ON THE SPECULAR REFLECTION FROM ROUGH SURFACES.

By T. K. Chinmayanandam.

IT is well known that matt surfaces behave nearly in the same way as polished reflectors, when the incident rays are of great wave-length or fall very obliquely on the surface. In a paper published in the Physical Review for January, 1916, A. F. Gorton has followed up the work of Lord Rayleigh[1] and T. J. Meyer[2] on this subject, and has given several curves showing the relative reflecting power of surfaces of ground

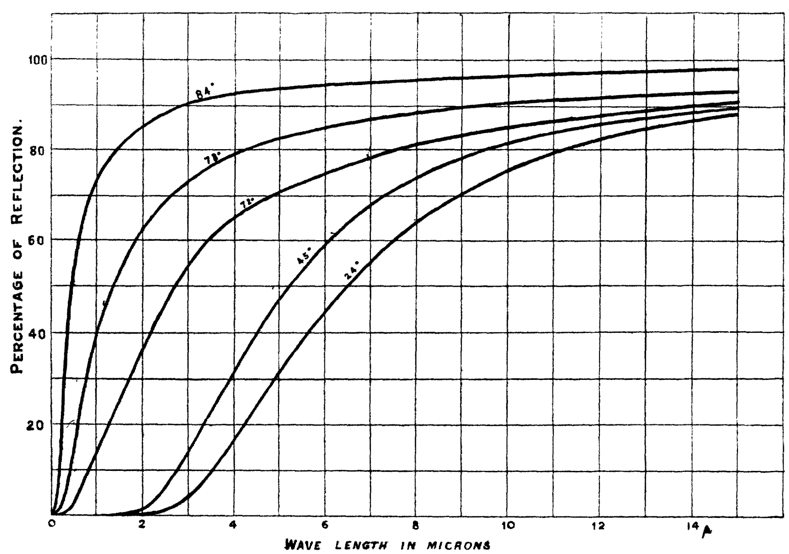

Fig. 1.

glass as compared with that of a polished surface of the same material for various angles of incidence and for various wave-lengths of incident radiation. Though the primary object of Gorton's investigation appears to have been the development of an infra-red screen, the experimental results obtained by him are also of interest from the point of view of the general theory of reflection from rough surfaces. In the present paper, it is proposed to record the results of an attempt made to develop a math-

[1] Rayleigh, Nature, 64, p. 385, 1901.

[2] T. J. Meyer, Verh. Deutsch. Phys. Ges., p. 126, Feb., 1914.

ematical formula which would quantitatively express the relative reflecting power of a rough surface over the range of incidences and wave-lengths studied by Gorton.

One important feature about the reflection from rough surfaces which is emphasized in Gorton's paper is the absence of decided interference minima in the reflection curves, such as would occur in the case of a plane grating or any surface of regular topography. This result is attributed to the fact that surfaces like those of groundglass are extremely irregular in structure. Starting on this idea, it seems possible, on certain hypotheses, to calculate the relative reflecting power of the surface for a given wave-length and a given incidence. It is convenient in the first place to make the simplifying assumption that the "relative reflecting power" of a rough surface depends only on its topography and not upon the nature of the material of which it is composed. *A priori*, it can be seen that such an assumption would not be very wide of the truth, particularly in the case of surfaces which, if polished, would have a fairly high coefficient of reflection at all incidences.[1] Further, the surface may be assumed to be "ideally rough," in other words, that different elements of it are distributed at different depths according to the probability law, the aggregate area of the elements lying between the distances x and $x + dx$ from a mean plane being proportional to $e^{-ax^2}dx$. The disturbance due to the reflected radiations is the resultant of the disturbances due to radiation from each of the surface elements, the proper phase of each being taken into account. Taking the mean plane as the reference plane, the relative phase of the disturbance sent out from an element of the surface at a depth x below it may be taken to be $2x \cos\theta \cdot 2\pi/\lambda$, where λ is the wave-length of the incident radiation, and θ the angle of incidence. Assuming now that, to a sufficient approximation, the surface is uniformly illuminated by the incident radiation, we get for the resultant disturbance due to the reflected radiation

$$y = \Sigma \cos(\omega t + 2x \cos\theta \cdot 2\pi/\lambda) ds, \tag{1}$$

the summation extending all over the elements. Since the distribution of the elements at different depths is assumed to be according to the probability law,

$$y = \frac{\int_{-\infty}^{\infty} e^{-ax^2} \cos(\omega t + [4\pi x \cos\theta]/\lambda) dx}{\int_{-\infty}^{\infty} e^{-ax^2} dx}. \tag{2}$$

Evaluating the integrals, the intensity of the reflected radiation is found to be given by

[1] See also Gorton's remarks, Phys. Rev., Vol. VII., ser. 2, p. 75.

$$I = e^{-(8\pi^2 \cos^2 \theta)/a\lambda^2}, \tag{3}$$

and this will also determine the relative reflecting power of the surface.

It is easily seen that this formula represents curves of the same general form as those obtained experimentally by Gorton. The curves have an inflection point, the inflectional gradient being proportional to $a^{\frac{1}{2}}/\cos\theta$, so that the curves became steeper (1) as a increases, *i. e.*, as the surface becomes finer, and (2) as the angle of incidence increases; these results are in general agreement with Gorton's observations. The actual results calculated from the formula are given below for the different surfaces used by him. The surfaces are denoted by A, B, C, D, where A refers to curves I and II in Fig. 6 (p. 73) of his paper, B refers to curve III in the same figure, and C and D refer respectively to the curves in Figs. 7 and 8. It will be seen from Table I. that for moderate incidences the agreement is fairly good.

TABLE I.

Surface.	Angle of Incidence.		Relative Reflecting Power (as Percentages) for Wave-lengths (in μ).										
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
A	23°	Obsd.	—	0.8	4.0	13.9	26.6	38.2	47.4	56.7	63.8	68.5	71.6
		Calcd.	—	0.0	2.0	10.6	23.8	36.9	47.1	57.0	64.2	69.8	74.3
B	45°	Obsd.	—	15.9	33.8	50.3	62.7	71.5	77.7	81.7	—	—	—
		Calcd.	—	7.1	30.8	51.6	65.5	74.5	80.5	84.7	—	—	—
C	24°	Obsd.	2.0	3.2	7.9	19.1	32.1	44.5	54.1	63.7	69.8	75.0	—
		Calcd.	0.0	0.0	4.1	16.6	31.6	45.0	55.6	63.8	70.1	75.0	—
	45°	Obsd.	3.2	5.3	17.3	33.2	48.0	59.0	67.1	73.1	77.4	79.5	82.3
		Calcd.	0.0	1.3	14.7	34.0	50.2	61.9	70.3	76.4	80.8	84.2	86.7
D	54°	Obsd.	—	8.2	28.0	45.6	59.5	69.9	75.2	—	—	—	—
		Calcd.	—	4.0	24.0	44.8	59.8	70.0	76.9	—	—	—	—

But at oblique incidences, the simple single-constant formula fails to express the results quantitatively, as can be seen from the figures in Table II. below.

TABLE II.

Surface.	Angle of Incidence.		Percentage of Reflection for Wave-lengths (in μ).										
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
A	70°	Obsd.	4.0	25.3	39.8	48.1	55.3	60.2	63.7	66.9	70.1	72.6	75.2
		Calcd.	0.7	28.9	57.5	73.1	82.0	87.1	90.4	92.5	94.1	95.1	96.0
C	72°	Obsd.	17.0	36.9	51.0	62.3	69.3	74.4	77.9	80.9	83.1	84.7	—
		Calcd.	3.7	43.9	69.4	81.4	87.6	91.3	93.5	95.0	96.0	96.7	—
	84°	Obsd.	73.3	85.7	90.4	93.3	95.1	95.9	96.5	97.0	97.5	97.8	98.1
		Calcd.	68.9	91.1	95.9	97.7	98.5	99.0	99.2	99.4	99.5	99.6	99.7
D	70°	Obsd.	11.0	46.3	66.9	77.3	83.4	85.1	85.1	—	—	—	—
		Calcd.	1.3	33.7	61.7	76.2	84.0	88.6	91.5	—	—	—	—
	81°	Obsd.	82.3	92.5	94.7	96.3	97.7	98.7	99.3	—	—	—	—
		Calcd.	41.0	80.0	90.5	94.6	96.5	97.5	98.2	—	—	—	—

The constants for the different surfaces are as follows:[1]

Surface.	*A*.	*B*.	*C*.	*D*.
$a\ 10^8 \times$	1.86	3.72	2.29	2.13
$d\ \mu$	0.61	0.43	0.55	0.56

the height above or depth below the mean plane for which the proportional frequency of occurrence is one half being given by d in the second line.

The failure of the simple theory given above to account for the phenomena observed at very oblique incidences could have been anticipated; for, the assumption made that the elements of the rough surface are all equally illuminated by the incident radiation would obviously be wide of the mark at such incidences. The extent of the discrepancy evidently depends on the degree of rugosity of the surface. If this is very marked, and the radiation is of relatively short wave-length and is incident very obliquely, hardly anything more than the upper parts of the surface would be illuminated. These would act as diffracting sources of radiation, the permanent phase differences between them being, of course, much smaller than the phase differences between the radiations from the highest and deepest parts of the surface. On the other hand, with greater wave-lengths, even the deeper parts of the surface would contribute to the reflection, but the increase due to this may to some extent be set off by the larger phase differences that come into play.[2] In view of these complications, it seems very difficult to give even an approximate theoretical treatment for the case of very oblique incidences. An empirical formula may however be devised to fit the experimental results. This should obviously satisfy the following conditions indicated by theory. (*a*) For very large wave-lengths, the relative reflecting power should be unity. (*b*) For moderate obliquities and over a range of incidences depending on the rugosity of the surface, it should reduce to the simple single-constant formula (3) above. (*c*) For very oblique incidences, it should give results, which, for short wave-lengths, should be greater, but which increase with the wave-length less rapidly than as suggested by the single-constant formula (3). In divising such a formula, preference has naturally been given to terms of the exponential type suggested by the simple theory. The formula which has been found suitable is

$$I = e^{-a\tan^3\theta}e^{-(b\cos^2\theta)/\lambda^2} + (1 - e^{-a\tan^3\theta})e^{-(c\cos\theta + d\cos^2\theta)/\lambda}.$$

[1] The surfaces *A* and *B* had been silvered, *C* had been platinized cathodically, but *D* was unsilvered.

[2] The expression $2x\cos\theta \cdot 2\pi/\lambda$ for the phase difference between the radiations from the mean plane and a plane at a distance x from it will not, also, be valid in these circumstances.

This formula evidently satisfies the conditions stated above. For, when $\lambda = \infty$, I is unity. When θ is small, the factor $e^{-a \tan^3 \theta}$ is practically unity, and the expression reduces to the formula (3). On the other hand, when θ approaches $\pi/2$, this factor becomes negligible, and the expression reduces to the form

$$I = e^{-(c \cos^{\frac{1}{2}}\theta + d \cos^2 \theta)/\lambda}$$

which approaches unity more slowly than (3) as λ is increased. The actual results calculated from this empirical formula have been plotted out in Fig. 1 for the surface C. The numerical values have also been given in Table III. for detailed comparison with Gorton's observations. The constants used have the following values. It will be seen that of the four constants, a is the one most sensitive to alterations in the character of the surface.

Surface.	*A*.	*C*.	*D*.
a	0.24	0.079	0.0115
b	42.4	34.5	32.9 ($\times 10^{-8}$)
c	2.5	1.43	0.0 ($\times 10^{-4}$)
d	17.8	15.75	7.89 ($\times 10^{-4}$)

TABLE III.

Surface.	Angle of Incidence.		Percentage of Reflection for Wave-lengths (in μ). 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
A	23°	Obsd.	—	0.8	4.0	13.9	26.6	38.2	47.4	56.7	63.8	68.5	71.6
		Calcd.	—	0.0	2.0	10.6	23.8	36.9	47.1	57.0	64.2	69.8	74.3
	70°	Obsd.	4.0	25.3	39.8	48.1	55.3	60.2	63.7	66.9	70.1	72.6	75.2
		Calcd.	5.3	23.0	37.5	48.0	55.6	61.3	65.7	69.2	72.1	74.5	76.5
C	24°	Obsd.	2.0	3.2	7.9	19.1	32.1	44.5	54.1	63.7	69.8	75.0	77.9
		Calcd.	0.0	0.0	4.1	16.6	31.6	44.9	55.6	63.8	70.1	75.0	78.8
	45°	Obsd.	3.2	5.3	17.3	33.2	48.0	59.0	67.0	73.1	77.4	79.5	82.3
		Calcd.	0.0	1.3	14.2	32.3	47.4	59.1	67.3	73.3	77.7	81.1	83.6
	72°	Obsd.	17.0	36.9	51.0	62.3	69.3	74.4	77.9	80.9	83.1	84.7	—
		Calcd.	14.5	36.1	55.5	64.9	71.0	75.3	78.4	80.9	82.9	84.5	—
	78°	Obsd.	38.2	55.6	64.7	70.3	73.3	76.8	79.6	81.7	83.8	85.8	87.7
		Calcd.	39.1	62.5	73.1	79.1	82.9	85.6	87.5	89.0	90.1	91.0	91.8
	84°	Obsd.	73.3	85.7	90.4	93.3	95.1	95.9	96.5	97.0	97.5	97.8	98.1
		Calcd.	73.3	85.6	90.2	92.5	94.0	95.0	95.6	96.2	96.6	97.0	97.2
D	54°	Obsd.	—	8.2	28.0	45.6	59.5	69.9	75.2	—	—	—	—
		Calcd.	—	4.5	24.0	44.5	59.5	69.6	76.3	—	—	—	—
	70°	Obsd.	11.0	46.3	66.9	77.3	83.4	85.1	85.1	—	—	—	—
		Calcd.	9.4	40.0	64.0	76.7	83.6	87.9	90.5	—	—	—	—
	75°	Obsd.	31.0	66.1	77.1	84.0	88.1	90.3	91.2	—	—	—	—
		Calcd.	32.6	66.4	80.8	87.3	90.8	92.9	94.2	—	—	—	—
	81°	Obsd.	82.3	92.5	94.7	96.3	97.7	98.7	99.3	—	—	—	—
		Calcd.	80.6	90.4	93.7	95.3	96.2	96.9	97.5	—	—	—	—

The agreement between the calculated and the observed values in Table III. seems fairly good, except in the case of Surface C for an incidence of 78° for which the corresponding experimental curve given by Gorton appears to be rather irregular. The fact that the empirical formula suggested is found to give fair agreement with observations on three different surfaces at different angles of incidence and for different wave-lengths over a wide range makes it probable that it is of fairly general application to rough surfaces.