&\equiv b_Y/S_Y,\\ \Delta_Y&\equiv m_Y-m_N, & S_Y&\equiv m_Y+m_N;\end{aligned}\quad(6)$$

$Y$ stands for $\Lambda$ or $\Sigma$.

The $\Sigma^+\to p+\pi^0$ amplitude is given by

$$T^0=(1/\sqrt{2})(T^+-T^-),\quad(7)$$

which follows from the $|\Delta T|=\frac{1}{2}$ rule.

[13] An estimate of the $K$ pole contribution to $Y\to N+\pi$ decays requires a knowledge of the $(K\to\pi)$ vertex and the $(KYN)$-coupling constant $(g_{KYN})$. A rough upper limit on the strength of the $(K\pi)$ vertex can be obtained by assuming [see S. Oneda, S. Hori, M. Nakagawa, and A. Toyoda, Physics Letters **2**, 243 (1962)] that the $K_1{}^0-K_2{}^0$ mass difference arises mainly through $K^0\to\pi^0\to\bar{K}^0$. With the value of the $(K\pi)$ vertex thus obtained, it is found that the ratio of the rates of $Y\to N+\pi$ given by the $K$-pole contribution to that found experimentally is nearly $(g_{KYN}{}^2/4\pi)(1/30$ or $1/80)$ depending upon whether $Y$ is $\Sigma$ or $\Lambda$.

[14] Such a calculation involves a knowledge of the $(K^*\to\pi)$ vertex and the $K^*YN$ coupling constant $(g_{K^*YN})$. The former may be evaluated by assuming that the transition $K^*\to\pi$ takes place via a baryon-antibaryon loop involving a $|\Delta T|=\frac{1}{2}-(N\to Y)$ vertex of the form $\rho m_p\bar{Y}(a+ib\gamma_5)N\sim\rho m_p\bar{Y}(1+i\gamma_5)N$. The contribution from the loop, treated as a black box, can be written apart from the relevant vertex factors, in terms of the baryon Compton wavelength. [Such calculations for various other $K$-meson form factors have been known (by comparison with experiments) to produce, at least, the right order of magnitude (see J. C. Pati, S. Oneda, and B. Sakita, Nucl. Phys. **18**, 318 (1960).] Assuming, then, that $(g_{K^*YN/4\pi}{}^2)\leq1$, one finds that the contribution of the baryon-pole term to $Y\to N+\pi$ decays is more that 40 times larger than that of the $K^*$-pole term.

[15] At the moment, there exists good indication for the 880-MeV $K^*$ to have spin 1. See W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Rev. Letters **9**, 330 (1962).

[16] G. Alexander, L. Jacobs, G. R. Kalbfleisch, D. H. Miller, G. A. Smith, and J. Schwartz, UCRL-10286. Other references are cited here.

### III. SOLUTIONS FOR THE PARAMETERS

We will now proceed to solve for the parameters introduced in the previous section by using the following experimental information:

(i) The rates of the three $\Sigma \to N+\pi$ decay modes are nearly equal[17] and that one may reasonably consider a value for their common decay rate,

$$W_\Sigma = 0.64\times 10^{10}\ \text{sec}^{-1}. \tag{8}$$

(ii) The sign of the pion asymmetry parameter in $\Sigma^+ \to p+\pi^0$ decay is positive and its magnitude is large[18] ($\alpha_{\Sigma^+\to p+\pi^0} = 0.79_{-0.09}^{+0.08}$).[19]

(iii) The $\Sigma^\pm \to n+\pi^\pm$ asymmetry parameters are very small[18] and could be consistent with zero. We will assume, to render our analysis simpler, that they are zero. This, together with Eq. (7), and (ii), mentioned above, imply that either $\Sigma^+ \to n+\pi^+$ proceeds via pure[20] $S$ wave and $\Sigma^- \to n+\pi^-$ via pure $P$ wave, or vice versa. These two alternatives, as said before, will be referred to as Cases A and B, respectively.

(iv) The pion asymmetry parameter[21] of $\Lambda \to p+\pi^-$ decay is

$$\alpha_{\Lambda\to p+\pi^-} = -(0.62\pm 0.07) \tag{9}$$

and the ratio of $P$- to $S$-wave amplitudes in this decay is

$$(P/S)_\Lambda = -(0.36_{-0.06}^{+0.05}). \tag{10}$$

(v) The rate[17] of $\Lambda \to p+\pi^-$ decay is

$$W(\Lambda \to p+\pi^-) \simeq 0.24\times 10^{10}\ \text{sec}^{-1}. \tag{11}$$

The above pieces of information imply the following sets of equations, depending upon whether Case A or Case B is true.

| Case A | | Case B | |
|---|---|---|---|
| $B^+ = A^- = 0$ | (12A) | $A^+ = B^- = 0$ | (12B) |
| $B^-/A^+ = 10x$ | (13A) | $B^+/A^- = 10x$ | (13B) |

and for either Case A or B

$$B_\Lambda^-/A_\Lambda^- = 10y, \tag{14}$$

$$W(\Lambda \to p+\pi^-) = \frac{m_\pi^2}{4\pi}(m_n+E_n)\left[\left(\frac{A_\Lambda^-}{\Delta_\Lambda}\right)^2 \langle\gamma_\mu\rangle^2{}_\Lambda + \left(\frac{B_\Lambda^-}{-S_\Lambda}\right)^2 \langle\gamma_\mu i\gamma_5\rangle^2{}_\Lambda\right](q/m_\Lambda) = 0.24\times 10^{10}\ \text{sec}^{-1}, \tag{15}$$

$$W(\Sigma^\pm \to n+\pi^\pm) = \frac{m_\pi^2}{4\pi}(m_n+E_n)\left[\left(\frac{A^\pm}{\Delta_\Sigma}\right)^2 \langle\gamma_\mu\rangle^2{}_{\Sigma^\pm} + \left(\frac{B^\pm}{-S_\Sigma}\right)^2 \langle\gamma_\mu i\gamma_5\rangle^2{}_\Sigma\right](q/m_\Sigma) = 0.64\times 10^{10}\ \text{sec}^{-1}. \tag{16}$$

By the equality of the rates of $\Sigma^\pm \to n+\pi^\pm$ decays, we have $|x|\simeq 1$. By the positive sign of the $\Sigma^+ \to p+\pi^0$ asymmetry parameter and Eq. (7), it follows that $x$ should be positive. So we take

$$x = +1. \tag{17}$$

The sign and magnitude of $y$ are determined by Eq. (10), which gives

$$y = 0.63_{-0.11}^{+0.07}. \tag{18}$$

We will confine our analysis to this[22] range of values of $y$.

The quantities $E_n$ and $q$ in Eqs. (15) and (16) are the total energy and momentum of the nucleon in the rest frame of the respective parent hyperon. The values of $\langle\gamma_\mu\rangle_Y$, $\langle\gamma_\mu i\gamma_5\rangle_Y$, $(E_n)_Y$, and $(q/m_Y)$ are taken from the table given by Bludman.[23]

Using Eqs. (5) and (12A,B) through (16), we get the following equations for Cases A and B, respectively:

Case A

$$\left.\begin{aligned} A_\Sigma &= (-10x)\left(\frac{h+1}{h-1}\right)B_\Sigma,\\ B_\Lambda &= (h/g)B_\Sigma,\\ A_\Lambda &= (10x)\left(\frac{h+1}{h-1}\right)\left(\frac{h-2}{g}\right)B_\Sigma,\\ B_\Sigma^2(g_N^2/4\pi)(h+1)^2 &\simeq 1.833/S_\Sigma^2,\end{aligned}\right\} \tag{19A}$$

$$\left.\begin{aligned} \frac{(h+1)(g^2+h-2)}{(h-1)(g^2+h)} &= (-y/x),\\ (g^2+h)/g(h+1) &= C.\end{aligned}\right\} \tag{20A}$$

[17] W. E. Humphrey and R. R. Ross, Phys. Rev. **127**, 1305 (1962).

[18] R. D. Tripp, M. B. Watson and M. Ferro-Luzzi, Phys. Rev. Letters **9**, 66 (1962). Other references are quoted here.

[19] Recently some remarks have been made (reference 18) against the $|\Delta T|=\frac{1}{2}$ rule, due to the fact that the observed rates and asymmetry parameters of $\Sigma^\pm \to n+\pi^\pm$ decays would be consistent with the $|\Delta T|=\frac{1}{2}$ rule and a large magnitude of the $\Sigma^+ \to p+\pi^0$ asymmetry parameter, if the latter would have been nearly unity, instead of 0.8 as is observed. We firstly wish to point out that the said asymmetry parameter should be measured better than it has been; and secondly, to the extent that a violation of the $|\Delta T|=\frac{1}{2}$ rule does occur in $K^+ \to \pi^+ + \pi^0$ decay, a similar violation of the triangular relationship of $\Sigma$ decay [Eq. (7)], predicted by the $|\Delta T|=\frac{1}{2}$ rule, could lead to a value of the $\Sigma^+ \to p+\pi^0$ asymmetry parameter smaller than unity by 0.1 or even more. This, therefore, in no way contradicts the hypothesis that at least the dominant parts of the transition amplitudes for the nonleptonic strange particle decays involve $|\Delta T|=\frac{1}{2}$. A real proof against the $|\Delta T|=\frac{1}{2}$ rule can come if both of $\Sigma^\pm \to n+\pi^\pm$ decays are found to proceed through the same angular momentum channel ($S$ or $P$).

[20] See Fig. 2 of reference 18 for the actual orientations of the three $\Sigma$-decay amplitudes in the $S$-$P$ plane with the presently known values of the three $\Sigma$ asymmetry parameters and decay rates.

[21] J. W. Cronin and O. E. Overseth (to be published). Other references are cited here.

[22] It may be noted that the value unity, used for the $|P/S|$ ratio in $\Lambda \to p+\pi^-$ decay by SU (reference 2), corresponds to $y\simeq 1.8$.

[23] S. A. Bludman, Phys. Rev. **115**, 468 (1960).

Case B

$$A_\Sigma=(-10x)\left(\frac{h+1}{h-1}\right)B_\Sigma,$$

$$B_\Lambda=-\left(\frac{h+2}{g}\right)B_\Sigma, \tag{19B}$$

$$A_\Lambda=(-10x)(h/g)\left(\frac{h+1}{h-1}\right)B_\Sigma,$$

$$B_\Sigma^2(g_N^2/4\pi)(h+1)^2\simeq 1.833/S_\Sigma^2,$$

$$\frac{(h+1)(g^2-h)}{(h-1)(g^2-h-2)}=(-y/x), \tag{20B}$$

$$(g^2-h-2)/(h+1)g=C,$$

where

$$g\equiv g_\Lambda/g_N,\quad h\equiv g_\Sigma/g_N, \tag{21}$$

and

$$C\equiv(\pm)1.168/(1+0.29y^2)^{1/2}. \tag{22}$$

It is clear from (20A) or (20B) that, changing the sign of $C$ just leads to a change in the sign of $g$, without altering the sign or the value of $h$. Thus, from this analysis, one *cannot* decide the sign of $g$ and hence that of $a_\Lambda$ and $b_\Lambda$ [see Eqs. (19A) and (19B)]. This is also clear from Eq. (5); if one alters the sign of $g_\Lambda$, simultaneously altering the signs of $a_\Lambda$ and $b_\Lambda$, all the amplitudes in Eq. (5) remain the same except for a change of absolute signs of $A_\Lambda^-$ and $B_\Lambda^-$, which are not known and which do not matter for our analysis.

The solutions obtained for the various parameters from Eqs. (19A) and (20A), and (19B) and (20B) are given in Table I. Here we give the values only for $x=1$ and $y=0.6$. The solutions and the discussions to follow essentially remain the same for other values of $x$ and $y$, subject to $x\approx 1$ and $0.5<y<0.7$.

TABLE I. The solutions for the parameters for $x\approx 1$ and $y\approx 0.6$. The $(\pm)$ signs of $g$ correspond to $(\pm)$ signs of $C$ [see Eq. (22)]. The $(\pm)$ signs of $b_\Sigma$ and hence that of $a_\Sigma$ arise, since we determine $b_\Sigma$ from $\Sigma$-decay rate [see Eqs. (19A and 19B)]. The double $(\pm)$ signs of $a_\Lambda$ and $b_\Lambda$ are due to those of $g$ and $b_\Sigma$.

| | | Case A | | | Case B |
|---|---|---|---|---|---|
| | | Solution (i)A | Solution (ii)A | Solution (iii)A | Solution B |
| $g\equiv(g_\Lambda/g_N)$ | $(\pm)$ | 1.33 | −1.20 | 0.71 | −0.947 |
| $h\equiv(g_\Sigma/g_N)$ | | 0.60 | −1.19 | 1.34 | 0.937 |
| $b_\Sigma$ | $(\pm)$ | 0.22 | 1.86 | 0.15 | 0.18 |
| $b_\Lambda(\pm)$ | $(\pm)$ | 0.095 | 1.78 | 0.27 | 0.537 |
| $a_\Sigma$ | $(\pm)$ | 1.05 | −0.19 | −1.24 | −6.68 |
| $a_\Lambda(\pm)$ | $(\pm)$ | 0.76 | 0.35 | −0.80 | 4.58 |

As mentioned in Sec. I, there exist three solutions for Case A:

$$\text{(i)A}\quad g_\Lambda^2\simeq 4g_\Sigma^2,\quad g_\Sigma/g_N\simeq 0.6,$$
$$\text{(ii)A}\quad g_\Lambda^2\simeq g_\Sigma^2,\quad g_\Sigma/g_N\simeq -1.2, \tag{23}$$

and

$$\text{(iii)A}\quad g_\Lambda^2\simeq g_\Sigma^2/4,\quad g_\Sigma/g_N\simeq 1.3,$$

and only one for Case B:

$$g_\Lambda^2\simeq g_\Sigma^2,\quad g_\Sigma/g_N\simeq +0.94. \tag{24}$$

The present status of our knowledge about the strong coupling constants does not permit us to choose between any of the above solutions to the extent that they *all* yield the three strong coupling constants being comparable with each other. Hence, one cannot draw any definite conclusions about the relationships between the various coupling constants; neither can one choose between Case A and Case B. This is, in a sense, unfortunate, because had the experimental value of the $|P/S|$ ratio in $\Lambda$ decay been different (for example, had it been nearly unity instead of 0.36), one would have had the lucky situation of a rather uniquely favored solution, as demonstrated by SU.[2]

At this stage, it may be worth mentioning that there exist some indications in favor of the $G^{(+)}$ global symmetry hypothesis: $g_\Lambda\simeq +g_\Sigma\simeq +g_N$, although the conclusions are not clear cut. These come[24] from a meson-theoretic calculation of $(\Lambda N)$ scattering lengths and $(\Sigma^- p)$ branching ratios and a comparison of the former with information from hyperfragment analysis and of the latter with direct experiments. More definitely it is found that $(g_\Lambda/g_N)$ should be nearly equal to one; $(g_\Sigma/g_N)$ should be nearly equal to one or smaller than one, while large negative values of $g_\Sigma/g_N$ (i.e., $g_\Sigma/g_N\approx -1$) cannot be tolerated. This would, therefore, favor our Solution B as well as (i)A over Solutions (ii)A and (iii)A. But apart from other ambiguities, such conclusions are quite sensitive to the choice of the hard-core radii in baryon-baryon interactions and we have hardly any understanding of their origin and of what their value should be.

At the moment, therefore, one has to leave the question of which is the most favored solution to be decided in future by some additional information (experimental and/or theoretical), which does not exist at present. For example, if the polarization measurements of $\Sigma^\pm\to n+\pi^\pm$ decays reveal that Case B holds, then one will have a unique prediction for the relationships between the strong coupling constants [Eq. (24)]. On the other hand, if they show that Case A holds, one could discard $G^{(+)}$ global symmetry [Eq. (24)], insofar as it applies to renormalized coupling constants, but

[24] J. J. deSwart and C. Dullemond, Ann. Phys. (N. Y.) **16**, 263 (1961); J. J. deSwart and C. Iddings (to be published). For a discussion of the work of the above authors and others, see R. H. Dalitz, in *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962).

one would still need further information to decide between Solutions (i), (ii), and (iii) of Case A.

In the next section, we apply all the four sets of solutions for the weak-vertex parameters, obtained in this section, to $Y \to N+\gamma$ decays. It will be shown that the observed and some yet unobserved properties of $\Sigma^+ \to p+\gamma$ decay could serve to choose the most favored solution.

## IV. $Y \to N+\gamma$ DECAYS

Let us consider the radiative decays[25]

$$Y \to N+\gamma. \tag{25}$$

By gauge invariance and reality of the photon ($k^2=k\cdot\epsilon=0$), the matrix element for the above process, in obvious notation, has the general form:

$$M(Y\to N+\gamma)=(2\pi)^4\delta^4(p_Y-p_N-k)(ie)\bar{u}_N \times[A_Y-i\gamma_5 B_Y]\sigma_{\mu\nu}u_Y k_\nu \epsilon_\mu, \tag{26}$$

where $A_Y$ and $B_Y$ are constants and

$$\sigma_{\mu\nu}\equiv(i/2)(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu). \tag{27}$$

The decay rate is given by

$$W(Y\to N+\gamma)=\frac{e^2(A_Y{}^2+B_Y{}^2)}{8\pi}\left(\frac{Y^2-N^2}{Y}\right)^3, \tag{28}$$

where $Y$ and $N$ on the right stand for the masses of the corresponding particles.

The angular distribution of the emitted nucleon in the rest system of the parent hyperon is given by

$$W(\theta)\propto 1+\alpha P_Y\cos\theta, \tag{29}$$

where $P_Y$ is the polarization of the hyperon, $\theta$ the angle between the direction of polarization of the hyperon and the momentum of the nucleon, and

$$\alpha=2A_YB_Y/(A_Y{}^2+B_Y{}^2). \tag{30}$$

Let us assume, as for the $Y\to N+\pi$ modes, that the radiative decays are also dominated by the baryon-pole term contributions. The relevant diagrams for $\Sigma^+\to p+\gamma$ decay are then given by Fig. 2.

The weak vertex ($\Sigma^+\to p$) will be represented, as in Fig. 1, by[26]

$$\sqrt{2}\rho m_p(a_\Sigma+ib_\Sigma\gamma_5)$$

and the electromagnetic vertices of the proton and $\Sigma^+$

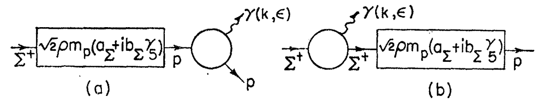


FIG. 2. Diagrams for $\Sigma^+\to p+\gamma$ decay in the baryon pole approximation.

will, respectively, be denoted by

$$ie\left[\gamma_\mu-\sigma_{\mu\nu}k_\nu\frac{X_p}{2m_p}\right]$$

and

$$ie\left[\gamma_\mu-\sigma_{\mu\nu}k_\nu\frac{X_{\Sigma^+}}{2m_\Sigma}\right], \tag{31}$$

where $X_p$ and $X_{\Sigma^+}$ denote the anomalous magnetic moments of the proton and $\Sigma^+$, respectively.[27]

$A_Y$ and $B_Y$, defined in Eq. (26), are then given for the process $\Sigma^+\to p+\gamma$, by[28]

$$\begin{aligned} A_{\Sigma^+}&=(\sqrt{2}\rho m_p)\frac{a_\Sigma}{\Sigma-N}\left(\frac{X_{\Sigma^+}}{2\Sigma^+}-\frac{X_p}{2N}\right),\\ B_{\Sigma^+}&=(\sqrt{2}\rho m_p)\frac{(-b_\Sigma)}{\Sigma+N}\left(\frac{X_{\Sigma^+}}{2\Sigma^+}+\frac{X_p}{2N}\right). \end{aligned} \tag{32}$$

The expressions for $A_\Lambda$ and $B_\Lambda$ for the process $\Lambda\to n+\gamma$ can be obtained from those for $A_{\Sigma^+}$ and $B_{\Sigma^+}$, respectively, by replacing $\Sigma$ or $\Sigma^+$ by $\Lambda$; $p$ by $n$; and $\sqrt{2}$ by 1 on the right-hand side of (32). But since $\Lambda\to n+\gamma$ is very hard to study experimentally, while $\Sigma^+\to p+\gamma$ decay has already been observed,[29] we will concentrate on a study of $\Sigma^+\to p+\gamma$ decay only.

We use the values of $a_\Sigma$ and $b_\Sigma$, given by each of the four solutions in Table I. The rate of $\Sigma^+\to p+\gamma$ decay can then be computed from Eqs. (28) and (32). The computed rates for the different solutions as a function of the as yet unknown quantity,[30]

$$\kappa\equiv(X_{\Sigma^+}/\Sigma^+)/(X_p/N), \tag{33}$$

are drawn in Figs. 3(a), (b), and (c). The solutions (i)A and (iii)A yield nearly the same rate; so only one figure [Fig. 3(a)] is drawn for both of them.

Schneps *et al.*[29] have seen nearly three events of $\Sigma^+\to p+\gamma$ decays, as compared to 264 ($\Sigma^+\to p+\pi^0$) events. Thus they estimate the branching ratio

[25] A general discussion of $Y\to N+\gamma$ decays has been given by many authors. See, for example, G. Calcuci and G. Furlan, Nuovo Cimento **21**, 679 (1961).

[26] In the analysis of $Y\to N+\pi$ decays, we have treated $a_\Sigma$ and $b_\Sigma$ as constants. In general, they are functions of the square of the four-momentum of either of the two fermions associated with the weak vertex ($\Sigma\to N$). For radiative decays $\Sigma^+\to p+\gamma$, in the present model, however, it is easy to show that gauge invariance requires $a_\Sigma(N^2)=a_\Sigma(\Sigma^2)$ and $b_\Sigma(N^2)=b_\Sigma(\Sigma^2)$.

[27] Strictly speaking, we should have included momentum-dependent charge and magnetic momentum form factors. For the present problem, however, we take these to be nearly unity.

[28] The particle symbols, when not used as subscripts, denote the masses of the corresponding particles.

[29] J. Schneps and Y. W. Kang, Nuovo Cimento **19**, 1218 (1961); G. Quareni, A. Quareni Vignudelli, G. Dascola, and S. Mora, *ibid.* **14**, 1179 (1959); R. G. Glasser, N. Seeman, Y. Prakash, G. A. Snow, and P. Steinberg, *ibid.* **19**, 1058 (1961).

[30] The value of $\kappa$ is predicted to be $+1$ in the framework of global symmetry [see S. N. Biswas, Phys. Rev. **127**, 1350 (1962); earlier references are cited here] or unitary symmetry [see S. Coleman and S. Glashow, Phys. Rev. Letters **6**, 423 (1961)].

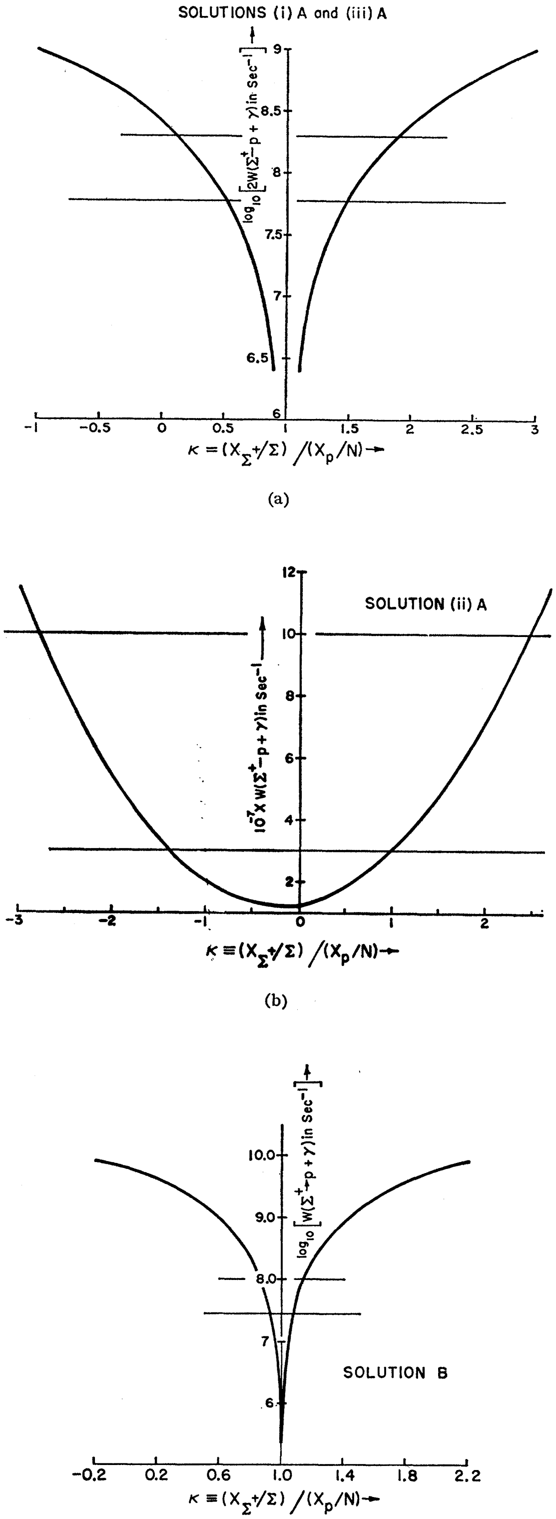


FIG. 3 (a), (b), and (c). Graphs, showing the rate of $\Sigma^+ \to p+\gamma$ decay as a function of $\kappa = (X_{\Sigma^+}/\Sigma)/(X_p/N)$ for the four different solutions. The horizontal lines on each of Figs. 3(a), (b), and (c) correspond to the limits on $W(\Sigma^+ \to p+\gamma)$ given by Eq. (34).

$W(\Sigma^+ \to p+\gamma)/W(\Sigma^+ \to p+\pi^0)$ to be nearly 1%. Quareni *et al.*[29] also report nearly 2 to 3 events of $(\Sigma^+ \to p+\gamma)$ and give the above branching ratio as nearly 1%. Glasser *et al.*,[29] on the other hand, report no $(\Sigma^+ \to p+\gamma)$ event as compared to 144 $(\Sigma^+ \to p+\pi^0)$ events. Until further data with better statistics are available, therefore, let us take the above branching ratio to lie between the limits 0.5 and 2%, i.e., the rate of $\Sigma^+ \to p+\gamma$ decay to lie roughly within the limits

$$3\times 10^7 \text{ sec}^{-1} < W(\Sigma^+ \to p+\gamma) < 10^8 \text{ sec}^{-1}. \tag{34}$$

The two horizontal lines in each of Figs. 3(a), (b), and (c) correspond to the above limits on $W(\Sigma^+ \to p+\gamma)$.

The following features emerge by an examination of Figs. 3(a), (b), and (c) and the separate contributions from the $|A_{\Sigma^+}|^2$ and $|B_{\Sigma^+}|^2$ terms.

1. The said limits on the rate of $\Sigma^+ \to p+\gamma$ decay impose the following limits on the value of $\kappa$ for the different solutions:

$$\left.\begin{array}{ll} 0.10<\kappa<0.5 & (L) \\ \text{or} \quad 1.5<\kappa<1.0 & (U) \end{array}\right\} \text{[Solutions (i) and (iii)A]}, \tag{35}$$

$$\left.\begin{array}{ll} -2.8<\kappa<-1.36 & (L) \\ \text{or} \quad 1.0<\kappa<2.46 & (U) \end{array}\right\} \text{[Solution (ii)A]}, \tag{36}$$

$$\left.\begin{array}{ll} 0.87<\kappa<0.94 & (L) \\ \text{or} \quad 1.06<\kappa<1.13 & (U) \end{array}\right\} \text{(Solution B)}. \tag{37}$$

2. $W(\Sigma^+ \to p+\gamma)$ is an extremely rapidly varying function of $\kappa$ for Solution B, especially within the region specified by the limits (34), while it is fairly slowly varying for Solutions (i)A and (iii)A and extremely slowly varying for Solution (ii)A. This is specially interesting since, unless $\kappa$ happens to lie rather "accidentally" within the narrow limits $L$ or $U$ [Eq. (37)], the observed rate of $\Sigma^+ \to p+\gamma$ decay will decidedly favor Case A over Case B. In this connection, a direct measurement of the $\Sigma^+$ magnetic moment will be very useful. If it is found that $\kappa$ lies *definitely* outside the range of values given by L or U in (37), then we could rule out Case B and predict that Case A should hold.

On the other hand, if by polarization measurements in $\Sigma^\pm \to n+\pi^\pm$ decays it is found that Case B holds, then one may predict (to the extent that the present model for hyperon decays is a good approximation) that $\kappa$ should lie in the range $L$ or $U$, given by (37). If, however, the same experiments reveal that Case A holds, then one will still be left with the task of deciding between the three solutions of Case A.

3(a). For Case A, Solutions (i) or (iii), the decay $\Sigma^+ \to p+\gamma$ proceeds mainly through the $|A_{\Sigma^+}|^2$ term and the asymmetry parameter $|\alpha|$, defined by Eq. (30), is very small (less than 0.2). Thus, the various properties of $\Sigma^+ \to p+\gamma$ decay are identical for Solutions (i)A and (iii)A, except for a difference in the sign of the asymmetry parameter (whose magnitude is small), which arises due to opposite signs of $(a_\Sigma b_\Sigma)$ for the two solutions (see Table I).

(b) For Case B, the decay $\Sigma^+ \to p+\gamma$ also proceeds mainly through the $|A_{\Sigma^+}|^2$ term and the asymmetry parameter $|\alpha|$ is also very small (less than 0.2).

(c) For Case A, Solution (ii), the decay $\Sigma^+ \to p+\gamma$ proceeds mainly through either the $|A_{\Sigma^+}|^2$ term or $|B_{\Sigma^+}|^2$ term depending upon whether $\kappa$ lies in the range $L$ or $U$ given by (36). Depending upon the exact value of $\kappa$, in the case of the former the asymmetry parameter $|\alpha|$ should lie between 0.3 and 0.8, and in case of the latter between 0 and 0.5.

Thus, eventually, when the very hard task of measuring the asymmetry parameter and the various polarization measurements[31] in $\Sigma^+ \to p+\gamma$ decay can be performed, they will shed some light on the problem. If it is found that the decay goes mainly through the $|B_{\Sigma^+}|^2$ term, then it will decidedly favor Solution (ii)A over all other solutions; the same will be the conclusion if the asymmetry parameter is found to be considerably larger than 0.2.

## V. CONCLUSION

By adopting the ($\Sigma$, $\Lambda$, and $N$)-pole approximation in the framework of the $|\Delta T| = \frac{1}{2}$ rule for the $Y \to N+\pi$ decays, it is found that the observed features of these decays lead to four possible sets of solutions for the four weak-vertex parameters $a_{\Sigma,\Lambda}$ and $b_{\Sigma,\Lambda}$ and the two strong-vertex parameters $g_\Sigma$ and $g_\Lambda$. One common feature of all these solutions is: The three strong coupling constants $g_\Sigma$, $g_\Lambda$, and $g_N$ turn out to be comparable with each other in magnitude within a factor of 2.

At present we are not in a position, as is clear from the discussion at the end of Sec. III, to choose the most favored solution. But an application of the above sets of solutions to the radiative $\Sigma^+ \to p+\gamma$ decay reveals that a host of information may emerge from a certain set of experiments, which we list below.

(I) Polarization measurements in $\Sigma^\pm \to n+\pi^\pm$ decays to determine the dominant angular momentum states involved in these two decays.

(II) Measurement of the anomalous magnetic moment of $\Sigma^+$.

(III) Accurate determination of the rate of $\Sigma^+ \to p+\gamma$ decay.

(IV) Measurement of asymmetry parameter and polarization properties in $\Sigma^+ \to p+\gamma$ decay to determine whether the $|A_{\Sigma^+}|^2$ term or the $|B_{\Sigma^+}|^2$ term make the dominant contributions.

It has been pointed out at the end of Secs. III and IV that the results of one or more of these experiments will suffice to choose the most favored solution and predict the results of the remaining ones, thus subjecting the present model to a direct test. For example, if (I) shows that Case B holds, apart from predicting that the $G^{(+)}$ global symmetry should hold, it predicts the results of (II) and (IV), given that of (III). A second example: If (II) shows that $\kappa$ lies outside either of the two narrow ranges $L$ and $U$, given by (37), then, in addition to ruling out $G^{(+)}$ global symmetry, it will predict that $\Sigma^+ \to n+\pi^+$ must proceed via $S$ wave and $\Sigma^- \to n+\pi^-$ via $P$ wave (Case A); i.e., it will predict the result of (I) and so on.

[31] For a discussion of the possible experimental measurements and their theoretical implications in $\Sigma^+ \to p+\gamma$ decay, see J. Dreitlein and H. Primakoff [Phys. Rev. **125**, 1671 (1962)], who discuss the implications of parity nonconservation in $\Sigma^0 \to \Lambda^0+\gamma$ decay.

## ACKNOWLEDGMENTS

Part of this work had been done while the author was the Richard C. Tolman Fellow at the California Institute of Technology. He wishes to thank Professor M. Gell-Mann for many discussions and the hospitality extended to him at the California Institute of Technology. He would like to thank Dr. J. Cronin, Dr. D. Keefe, and Dr. G. Alexander for discussions on the status of various experiments. He also wishes to thank Professor J. R. Oppenheimer and other members of the Institute for Advanced Study for their kind hospitality.