and for the configuration $(1f_{7/2})^3$, the magnitudes of $Q_J{}^s/\langle r^2\rangle$ in (A22) remain the same but all signs must be changed.

As mentioned previously, the sign of $Q_{3/2}{}^s$ for the configuration $(1f_{7/2})^{-3}$ in Table VIII.3 of reference 4 is incorrect. This can be alternatively checked by using the shell-model wave function expressed in terms of Slater determinants. For three particles (not for three holes), this wave function can be easily constructed:

$$\Psi_{J=3/2}{}^{M=3/2}=(3/14)^{1/2}|\psi_{7/2}{}^{5/2}\psi_{7/2}{}^{3/2}\psi_{7/2}{}^{-5/2}|+(3/14)^{1/2}|\psi_{7/2}{}^{3/2}\psi_{7/2}{}^{1/2}\psi_{7/2}{}^{-1/2}|+(3/10)^{1/2}|\psi_{7/2}{}^{7/2}\psi_{7/2}{}^{-1/2}\psi_{7/2}{}^{-3/2}|$$
$$-(1/10)^{1/2}|\psi_{7/2}{}^{7/2}\psi_{7/2}{}^{1/2}\psi_{7/2}{}^{-5/2}|-(6/35)^{1/2}|\psi_{7/2}{}^{5/2}\psi_{7/2}{}^{1/2}\psi_{7/2}{}^{-3/2}|, \quad \text{(A23)}$$

where $|\psi_j{}^{m_j}\psi_j{}^{m_j'}\psi_j{}^{m_j''}|$ is the totally antisymmetric normalized Slater determinant.

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Some Regularities of the Magnetic Moment Distribution of Odd-A Nuclei

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Some systematic features of the magnetic moment distribution of odd-A nuclei are established and theoretical explanation for them is given. The explanation suggested here is based on the idea of quenching of the intrinsic magnetic moments of nucleons in nuclei. This idea was created independently by several investigators many years ago and has been re-examined in detail recently. Essentially all these investigations predict that the magnetic moment of an odd-A nucleus should be somewhere between the so-called Schmidt and Dirac limits. However, if the exclusion principle is the only reason for the quenching, the magnitude of the quenching turns out to be too small to explain the large magnetic moment deviations from the Schmidt limit. Therefore, the question is whether this idea is valid or whether, even if it is valid, other factors such as configuration mixing, core excitation, etc., are more important. In this paper, these questions are attacked empirically. Furthermore, it is shown that the parity rule may also be an important factor in quenching the intrinsic magnetic moments of nucleons in nuclei.

I. INTRODUCTION

It has been shown by Bow that the magnetic moment ratio of two odd-Z–even-N (or odd-N–even-Z) nuclei with the configurations $(nlj)^\nu$ and $(nlj)^{\nu'}$ for the incomplete-shell nucleons of the odd parts, respectively, is given by the following relation (assuming j-j coupling and pure configuration)[1]:

$$\mu_J[(nlj)^\nu]/\mu_{J'}[(nlj)^{\nu'}]=(J'+1)J^2/(J+1)J'^2, \quad |\nu|=|\nu'|, \quad (1)$$

where ν and ν' are positive or negative odd integers (absolute value $\leqslant j+\frac{1}{2}$). When they are positive they represent the number of protons (or neutrons), and when negative the number of holes. The total angular momentum of the ground states of these two nuclei are J and J', respectively.

The result of Eq. (1) is derived on the basis of the semiatomic model which is an extremely weak-coupling case of the unified model given by Bohr and Mottelson. The counterpart of Eq. (1) in the shell model (or Schmidt model) is as follows (also assuming j-j coupling and pure configuration)[2]:

$$\mu_J[(nlj)^\nu]/\mu_{J'}[(nlj)^{\nu'}]=J/J', \quad \text{no restriction on } \nu,\ \nu',\ n, \text{ and } n'. \quad (2)$$

Here "no restriction" means that n and n' can be either different or equal for arbitrary ν and ν' (absolute value $\leqslant j+\frac{1}{2}$).

A test for the comparative validity of Eqs. (1) and (2) has been made in the region of $(1f_{7/2})$ shell.[1] The results show that Eq. (1) is better. It is the purpose of this paper to extend this investigation to the whole range of the nuclear chart and, furthermore, to see if there is any regularity (in addition to the Schmidt and Dirac limits) in the magnetic moment distribution of odd-A nuclei. Theoretical explanation for the systematic features found in this investigation is also attempted.

When $J=J'$, Eqs. (1) and (2) become identical except for the more strict restrictions on ν and n in

[1] Y. F. Bow, preceding paper, Phys. Rev. **130**, 1931 (1963).

[2] M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York, 1955).

TABLE I. Comparison of magnetic moments of odd-Z–even-N nuclei. The experimental data are taken from the compilation of D. Strominger, J. M. Hollander, and G. T. Seaborg [Rev. Mod. Phys. **30**, 585 (1958)]. All configuration assignments (except those with a question mark) are given in reference 2. In each of the exceptional cases, the first configuration is assigned by the present author and the second one by Mayer and Jensen. In making our configuration assignments, the following rules (and/or assumptions) are followed: (a) each assignment should be consistent with the spin and parity of the ground state; (b) the single-particle levels between the major closed shells (magic numbers) can be shifted around, keeping the level order of $j=l+\frac{1}{2}$ and $j'=l-\frac{1}{2}$ (same l) unchanged, for example, the configuration of the 43 protons in $_{43}Tc^{99}$ is assigned as 28-closed-shell plus $(2p_{3/2})^4(2p_{1/2})^2(1g_{9/2})^{-1}$ according to this rule; (c) the level $j'=l-\frac{1}{2}$ can never be filled before the level $j=l+\frac{1}{2}$ (same l) has been completely filled. These rules (a)–(c) have also been implicitly followed by Mayer and Jensen. However, we have applied them more strongly in some cases.

Two cases, $_{33}As^{75}$ and $_{63}Eu^{153}$, have been rejected in the calculation of the standard deviation σ, on the ground that they may have different "configurations" from those in their groups. It is interesting to point out that the assignment of $(1g_{7/2})^3$ to $_{63}Eu^{153}$ is not only consistent with parity and spin of the ground state but also in conformity with the magnetic moments of $_{71}Eu^{175}$ and $_{73}Ta^{181}$. However, in this case, the experimental data are still not accurate enough for concluding whether Eq. (1) or Eq. (2) is better.

Z	Element	A	$J(\hbar)$	$(nlj)^\nu$	μ_{exp} (nuclear magneton)	σ
1	H	1	1/2	$s_{1/2}$	+2.793	...
1	H	3	1/2	$(1s_{1/2})^{\pm1}$	+2.799	...
3	Li	7	3/2	$(1p_{3/2})^1$	+3.256	0.401
5	B	11	3/2	$(1p_{3/2})^{-1}$	+2.689	
9	F	19	1/2	$(2s_{1/2})^{\pm1}$	+2.628	1.058
15	P	31	1/2	$(2s_{1/2})^{\pm1}$	+1.131	
17	Cl	35	3/2	$(1d_{3/2})^1$	+0.821	0.303
17	Cl	37	3/2	$(1d_{3/2})^1$	+0.683	
19	K	39	3/2	$(1d_{3/2})^{-1}$	+0.391	
19	K	41	3/2	$(1d_{3/2})^{-1}$	+0.215	
29	Cu	63	3/2	$(2p_{3/2})^1$	+2.221	0.279
29	Cu	65	3/2	$(2p_{3/2})^1$	+2.238	
31	Ga	69	3/2	$(2p_{3/2})^{-1}$	+2.011	
31	Ga	71	3/2	$(2p_{3/2})^{-1}$	+2.555	
33	As	75	3/2	$(1f_{5/2})^3$? $(2p_{3/2})^{-1}$?	+1.435	
35	By	79	3/2	$(2p_{3/2})^{-1}$	+2.099	
35	By	81	3/2	$(2p_{3/2})^{-1}$	+2.263	
37	Rb	87	3/2	$(2p_{3/2})^{-1}$	+2.741	
39	Y	89	1/2	$(2p_{1/2})^{\pm1}$	−0.134	0.023
45	Rh	103	1/2	$(2p_{1/2})^{\pm1}$	−0.088	
47[a]	Ag	107	1/2	$(2p_{1/2})^{\pm1}$	−0.113	
47[a]	Ag	109	1/2	$(2p_{1/2})^{\pm1}$	−0.130	
41	Nb	93	9/2	$(1g_{9/2})^3$? $(1g_{9/2})^1$?	+6.144	...
43	Tc	99	9/2	$(1g_{9/2})^{-1}$? $(1g_{9/2})^3$?	+5.657	0.092
49[a]	In	113	9/2	$(1g_{9/2})^{-1}$	+5.496	
49[a]	In	115	9/2	$(1g_{9/2})^{-1}$	+5.508	
51	Sb	121	5/2	$(2d_{5/2})^1$	+3.342	0.121
53[b]	I	127	5/2	$(1g_{7/2})^3$? $(2d_{5/2})^1$?	+2.794	
55	Cs	131	5/2	$(2d_{5/2})^{-1}$	+3.48	
59	Pr	141	5/2	$(2d_{5/2})^1$	+3.8	
63	Eu	151	5/2	$(2d_{5/2})^{-1}$	+3.4	
63	Eu	153	5/2	$(1g_{7/2})^3$? $(2d_{5/2})^1$?	+1.5	
75	Re	185	5/2	$(2d_{5/2})^{-1}$	+3.144	
75	Re	187	5/2	$(2d_{5/2})^{-1}$	+3.176	
51	Sb	123	7/2	$(1g_{7/2})^{-1}$	+2.533	0.118
53	I	129	7/2	$(1g_{7/2})^1$? $(1g_{7/2})^3$?	+2.603	
55[a]	Cs	133	7/2	$(1g_{7/2})^1$? $(1g_{7/2})^{-3}$?	+2.564	
55[a]	Cs	135	7/2	$(1g_{7/2})^1$? $(1g_{7/2})^{-3}$?	+2.713	
55[a]	Cs	137	7/2	$(1g_{7/2})^1$? $(1g_{7/2})^{-3}$?	+2.822	
57	La	139	7/2	$(1g_{7/2})^{-1}$	+2.761	
71	Lu	175	7/2	$(1g_{7/2})^{-3}$	+2.0	0.071
73	Ta	181	7/2	$(1g_{7/2})^{-3}$	+2.1	
77	Ir	191	3/2	$(2d_{3/2})^{-1}$	+0.16	0.019
77	Ir	193	3/2	$(2d_{3/2})^{-1}$	+0.17	
79	Au	197	3/2	$(2d_{3/2})^{-1}$	+0.136	
81[a]	Tl	203	1/2	$(3s_{1/2})^{\pm1}$	+1.596	0.011
81[a]	Tl	205	1/2	$(3s_{1/2})^{\pm1}$	+1.612	
95	Am	241	5/2	$(2f_{7/2})^3$	+1.4	0.000
95	Am	243	5/2	$(2f_{7/2})^3$	+1.4	

[a] The fact that almost exactly identical magnetic moments in pairs or triplets of isotopes of equal spin happen in such cases has long been recognized as an indication for the validity of the independent-particle model [see J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 773].

[b] In this case, probably a mixture of the two possible configurations $(1g_{7/2})^3$ and $(1g_{7/2})^2(2d_{5/2})^1$ occurs. This is unlike the other cases in which only one configuration seems dominating.

the former case. These restrictions originate from the general idea that the gyromagnetic ratio of a nucleon (in the nucleus) is in general a function of the configuration (in the Schmidt model it is a function of l and j only). Obviously the most efficient way to compare the relative validity of Eqs. (1) and (2) is to consider all cases in which $J\neq J'$. It is unfortunate, however, that only limited experimental data for such cases are available [see Table I of reference 1 and the captions to Tables I and II]. Nevertheless, a detailed investigation of those cases in which $J=J'$ may provide some empirical verification for the general idea that the gyromagnetic ratio of a nucleon in the nucleus is in general a function of n, l, j, and ν.

It is noted that Eqs. (1) and (2) do not depend on the gyromagnetic ratio of the nucleon, but rather on the coupling scheme. Therefore, our investigation also provides some indication of the validity of the j-j coupling scheme.

In considering the general trend of the magnetic moment distribution, either the Schmidt model or the semiatomic model can be chosen as the basis of our discussion. The latter is a modification of the former by including the residual deformation of the core.[1] However, in this paper, we ignore all possible contributions to the nuclear magnetic moment except the intrinsic motion of the incomplete-shell nucleons of the odd-part of the nucleus. For detailed calculation of some specific case, this is, in general, too crude. However, the dominating factor which controls the general trend of the magnetic moment distribution can be more easily discerned in this crude treatment. Therefore, the simple Schmidt model will be chosen as the basis of our discussion [including the restrictions on n and ν made in Eq. (1)].

II. EXPERIMENTAL

The magnetic moments of odd A nuclei are collected in Tables I and II. (See also Table I of reference 1.) In these tables all nuclei with the same configuration in the sense of Eq. (1) are grouped together. In accordance with our arguments, all nuclei in each group should have the same magnetic moment. However, except for a few cases, they deviate from this "hypothetical magnetic moment." In order to measure this deviation, a statistical point of view will be taken. Suppose that the "hypothetical magnetic moment" exists in each group and, in other words, our theory is perfect. Then the usual standard deviation, which is approximately the range (maximum value minus minimum value) of the magnetic moments in the group divided by the square root of the number of nuclei in the same group, would be solely a measure of the experimental accuracy. On the other hand, if the experimental magnetic moments are accurate enough, then the same standard deviation may be taken as an indication for the inaccuracy of our theory. Obviously, the more inaccurate our theory, the bigger the standard deviation. The standard deviations (σ) are arranged in the last columns of Tables I and II, and are interpreted as a measure of the inadequacy of the theory.

From the trend of the magnitude of the standard deviation, we may draw the following conclusions. (1) In the region of medium weight and heavy nuclei, j-j coupling is a good approximation. In the region of light nuclei, however, it may not be adequate. This confirms the conclusion of previous investigations.[3] (2) The magnetic moment variation in each group may be explained by the local irregularities such as core excitation,[4] configuration mixing,[5] deviation from j-j coupling,[3] etc., but the sensitive dependence of the magnetic moments on ν (and n) is hardly explainable by these local irregularities. (3) It seems impossible, by assigning one single-particle gyromagnetic ratio for each shell, to explain the whole range of the experimental data of the magnetic moments for odd-A nuclei.

The results of Tables I and II also show that the experimental magnetic moment deviations from the Schmidt limits are mainly due to the wrong value assigned to the single-particle gyromagnetic ratio in the theory. Now our question is whether the orbital part or the intrinsic part of the single-particle gyro-

[3] A. M. Lane, Proc. Phys. Soc. (London) **A66**, 977 (1953); **A68**, 189, 197 (1955).
[4] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953).
[5] A. Arima and H. Hovie, Progr. Theoret. Phys. (Kyoto) **11**, 509 (1954); R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **A66**, 1158 (1953).

TABLE II. Comparison of magnetic moments of odd-N–even-Z nuclei. See the caption to Table I for a detailed explanation. The assignments of $(1h_{9/2})^3$ to ${}_{68}Dy^{163}$ and $(2f_{5/2})^3$ to ${}_{76}Os^{189}$ are based on the sign of their magnetic moments, besides the requirements of parity and spin of their ground states. It is important to observe that the signs of the nuclear magnetic moments are in general correctly predicted by the Schmidt model. Here we have another example which seems to favor Eq. (1); the magnetic moments of ${}_{64}Dy^{161}$, ${}_{60}Nd^{145}$, ${}_{62}Sm^{147}$, ${}_{62}Sm^{149}$, and Er^{167} are more accurately related by Eq. (1) than by Eq. (2).

N	Element	A	$J(\hbar)$	$(nlj)^\nu$	μ_{exp} (nuclear magneton)	σ
1	n	1	1/2	$s_{1/2}$	−1.913	...
1	He	3	1/2	$(1s_{1/2})^{\pm 1}$	−2.127	...
9	O	17	5/2	$(1d_{5/2})^1$	−1.893	0.734
13	Mg	25	5/2	$(1d_{5/2})^{-1}$	−0.855	
51	Zr	91	5/2	$(2d_{5/2})^1$	−1.298	
53	Mo	95	5/2	$(2d_{5/2})^1$	−0.929	0.213
55	Mo	97	5/2	$(2d_{5/2})^{-1}$	−0.949	
55	Ru	99	5/2	$(2d_{5/2})^{\pm 3}$? $(2d_{5/2})^{-1}$?	−0.6	
57	Ru	101	5/2	$(2d_{5/2})^{\pm 3}$? $(2d_{5/2})^{-1}$?	−0.7	0.065
59	Pd	105	5/2	$(2d_{5/2})^{\pm 3}$? $(2d_{5/2})^{-1}$?	−0.57	
63	Cd*	111	5/2	$(2d_{5/2})^{\pm 3}$ $(2d_{5/2})^{-1}$?	−0.7	
63	Cd	111	1/2	$(3s_{1/2})^{\pm 1}$	−0.592	
65	Cd	113	1/2	$(3s_{1/2})^{\pm 1}$	−0.620	
65	Sn	115	1/2	$(3s_{1/2})^{\pm 1}$	−0.913	
67	Sn	117	1/2	$(3s_{1/2})^{\pm 1}$	−0.995	0.159
69	Sn	119	1/2	$(3s_{1/2})^{\pm 1}$	−1.041	
71	Te	123	1/2	$(3s_{1/2})^{\pm 1}$	−0.732	
73	Te	125	1/2	$(3s_{1/2})^{\pm 1}$	−0.882	
75	Xe	129	1/2	$(3s_{1/2})^{\pm 1}$	−0.773	
77	Xe	131	3/2	$(2d_{3/2})^1$	+0.687	
99	Ba	135	3/2	$(2d_{3/2})^1$	+0.832	0.141
81	Ba	137	3/2	$(2d_{3/2})^{-1}$	+0.931	
85	Nd	145	7/2	$(2f_{7/2})^3$	−0.62	
85	Sm	147	7/2	$(2f_{7/2})^3$	−0.76	0.081
87	Sm	149	7/2	$(2f_{7/2})^{-3}$	−0.64	
99	Er	167	7/2	$(2f_{7/2})^{-3}$	(±0.5)	
95	Dy	163	5/2	$(1h_{9/2})^3$? $(2f_{7/2})^{-3}$?	+0.51	...
79	Dy	161	5/2	$(2f_{7/2})^{-3}$	−0.37	0.212
103	Yb	173	5/2	$(2f_{7/2})^{-3}$? $(2f_{5/2})^3$?	−0.67	
101	Yb	171	1/2	$(3p_{1/2})^{\pm 1}$	+0.45	
109	W	183	1/2	$(3p_{1/2})^{\pm 1}$	+0.115	
111	Os	187	1/2	$(3p_{1/2})^{\pm 1}$	+0.12	0.198
117	Pt	195	1/2	$(3p_{1/2})^{\pm 1}$	+0.600	
119	Hg	199	1/2	$(3p_{1/2})^{\pm 1}$	+0.499	
125	Pb	207	1/2	$(3p_{1/2})^{\pm 1}$	+0.584	
113[a]	Os	189	3/2	$(2f_{5/2})^{\pm 3}$	+0.651	...
121	Hg	201	3/2	$(3p_{3/2})^{-1}$	−0.607	...

[a] The configuration for this nucleus was incorrectly assigned as $(3p_{1/2})^{\pm 1}$ in reference 2, probably due to the fact that the experimental value of the spin of the ground state was uncertain at that time.

magnetic ratio is incorrect. To answer this question, the following regularities of the magnetic moment distribution are helpful:

(A) In the usual Schmidt plot, the experimental magnetic moments are bound between the Dirac and Schmidt limits. The magnetic moment deviation for each experimental value from the Schmidt limit can

TABLE III. Average value of the percentage reduction (η) of the intrinsic magnetic moments of nucleons in nuclei. In making this table, Table VI of reference 2 is used.

	odd-Z–even-N		odd-N–even-Z	
	$j=l+\frac{1}{2}$	$j=l-\frac{1}{2}$	$j=l+\frac{1}{2}$	$j=l-\frac{1}{2}$
Average value of η (%):	77	40	66	25

be conveniently measured by a parameter η which is the percentage reduction of the intrinsic magnetic moment of the nucleon. These parameters have been calculated in reference 2 and are used in Table III, in which the average values of η are tabulated for different types of nuclei. From Table III, it is clear that the magnetic moment deviations of odd-N–even-Z nuclei are, in general, smaller than those of odd-Z–even-N nuclei. It is also true that nuclei with their incomplete-shell protons (or neutrons) in the shell with $j=l-\frac{1}{2}$, in general, have less magnetic moment deviations than those nuclei with their incomplete-shell protons (or neutrons) in the shell with $j=l+\frac{1}{2}$.

(B) Except for ${}_{109}W^{183}$ and ${}_{77}Xe^{131}$ the range of the parameter η for those nuclei with their incomplete-shell protons (or neutrons) in the shells $(np_{1/2})$ and $(nd_{3/2})$ $(n>1)$ is approximately 3–25%, which is much smaller than other cases ($>50\%$).

(C) For an odd-Z–even-N nucleus, the magnitude of the magnetic moment tends to increase with increasing number of protons (or proton holes) in the incomplete shell for the case $j=l+\frac{1}{2}$, and decrease for the case $j=l-\frac{1}{2}$. However, for an odd-N–even-Z nucleus, the magnitude of the magnetic moment tends to decrease with increasing number of neutrons (or neutron holes) in the incomplete shell for the case $j=l+\frac{1}{2}$, but no experimental information is available for making similar statement for the case $j=l-\frac{1}{2}$.

The peculiar regularities discussed above would not be expected, if the magnetic moment deviations were entirely due to core excitation, configuration mixing, and deviation from j-j coupling, because these factors are more or less local in nature. Furthermore, in view of the large intrinsic magnetic moments of proton and neutron, an error in the orbital part of the gyromagnetic ratio would not be very decisive in accounting for these peculiar regularities. Therefore, there would seem to remain only one possibility, namely, that the intrinsic magnetic moment of a bound nucleon can be rather different from its free-particle value. A more detailed investigation of this possibility is presented in the next section.

III. THEORETICAL EXPLANATION

The intrinsic magnetic moment of a nucleon, say, a proton, consists of two parts, namely, the normal part and the anomalous part. The Dirac equation of the proton automatically gives the normal part (one nuclear magneton) but fails to account for the anomalous part (+1.793 nm). This has been explained in the following well-known fashion: The interaction between the proton field and the positive pion field leads the proton to the virtual emission and re-absorption of positive pions; the anomalous part of the intrinsic magnetic moment of proton is then due to such an interplay of these two fields. Similar arguments also hold for neutrons.

Since virtual process is a second-order effect, intermediate states are necessary. For a free nucleon, say, a proton, the exclusion principle does not affect the intermediate states (because no other neutron is present), so the virtual emission and reabsorption of positive pions gives the full amount of the anomalous magnetic moment of the proton. However, for a proton bound in the nucleus, many intermediate states are blocked by the exclusion principle, so the full amount of the anomalous moment cannot be reached. In other words, the anomalous part of the magnetic moment of a nucleon is, in general, quenched in the nucleus. These arguments have been used earlier[6–10] to explain the magnetic moment deviations. Now let us see if they are consistent with the empirical observations made in Sec. II.

Roughly speaking, the reduction of the anomalous magnetic moment of a nucleon in the nucleus is determined by the level density of the intermediate states permitted by the exclusion principle (and by the parity rule as we shall see later). It seems reasonable to assume that the level density of these possible intermediate states is proportional to the "total" single-particle level density of the nucleus. Taking the cold gas model as first approximation, the "total" single-particle level density is given by $\exp(\alpha E^{1/2})$, where α is a parameter and E is the excitation energy.[11] Since the single-particle levels with $j=l-\frac{1}{2}$ are always comparatively higher in energy than those with $j'=l+\frac{1}{2}$ (because of strong spin-orbit interaction) and since the topmost neutron levels in a nucleus are, in general, higher in energy than the topmost proton levels in neighboring nuclei (because of Coulomb interaction), the general feature of the empirical observation (A) is completely explained.

Now let us consider the single-particle level scheme proposed by Mayer and Jensen[2] more literally. We notice that all levels between the major closed shells (magic numbers except 2, 8, 20, and 28) have the same parity except for the last (highest) one whose parity is always opposite to the others. For simplicity, let us neglect all those (nucleon) states which are not between the two major closed shells in which the original proton, say, is situated. Consider a proton in the state (nlj) and the virtual neutron in the state $(n'l'j')$. If these

[6] F. Bloch, Phys. Rev. **83**, 839 (1951).
[7] H. Miyazawa, Progr. Theoret. Phys. (Kyoto) **5**, 801 (1951).
[8] A. de-Shalit, Helv. Phys. Acta **24**, 296 (1951).
[9] C. Candler, Proc. Phys. Soc. (London) **A64**, 999 (1951).
[10] S. D. Drell and J. D. Walecka, Phys. Rev. **120**, 1069 (1960).
[11] E. Fermi, *Nuclear Physics* (University of Chicago Press, Chicago, 1950).

two (nucleon) states have the same parity, then all virtual states in which the positive pion emitted or reabsorbed has even orbital angular momentum are forbidden, because the intrinsic parity of a pion is odd. On the other hand, if these two (nucleon) states have the opposite parity, then all virtual states in which the positive pion has odd orbital angular momentum are forbidden.

Now taking into account the exclusion principle, we note that all states below the state (nlj) usually have less chance to be considered as intermediate (virtual neutron) states, because all these states may be already completely filled up by neutrons. However, it may happen that in the vicinity of the state (nlj), states with both parities exist and are not completely filled with neutrons. Then the level density of the intermediate states would be greatly enhanced and consequently the anomalous magnetic moment of the proton would approximate its free-particle value. Such a situation is likely to happen near the end of a major closed shell, because only there do (nucleon) states with both parities exist. A glance at the level scheme will show that when the proton is in the state $(np_{1/2})$ or $(nd_{3/2})$ $(n>1)$,[12] the chance for the reduction of its anomalous magnetic moment would be greatly decreased. This is exactly what we have observed. Of course, similar arguments can be carried through for a neutron, and good agreement is found in this case also. This completes the detailed explanation of the empirical observation (B).

Now let us consider the explanation for the empirical observation (C). First of all, it is noted that increasing the number of nucleons in the single-particle levels with either $j=l+\frac{1}{2}$ or $j'=l-\frac{1}{2}$, increases the level density in the vicinity of this level and therefore decreases the reduction of the anomalous magnetic moments of the nucleons. Since a virtual neutron (presumably a Dirac neutron with vanishing intrinsic magnetic moment) has no orbital magnetic moment contribution, the Schmidt formulas for odd-Z–even-N nuclei[2] $\{\mu_{\text{odd-}Z\text{–even-}N}=j+[j/(j+1)](\frac{1}{2}-\mu_p{}^0)$, where $j=l-\frac{1}{2}$; and $\mu_{\text{odd-}Z\text{–even-}N}=j-\frac{1}{2}+\mu_p{}^0$, where $j=l+\frac{1}{2}\}$ can be expected more valid as the magnetic moment of proton (in the nucleus) approaches $\mu_p{}^0$ (the free-particle magnetic moment of proton). However, since the magnetic moment of proton (in the nucleus) can vary from one nuclear magnetron to $\mu_p{}^0$, the experimental magnetic moment distribution spreads out widely between the Dirac and Schmidt limits. Finally, the sign difference of $\mu_p{}^0$ in these two formulas gives the complete explanation for the case of odd-Z–even-N nuclei.

However, for odd-N–even-Z nuclei, the orbital magnetic moment contribution of the virtual proton complicates the situation. For simplicity, let us consider one intermediate (virtual proton) state only, say $j'=l'+\frac{1}{2}$. (The virtual proton is presumed to be a Dirac proton with magnetic moment of one nuclear magneton.) Now suppose that for a certain fraction of time τ the neutron (in the state $j=l+\frac{1}{2}$) is a virtual proton and negative pion. As τ increases the magnitude of the anomalous magnetic moment of the neutron increases, and at the same time the orbital magnetic moment contribution due to the virtual proton also increases. Since the increments of both are not necessarily linear in τ, we shall assume that $\mu_N{}^0 f_1(\tau)$ is the intrinsic magnetic moment of the neutron in the state $j=l+\frac{1}{2}$, and $g_1(\tau)(j'+\frac{1}{2})$ is the orbital contribution of the virtual proton in the state $j'=l'+\frac{1}{2}$, where $f_1(\tau)$ and $g_1(\tau)$ are functions of τ. ($\mu_N{}^0$ is the free-particle magnetic moment of neutron.) Combining these two contributions to the magnetic moment, we have

$$\mu_{\text{odd-}N\text{–even-}Z}=\mu_N{}^0 f_1(\tau)+g_1(\tau)(j'+\tfrac{1}{2}), \quad (j=l+\tfrac{1}{2}). \quad (3)$$

The fine structures of the two functions in Eq. (3) are still unknown, except for the fact that these two functions should be non-negative. [Also, obviously, $f_1(0)=g_1(0)=0$.] However, with this single piece of information, it is possible to explain the general features of the magnetic moment distribution of odd-N–even-Z nuclei. Since $f_1(\tau)$ and $g_1(\tau)$ are non-negative, the two terms in Eq. (3) are always opposite in sign. Therefore, if these functions are reasonably well-behaved, then it can be expected that the magnetic moment will decrease as τ increases (that is to say, as the number of neutrons in the state $j=l+\frac{1}{2}$ increases). Furthermore, even if $f_1(\tau)\approx 1$, the second term always leads to a deviation. Actually, $f_1(\tau)$ can vary from one to zero, so the distribution spreads out widely with more weight given to the Dirac limit. This completes the explanation for the case $j=l+\frac{1}{2}$.

Now consider the case $j=l-\frac{1}{2}$. By similar arguments, we may define two more non-negative functions, $f_2(\tau)$ and $g_2(\tau)$, and have

$$\mu_{\text{odd-}N\text{–even-}Z}=-\frac{j}{j+1}\mu_N{}^0 f_2(\tau)+g_2(\tau)(j'+\tfrac{1}{2}), \quad (j=l-\tfrac{1}{2}). \quad (4)$$

It is noted that the two terms in Eq. (4) always have the same sign. Therefore, even when the first term cannot attain the Schmidt value [that is, $f_2(\tau)=1$], the second term always makes a compensation for this reduction. Consequently, the range of the magnetic moment distribution is narrower in this case. From Eq. (4) we can also conclude that the magnetic moment tends to increase with increasing number of neutrons

[12] The orbital angular momentum L of the pion is determined by the conservation of angular momentum: (1) $j=l\pm\frac{1}{2}$, $j'=l'\pm\frac{1}{2}$ and $l, l'\neq 0$ then $|l'-l|\leq L\leq|l'+l\pm 1|$; (2) if $j=l\pm\frac{1}{2}$, $j'=l'\mp\frac{1}{2}$ and $l, l'\neq 0$, then $|l'-l\pm 1|\leq L\leq|l'+l|$; (3) if $l=0$, $l'\neq 0$, then $l'\leq L\leq l'+1$ or $l'-1\leq L\leq l'$; (4) if $l=0$, then $L=0$ or 1. From this simple calculation, we see that although the single-particle level $3s_{1/2}$ is also situated near the end of the magic number 82, the intermediate states for a nucleon in this level is still quite limited.

in the state $j=l-\frac{1}{2}$ (that is, as τ increases). However, no empirical example for this statement has been found.

In Eqs. (3) and (4), the second terms are assumed to vary more slowly and to be smaller in magnitude than the first terms. Otherwise the magnetic moment distribution would extend beyond the Schmidt limits. However, if the first term in Eq. (4) nearly attains the Schmidt value, then the contribution from the second term would move the magnetic moment upward (in the Schmidt plot) and eventually beyond the Schmidt limit (η is negative). As pointed out before, such a situation is likely to happen near the end of a major closed shell. A glance at the Schmidt plot will show that this prediction is indeed fulfilled. In passing, it is noted that for $J=\frac{1}{2}$, semiatomic model and shell model coincide, because the residual deformation of the core vanishes in this case.[1] This may partially explain the fact that negative value of η only happens in such cases.

IV. GENERAL DISCUSSION

From the above discussion, it becomes clear that the reduction of the anomalous magnetic moment of a nucleon in a nucleus can be separated into two parts, one of which results from the exclusion principle and the other from the parity rule. The exclusion principle limits the virtual nucleon states, and the parity rule restricts the virtual emission and absorption of pions.

The regularities of the magnetic moment distribution discussed in this paper originate from a single source (except for the Coulomb perturbation), namely, the strong spin-orbit interaction. In our treatment, the spin-orbit interaction has been considered phenomenologically, that is to say, we first assume the single-particle energy level scheme of the j-j coupling shell model, and then consider the virtual emission (and absorption) of a pion in this scheme. Here the parallel analog between the photon emission (and absorption) in the atomic case and our present situation is obvious. The main differences between them are that, firstly, the former process is real but the latter is virtual, and secondly there is only one type of photon but there are three types of pions. Consequently, the rigorous way to attack our problem may be considering the three types of pions together in the so-called symmetrical theory. However, our simple phenomenological treatment may give some clue to the more rigorous formulation. In conclusion, we may note that it is possible that the spin-orbit interaction itself may be accounted for by the so-called three-pion resonance (in the $T=0$, $J=1$ state).[13] If this is indeed the case, then the regularities discussed here would ultimately be the manifestation of the interplay of the three pion fields.

[13] J. J. Sakurai, Phys. Rev. **119**, 1784 (1960).