Nitrogen-14 and the Shell Model

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A shell-model calculation has been done of the lower lying levels of N^{14} under the assumption that the C^{12} nucleus forms an inert core about which two nucleons can move in $1p_{1/2}$, $2s_{1/2}$, $1d_{5/2}$, or $1d_{3/2}$ orbitals. It has been possible to give shell-model assignments to practically all of the observed levels in N^{14} below 10.50 MeV, and these assignments are in good agreement with the experimental data and with other theoretical calculations. Several levels not predicted by this model are expected to arise from the excitation of $1p_{3/2}$ particle(s) out of the C^{12} core into higher orbitals.

I. INTRODUCTION

THERE has been a great deal of experimental work and theoretical work done on the nuclear properties of nitrogen-14 and neighboring nuclei in the past decade. Nitrogen-14 is at the end of the $1p$ shell and just before the $2s,1d$ shell. The nitrogen-14 nucleus still has few enough nucleons that it is amenable to a more or less detailed shell-model calculation such as those calculations done by Inglis,[1] Kurath,[2] Visscher and Ferrell,[3] Elliott,[4] Skyrme,[5] and others.[6–9] Talmi and Unna[8] and Warburton and Pinkston[9] have considered more specific models in their calculations and have met with varying degrees of success depending on the model or models assumed. These latter calculations indicate that in the jj-coupling notation, a calculation should include the $1p_{3/2}$ and $1p_{1/2}$ particles in the $1p$ shell as well as the $2s_{1/2}$, $1d_{5/2}$, and $1d_{3/2}$ particles in the $2s,1d$ shell. It is unlikely that the structure and properties of the lower lying states of N^{14} are affected appreciably by excitation of the $1s_{1/2}$ particles. There are, however, certain properties of the lower lying states which can be explained by exciting particles into the $2p,1f$ shell.[7]

The earlier calculations[1–6] considered configurations in the $1p$ shell only and were intermediate-coupling calculations. For nuclei between mass 5 and mass 16, the coupling scheme appears to be nearly a LS coupling scheme near the beginning of the $1p$ shell and progresses steadily towards a jj coupling scheme as one approaches the end of the $1p$ shell. Many of the low-lying levels of N^{14} result from exciting one or more particles from the $1p$ shell into the $2s,1d$ shell. Consequently these calculations could not hope to explain all the lower lying levels in N^{14}.

Warburton and Pinkston[9] used experimental data such as cross sections, electric quadrupole and magnetic dipole transition rates, branching ratios of electromagnetic transitions, spin, parity, etc., as a guide to determine what the configurations and their admixtures were for practically all the levels in N^{14} up to 10.50 MeV. The results of Warburton and Pinkston will be compared with the results of this paper in Sec. III.

Talmi and Unna[8] have taken an entirely different approach and assumed that the states are given by pure jj configurations with very little mixing between these configurations. The method of Talmi and Unna is to adjust several parameters which describe an effective two-body force between the particles so that the best agreement with the experimental levels is obtained. By adjusting their force parameters in this way, Talmi and Unna are able to include for the most part the effects of configuration mixing on the energy levels.[8,10] Talmi and Unna are not restricted to one nucleus but are able to apply their method with the same parameters with a great deal of success to the several nuclei in one part of the periodic table whose basic configurations come from the same set of shell-model levels. The results of Talmi and Unna for N^{14} will be compared with the results in this paper in Sec. III.

In this paper, a conventional two-particle shell-model calculation of N^{14} was done with the assumption that C^{12} was an inert core with a configuration of $(1s_{1/2})^4(1p_{3/2})^8$. With this assumption, the lower lying states of N^{14} would then result from two particles in $1p_{1/2}$, $2s_{1/2}$, $1d_{5/2}$, or $1d_{3/2}$ orbitals. Since C^{12} does not really form an inert core, it is expected, as the p-shell calculations[1–5] and this calculation confirm, that some of the lower levels in N^{14} will result from $1p_{3/2}$ particles being excited out of the C^{12} core into higher orbitals. The levels in N^{14} which arise in this way will be called

* Research supported in part by the United States Atomic Energy Cioonmission.
[1] D. R. Inglis, Phys. Rev. **87**, 915 (1952); Rev. Mod. Phys. **25**, 390 (1953).
[2] D. Kurath, Phys. Rev. **101**, 216 (1956); **106**, 975 (1957); D. Kurath and L. Picman, Nucl. Phys. **10**, 313 (1959).
[3] W. M. Visscher and R. A. Ferrell, Phys. Rev. **107**, 781 (1957).
[4] J. P. Elliott, Phil. Mag. **1**, 503 (1956).
[5] Skyrme, in *Proceedings of the Rehoveth Conference on Nuclear Structure*, edited by M. H. Lipkin (North-Holland Publishing Company, Amsterdam, 1958).
[6] A quite complete list of references for experimental and theoretical work done on nitrogen-14 can be found in F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959).
[7] S. Fallieros, Ph.d. thesis, University of Maryland, 1959, Physics Department Technical Report No. 128 (unpublished).
[8] I. Talmi and I. Unna, Ann. Rev. Nucl. Sci. **10**, 353 (1960); Phys. Rev. **112**, 452 (1958).
[9] E. K. Warburton and W. T. Pinkston, Phys. Rev. **118**, 733 1960).
[10] W. W. True and E. K. Warburton, Nucl. Phys. **22**, 426 (1961).

core-excited levels and, wherever possible, these states will be pointed out in Sec. III.

Section II of this paper will discuss the calculation of the energy levels of N^{14}, the parameters used, and the results. In Sec. III the results of this calculation will be compared with other theoretical calculations and with various experimental information.[11–15]

II. CHOICE OF PARAMETERS AND RESULTS

It is desirable to choose the parameters used in any calculation from first principles as much as possible. In addition, one must be guided by the parameters which have worked well in the past and finally make a compromise between the above two sets of parameters or rationalize on the set of parameters used. Since neither the force between nucleons nor the many particle system is understood in detail, the final justification is the comparison of the theoretical results with the experimental data.

As pointed out in Sec. I, the C^{12} nucleus will be considered as an inert core which forms a central attractive potential in which two nucleons, a proton and a neutron, move. It will further be assumed that these nucleons can only move in the $1p_{1/2}$, $2s_{1/2}$, $1d_{5/2}$, and $1d_{3/2}$ orbitals. Since no confusion will arise in this paper, the radial quantum number shall be omitted and $1p_{1/2}$, $2s_{1/2}$, $1d_{5/2}$, and $1d_{3/2}$ will be written as $p_{1/2}$, $s_{1/2}$, $d_{5/2}$, and $d_{3/2}$, respectively.

The states of N^{14} can be described by the energy, total angular momentum, parity, total isotopic spin, and z component of isotopic spin. In the case of N^{14}, the z component of isotopic spin is zero. The nuclear force is essentially charge independent and it will, therefore, be assumed that the total isotopic spin of a state is a good quantum number. The Coulomb force does not conserve isotopic spin, but tends to mix states with different values of the isotopic spin, T. For example, the proton in N^{14} interacting with the C^{12} core will mix a small amount of $T=1$ states into the $T=0$ states and a small amount of $T=0$ states into the $T=1$ states. This admixture in N^{14} will be 1% or less and will be neglected. Note that our wave functions will be antisymmetric under interchange of the two particles in this space, spin, and isotopic spin space.

A. Single Particle Parameters

The difference in the interactions of neutrons in the various orbitals with the C^{12} core can be determined from the experimental energy levels of C^{13} if the states listed in Table I are assumed to be pure single particle states as is done in this paper. These single particle interactions are normalized so that a $p_{1/2}$ particle has zero interaction with the core. It is then necessary to add an arbitrary energy normalization to the resulting energy levels in N^{14} in order to have the lowest calculated $J^\pi,T=1^+,0$ level coincide with the energy of the ground state of N^{14} which is at zero MeV. It is possible to calculate what the absolute interaction energy of a $p_{1/2}$ neutron with the core is by using the $C^{12}-C^{13}$ mass difference.

TABLE I. Levels in C^{13} and N^{13} assumed to be pure single-particle levels.

Configuration	C^{13} levels (MeV)	N^{13} levels (MeV)
$1p_{1/2}$	0	0
$2s_{1/2}$	3.09	2.367
$1d_{5/2}$	3.85	3.56
$1d_{3/2}$	8.33	8.08

Note that this interaction energy of a $p_{1/2}$ neutron with the core is based on the assumption that the other 12 nucleons form an inert spherical core. That is, in this paper, the deformation of the C^{12} core and the fact that the ground state of C^{12} is not a pure $(1s_{1/2})^4(1p_{3/2})^8$ configuration have been neglected.[2] According to Thomas,[16] and Lane and Thomas,[17] the resonance energies observed by scattering experiments will be shifted from the true energy eigenvalues of the system. Calculation of these energy shifts require the adoption of specific nuclear models for C^{13} and N^{13}. For the purposes of this paper, it will be assumed that these shifts are negligible.

The Coulomb interaction energy of a proton with the C^{12} core can be determined by assuming that the Coulomb interaction energy is the difference between the interaction energy of a neutron in a given orbital with the C^{12} core and the interaction energy of a proton in the same orbital with the C^{12} core. These Coulomb and nuclear interaction energies with the C^{12} core for the four orbitals considered are listed in Table II.

Harmonic oscillator wave functions[18] were used to

TABLE II. Single-particle interaction energies with the C^{12} core. These interaction energies are normalized as discussed in the text.

Configuration	Nuclear interaction energy (MeV)	Coulomb interaction energy (MeV)
$1p_{1/2}$	0	3.005
$2s_{1/2}$	3.09	2.280
$1d_{5/2}$	3.85	2.715
$1d_{3/2}$	8.33	2.755

[11] Reference 6 does an excellent job of reviewing the experimental information up to about 1959. See references 12–15 for later experimental information.

[12] B. G. Harvey, J. Cerny, R. H. Pehl, and E. Rivet, Nucl. Phys. **39**, 160 (1962); B. G. Harvey and J. Cerny, Phys. Rev. **120**, 2162 (1960).

[13] B. G. Harvey, J. Cerny, R. H. Pehl, E. Rivet, and W. W. True (to be published).

[14] R. E. Segel, J. W. Daughtry, and J. W. Olness, Phys. Rev. **123**, 194 (1961).

[15] E. Kashy, R. R. Perry, and J. R. Risser, Phys. Rev. **117**, 1289 (1960).

[16] R. G. Thomas, Phys. Rev. **97**, 224 (1955).

[17] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

[18] I. Talmi, Helv. Phys. Acta **25**, 185 (1952).

represent the single-particle wave functions and their space dependence is

$$\Psi_{nlm}\sim r^{l}e^{-\nu r^{2}/2}L_{n+l-1/2}{}^{l+1/2}(\nu r^{2})Y_{lm}(\theta,\phi), \quad (1)$$

where $L_q{}^p(x)$ is an associated Laguerre polynomial.[19] The value of ν used in this calculation will be discussed in Sec. IIC.

B. Nuclear Force Parameters

Since N^{14} consists of a neutron and a proton outside the C^{12} core, the nuclear interaction between these two particles, assumed to be a central interaction, can be separated into four separate forces, a singlet-even force, a triplet-even force, a singlet-odd force, and a triplet-odd force. Analysis of experimental results[20] and other theoretical calculations[21,22] indicate that the singlet-odd and triplet-odd forces play a less dominant role than the singlet-even and triplet-even forces in the two-particle interaction. For this reason, the neutron-proton force is assumed to consist of a singlet-even force and a triplet-even force and that both of these forces have the same radial dependence. A Gaussian well was taken for this radial dependence for calculational simplicity. The nuclear force is, therefore, of the form

$$V(r)=V_0e^{-\beta r^2}[(1+\alpha)+(1+\alpha)P^r+(\alpha-1)P^\sigma+(\alpha-1)P^rP^\sigma], \quad (2)$$

where P^r and P^σ are the space exchange and spin exchange operators, respectively. α is the ratio of the triplet-even force strength to the singlet-even force strength. The value of V_0 and β were taken to be -8.125 MeV and 0.2922 F^{-2}, which makes this force have a singlet-even strength the same as the singlet-even force[21] used successfully in Pb^{206}. This singlet-even force has an effective range, $r_{0s}=2.65$ F, and a bound state at zero energy.[23] The choice of the parameters α and ν is discussed below.

C. Parameters ν and α

The choice for the two parameters ν and α was not discussed in the preceding two sections because they cannot be determined with as much confidence as the other parameters in the calculation. These parameters were chosen partially by heuristic calculations and partially by the best agreement with the experimental energy levels of N^{14} as will be discussed below.

From the relative position of the singlet and triplet states of the deuteron, one expects[22] that the ratio of the triplet-even strength to the singlet-even strength, α, to be about 1.5. Calculations have been made of the excited energy levels of N^{14} with the parameter α varying from 1 to 3 in steps of 0.5 for various choices of ν. The energies of the $T=1$ and $T=0$ states varied quite rapidly with α. For example, for $\nu=0.3$ F^{-2} and with the lowest $J^\pi,T=1^+,0$ level normalized to zero MeV, the lowest $0^+,1$ level varied linearly from about 0.5 to about 7 MeV as α was changed from 1 to 3. The other $1^+,0$ levels changed at a slightly slower rate when α was varied.

The harmonic oscillator parameter ν can be determined in several ways. Following Redlich's approach,[24] the expectation value of r^2, $\langle r^2\rangle$, in the p shell is $5/2\nu$ and the expectation value of r^2 in the s,d shell is $7/2\nu$. Picking a fixed ν for all particles and assuming that it is the latter value of $\langle r^2\rangle$ which is important, one can determine ν by using

$$\langle r^2\rangle=(1.40A^{1/3}\text{ F})^2. \quad (3)$$

Taking $A=14$, this value of $\langle r^2\rangle$ yields $\nu\approx0.3$ F^{-2} and

TABLE III. Energy levels and dominant configurations for N^{14} with $\nu_p=\nu_{s,d}=0.3$ F^{-2} and $\alpha=1.6$.

J^π,T	Energy (MeV)	Dominant configuration(s)
$0^-,0$	2.96	$p_{1/2}s_{1/2}$
$0^+,1$	2.72	$p_{1/2}{}^2$
	7.91	$s_{1/2}{}^2$
	10.49	$d_{5/2}{}^2$
	20.54	$d_{3/2}{}^2$
$0^-,1$	8.12	$p_{1/2}s_{1/2}$
$1^+,0$	0	$p_{1/2}{}^2$
	5.54	$s_{1/2}{}^2$
	9.34	$s_{1/2}{}^2+d_{5/2}{}^2+d_{3/2}d_{5/2}$
	11.44	$s_{1/2}d_{3/2}$
	14.28	$d_{5/2}{}^2+s_{1/2}d_{3/2}+d_{3/2}d_{5/2}$
	20.16	$d_{3/2}{}^2$
$1^-,0$	4.58	$p_{1/2}s_{1/2}$
	11.78	$p_{1/2}d_{3/2}$
$1^+,1$	16.32	$s_{1/2}d_{3/2}$
	17.30	$d_{3/2}d_{5/2}$
$1^-,1$	6.99	$p_{1/2}s_{1/2}$
	12.00	$p_{1/2}d_{3/2}$
$2^+,0$	8.71	$s_{1/2}d_{5/2}$
	13.82	$d_{3/2}d_{5/2}+s_{1/2}d_{5/2}+s_{1/2}d_{3/2}$
	15.45	$s_{1/2}d_{3/2}+d_{3/2}d_{5/2}$
$2^-,0$	4.50	$p_{1/2}d_{5/2}$
	7.53	$p_{1/2}d_{3/2}$
$2^+,1$	9.57	$s_{1/2}d_{5/2}$
	11.95	$d_{5/2}{}^2$
	15.59	$s_{1/2}d_{3/2}$
	16.93	$d_{3/2}d_{5/2}$
	21.36	$d_{3/2}{}^2$
$2^-,1$	8.99	$p_{1/2}d_{5/2}$
	13.45	$p_{1/2}d_{3/2}$
$3^+,0$	6.77	$s_{1/2}d_{5/2}$
	11.10	$d_{5/2}{}^2$
	15.21	$d_{3/2}d_{5/2}$
	18.98	$d_{3/2}{}^2$
$3^-,0$	5.28	$p_{1/2}d_{5/2}$
$3^+,1$	11.82	$s_{1/2}d_{5/2}$
	17.30	$d_{3/2}d_{5/2}$
$3^-,1$	7.43	$p_{1/2}d_{5/2}$
$4^+,0$	13.10	$d_{3/2}d_{5/2}$
$4^+,1$	11.94	$d_{5/2}{}^2$
	15.54	$d_{3/2}d_{5/2}$
$5^+,0$	8.60	$d_{5/2}{}^2$

[19] W. H. Shaffer, Rev. Mod. Phys. **16**, 245 (1944).
[20] Gammel, Christian, and Thaler, Phys. Rev. **105**, 311 (1957).
[21] W. W. True and K. W. Ford, Phys. Rev. **109**, 1675 (1958).
[22] M. H. L. Pryce, Proc. Phys. Soc. (London) **A65**, 773 (1952).
[23] H. A. Bethe and P. Morrison, *Elementary Nuclear Theory* (John Wiley & Sons, Inc., New York, 1956), 2nd ed., p. 94.
[24] M. G. Redlich, thesis, Princeton University, 1954 (unpublished); Phys. Rev. **99**, 1427 (1955).

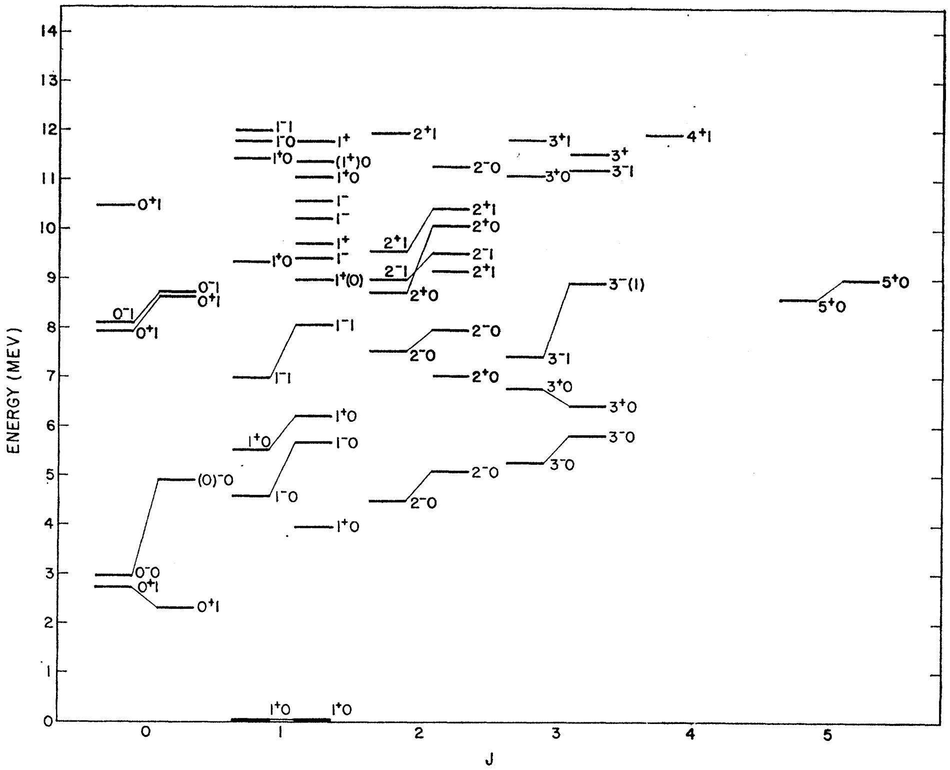

FIG. 1. Energy levels of N^{14}. For each spin, the first column gives the energy levels, spins, parities, and isotopic spins calculated with $\nu_p=\nu_{s,d}=0.3$ F^{-2} and $\alpha=1.6$. The second column gives the experimentally observed energy levels, spins, parities, and isotopic spins.

$\hbar\omega\approx 13$ MeV. This is in agreement with the calculations of Redlich[24] and Talmi and Unna.[8]

Another possible way to determine ν is that done by True and Ford.[21] In this case, the classical turning point of a particle in the third oscillator level is set equal to a suitable nuclear radius, $\bar{r}$. Picking the same radius as above, one has

$$\nu=7/\bar{r}^2=7/(1.40A^{1/3}\text{ F})^2. \tag{4}$$

This method fixes ν to be just twice the ν above. One has in this case, $\nu\approx 0.6$ F^{-2} and $\hbar\omega\approx 26$ MeV. This value of $\hbar\omega$ appears to be too large on the basis of other evidence.[8,24]

Calculations were carried out on the energy levels of N^{14} for $\nu=0.2$, 0.3, 0.4, 0.5, and 0.6 F^{-2} for all five values of α quoted above. Note that only the ratio of ν/β enters in the calculation, and so varying ν with β fixed is equivalent to varying β with ν fixed.

Comparing these calculated results with the experimental energy levels of N^{14} indicated that the best fit to the experimental data was with a ν of about 0.3 F^{-2} and an α of about 1.6. The calculated levels are listed in Table III and are compared with the experimental levels in a Grotrian diagram in Fig. 1. This value of α is approximately the same as that determined by other people.[3,22,25,26] For example, with $\alpha=1.6$, the central force of (2) is

$$V(r)=\left(-\frac{32.5}{4}\text{ MeV}\right)e^{-0.3r^2}\times(2.6+2.6P^r+0.6P^\sigma+0.6P^rP^\sigma)$$

while the central force used by Visscher and Ferrell[3] is

$$V(r)=\left(-\frac{32.5}{4}\text{ MeV}\right)e^{-0.334r^2}(2.606+2.606P^r+0.588P^\sigma+0.588P^rP^\sigma).$$

Except for the lowest 1^+,0 and 0^+,1 states and two other states, the calculated levels are in general about 1 MeV too low as can be seen by Fig. 1. These two lower levels are predominantly $p_{1/2}{}^2$ configurations. If these two states were depressed by about 1 MeV relative to the others, and the resulting energy spectrum renormalized so that the lowest 1^+,0 state was at zero energy, much better agreement between theory and experiment would be obtained. It is quite reasonable that including the possibility of core excitations would eliminate most of these discrepancies.

Another possibility is that the p-shell particles have a smaller $\langle r^2\rangle$ than the s,d-shell particles do. Note that the ν for the p shell is smaller than the ν for the s,d shell as shown by (5) if the same $\langle r^2\rangle$ is used:

$$\nu_p=\frac{5}{2\langle r^2\rangle}\quad\text{and}\quad\nu_{s,d}=\frac{7}{2\langle r^2\rangle}. \tag{5}$$

[25] N. Newby, Jr., and E. J. Konopinski, Phys. Rev. **115**, 434 (1959).
[26] N. Glendenning, Phys. Rev. **127**, 923 (1962).

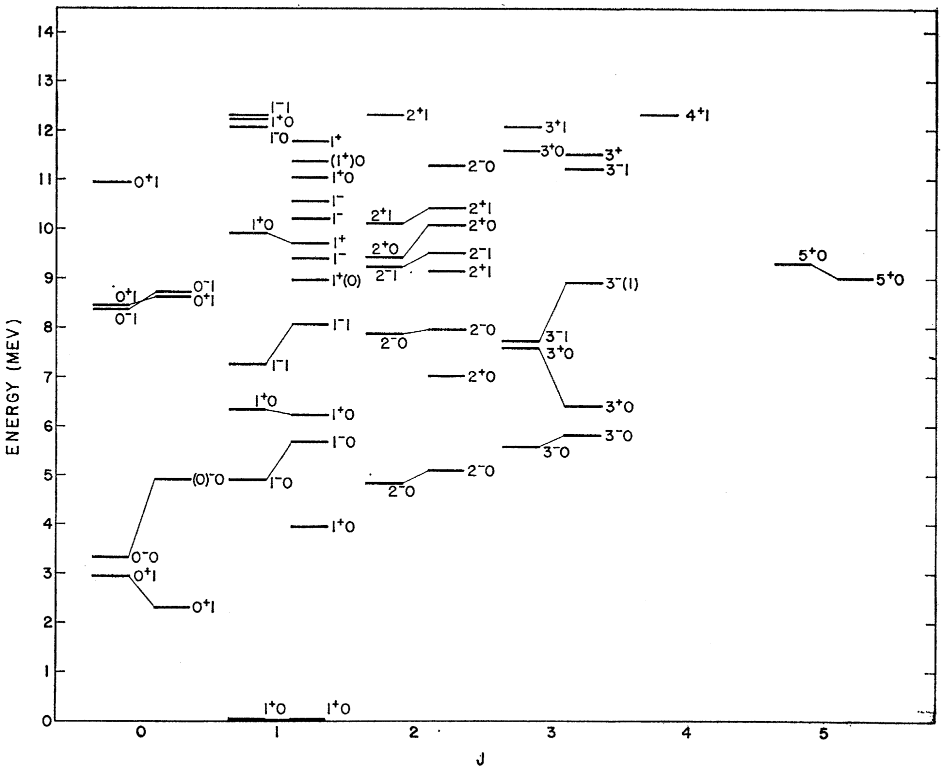

FIG. 2. Energy levels of N^{14}. For each spin, the first column gives the energy levels, spins, parities, and isotopic spins calculated with $\nu_p=0.32$ F^{-2}, $\nu_{s,d}=0.27$ F^{-2}, and $\alpha=1.6$. The second column gives the experimentally observed energy levels, spins, parities, and isotopic spins.

This would indicate that the radial wave function of the p particles falls off less rapidly than the radial wave functions of the s,d particles.

Even if one took $A=13$ in (3) for the p shell and $A=17$ in (3) for the s,d shell, the ν_p would still be smaller than the $\nu_{s,d}$, viz. $\nu_p\approx 0.23$ F^{-2} and $\nu_{s,d}\approx 0.27$ F^{-2}. Taking $\nu_p<\nu_{s,d}$ has the effect of moving the ground state up with respect to the other levels which is not in the desired direction to remove the discrepancies between these calculations and the experimental results.

To see what the effect of having $\nu_p>\nu_{s,d}$ would have on the energy levels, ν_p and $\nu_{s,d}$ were determined so that $\nu_{s,d}=0.27$ F^{-2} (see above) and ν_p was fixed by requiring that $\nu_p\langle r^2\rangle_p/\nu_{s,d}\langle r^2\rangle_{s,d}=1$, where $\langle r^2\rangle_p$ is given by (3) with $A=13$ and $\langle r^2\rangle_{s,d}$ is given by (3) with $A=17$. This procedure requires that the radial wave function of the p particles falls off more rapidly than that of the s,d particles and effectively causes the p particles to be closer to the core than the s,d particles. ν_p determined in this manner is 0.3226 F^{-2}.

The energy levels were calculated for N^{14} with $\nu_p=0.32$ F^{-2}, $\nu_{s,d}=0.27$ F^{-2}, and $\alpha=1.6$. These calculated energy levels are compared with the experimental levels in Fig. 2 and are listed in Table IV. There is an over-all improvement in the agreement between theory and experiment. However, the improvement is not as good as one might expect. This point will be discussed further later in this paper.

III. DISCUSSION OF RESULTS

In this section, all remarks will refer to the calculation with $\nu_p=0.32$ F^{-2}, $\nu_{s,d}=0.27$ F^{-2}, and $\alpha=1.6$. The results of this calculation are listed in Table IV and compared with experiment in Fig. 2 and in Table V. All remarks could equally well be applied to the calculation with $\nu_p=\nu_{s,d}=0.3$ F^{-2} and $\alpha=1.6$ where the results of this calculation are compared with experiment in Fig. 1 (see Table III also). The wave functions in both cases are practically the same and the same conclusion about the energy levels can be drawn. In fact, the results are quite insensitive to small variations in the parameters ν_p and $\nu_{s,d}$.

As pointed out in the Introduction, the levels which arise predominantly from core excitation cannot be explained with the model for N^{14} used in this paper. The levels which are expected[1–9] to be core-excited levels are the 3.95-MeV $1^+,0$, 7.03-MeV $2^+,0$, and the 9.17-MeV $2^+,1$ levels. Figure 2 indicates that these levels are not predicted with this model of N^{14}. These conclusions are also supported by the works of Harvey and Cerny.[12] The assignment of core-excited levels to these levels is essentially in agreement with the calculations of Talmi and Unna[8] and Warburton and Pinkston[9] (see Table V).

One expects a rather large amount of the core-excited states to be admixed with the $1^+,0$ ground state and the $0^+,1$ state at 2.312 MeV which are calculated to be 93 and 90% $p_{1/2}{}^2$, respectively (see Table IV). Since

TABLE IV. Energy levels, dominant configuration(s), and eigenfunctions for N^{14} with $\nu_p=0.32$ F^{-2}, $\nu_{s,d}=0.27$ F^{-2}, and $\alpha=1.6$.[a]

Energy (MeV)	Dominant configuration(s)	Eigenfunctions					
$J^\pi,T=0^-,0$		$p_{1/2}s_{1/2}$					
3.31	$p_{1/2}s_{1/2}$	1.000					
$J^\pi,T=0^+,1$		$p_{1/2}^2$	$s_{1/2}^2$	$d_{5/2}^2$	$d_{3/2}^2$		
2.94	$p_{1/2}^2$	−0.9501	0.1219	0.2635	0.1139		
8.46	$s_{1/2}^2$	−0.2056	−0.9360	−0.2754	−0.0760		
10.93	$d_{5/2}^2$	−0.2262	0.3278	−0.9063	−0.1414		
20.86	$d_{3/2}^2$						
$J^\pi,T=0^-,1$		$p_{1/2}s_{1/2}$					
8.37	$p_{1/2}s_{1/2}$	1.000					
$J^\pi,T=1^+,0$		$p_{1/2}^2$	$s_{1/2}^2$	$d_{5/2}^2$	$s_{1/2}d_{3/2}$	$d_{3/2}d_{5/2}$	$d_{3/2}^2$
0	$p_{1/2}^2$	0.9666	0.0643	0.1839	0.1012	0.0105	−0.1318
6.34	$s_{1/2}^2$	0.1303	−0.8732	−0.3268	0.0201	−0.3308	0.0628
9.92	$d_{5/2}^2$	0.1346	0.4713	−0.7457	−0.0398	−0.4357	0.1108
12.24	$s_{1/2}d_{3/2}$	0.1483	−0.0759	−0.2193	−0.8379	0.4505	0.1378
14.75	$d_{3/2}d_{5/2}$						
20.57	$d_{3/2}^2$						
$J^\pi,T=1^-,0$		$p_{1/2}s_{1/2}$	$p_{1/2}d_{3/2}$				
4.90	$p_{1/2}s_{1/2}$	0.9931	0.1175				
12.07	$p_{1/2}d_{3/2}$	0.1175	−0.9931				
$J^\pi,T=1^+,1$		$s_{1/2}d_{3/2}$	$d_{3/2}d_{5/2}$				
16.58	$s_{1/2}d_{3/2}$						
17.55	$d_{3/2}d_{5/2}$						
$J^\pi,T=1^-,1$		$p_{1/2}s_{1/2}$	$p_{1/2}d_{3/2}$				
7.26	$p_{1/2}s_{1/2}$	−0.9945	0.1050				
12.27	$p_{1/2}d_{3/2}$	−0.1050	−0.9945				
$J^\pi,T=2^+,0$		$s_{1/2}d_{5/2}$	$s_{1/2}d_{3/2}$	$d_{3/2}d_{5/2}$			
9.45	$s_{1/2}d_{5/2}$	−0.8729	0.3935	0.2885			
14.28	$d_{3/2}d_{5/2}$						
15.90	$s_{1/2}d_{3/2}+d_{3/2}d_{5/2}$						
$J^\pi,T=2^-,0$		$p_{1/2}d_{5/2}$	$p_{1/2}d_{3/2}$				
4.83	$p_{1/2}d_{5/2}$	0.9829	0.1842				
7.89	$p_{1/2}d_{3/2}$	0.1842	−0.9829				
$J^\pi,T=2^+,1$		$s_{1/2}d_{5/2}$	$d_{5/2}^2$	$s_{1/2}d_{3/2}$	$d_{3/2}d_{5/2}$	$d_{3/2}^2$	
10.12	$s_{1/2}d_{5/2}$	−0.8981	−0.3599	−0.2219	0.1003	−0.0675	
12.31	$d_{5/2}^2$	−0.3857	0.9163	0.0171	−0.0920	0.0535	
15.90	$s_{1/2}d_{3/2}$						
17.22	$d_{3/2}d_{5/2}$						
21.65	$d_{3/2}^2$						
$J^\pi,T=2^-,1$		$p_{1/2}d_{5/2}$	$p_{1/2}d_{3/2}$				
9.25	$p_{1/2}d_{5/2}$	−0.9997	0.0260				
13.71	$p_{1/2}d_{3/2}$	−0.0260	−0.9997				
$J^\pi,T=3^+,0$		$s_{1/2}d_{5/2}$	$d_{5/2}^2$	$d_{3/2}d_{5/2}$	$d_{3/2}^2$		
7.61	$s_{1/2}d_{5/2}$	−0.8969	−0.4082	−0.1673	0.0312		
11.60	$d_{5/2}^2$	−0.4307	0.8919	0.1203	−0.0675		
15.76	$d_{3/2}d_{5/2}$						
19.46	$d_{3/2}^2$						
$J^\pi,T=3^-,0$		$p_{1/2}d_{5/2}$					
5.60	$p_{1/2}d_{5/2}$	1.000					
$J^\pi,T=3^+,1$		$s_{1/2}d_{5/2}$	$d_{3/2}d_{5/2}$				
12.07	$s_{1/2}d_{5/2}$	1.000	0				
17.55	$d_{3/2}d_{5/2}$						
$J^\pi,T=3^-,1$		$p_{1/2}d_{5/2}$					
7.71	$p_{1/2}d_{5/2}$	1.000					
$J^\pi,T=4^+,0$		$d_{3/2}d_{5/2}$					
13.82	$d_{3/2}d_{5/2}$	1.000					
$J^\pi,T=4^+,1$		$d_{5/2}^2$	$d_{3/2}d_{5/2}$				
12.33	$d_{5/2}^2$	−0.9636	0.2674				
15.95	$d_{3/2}d_{5/2}$						
$J^\pi,T=5^+,0$		$d_{5/2}^2$					
9.32	$d_{5/2}^2$	1.000					

[a] The wave functions have been given only for the levels below 14 MeV.

these two states are predominantly in the p shell, one expects a greater amount of admixture with the core-excited states than one would expect for the other states of N^{14}. Consequently, any calculation of the quadrupole moment or the magnetic moment of the ground state or the transition rates to either of these states would be questionable because of these unknown admixtures. For example, a $p_{1/2}^2$ configuration does not contribute to the quadrupole moment. So a calculation of the quadrupole moment of the ground state from the results of this paper would only have contributions from the small admixtures of $d_{5/2}^2$, $s_{1/2}d_{3/2}$, $d_{3/2}d_{5/2}$, and $d_{3/2}^2$ configurations in the ground state. These admixtures are expected to be a great deal less than the admixtures

TABLE V. Comparison of the results of this paper with those of Warburton and Pinkston and Talmi and Unna for levels below 10.50 MeV in N^{14}.

Energy (MeV)	Experimental[a] J^π,T	J^π,T, and dominant configurations[b]	Warburton and Pinkston[c]	Talmi and Unna[d]
0	$1^+,0$	$1^+,0;\ p_{1/2}^2$	$1^+,0;\ p_{1/2}^2$	$p_{1/2}^2$ plus strong $p_{3/2}^{-1}p_{1/2}^{-1}$ admixture
2.312	$0^+,1$	$0^+,1;\ p_{1/2}^2$	$0^+,1;\ p_{1/2}^2$	almost pure $p_{1/2}^2$
3.945	$1^+,0$	core excited	$1^+,0;\ p_{3/2}^{-1}p_{1/2}^{-1}$	core excited plus strong $p_{1/2}^2$ admixture
4.91[e]	$(0)^-,0$	$0^-,0;\ p_{1/2}s_{1/2}$	$0^-,0;\ p_{1/2}s_{1/2}$	$p_{1/2}s_{1/2}$
5.10[e,f]	$2^-,0$	$2^-,0;\ p_{1/2}d_{5/2}$	$2^-,0;\ p_{1/2}d_{5/2}$	$p_{1/2}d_{5/2}$
5.69[e]	$1^-,0$	$1^-,0;\ p_{1/2}s_{1/2}$	$1^-,0;\ p_{1/2}s_{1/2}$	$p_{1/2}s_{1/2}$
5.83[e]	$3^-,0$	$3^-,0;\ p_{1/2}d_{5/2}$	$3^-,0;\ p_{1/2}d_{5/2}$	$p_{1/2}d_{5/2}$
6.05[g]	?	?	?	
6.23[h]	$1^+,0$	$1^+,0;\ s_{1/2}^2$	$1^+,0;\ (s,d)$	
6.44[i]	$3^+,0$	$3^+,0;\ s_{1/2}d_{5/2}$	$3^+,0;\ (s,d)$ or p^9d(?)	
6.70[g]	?	?	?	
7.03[e,j]	$2^+,0$	core excited	$2^+,0;\ p_{3/2}^{-1}p_{1/2}^{-1}$	core excited
7.40[g,k]	?	?	?	
7.60[g,k]	?	?	?	
7.97	$2^-,0$	$2^-,0;\ p_{1/2}d_{3/2}$	?	
8.06	$1^-,1$	$1^-,1;\ p_{1/2}s_{1/2}$	$1^-,1;\ p_{1/2}s_{1/2}$	$p_{1/2}s_{1/2}$
8.45[g]	?	?	?	
8.63	$0^+,1$	$0^+,1;\ s_{1/2}^2$	$0^+,1;\ (s,d)$	
8.71	$0^-,1$	$0^-,1;\ p_{1/2}s_{1/2}$	$0^-,1;\ p_{1/2}s_{1/2}$	$p_{1/2}s_{1/2}$
8.91	$3^-,(1)$	$3^-,1;\ p_{1/2}d_{5/2}$	$3^-,1;\ p_{1/2}d_{5/2}$	$p_{1/2}d_{5/2}$
8.99	$1^+,(0)$	core excited	?	
9.00[e]	$5^+,0$	$5^+,0;\ d_{5/2}^2$		
9.17	$2^+,1$	core excited	$2^+,1;\ (s,d)+p_{3/2}^{-1}p_{1/2}^{-1}$	
9.41	$1^-,?$	?	$p_{1/2}d_{3/2}$(?)	
9.51	$2^-,1$	$2^-,1;\ p_{1/2}d_{5/2}$	$2^-,1;\ p_{1/2}d_{5/2}$	$p_{1/2}d_{5/2}$
9.71	$1^+,?$	$1^+,0;\ d_{5/2}^2$	?	
10.09	$2^+,0$	$2^+,0;\ s_{1/2}d_{5/2}$	?	
10.22	$1^-,?$	?	p^9s and/or p^9d	
10.42	$2^+,1$	$2^+,1;\ s_{1/2}d_{5/2}$	$2^+,1;\ p_{3/2}^{-1}p_{1/2}^{-1}+(s,d)$	core excited

[a] These energies and J^π,T assignments, unless otherwise noted, were taken from F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959); and a Technical Report of August 1960 by these same authors.
[b] The J^π,T and dominant configurations in this column are those calculated in this paper (See Table IV and Fig. 2).
[c] The results of Warburton and Pinkston quoted in this Table were taken from E. K. Warburton and W. T. Pinkston, Phys. Rev. **118**, 733 (1960), Table VII, p. 752.
[d] The results of Talmi and Unna quoted in this Table were taken from I. Talmi and I. Unna, Ann. Rev. Nucl. Sci. **10**, 353 (1960); Phys. Rev. **112**, 452 (1958).
[e] The parity assignments for these levels were taken from Harvey *et al.* Nucl. Phys. **39**, 160 (1962).
[f] E. K. Warburton has measured a negative parity for this level (private communication).
[g] These experimental levels are not included in Fig. 1 and Fig. 2.
[h] The + parity is assigned to this level on the basis of the work of W. W. True and E. K. Warburton, Nucl. Phys. **22**, 426 (1961), and this assignment is supported by the work in this paper.
[i] A positive parity for this level has been experimentally measured by E. K. Warburton (private communication).
[j] A $J=2$ for this level has been experimentally measured by H. J. Rose, Nucl. Phys. **19**, 113 (1960).
[k] These levels are not seen (α,d), (He^3,p), and (α,α') scattering experiments. See reference 12.

of the core-excited states and so one could not expect to get the correct value for the quadrupole moment.

It is expected that the omission of the core-excited states in these calculations is the most important single reason why the calculated and observed energy levels do not agree better even though various ν's and α's were used.

The predictions of this calculation for the shell-model assignments for the energy levels of N^{14} up to 10.50 MeV excitation energy are compared with the predictions of Warburton and Pinkston[9] and those of Talmi and Unna[8] in Table V. There are several $J=1$ levels immediately above 9 MeV which cannot be given assignments from this calculation and consequently the assignments are omitted in Table V.

There is excellent agreement between the assignments of this paper for the spin, parity, isotopic spin, and shell-model configurations of the levels in N^{14} with the assignments of Warburton and Pinkston as can be seen in Table V.

The comparison between the predictions of Talmi and Unna and those of this paper are also compared in Table V. Except for the 10.42 MeV 2^+,1 level, this paper is also in agreement with Talmi and Unna. It is quite possible that the calculated 10.12-MeV 2^+,1 level should be associated with the observed 9.17-MeV 2^+,1 level and not the observed 10.42-MeV 2^+,1 level. Warburton and Pinkston imply that both the 9.17-MeV 2^+,1 level and 10.42-MeV 2^+,1 level consist of an admixture of a core-excited level and a level with two particles in the s,d shell. Consequently, it would not be inconsistent with their results to associate the calculated 10.12-MeV 2^+,1 level with either one of these known 2^+,1 levels.

IV. CONCLUSIONS

In view of the simple model taken for N^{14} which neglects the deformation and core-excitation of the C^{12} core, it is heartening that the agreement between the calculated energies, spins, and parity, and those of the observed levels is so good. Also, the fact that this model agrees quite well with three other quite different types of calculations, the pure p-shell calculations, the approach of Warburton and Pinkston, and the approach of Talmi and Unna, gives strength to the shell-model assignments of the energy levels which are given in this paper.

It should be stressed that the disagreement between the positions of the calculated energy levels and the positions of the experimentally observed energy levels is most probably due to the neglect of the deformation and core-excitation of the C^{12} core and not due to ignorance of the parameters ν_p, ν_s, and ν_d of the harmonic oscillator wave functions.

It is to be noted from Table IV that the eigenfunctions for practically all the states are quite pure jj two-particle wave functions. This fact is also true of the unlisted eigenfunctions. This purity of the eigenfunctions appears to have a direct connection with the conjecture of Talmi and Unna[8] that it is possible to use pure jj wave functions and an effective potential to calculate energy eigenvalues. That is, in some manner which is not completely clear, the effective potential seems to include some of the more important aspects of configuration mixing.

Sebe[27] has recently calculated the positions and nuclear properties of the low-lying negative-parity states in N^{14} using a model in which a proton is coupled to a C^{13} core. The C^{13} core was assumed to exist in either the ground state or first excited state of C^{13} and the wave functions for these "basic" core states were obtained from an intermediate shell-model calculation.

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[27] T. Sebe (to be published).

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Low-Energy Process $\gamma+p\to K^+ + \Sigma^{0}$*

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The reaction $\gamma+p\to\Sigma^0+K^+$ is discussed using the model developed in a previous paper. For odd-$K\Sigma$ parity the differential cross section can be accounted for by the one-nucleon pole term and the K and K^* exchange terms. With this model it is very difficult to fit the data for even-$K\Sigma$ parity. The coupling constant found for odd-$K\Sigma$ parity is $g_{\Sigma NK}^2/4\pi\approx 4.5$, very close to the value $g_{\Lambda NK}^2/4\pi\approx 4.0$ found in the previous paper.

I. INTRODUCTION

In a previous paper[1] a model was constructed for the photoproduction process $\gamma+p\to K^+ + \Lambda^0$ at low energies. This model was based on the approximation of neglecting faraway singularities as viewed from the "physical region." The close resemblance in kinematics of the class of strange particle production processes, viz., $\gamma+N\to K+Y$ and $\pi+N\to K+Y$, suggests that the same model should hold for all of them. In the following the model is applied to $\gamma+p\to K^+ + \Sigma^0$.

The terms to be taken in our calculation would thus be the one-nucleon term in the direct channel (s channel),[2] as well as the K^+ and K^* exchange terms. Since there is no evidence to date of any enhancement in a particular multipole state of the $K\Sigma$ system above the production threshold, we shall not have contributions due to such enhancements. It cannot be overemphasized that this very simple model would not be adequate as the energy gets higher. It is our hope, however, that it will give a description of what is happening in the low-energy region and serves as a guide in the high-energy region.

Now let us turn to the experimental side. Up to the present only very scanty data exist for $\gamma+p\to K^+ + \Sigma^0$. Several measurements of this process were made before 1960 at California Institute of Technology and at Cornell.[3] Recently new data became available from the work done at Cornell.[4] We will compare our model with the new data. The experiments are still proceeding and

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[1] T. K. Kuo, Phys. Rev. **129**, 2264 (1963). This paper will hereafter be referred to as I.

[2] The 3−3 resonance, for simplicity, is neglected. When more experimental information becomes available, we should put in its contribution.

[3] A summary of these can be found in F. Turkot, in *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 369.

[4] R. L. Anderson, E. Gabathuler, D. Jones, B. D. McDaniel, and A. J. Sadoff, Phys. Rev. Letters **9**, 131 (1962).