

Collective Excitations in  $\text{Ni}^{58,60,62}$  and  $\text{Zn}^{64,66,68}\dagger$ 

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Collective levels in  $\text{Ni}^{58,60,62}$  and  $\text{Zn}^{64,66,68}$  have been studied by inelastic scattering of 43-MeV alpha particles. The relative strengths of the  $2^+$  first excited level are proportional to values of  $\beta_2^2$  (derived from Coulomb excitation) for isotopes of the same element. The ratio of strength to  $\beta_2^2$  shows no significant variation with  $N$  for the nuclei studied, but is strongly dependent upon  $Z$ . Probable two-phonon triplets can be identified in  $\text{Ni}^{60}$  and  $\text{Ni}^{62}$ . The two-phonon groups in  $\text{Ni}^{58,60,62}$  reach maxima in their angular distributions about 0.2 cycle later than in elastic scattering, while the two-phonon groups in  $\text{Zn}^{64,66,68}$  reach the maxima 0.2 cycle earlier than in elastic scattering. Thus, the two-phonon groups from Ni are out of phase with those from Zn. A proposed scheme of two-phonon levels is given. Levels with three-phonon properties are found near 3.24 MeV in  $\text{Ni}^{62}$  and probably near this energy in  $\text{Ni}^{58}$  and  $\text{Ni}^{60}$ . The cross section for the  $3^-$  collective level is about 1.8 times as large in Zn as in Ni, and varies rather slowly with neutron number. It is likely that the  $3^-$  collective levels in Zn contain a large proportion of configurations with two unpaired protons. In all six targets groups were seen in the energy range expected for combination frequencies of a  $3^-$  phonon and a  $2^+$  phonon. A possible second  $3^-$  level was seen in all six targets at slightly more than 1.5 times the energy of the first  $3^-$  collective level.

## I. INTRODUCTION

**T**HOROUGH studies of inelastic scattering of medium-energy particles by nuclei in the Ti-Zn region have been made by several authors.<sup>1-14</sup> Isotopes of Ti,<sup>1,2</sup> Cr,<sup>2</sup> Fe,<sup>2,4-6,10</sup> Co,<sup>7</sup> Ni,<sup>2,3,5-10</sup> Cu,<sup>6,10</sup> and Zn<sup>2,6,11-14</sup> have been investigated. In the case of even-even nuclei, these studies show very strong excitation of the  $2^+$  first excited level (whose angular distribution is out of phase with elastic scattering as predicted<sup>15</sup> by the phase rule), strong excitation of the  $3^-$  group<sup>16,17</sup> which is in phase, and relatively weak excitation of one or two presumed two-phonon states<sup>1-3,5-8,11,13,14</sup> which are usually in phase, in contradiction to the phase rule

for one-phonon transitions, but which can be explained as two-phonon transitions.<sup>18</sup> Several other excited groups are also seen, including<sup>2,4,8,14</sup> possible second  $3^-$  groups. For the odd- $A$  nuclei, levels analogous to the  $2^+$  and  $3^-$  levels of even nuclei have been reported.<sup>6,10</sup>

Data have been taken at this laboratory on  $\text{Ni}^{58}$  and  $\text{Ni}^{60}$ ,<sup>8</sup> and on  $\text{Zn}^{64}$  and  $\text{Zn}^{68}$ .<sup>14</sup> Many similarities in the collective spectra of these nuclei have already been noted. In this experiment the nuclides  $\text{Ni}^{62}$  and  $\text{Zn}^{66}$  were studied. Comparison of the  $\text{Ni}^{62}$  data with the available  $\text{Zn}^{64}$  data shows the changes in collective properties produced by a change in  $Z$  at constant  $N$ .

## II. DESCRIPTION OF EXPERIMENT

This experiment used the 43-MeV beam of alpha particles from the Argonne 60-in. cyclotron. The thickness of the  $\text{Ni}^{62}$  target was 2.01 mg/cm<sup>2</sup> and that of the  $\text{Zn}^{66}$  target was 5.22 mg/cm<sup>2</sup>. The targets were placed at an angle of  $45^\circ \pm 0.1^\circ$  to the beam. Experimental techniques were similar to those of previous work except that the detector was a lithium-drifted solid-state detector<sup>19</sup> instead of the silicon surface-barrier type used previously. An absorber of 31.0 mg/cm<sup>2</sup> thickness was used to improve the peak separation. Some of these runs were repeated without absorber and these repeated runs were used to derive  $Q$  values. The energy calibration was made by comparison with scattering from known levels in  $\text{C}^{12}$ .

## III. ANALYSIS OF DATA BY COMPUTER

A large amount of extremely tedious and repetitive handling of numbers is necessary in order to obtain angular distributions over a wide angular range. Most of this routine numerical analysis can be done by computer. Therefore, a set of computer programs was

<sup>†</sup> Work performed under the auspices of the U. S. Atomic Energy Commission.

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<sup>2</sup> K. Matsuda, Nucl. Phys. **33**, 536 (1962).

<sup>3</sup> R. K. Jolly, E. K. Lin, and B. L. Cohen, Phys. Rev. **128**, 2292 (1962).

<sup>4</sup> J. Bellicard and P. Barreau, Nucl. Phys. **36**, 476 (1962).

<sup>5</sup> R. Beurtey, P. Catillon, R. Chaminade, M. Crut, H. Faraggi, A. Papineau, J. Saudinos, and J. Thirion, Compt. Rend. **252**, 1756 (1961).

<sup>6</sup> J. Saudinos, thesis, University of Paris, 1961 (unpublished).

<sup>7</sup> H. Crannell, R. Helm, H. Kendall, J. Oeser, and M. Yearian, Phys. Rev. **123**, 923 (1961).

<sup>8</sup> H. W. Broek, T. H. Braid, J. L. Yntema, and B. Zeidman, Phys. Rev. **126**, 1514 (1962).

<sup>9</sup> R. Beurtey, P. Catillon, R. Chaminade, M. Crut, H. Faraggi, A. Papineau, J. Saudinos, and J. Thirion, J. Phys. Radium **21**, 399 (1960).

<sup>10</sup> J. Saudinos, R. Beurtey, P. Catillon, R. Chaminade, M. Crut, H. Faraggi, A. Papineau, and J. Thirion, Compt. Rend. **252**, 96 (1961).

<sup>11</sup> R. Beurtey, P. Catillon, R. Chaminade, H. Faraggi, A. Papineau, and J. Thirion, Nucl. Phys. **13**, 397 (1959).

<sup>12</sup> R. Beurtey, P. Catillon, R. Chaminade, H. Faraggi, A. Papineau, and J. Thirion, Compt. Rend. **249**, 2189 (1959).

<sup>13</sup> R. Chaminade, M. Crut, H. Faraggi, D. Garreta, J. Saudinos, and J. Thirion, J. Phys. Radium **22**, 607 (1961).

<sup>14</sup> H. W. Broek, T. H. Braid, J. L. Yntema, and B. Zeidman, Nucl. Phys. **38**, 305 (1962).

<sup>15</sup> J. S. Blair, Phys. Rev. **115**, 928 (1959).

<sup>16</sup> B. L. Cohen, Phys. Rev. **105**, 1549 (1957).

<sup>17</sup> M. Crut and N. S. Wall, Phys. Rev. Letters **3**, 520 (1959).

<sup>18</sup> B. Buck, Phys. Rev. **127**, 940 (1962).

<sup>19</sup> Made by H. M. Mann, Electronics Division, Argonne National Laboratory.

written and used in the analysis of the  $\text{Ni}^{62}$  and  $\text{Zn}^{66}$  data. Analysis by computer has several advantages: (1) The experimenter is freed from the most routine parts of the analysis; (2) more accuracy is possible since more exact calculations are practical; (3) the analysis is done consistently by the same method; (4) a given mass of data is analyzed more quickly; and (5) the probability of error is reduced.

Data are taken from the R.I.D.L. pulse-height analyzer either on a paper tape (tally punch) or by a typewriter or both. If runs are less than about 5 min in duration, appreciable time can be saved by omitting the type-out which requires about 5 times as much time as the punchout.

The computation (programmed in Fortran for the IBM-704 computer) is divided into three stages, the first of which is computation of the over-all constant of proportionality of the system (i.e., the number of keV of particle energy at the detector per channel on the analyzer). This calibration computation uses the scattering from known levels, usually ones in  $\text{C}^{12}$ . The assumption is made (and checked) that the detector system is linear in energy, but zero energy need not correspond to channel zero. If an absorber is used, the corrections for energy lost in the absorber are made in terms of a Taylor expansion of the range-energy relation which is consistent to within 0.5% with the known range-energy relation<sup>20</sup> (for Al absorber) from 2 MeV/nucleon to 20 MeV/nucleon.

The second step is to use the known constant of proportionality and the kinematical relationships in order to derive accurate  $Q$  values. The  $Q$  value of each group is derived at whatever angles the particular group shows to best advantage and a least-squares average is taken of all the determinations.

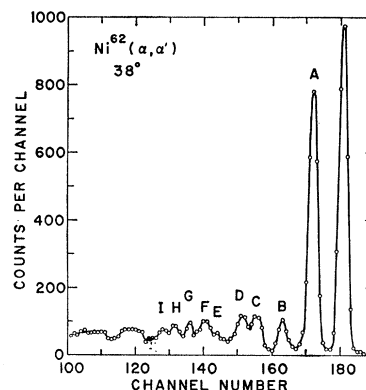
In the third stage the angular distributions are computed. The computer is given the relevant information: mass numbers, charge numbers,  $Q$  values of groups to be analyzed, absorber thickness, constant of proportionality (keV/channel), number of channels to take in summing the peaks, target thickness, background, estimated error in background, etc. The computer then proceeds to sum each peak, subtract the estimated background, convert to a lab cross section, convert to a c.m. cross section, and (for the elastic peak only) find the ratio to Rutherford scattering. A detailed report on these programs is in preparation.

#### IV. RESULTS

##### A. Pulse-Height Spectra

Figure 1 shows the spectrum from  $\text{Ni}^{62}$  at  $38^\circ$ . The angle was chosen for display because the nine lettered groups are all evident there. Group *A* is from the  $2^+$  first excited level at  $1.15 \pm 0.02$  MeV; group *B* at  $2.30 \pm 0.02$  MeV is in phase with elastic scattering and is the two-phonon group; and group *C* at  $3.24 \pm 0.04$

FIG. 1. Pulse-height spectrum of alpha particles scattered by  $\text{Ni}^{62}$  at  $38^\circ$ . Energies of the groups are given in the text.



MeV is an out-of-phase group interpreted as the three-phonon group. Group *D* is the strong  $3^-$  group at  $3.71 \pm 0.02$  MeV, but appears weak here because its angular distribution reaches a minimum near  $38^\circ$ . Other groups are a weak group *E* at  $4.6 \pm 0.1$  MeV, group *F* at  $4.93 \pm 0.03$  MeV, in-phase group *G* at  $5.57 \pm 0.03$  MeV, group *H* at  $6.07 \pm 0.04$  MeV, and in-phase group *I* at  $6.48 \pm 0.05$  MeV. Many of these groups have obvious analogs in  $\text{Ni}^{58}$ ,  $\text{Ni}^{60}$ , and the zinc isotopes. The  $Q$  values given in this section are those derived in this experiment. These  $Q$  values are compared to those of other experiments in Tables I through VI.

Figure 2 shows the groups from  $\text{Zn}^{66}$  at  $34^\circ$  (where the in-phase groups are near maxima in their angular distributions). Group *A* is the  $2^+$  first excited level at  $1.04 \pm 0.02$  MeV group *B* at  $1.8 \pm 0.1$  MeV is in phase and is presumably two-phonon, group *C* is the strong  $3^-$  group at  $2.81 \pm 0.02$  MeV, group *D* at  $3.76 \pm 0.03$  MeV is probably in phase, group *E* lies at  $4.40 \pm 0.05$  MeV, and group *F* is a broad group at  $6.5 \pm 0.15$  MeV. Other groups probably exist at 5.4 and 5.8 MeV. By way of comparison, previously obtained<sup>14</sup> spectra from  $\text{Zn}^{64}$  and  $\text{Zn}^{68}$  at the same lab angle ( $34^\circ$ ) are shown in Figs. 3 and 4. Groups *A* through *E* in  $\text{Zn}^{66}$  have analogs in  $\text{Zn}^{64}$  and  $\text{Zn}^{68}$  but above group *E* there is no obvious similarity.

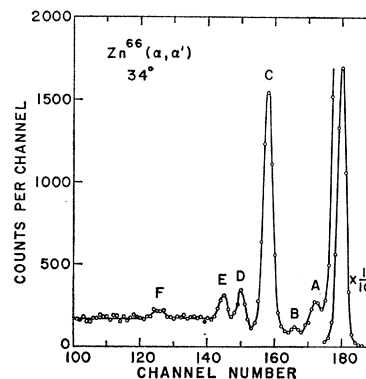


FIG. 2. Pulse-height spectrum of alpha particles scattered by  $\text{Zn}^{66}$  at  $34^\circ$ . Energies of the groups are given in the text.

<sup>20</sup> R. M. Sternheimer, Phys. Rev. **115**, 137 (1959).

### B. Comparative Elastic Scattering

Figure 5 shows the ratio of elastic scattering to the Rutherford cross section for the nuclides  $\text{Ni}^{58}$ ,  $\text{Ni}^{60}$ ,  $\text{Ni}^{62}$ , and  $\text{Zn}^{66}$ . The abscissa is not the c.m. angle but rather is the c.m. angle multiplied by  $(A/68)^{1/3}$ . This factor has been included in order to compare points at the same position in the oscillation pattern and to provide a rigorous test of the rule that the corresponding maxima and minima occur at angles inversely

proportional to  $kR$ . In previous unpublished work it was found that the ratios to Rutherford of  $\text{Cu}^{63,65}$ ,  $\text{Zn}^{64}$ , and  $\text{Zn}^{68}$  are very nearly the same when plotted in the manner of Fig. 5. The curve through the points for these four isotopes is included in Fig. 5 for comparison.

### C. The First Excited Level

Figure 6 shows the angular distributions of the  $2^+$  first excited states of  $\text{Ni}^{62}$  and  $\text{Zn}^{66}$ . The solid curve is

TABLE I. Level energies of  $\text{Ni}^{68}$  (keV).

| Spins  |           | $2^+$          | $4^+$         |               |                |                |                      |      |               |
|--------|-----------|----------------|---------------|---------------|----------------|----------------|----------------------|------|---------------|
| Source | Reference |                |               |               |                |                |                      |      |               |
| ANL    | 8         | $1420 \pm 30$  | $2470 \pm 60$ |               |                |                |                      |      |               |
| INS    | 2         | 1449           | 2459          |               | 2780           | 2905           | 2905 to 3500 (broad) | 3045 | 3260          |
| MIT    | 24        | 1452           | 2458          |               | 2772           | 2899           | 2939                 | 3035 | 3260          |
| Rice   | 25        | 1453           | 2456          | 2483          | 2779           | 2905           | 2945                 | 3041 | 3418          |
| Pitt.  | 3         | 1450           | 2460          |               | 2770           |                |                      | 3040 | 3270          |
| Spins  |           |                |               |               |                |                |                      |      | $3^-$         |
| ANL    | 8         |                |               |               |                |                |                      |      | $4450 \pm 50$ |
| INS    | 2         | 3526           |               | 3630          | 3773           | 3906           | 4103                 | 4400 | 4450          |
| MIT    | 24        | 3528           | 3592          | 3630          | 3774           | 3898           | 4106                 |      | 4750          |
| Pitt.  | 3         | 3520           | 3570          |               | 3870           | 3900           | 4100                 | 4430 | 4500          |
| ANL    | 8         | $5100 \pm 100$ | $5550 \pm 50$ | $5950 \pm 50$ | $6800 \pm 100$ | $7100 \pm 100$ | $8100 \pm 200$       |      |               |
| INS    | 2         |                | 5590          | 6020          |                |                |                      |      |               |

TABLE II. Level energies of  $\text{Ni}^{60}$  (keV). In these tables, a parenthesis around a spin assignment means that the assignment is not certain. Similarly, a parenthesis around an energy means that it is not absolutely certain that the level was observed.

| Spins  |           | $2^+$         | $2^{(+)}$      | $(0^+)$ | $4^+$         |                | $2^{(+)}$      |                |                |                |
|--------|-----------|---------------|----------------|---------|---------------|----------------|----------------|----------------|----------------|----------------|
| Source | Reference |               |                |         |               |                |                |                |                |                |
| ANL    | 8         | $1310 \pm 30$ | $2200 \pm 100$ |         | $2500 \pm 60$ |                |                | $3200 \pm 100$ |                |                |
| INS    | 2         | 1333          | 2158           | 2287    | 2503          | 2640           | 3110           | (3170)         |                | (3280)         |
| MIT    | 24        | 1333          | 2159           | 2285    | 2504          | 2624           | 3120           | 3184           | 3191           | 3268           |
| Rice   | 25        | 1330          | 2161           | 2287    | 2512          | 2629           |                |                |                |                |
| Pitt.  | 3         | 1330          | 2160           | 2290    | 2520          | 2640           | 3140           |                |                |                |
| INS    | 2         |               | (3390)         |         |               | 3670           | 3726           | (3860)         |                | (3920)         |
| MIT    | 24        | 3316          | 3391           | 3587    | 3618          | 3670           | 3732           | 3869           | 3886           | 3925           |
| Pitt.  | 3         | 3310          | 3380           |         |               | 3700           | 3750           |                |                | 3920           |
| Spins  |           | $(1^+, 2^+)$  | $3^-$          |         |               |                |                |                |                |                |
| ANL    | 8         |               | $4050 \pm 50$  |         |               | $5100 \pm 100$ | $5700 \pm 100$ | $6200 \pm 100$ | $7000 \pm 100$ | $7600 \pm 200$ |
| INS    | 2         |               | 4038           |         | 4310          | 5090           |                |                |                | $8900 \pm 200$ |
| MIT    | 24        | 4005          | 4038           | 4076    |               |                |                |                |                |                |
| Pitt.  | 3         |               | 4070           |         | 4333          |                |                |                |                |                |

TABLE III. Level energies of  $\text{Ni}^{62}$  (keV).

| Spins  |              | $2^+$         | $0^+$         | $2^+$         | $4^+$         |               |      |      |      |                |
|--------|--------------|---------------|---------------|---------------|---------------|---------------|------|------|------|----------------|
| Source | Reference    |               |               |               |               |               |      |      |      |                |
| ANL    | (This paper) | $1150 \pm 20$ |               | $2300 \pm 20$ |               |               |      |      |      | $3240 \pm 40$  |
| MIT    | 24           | 1172          | 2048          | 2302          | 2336          | 2888          | 3055 | 3155 | 3175 | 3254           |
| Rice   | 25           | 1172          | 2047          | 2304          |               |               |      |      |      |                |
| Pitt.  | 3            | 1170          | 2040          |               | 2330          | 2900          |      | 3160 |      |                |
| Spins  |              |               |               |               |               | $3^-$         |      |      |      |                |
| ANL    | (This paper) |               |               |               |               | $3710 \pm 20$ |      |      |      | $4600 \pm 100$ |
| MIT    | 24           | 3267          | 3367          | 3467          | 3516          |               |      |      |      |                |
| Pitt.  | 3            | 3290          |               |               | 3520          | 3770          | 3880 | 4190 | 4240 |                |
| ANL    | (This paper) | $4930 \pm 30$ | $5570 \pm 30$ | $6070 \pm 40$ | $6480 \pm 50$ |               |      |      |      |                |

TABLE IV. Level energies of  $\text{Zn}^{64}$  (keV).

| Spins  |           | 2 <sup>+</sup> | 2 <sup>+</sup> | (0 <sup>+</sup> ) | (4 <sup>+</sup> ) | (0 <sup>+</sup> ) |          | 3 <sup>-</sup> |        |           |
|--------|-----------|----------------|----------------|-------------------|-------------------|-------------------|----------|----------------|--------|-----------|
| Source | Reference |                |                |                   |                   |                   |          |                |        |           |
| ANL    | 14        | 990±20         | 1790±30        |                   | 2300±30           |                   | (2700)   | 3010±20        |        |           |
| INS    | 2         | 997            | 1805           |                   | 2305              |                   |          | 2971           | 3144   |           |
| Bartol | 26        | 991            | 1799           | 1910              | 2330              | 2630              |          |                |        |           |
| LRL    | 27        | 980            | 1780           | 1920              | 2280              | 2590              | 2730     | 2980           |        | 3270 3530 |
| Saclay | 11        | 995            | (1790)         |                   | (2290)            |                   |          | 2930           |        |           |
| Texas  | 28        | 970            |                |                   | 2270              |                   |          | 3040           |        | 3430      |
| ORNL   | 31        | 1020           | 1830           |                   | 2130              |                   |          | 3000           |        |           |
| ANL    | 14        | 3800±60        | (4000)         | 4150±100          |                   |                   | 4700±100 | 5200±200       |        |           |
| LRL    | 27        | 3840           |                | 4120              | 4270              | 4490              | 4660     |                |        |           |
| Texas  | 28        | (3840)         |                |                   |                   |                   |          |                | (5640) |           |

TABLE V. Level energies of  $\text{Zn}^{66}$  (keV).

| Spins    |              | 2 <sup>+</sup> | 2 <sup>+</sup> | (0 <sup>+</sup> ) | (4 <sup>+</sup> ) | 3 <sup>-</sup> |           |         |           |         |
|----------|--------------|----------------|----------------|-------------------|-------------------|----------------|-----------|---------|-----------|---------|
| Source   | Reference    |                |                |                   |                   |                |           |         |           |         |
| ANL      | (This paper) | 1040±20        | 1800±100       |                   |                   | 2810±20        |           | 3760±30 |           | 4400±50 |
| Bartol   | 26           | 1038           | 1870           | 2380              | 2470              |                |           |         |           |         |
| Texas    | 28           | 1040           |                | 2410              | 2750              | 3240           |           | 3760    | 4120 4330 | (4520)  |
| Saclay   | 11           | 1055           | (1890)         | (2410)            |                   | 2870           |           |         |           |         |
| INS      | 2            |                |                |                   |                   | 2830           |           |         |           |         |
| Stanford | 29           | 1040           |                | 2400              | 2750              |                | 3240 3410 | 3780    | 4120 4330 |         |
| BNL      | 30           | 1037           | 1865           | 2370              |                   |                | 3220 3400 | 3785    | 4100 4300 | 4450    |
| ANL      | (This paper) | (5400)         | (5800)         | 6500±150          |                   |                |           |         |           |         |
|          |              | ±150           | ±150           |                   |                   |                |           |         |           |         |
| Stanford | 29           | 4830           |                |                   |                   |                |           |         |           |         |
| BNL      | 30           | 4830           |                |                   |                   |                |           |         |           |         |

TABLE VI. Level energies of  $\text{Zn}^{68}$  (keV).

| Spins    |           | 2 <sup>+</sup> | (2) <sup>+</sup> | 2 <sup>+</sup> |      | 3 <sup>-</sup> |         |          |          |           |
|----------|-----------|----------------|------------------|----------------|------|----------------|---------|----------|----------|-----------|
| Source   | Reference |                |                  |                |      |                |         |          |          |           |
| ANL      | 14        | 1070±20        | (1660)±40        | 1890±30        |      | (2420)±60      | 2760±20 | 3600±100 | 3800±100 | 4330±50   |
| ORNL     | 31        | 1070           |                  | 1980           | 2340 |                | 2680    | 3470     | 3800     | 4100 4500 |
| Bartol   | 26        | 1078           |                  |                |      |                |         |          |          |           |
| Saclay   | 11        | 1075           | (1600)           |                |      | (2400)         | 2760    |          |          |           |
| Wash. U. | 23        | 1100           |                  | 1880           |      |                |         | 3490     |          |           |
| INS      | 2         |                |                  |                |      |                | 2740    |          |          |           |
| Stanford | 32        | 1070           |                  | 1880           | 2310 |                |         |          |          |           |
| ANL      | 14        | (4800)         |                  | 5400±50        |      | (5900)         |         |          |          |           |
| ORNL     | 31        | 4900           | 5100             | 5400           |      |                |         |          |          |           |

the theoretical fit for the case of  $\text{Ni}^{58}$  as computed by Satchler, Bassel, and Drisko<sup>8,21,22</sup> by means of the distorted-wave Born approximation (DWBA). The theoretical curve fits the  $\text{Ni}^{58}$  data quite well at angles less than 60°, with the possible exception that the observed peak at 21° c.m. is roughly 20% higher than the theory indicates. The cross sections for  $\text{Ni}^{62}$  and  $\text{Zn}^{66}$  are somewhat larger than that for  $\text{Ni}^{58}$ . Also, the peak positions occur slightly earlier because they are inversely proportional to the nuclear radius.

#### D. The Collective 3<sup>-</sup> Level

Figure 7 shows the angular distributions of the collective octupole levels of  $\text{Ni}^{62}$  and  $\text{Zn}^{66}$ , compared with the theoretical fit<sup>8,21</sup> for the case of  $\text{Ni}^{58}$ . The 3<sup>-</sup> angular distributions for these nuclides (and for the rest of the even Ni and Zn nuclides) show the same general shape, but the positions of corresponding maxima are proportional to  $A^{-1/3}$ . The magnitude of the cross section is larger for zinc isotopes than for nickel isotopes, as Fig. 7 shows.

#### E. The Two-Phonon Groups

Figure 8 shows the angular distribution of the two-phonon group from  $\text{Ni}^{62}$  at 2.3-MeV excitation, and

<sup>21</sup> R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. **128**, 2693 (1962).

<sup>22</sup> E. Rost, Phys. Rev. **128**, 2708 (1962).

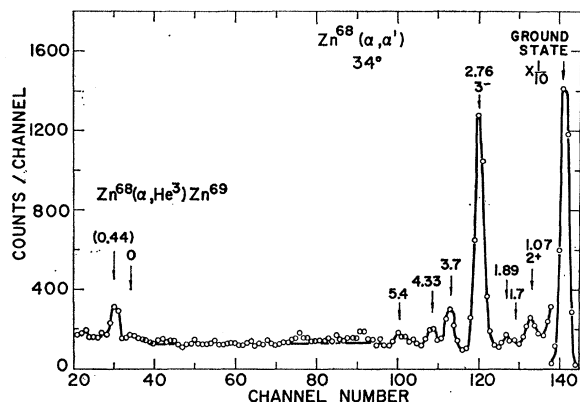


FIG. 3. Pulse-height spectrum of alpha particles and  $\text{He}^3$  particles scattered by  $\text{Zn}^{64}$  at  $34^\circ$ .

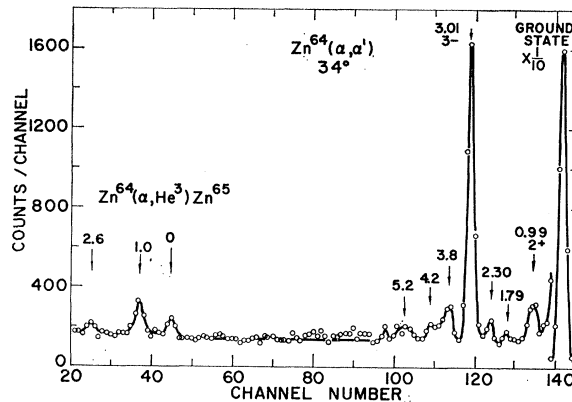


FIG. 4. Pulse-height spectrum of alpha particles and  $\text{He}^3$  particles scattered by  $\text{Zn}^{66}$  at  $34^\circ$ .

that from  $\text{Zn}^{66}$  at 1.8-MeV excitation. Both angular distributions are nearly in phase with elastic scattering but an appreciable phase difference exists between the two-phonon angular distribution and that for elastic scattering. This phase difference is about 0.2 cycle and is in a different sense for  $\text{Ni}^{62}$  than for  $\text{Zn}^{66}$ . Thus, the one-phonon groups in Fig. 8 appear to be nearly  $\frac{1}{2}$  cycle out of phase with one another.

#### F. Other Groups from $\text{Ni}^{62}$ and $\text{Zn}^{66}$

Angular distributions for groups at 4.93, 5.57, and 6.48 MeV in  $\text{Ni}^{62}$  and at 3.76 MeV in  $\text{Zn}^{66}$  are shown in Figs. 9 and 10. The 5.57- and 6.48-MeV groups of  $\text{Ni}^{62}$  show a strong resemblance to the  $3^-$  collective group.

The groups illustrated in Fig. 9 are probably out of phase. Angular distributions were derived for certain groups not illustrated in Figs. 6–11, but since no significant oscillation pattern could be seen for these groups all that is reported for them is the  $Q$  value and the strength. Figure 11 compares the 3.24-MeV three-phonon group of  $\text{Ni}^{62}$  with the two-phonon (2.30 MeV) and one-phonon (1.17 MeV) groups of  $\text{Ni}^{62}$ .

#### G. The $\text{He}^3$ Peaks

In experiments on inelastic scattering of alpha particles, peaks are seen at energies considerably lower than the energies of the prominent groups of alpha particles. These peaks are almost certainly due to  $\text{He}^3$

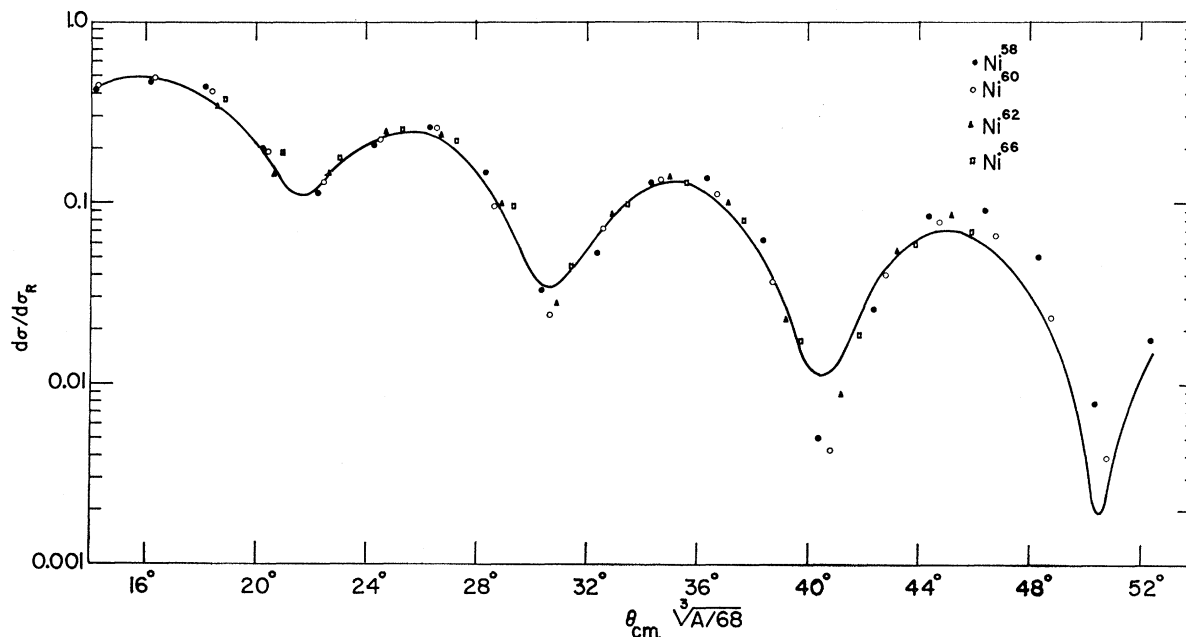


FIG. 5. Ratio of elastic scattering to the Rutherford scattering cross section. The abscissa is the c.m. angle multiplied by  $(A/68)^{1/3}$ . The solid line passes through the average position of experimental points for the nuclides  $\text{Cu}^{63}$ ,  $\text{Cu}^{65}$ ,  $\text{Zn}^{64}$ , and  $\text{Zn}^{68}$ .

particles because the peaks occur at the energies expected for particles from the  $(\alpha, \text{He}^3)$  reaction. The  $Q$  values determined for the groups are consistent with the assumption that these are  $\text{He}^3$  peaks. (No strong, sharp peaks are expected in the inelastic alpha spectrum at the highly negative  $Q$  values involved, and energetic particles with  $Z=1$  would not be stopped by the solid-

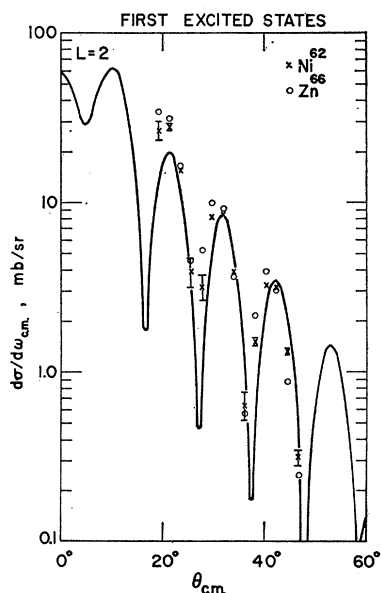


FIG. 6. Cross sections for excitation of the first excited level. Errors are shown for the  $\text{Ni}^{62}$  data except when smaller than the size of the points. Errors for the  $\text{Zn}^{66}$  data are about the same as for the  $\text{Ni}^{62}$  data. The curve is a DWBA fit to the data for  $\text{Ni}^{58}$  (reference 21).

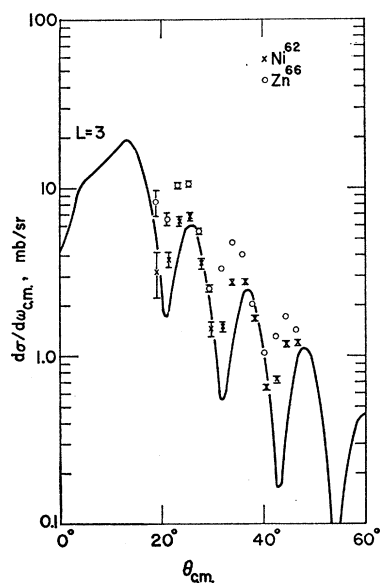


FIG. 7. Cross sections for excitation of the  $3^-$  collective level in  $\text{Ni}^{62}$  and  $\text{Zn}^{66}$ . Errors are shown on Figs. 7-9 except when smaller than the size of the points. The curve is a DWBA fit to the data for  $\text{Ni}^{58}$  (reference 21).

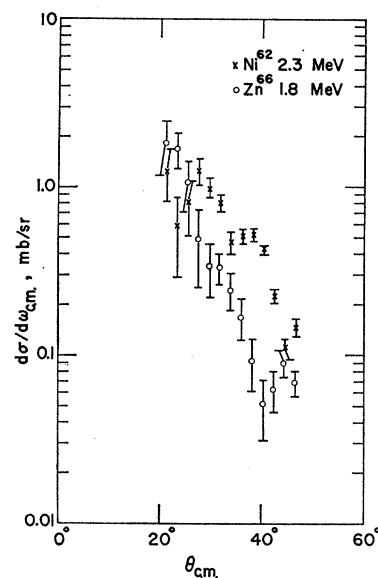


FIG. 8. Cross sections for excitation of the two-phonon groups at 2.3 MeV in  $\text{Ni}^{62}$  and at 1.8 MeV in  $\text{Zn}^{66}$ .

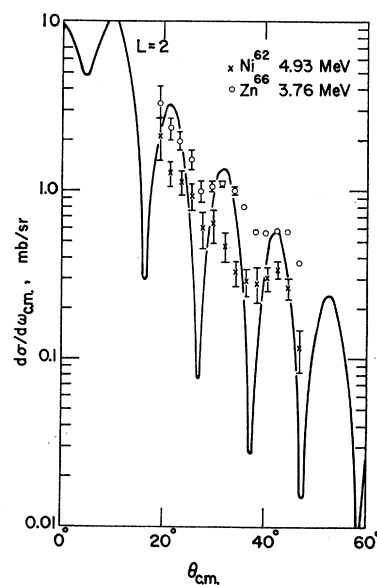


FIG. 9. Cross section for excitation of groups at 4.93 MeV in  $\text{Ni}^{62}$  and at 3.76 MeV in  $\text{Zn}^{66}$ . The points are compared with the  $L=2$  curve for  $\text{Ni}^{58}$ , which has been reduced by a factor of 6. Cross sections for groups above 4 MeV in  $\text{Ni}^{62}$  may be underestimated because of oversubtraction of background. The groups in this figure may result from combination of a  $2^+$  phonon and a  $3^-$  phonon.

state detector.) Excitation energies for groups observed in  $(\alpha, \text{He}^3)$  reactions are tabulated in Table VII. Errors are 100 to 200 keV.

The  $\text{Ni}^{62}(\alpha, \text{He}^3)$  spectrum shows a peak at approximately the energy expected for the ground-state group. Two strong groups are present at excitations of  $1.08 \pm 0.20$  MeV and  $2.04 \pm 0.20$  MeV. Their cross sections are about twice that of the group at the ground-state energy. A weak group is probably seen at 0.65 MeV.

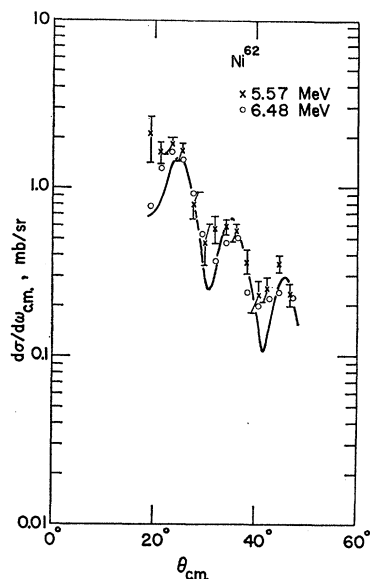


FIG. 10. Cross section for excitation of groups at 5.57 and 6.48 MeV in  $\text{Ni}^{62}$ . Errors for the group at 6.48 MeV are about the same as for the group at 5.57 MeV. The curve was drawn through the experimental points for the  $3^-$  peak for  $\text{Ni}^{62}$  (shown on Fig. 7) and then was reduced by a factor of 4.7 to facilitate comparison.

The  $\text{Zn}^{66}(\alpha, \text{He}^3)$  spectrum shows a small peak near the calculated ground-state energy, a strong but broad peak which extends from 0.4- to 0.9-MeV excitation, and two other strong peaks at 1.7- and 2.4-MeV excitation. The uncertainty in the  $Q$  values is about 0.2 MeV, and is mostly due to uncertainty in the location of the ground-state group. Thus, the levels in  $\text{Zn}^{67}$  observed by Shull and Elwyn<sup>23</sup> at 0.38, 0.88, and 1.61 MeV are probably seen in this experiment.

### H. Level Energies

Tables I through VI give the energy levels of the six nuclides as determined by several laboratories.<sup>2,3,8,11,14,23-32</sup> The agreement between the various determinations is generally consistent with quoted

<sup>23</sup> F. B. Shull and A. J. Elwyn, Phys. Rev. **112**, 1667 (1958).

<sup>24</sup> C. H. Paris and W. W. Buechner, in *Proceedings of the International Conference on Nuclear Physics, Paris, July 1958* (Crosby Lockwood and Son, London, 1959), p. 515.

<sup>25</sup> R. R. Spencer, G. C. Phillips, and T. E. Young, Phys. Rev. **108**, 69 (1957).

<sup>26</sup> D. M. Van Patter, M. A. Rothman, W. C. Porter, and C. E. Mandeville, Phys. Rev. **107**, 171 (1957); W. C. Porter, D. M. Van Patter, M. A. Rothman, and C. E. Mandeville, *ibid.* **112**, 468 (1958); A. K. Sen Gupta and D. M. Van Patter, Bull. Am. Phys. Soc. **8**, 375 (1963).

<sup>27</sup> J. Benveniste, A. C. Mitchell, and C. B. Fulmer (to be published).

<sup>28</sup> C. E. Weller and J. C. Grosskreutz, Phys. Rev. **102**, 1149 (1956).

<sup>29</sup> L. G. Mann, W. E. Meyerhof, and H. I. West, Jr., Phys. Rev. **92**, 1481 (1953).

<sup>30</sup> A. Schwarzschild and L. Grodzins, Phys. Rev. **119**, 276 (1960).

<sup>31</sup> B. L. Cohen and A. G. Rubin, Phys. Rev. **111**, 1568 (1958).

<sup>32</sup> D. J. Horen, Phys. Rev. **113**, 572 (1959).

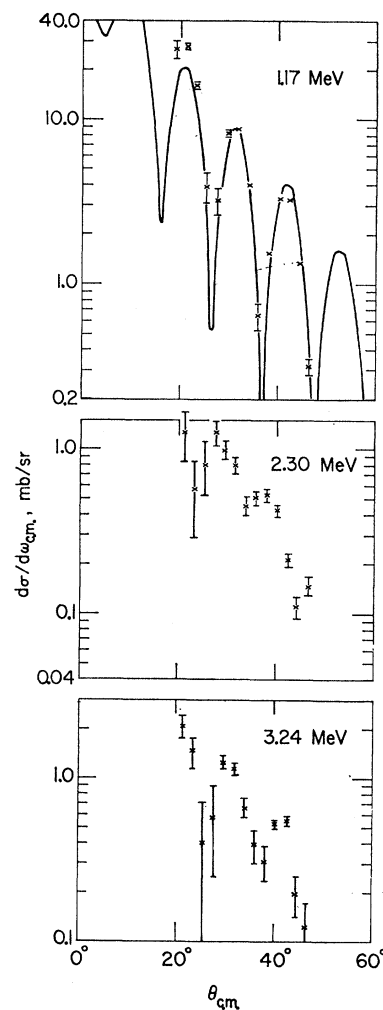


FIG. 11. Cross sections for excitation of one-phonon, two-phonon, and three-phonon groups in  $\text{Ni}^{62}$ .

uncertainties. The uncertainties in the  $Q$  values of the ANL experiments are given in the tables. In the INS work at Tokyo, errors are 20 keV or less.<sup>2</sup> The MIT work claims an accuracy of 5 keV except above 3.7 MeV in  $\text{Ni}^{60}$  where 7 keV is quoted.<sup>24</sup> The Rice work claims an accuracy of 4 to 10 keV,<sup>25</sup> the LRL data 20 to 40 keV,<sup>27</sup> and the Saclay data 30 to 50 keV.<sup>11</sup> Generally, good agreement between the determinations is found.

### V. CORRESPONDING LEVELS

#### A. Ratios to the $2^+$ Energy

If collective behavior exists in a neighboring group of nuclei, the energies and cross sections of the collective levels should vary relatively slowly and regularly with  $Z$  and  $N$ , provided no major shell closings are involved. To compare energies, the energies of the collective groups were arranged on a diagram in units of the energy of the  $2^+$  first excited level (Fig. 12). This diagram tests whether the  $3^-$  and other excitation energies have a definite ratio to the  $2^+$  energy. In this diagram, groups that have analogous properties are

TABLE VII. Excitation energies of groups seen in  $(\alpha, \text{He}^3)$  reactions (keV).

|                  |     |       |                    |      |
|------------------|-----|-------|--------------------|------|
| $\text{Ni}^{58}$ | 0   | 360   | 1610               | 3060 |
| $\text{Ni}^{60}$ | 0   | 1000  | 2000               | 3400 |
| $\text{Ni}^{62}$ | 0   | (650) | 1080               | 2040 |
| $\text{Zn}^{64}$ | 0   | 1000  | 2600               |      |
| $\text{Zn}^{66}$ | 0   |       | 400 to 900 (broad) | 1700 |
| $\text{Zn}^{68}$ | (0) | 440   |                    | 2400 |

connected together. The numbers on the diagram are the level energies in MeV. The signs refer to the phases of the angular distributions and are not always the same as the parity of the level or levels involved.

### B. Intensities and Phases

To help in the identification of corresponding groups from different nuclei, a diagram was made in which the strength of the transitions is compared. The strength of a transition was taken as the value read at  $35^\circ$  c.m. on a line passing through the maxima of the angular distribution. The strength is given in mb/sr except for the elastic peaks, in which cases ratio-to-Rutherford is given. These strengths are shown in Fig. 13, where the first number above each level is the strength and the second is called the phase position. There is need for a quantitative measure of the phases of the angular distributions, since the in-phase groups are not exactly in phase with each other, and since intermediate cases are found (e.g., the two-phonon group). Now the oscillation period of the angular distributions observed is quite close to  $10^\circ$ . Thus, a group might peak at  $27^\circ$ ,  $37^\circ$ ,  $47^\circ$ , etc. Such a peak would be given a phase position of 7 on Fig. 13. Alternatively a phase difference in degrees can be defined as

$$\delta = 360^\circ [(\phi - \phi_0)/P \pm N],$$

where  $P$  is the oscillation period in degrees,  $\phi$  is the phase position of the group,  $\phi_0$  is the phase position of elastic scattering, and  $N$  is whatever integer is needed to put  $\phi$  in the proper domain of definition (for example,  $|\delta| \leq 180^\circ$ ). Phase positions on Fig. 13 are given only to the nearest degree.

For elastic scattering the strength shown on Fig. 13 decreases steadily with  $A$  at  $35^\circ$  c.m. This effect is explained as follows. As  $A$  increases, the nuclear radius increases and the position of corresponding maxima shifts to smaller angles. But the ratio-to-Rutherford at corresponding maxima is relatively insensitive to  $A$  for the targets studied (as shown by Fig. 5). Hence, as  $A$  increases, a given angle occurs farther out in the diffraction pattern and, because the "envelope" drawn through the maxima falls uniformly with increasing angle, the strength represented appears to decrease with increasing  $A$ . (Similarly, the phase position decreases uniformly with increasing  $A$  because the increase of nuclear radius shifts the corresponding maxima to smaller angles.) Therefore, the strength of

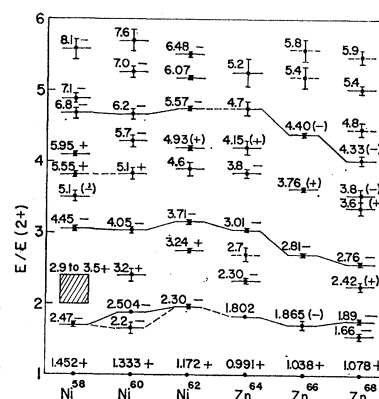


FIG. 12. Excitation energies of groups observed, in units of the energy of the  $2^+$  first excited level. The signs indicate whether a group is in phase (minus sign) or out of phase (plus sign) with elastic scattering. The signs do not necessarily indicate the parity of levels. The numbers in the figure are level energies in MeV. Groups with similar properties are connected by lines.

the elastic peak is seen to decrease with  $A$  if measured at the same c.m. angle and to be nearly constant with  $A$  if measured at the same point in the oscillation pattern (same c.m. angle multiplied by  $A^{-1/3}$ ). The question arises as to whether the strengths of inelastic peaks should be compared at the same c.m. angle or at the same position in the oscillation pattern. Fortunately, the difference is relatively small. In Fig. 13 the "envelopes" drawn through the maxima are compared at the same c.m. angle ( $35^\circ$ ) for the sake of convenience.

### C. The $2^+$ Transitions

The transitions to the  $2^+$  first excited states are almost completely out of phase with elastic scattering. As Fig. 13 shows, the phase difference averages about

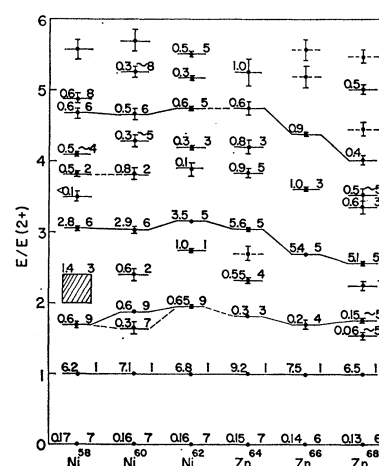


FIG. 13. Same as Fig. 12 except that the number on the left at each level is the strength (the value at  $35^\circ$  on the "envelope" of the angular distribution) in mb/sr and the number on the right is the phase position in degrees. For elastic scattering the value at  $35^\circ$  of the "envelope" of  $d\sigma/d\sigma_R$  is quoted.



+160° for these transitions. This transition has its greatest strength in Zn<sup>64</sup>. The strength decreases with increasing  $A$  in the Zn isotopes, but is larger in Ni<sup>60</sup> and Ni<sup>62</sup> than in Ni<sup>58</sup>. The two-proton difference between Ni<sup>62</sup> and Zn<sup>64</sup> has a greater effect upon the strength than do any of the two-neutron differences.

The small deformability of the 28-neutron shell is related<sup>33</sup> to the fact that the first excited levels of the 28-neutron isotopes of Ti, Cr, and Fe lie at considerably higher energies than the first excited states in the other isotopes of these elements. A similar effect is found at the 28-proton shell, since the first excited levels in even- $A$  Ni isotopes lie consistently higher than those in even Cr, Fe, or Zn isotopes with 30 to 36 neutrons. An effect for neutrons may also be seen in the Ni isotopes. The  $N=34$  minimum in the energy of the 2<sup>+</sup> collective level for even- $A$  Ni isotopes was explained by Kisslinger and Sorenson<sup>34</sup> in terms of a pairing force. Also in Zn<sup>64</sup>, Zn<sup>66</sup>, and Zn<sup>68</sup> the energy of the first excited level has its minimum value at  $N=34$ , in the middle of the  $f$ - $p$  shell, where only 6 of the 12 particles needed to fill the  $1f_{5/2}$ ,  $2p_{3/2}$ , and  $2p_{1/2}$  shell-model states are present. The lowest 2<sup>+</sup> state of Ge<sup>70</sup>, Ge<sup>72</sup>, and Ge<sup>74</sup> are at 1.04, 0.835, and 0.56 MeV, respectively. Thus, the energy of the first excited level appears to decrease again as  $N$  increases from 38 to 40 to 42, and there is evidence for a very weak closed shell at  $N=38$ .

If the optical-model wells are nearly the same for the target nuclei, the fact that the theoretical cross sections vary rather little with  $Q$  value (Fig. 5 of reference 21) means that ratios of cross sections are essentially ratios of  $\beta_2^2$ , the square of the nuclear quadrupole deformation. Hence a constant ratio should exist between the strength observed in this experiment and the  $\beta_2^2$  observed in Coulomb excitation by Stelson and McGowan.<sup>35</sup> The values of this ratio are 177 for Ni<sup>58</sup>, 171 for Ni<sup>60</sup>, 188 for Ni<sup>62</sup>, 145 for Zn<sup>64</sup>, 146 for Zn<sup>66</sup>, and 153 for Zn<sup>68</sup> (all in mb/sr). The values for two isotopes of the same element are the same within experimental error, but a significant difference occurs between  $Z=28$  and  $Z=30$ .

### D. The Two-Phonon Groups

Whereas the two-phonon groups in the nickel isotopes peak 2° later than for elastic scattering, those from the zinc isotopes peak about 2° ahead of elastic scattering. Thus, the two-phonon groups of nickel isotopes are almost completely out of phase with two-photon groups from zinc isotopes, as Fig. 8 shows for Ni<sup>62</sup> and Zn<sup>66</sup>. The angular distributions of two-phonon groups from Zn<sup>64</sup>,<sup>66</sup>,<sup>68</sup> resemble the angular distributions for the multiple excitation of a two-phonon level as computed

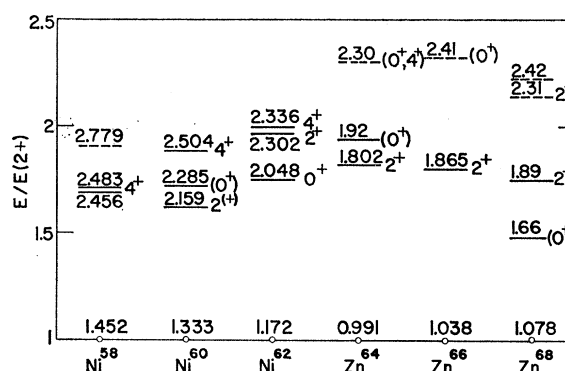


FIG. 14. Proposed scheme of two-phonon levels. Energies are plotted in units of the energy of the 2<sup>+</sup> first excited level. The level energies in MeV and the spins are given in the figure. It is uncertain whether or not the dotted levels are part of the two-phonon triplets. The level at 2.30 MeV in Zn<sup>64</sup> is probably 4<sup>+</sup> and the level at 2.41 MeV in Zn<sup>66</sup> is actually a doublet (reference 26).

by Buck.<sup>18</sup> Figure 14 shows a proposed scheme of two-phonon levels.

In Ni<sup>58</sup>, one two-phonon group was seen at 2.47 MeV. This group occurs at the position of a known 4<sup>+</sup> level.<sup>7</sup> There is evidence of a close doublet at this energy.<sup>25</sup> If so, then the group observed in the ANL experiments may be the combined scattering from two of the levels of the two-phonon triplet.

The Ni<sup>60</sup> data are consistent with the known spins,<sup>36</sup> namely, 4<sup>+</sup> for the 2.50-MeV level, 2<sup>+</sup> for the 2.16-MeV level, and 0<sup>+</sup> for the 2.29-MeV level. For Ni<sup>60</sup> two levels of the two-phonon triplet were resolved, whereas only one such level was resolved for Ni<sup>58</sup> and Ni<sup>62</sup>. The level at lower excitation in Ni<sup>60</sup> peaks slightly earlier than the other, and is only about half as strong. This factor of 2 has a possible interpretation in terms of statistical weights. If the cross sections of the two-phonon levels in a given target are proportional to  $2I+1$ , then the strengths of the 0<sup>+</sup>, 2<sup>+</sup>, and 4<sup>+</sup> levels are proportional to 1, 5, and 9, respectively. Thus, the stronger two-phonon group is from the 4<sup>+</sup> level and the weaker from the 2<sup>+</sup> level. (The 0<sup>+</sup> level is presumably so weak that it is obscured by the other two.) This explanation is consistent with the known spins.

In Ni<sup>62</sup> the known spins<sup>37</sup> are 0<sup>+</sup> at 2.048 MeV, 2<sup>+</sup> at 2.302 MeV, and 4<sup>+</sup> at 2.336 MeV. In this experiment only one group was seen in this region, at  $2.30 \pm 0.02$  MeV. The known level at 2.048 MeV was not observed and hence is definitely a 0<sup>+</sup> level, since the cross section to a 0<sup>+</sup> two-phonon level is expected to be smaller than the other cross sections.

In the Zn isotopes the situation is quite unsettled. The main difficulty is the lack of level-finding experiments with high resolution, such as have been performed

<sup>33</sup> O. Hansen, Nucl. Phys. **28**, 140 (1961).

<sup>34</sup> L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **32**, No. 9 (1960).

<sup>35</sup> P. H. Stelson and F. K. McGowan, Nucl. Phys. **32**, 652 (1962).

<sup>36</sup> Nuclear Data Sheets, compiled by K. Way et al. (Printing and Publishing Office, National Academy of Sciences-National Research Council, Washington 25, D. C.), NRC 60-5-25.

<sup>37</sup> A. K. Sen Gupta, P. N. Trehan, and D. M. Van Patter, Bull. Am. Phys. Soc. **7**, 81 (1962); D. M. Van Patter and A. K. Sen Gupta, *ibid.* **8**, 49 (1963).

on the nickel isotopes. It is difficult to interpret the observed groups when the number of levels that could produce them is uncertain. Also it is possible that different types of experiments may see different levels in the same energy region.

In  $\text{Zn}^{64}$  the experiments have revealed groups at 1.79 and 2.30 MeV. The two levels have very closely the intensity ratio 5:9 suggested by the  $2I+1$  factor. Hence, the 1.79-MeV level may be  $2^+$  and the 2.30-MeV level may be  $4^+$ . Van Patter *et al.*<sup>26,38</sup> have found a  $2^+$  level at 1.799 MeV and a  $4^+$  level at 2.33 MeV. A weak group at  $1.92 \pm 0.04$  MeV was seen by Benveniste, Mitchell, and Fulmer.<sup>27</sup> Possibly this level may be the  $0^+$  member of the triplet.

In  $\text{Zn}^{66}$  a weak two-phonon group is seen at 1.8 MeV and may be due primarily to the known  $2^+$  level at 1.865 MeV.<sup>26</sup> The known  $0^+$  level at 2.41 MeV was not seen in this experiment. It is possible that the very strong  $3^-$  level at 2.81 MeV may have obscured it.

In  $\text{Zn}^{68}$ , peaks were seen at 1.66, 1.89, and 2.42 MeV. The 1.89-MeV level is known to be a  $2^+$  level.<sup>38</sup> The 1.66-MeV group is weaker than this  $2^+$  level and hence is probably due to a  $0^+$  level. Systematics also makes this assignment plausible: In  $\text{Ge}^{70}$  the  $0^+$  second excited state occurs<sup>36</sup> at 1.21 MeV, which is 1.16 times the one-phonon energy; in  $\text{Ge}^{72}$  the  $0^+$  state occurs at 0.69 MeV, which is 0.83 times the one-phonon energy. Thus, it is not surprising to find a  $0^+$  level in  $\text{Zn}^{68}$  at 1.66 MeV (1.5 times the one-phonon energy). In  $\text{Zn}^{68}$  a  $2^+$  level is known to exist<sup>39</sup> at  $2.31 \pm 0.02$  MeV. This level is presumably not a part of the two-phonon triplet, and is probably distinct from the level found at  $2.42 \pm 0.06$  MeV in this experiment and at  $2.40 \pm 0.05$  MeV in the Saclay experiment.<sup>11</sup>

### E. Three-Phonon Groups

A group at 3.24 MeV in  $\text{Ni}^{62}$  has been found<sup>40</sup> to have some of the properties expected of a three-phonon group.

The vibrational model of the nucleus<sup>41</sup> predicts a  $2^+$  one-phonon first excited state, a  $0^+$ ,  $2^+$ ,  $4^+$  two-phonon triplet, and a  $0^+$ ,  $2^+$ ,  $3^+$ ,  $4^+$ ,  $6^+$  three-phonon quintet.<sup>42</sup> Furthermore, the simple harmonic oscillator predicts that the energies of these states are  $n\hbar\omega$ , where  $n$  is the number of phonons and  $\hbar\omega$  is the energy of the first excited state. However, in nuclei the presumed two-phonon states do not lie at exactly  $2\hbar\omega$  and are not degenerate. Thus there is need for a model that

TABLE VIII. Theoretical and experimental levels in the three-phonon region of  $\text{Ni}^{62}$ .

|                             | Spin  | Energy (MeV)      |
|-----------------------------|-------|-------------------|
| Theory (reference 43)       | $2^+$ | 2.93              |
|                             | $0^+$ | 3.20              |
|                             | $3^+$ | 3.22              |
|                             | $4^+$ | 3.23              |
|                             | $6^+$ | 3.26              |
|                             |       | 2.888 $\pm$ 0.005 |
| Experimental (reference 24) |       | 3.055 $\pm$ 0.005 |
|                             |       | 3.155 $\pm$ 0.005 |
|                             |       | 3.175 $\pm$ 0.005 |
|                             |       | 3.254 $\pm$ 0.005 |
|                             |       | 3.267 $\pm$ 0.005 |
|                             |       | 3.24 $\pm$ 0.04   |
| This experiment             |       | 2.90              |
| Experiment (reference 3)    |       | 3.16              |
|                             |       | 3.29              |
|                             |       |                   |

can predict the energies of the two-phonon states more accurately than the simple harmonic oscillator can. Such a model has recently been proposed by Kerman and Shakin<sup>43</sup> who have included cubic terms in the nuclear vibrational Hamiltonian. These authors achieved a good fit to the energies of the three two-phonon levels of  $\text{Ni}^{62}$  in terms of two parameters. Their formulas and parameters have been used to compute the energies of the three-phonon quintet of  $\text{Ni}^{62}$ . The results are shown in Table VIII together with the six levels found experimentally in this region by Paris and Buechner.<sup>24</sup> The agreement is very good. In this experiment an energy of  $3.24 \pm 0.04$  was found for the possible three-phonon group. The linewidth (0.29 MeV full-width at half-maximum) was larger than the theoretical peak separation. Hence, the experimental three-phonon energy should be compared with an average over the theoretical energies. A weighting factor of  $2I+1$  was used in computing the average. The average of the theoretical energies is 3.20 MeV.

It is possible to object that the observed group may be primarily from a  $4^+$  one-phonon level. This interpretation seems unlikely for three reasons. First, if the hexadecapole oscillation were solely responsible for this group, it would presumably produce sharp peaks in the  $\text{Ni}^{58}$  and  $\text{Ni}^{60}$  spectra at roughly this same energy. However, the observed peaks<sup>8</sup> at 3.2 MeV in  $\text{Ni}^{58}$  and  $\text{Ni}^{60}$  are much wider than the peak from a single level would be. Secondly, the strongest  $E4$  excitations found in  $\text{Ni}^{58}$  and  $\text{Ni}^{60}$  by inelastic electron scattering<sup>7</sup> are at considerably higher energies: 7.55 MeV in  $\text{Ni}^{58}$  and 5.1 MeV in  $\text{Ni}^{60}$ . Thirdly, the deuteron-scattering experiment of Jolly *et al.*<sup>3</sup> reveals three inelastic groups in  $\text{Ni}^{62}$  in the three-phonon region. Their experiment has a much smaller linewidth (40 keV) than the present experiment, but data are given for only one angle. The groups found by Jolly *et al.* are at 2.90, 3.16, and 3.29 MeV. These values are consistent with the other experiments and the theoretical position of the three-

<sup>38</sup> D. M. Van Patter, N. Nath, and M. A. Rothman, Bull. Am. Phys. Soc. **5**, 266 (1960).

<sup>39</sup> M. K. Ramaswamy and P. S. Jastram, Nucl. Phys. **16**, 113 (1960).

<sup>40</sup> Most of the material in this section on three-phonon groups has been reported by H. W. Broek, Phys. Letters **3**, 132 (1962).

<sup>41</sup> G. Scharff-Goldhaber and J. Weneser, Phys. Rev. **98**, 212 (1955).

<sup>42</sup> L. J. Tassie, Australian J. Phys. **15**, 135 (1962).

<sup>43</sup> A. K. Kerman and C. M. Shakin, Phys. Letters **1**, 151 (1962).

phonon levels. Therefore, levels with three-phonon properties are involved in the group at  $3.24 \pm 0.04$  MeV in  $\text{Ni}^{62}$  and probably in the broad groups at 3.2 MeV in  $\text{Ni}^{58}$  and  $\text{Ni}^{60}$ .

### F. The Collective Octupole Groups

The strong  $3^-$  groups were observed in all six targets. As Fig. 13 shows, the strength of the  $3^-$  groups increases slightly with  $A$  in nickel, decreases slightly with  $A$  in zinc, and is roughly 1.8 times as great in zinc as in nickel. The phase difference averages about  $-50^\circ$ . Thus, the  $3^-$  groups have maxima in their cross sections about  $\frac{1}{8}$  cycle ahead of those of elastic scattering.

The ratios of  $3^-$  excitation energy to  $2^+$  excitation energy are  $3.06 \pm 0.03$  for  $\text{Ni}^{58}$ ,  $3.04 \pm 0.03$  for  $\text{Ni}^{60}$ ,  $3.16 \pm 0.02$  for  $\text{Ni}^{62}$ ,  $3.04 \pm 0.02$  for  $\text{Zn}^{64}$ ,  $2.71 \pm 0.02$  for  $\text{Zn}^{66}$ , and  $2.56 \pm 0.02$  for  $\text{Zn}^{68}$ .

In the nickel data the tendency is for the  $3^-$  cross section to increase with increasing neutron number while the  $3^-$  energy decreases. The assumption that the optical-model wells for the target nuclei are nearly the same and the fact that the theoretical calculations are insensitive to the  $Q$  value imply that the  $3^-$  cross sections are proportional to  $\beta_3^2$ , the square of the nuclear octupole deformation. Hence, the nuclear octupole deformation increases with increasing neutron number for the Ni isotopes studied. And the octupole excitation energy decreases in these isotopes as the octupole deformation increases, in analogy to the results for the quadrupole excitation. But in the zinc isotopes the situation is paradoxical in that both the  $3^-$  cross section and the  $3^-$  energy decrease with increasing neutron number. The decrease in cross section is small and may be an effect of the increasing nuclear radius, because the ratio-to-Rutherford decreases with neutron number by about the same percentage. Hence,  $\beta_3$  is about the same in all the zinc isotopes observed. This equality is presumably related to the large neutron pairing energy: The  $3^-$  level lies below the neutron pairing energy in  $\text{Zn}^{66}$  and  $\text{Zn}^{68}$ . Neutron pairing energy per pair<sup>44</sup> is defined as

$$P_n(Z, N) = 2M(Z, N-1) - M(Z, N) - M(Z, N-2).$$

The neutron pairing energy is  $3.04 \pm 0.01$  MeV for  $\text{Zn}^{66}$  and  $3.14 \pm 0.02$  MeV for  $\text{Zn}^{68}$ . Hence, the  $3^-$  octupole excitation in Zn must contain a large proportion of shell-model states in which protons, rather than neutrons, are excited. (Proton pairing energies are only about 1.5 MeV for these nuclei.)

By way of contrast, the neutron pairing energies per pair in nickel ( $2.39 \pm 0.02$  MeV for  $\text{Ni}^{60}$ , and  $2.85 \pm 0.02$  MeV for  $\text{Ni}^{62}$ ) are smaller than the  $3^-$  energy. The belief that states with excited protons are involved in the octupole levels in the Zn isotopes studied is further

strengthened by the fact that the cross section is considerably higher for these zinc isotopes than for any of the nickel isotopes studied.

### G. Possible Second $3^-$ Collective Levels

In each nickel isotope, a group whose angular distribution resembles that of the collective  $3^-$  level (except for a constant factor) is seen at a higher excitation. The strength is 0.6 mb/sr at  $35^\circ$  in all three cases. These groups were at  $6.8 \pm 0.1$  MeV in  $\text{Ni}^{58}$ ,  $6.2 \pm 0.1$  MeV in  $\text{Ni}^{60}$ , and  $5.57 \pm 0.03$  MeV in  $\text{Ni}^{62}$ . The ratios of the energies of these groups to that of the  $3^-$  collective level are  $1.53 \pm 0.03$  for  $\text{Ni}^{58}$ ,  $1.53 \pm 0.03$  for  $\text{Ni}^{60}$ , and  $1.50 \pm 0.01$  for  $\text{Ni}^{62}$ . It seems unlikely that it can be merely coincidental that these ratios, cross sections, and angular distributions should be so similar. Also, the energy ratios are very close to the simple ratio  $3/2$ .

A search for similar groups in zinc leads to the groups at  $4.7 \pm 0.1$  MeV in  $\text{Zn}^{64}$ ,  $4.40 \pm 0.05$  MeV in  $\text{Zn}^{66}$ , and  $4.33 \pm 0.05$  MeV in  $\text{Zn}^{68}$ . Their ratios to the  $3^-$  energy are  $1.56 \pm 0.04$  for  $\text{Zn}^{64}$ ,  $1.57 \pm 0.03$  for  $\text{Zn}^{66}$ , and  $1.57 \pm 0.03$  for  $\text{Zn}^{68}$ . The similarity in energy ratio is striking, although the strengths show considerable variation (Fig. 13).

These groups do not constitute the  $l=5$  surface vibration because their angular distributions show (for  $\text{Ni}^{58}$  and  $\text{Ni}^{60}$ ) deep minima at  $20^\circ$ , and a minimum at  $20^\circ$  is expected<sup>45</sup> for  $l=1$  or  $l=3$  but not  $l=5$ . Other possible explanations are: (a) existence of more than one collective  $3^-$  level,<sup>46</sup> (b) a two-phonon level consisting of one  $2^+$  phonon and one  $3^-$  phonon, and (c) a two-phonon level consisting of two  $3^-$  phonons. The observed angular distribution is obviously consistent with (a). However, the other possibilities cannot be ruled out. Nevertheless, possibility (b) seems unlikely because of other observed groups whose energies are closer to the sum of the  $2^+$  energy and the  $3^-$  energy (as discussed more fully below).

A group at 6.9 MeV in  $\text{Ni}^{58}$  excited by an  $E4$  process has been observed in electron scattering.<sup>47</sup> The angular distribution seen for the group at  $6.8 \pm 0.1$  MeV in the present experiment is incompatible with a  $4^+$  one-phonon excitation.

The existence of more than one  $3^-$  level has also been reported in Fe isotopes<sup>4</sup> and other nuclei.<sup>2</sup>

### H. Possible $4^+$ and $5^-$ Vibrations

Possible  $4^+$  states were found<sup>8</sup> in  $\text{Ni}^{58}$  at 5.55 MeV and in  $\text{Ni}^{60}$  at 5.1 MeV. The angular distributions of these levels show a good resemblance to the DWBA

<sup>45</sup> J. Saudinos, *Compt. Rend.* **252**, 260 (1961).

<sup>46</sup> E. A. Sanderson and N. S. Wall, *Phys. Letters* **2**, 173 (1962).

<sup>44</sup> Pairing energy per nucleon has been tabulated according to an alternative definition by R. C. Barrett, *Phys. Rev.* **127**, 1670 (1962).

<sup>47</sup> H. W. Kendall, in *Electromagnetic Lifetimes and Properties of Nuclear States*, Rept. No. 37, Nuclear Science Series, National Academy of Sciences-National Research Council (U. S. Government Printing Office, Washington, D. C., 1962), p. 168.

calculation for a one-phonon  $4^+$  collective level. However, the observed maxima tend to occur at slightly larger angles than the calculated ones (Figs. 9 and 11, reference 8), and so at least an admixture of other states must be present. Such an admixture might be from a  $5^-$  level, since the electron-scattering experiments<sup>7</sup> gave a probable  $4^+$ , possible  $5^-$  assignment to the 5.1-MeV group of  $\text{Ni}^{60}$ . The ratio to the  $2^+$  energy is  $3.82 \pm 0.04$  for the 5.55-MeV level in  $\text{Ni}^{58}$ , and  $3.82 \pm 0.08$  for the 5.1-MeV level in  $\text{Ni}^{60}$ . However, no similar level is seen near  $E/E(2^+) = 3.82$  in the other isotopes:  $\text{Ni}^{62}$  has only a weak level here,  $\text{Zn}^{64}$  has a group with opposite phase, and  $\text{Zn}^{66}$  and  $\text{Zn}^{68}$  have no peak at all in this range. Thus, it is not certain that the collective hexadecapole oscillation for  $\text{Ni}^{58}$  and  $\text{Ni}^{60}$  has been observed. It is possible that these levels may be part of the two-phonon quintet arising from one  $2^+$  phonon and one  $3^-$  phonon.

Similarly, no set of corresponding one-phonon  $5^-$  levels is evident from the data. In the shell model the configurations  $1f1g_{9/2}$  and  $2p_{3/2}1g_{9/2}$  are the lowest configurations useful for constructing  $3^-$  states. Such configurations also produce  $5^-$  states, and thus the existence of a collective duokaitriakontapole (32-pole) oscillation might be suspected. However, the data show that such a level is either nonexistent or else weak and obscured by neighboring levels in this experiment. Even so, a two-phonon  $5^-$  level may exist (as explained in the following section).

### I. Other Groups

Of great interest is the region in which  $E \approx E(3^-) + E(2^+)$ . In this region a quintet of combination frequencies is expected from the combination of a  $2^+$  and a  $3^-$  phonon, which should produce<sup>42</sup> observable  $1^-$ ,  $2^-$ ,  $3^-$ ,  $4^-$ , and  $5^-$  levels. In all six targets, groups have been observed near this sum energy. These groups peak between  $1^\circ$  and  $5^\circ$  earlier than elastic scattering in all cases. Table IX compares the predicted energy with the energies of the observed groups and with a weighted average of the observed energies. The weighting factor used was the strength of the group as given in Fig. 13. The average of the observed  $Q$  values is between 0.03 and 0.15 MeV less than the sum in each case. The predicted energy is  $E(3^-) + E(2^+)$ .

Of course, it is impossible to be sure whether or not these groups represent the combination of a  $2^+$  and a  $3^-$  phonon until a measurement of the spins and parities is made.

### J. Energy Ratios

The ratio of the energy  $E(3^-)$  of the  $3^-$  collective level to the energy  $E(2^+)$  of the lowest collective  $2^+$  level is quite close to 3 for  $\text{Ni}^{58}$ ,  $\text{Ni}^{60}$ , and  $\text{Zn}^{64}$ . However, the energy of the lowest  $2^+$  level is sensitive to shell

TABLE IX. Energies of groups in the region in which  $E \approx E(3^-) + E(2^+)$ .

|                  | Theory | Experiment |      | Experiment averaged |
|------------------|--------|------------|------|---------------------|
| $\text{Ni}^{58}$ | 5.90   | 5.55       | 5.95 | 5.75                |
| $\text{Ni}^{60}$ | 5.38   | 5.1        | 5.7  | 5.26                |
| $\text{Ni}^{62}$ | 4.88   | 4.6        | 4.93 | 4.85                |
| $\text{Zn}^{64}$ | 4.00   | 3.8        | 4.15 | 3.97                |
| $\text{Zn}^{66}$ | 3.85   | 3.76       |      | 3.76                |
| $\text{Zn}^{68}$ | 3.84   | 3.6        | 3.8  | 3.69                |

closures and is a well-known indicator of nuclear type.<sup>41</sup> Sheline<sup>48</sup> has pointed out that for even-even nuclei with  $AE(2^+) < 40$  the quantity  $AE(2^+)$  is directly proportional to  $\gamma$ , the parameter describing the deviation from axial symmetry. A plot of  $AE(2^+)$  vs  $A$  for all known cases reveals that  $AE(2^+)$  lies between 12 and 50 MeV for deformed nuclei, lies above 110 MeV for nuclei with two closed shells, lies between 70 and 250 MeV for nuclei with single closed shells, and lies between 40 and 160 MeV for nuclei describable by vibrational, shell, or pairing models. Furthermore nuclei with  $Z=28$  or  $N=28$  have values of  $AE(2^+)$  which lie between 72 and 86 MeV, which are higher than those for neighboring even-even nuclei, and which therefore indicate weak closed shells at  $Z=28$  and  $N=28$ . Similarly, very weak shell effects may be indicated at  $N=38$  and  $Z=40$ .

But the energy  $E(3^-)$  of the  $3^-$  level varies with  $A$  in relatively smooth fashion.<sup>49</sup> The quantity  $E(3^-)$  is here defined as the energy of the  $3^-$  collective level, and is assumed to be the energy of the lowest  $3^-$  level in cases in which the collective character has not been established. The significance of the quantity<sup>48</sup>  $AE(2^+)$  leads one to examine  $AE(3^-)$ . Except for  $\text{O}^{16}$  and  $\text{Pb}^{208}$ , the quantity  $AE(3^-)$  lies between 140 and 300 MeV (for nuclei in which a  $3^-$  level is known). The quantity  $AE(3^-)$  appears to be insensitive to single closed shells or to ellipsoidal deformations. Values of  $AE(3^-)$  for  $\text{Ni}^{58, 60, 62}$  and  $\text{Zn}^{64, 66, 68}$  are, respectively, 258, 242, 230, 192, 185, and 188 MeV. The constancy among the Zn isotopes is interesting in view of the suggestion made above that configurations with unpaired neutrons are "frozen out" of the  $3^-$  collective level in the Zn isotopes.

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<sup>48</sup> R. K. Sheline, Nucl. Phys. **31**, 335 (1962).

<sup>49</sup> A. M. Lane and E. D. Pendlebury, Nucl. Phys. **15**, 39 (1960).