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THE FUNCTION OF PHASE DIFFERENCE IN THE BINAURAL LOCATION OF PURE TONES.

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Synopsis.—This paper offers an explanation of the sound images observed when pure tones of the same frequency and intensity but of different phase are applied to the two ears. Use is made of theoretical curves calculated by Stewart and Fry giving the relation between the position of an actual source and the resulting phase difference P of the sound at the two ears. On the assumption that the listener subconsciously perceives the phase difference and places the image in the position that experience has taught him to associate with that phase difference, most of the observed phenomena follow directly from the curves.

For frequencies below about 600 cycles they account for (1) the observed rotation of the image with continuously varying P; (2) its motion toward the ear followed by a range of uncertainty as P approaches 180°; (3) the increase with frequency of the value of P for maximum lateral displacement of the image. For higher frequencies they explain the progressive decrease in definiteness of the image with increase in frequency as due to the presence of a number of simultaneous images. These multiple images as given by the curves are in very good agreement with those observed in the rather unique experiments of Bowlker.

The completeness with which the assumed theory of direct perception of phase difference explains the observed phenomena is strong evidence in its favor against the rival theories based on cross conduction through the head.

Introduction.

The problem of the location of a source emitting a pure tone presents features which differentiate it quite sharply from the general problem of sound location. In the first place, pure tones are much less easily localized than are complex sounds. And in the second place, persons who are deaf in one ear have almost no sense of location for pure tones (provided the head is fixed), even though they may locate other sounds almost as well as a person with normal hearing.

Hence, while both monaural and binaural effects may be utilized in locating complex sounds, the location of a pure tone is essentially binaural, and so must depend upon differences in the characteristics of the sound reaching the two ears. A sustained pure tone is completely determined by its frequency, amplitude and phase, and of these characteristics only the last two are subject to variation with the position of the source. It follows therefore that the sense of location can be dependent only on differences of intensity or phase or both. The existence of a connection between relative intensity and location was probably recognized before that of phase difference, but both have been known for a comparatively long time. In what follows it will be assumed that the effect of relative intensity is reduced to a minimum by keeping the intensities equal at the two ears.

Stewart,[1] in his article on Binaural Beats, includes a very good review of the literature on phase difference as affecting location. The experimental results may be very briefly summarized here. Two general methods have been used. In one, a particular phase difference is maintained by means of an adjustable difference between the paths from a single source to the two ears. In the other the phase difference is made to vary continuously by exciting the two ears from sources of slightly different frequency. The rate of variation is determined by the difference in frequency.

It is commonly observed that with zero phase difference, the apparent source of the sound, or for brevity the image, is in the median plane. Its location varies with different experimenters but more often appears to be in front. For frequencies below about 600 cycles it is agreed that the image lies on the side for which the phase at that ear leads. With continuously varying phase difference the image is described as starting from a position straight in front for zero phase difference, moving more or less in a circle to one side, till opposite the ear, then moving in toward the ear, entering the head, jumping quickly to the other ear for 180°, and continuing on around to the front along a similar path on that side. For higher frequencies most observers report a falling off in the definiteness of the location and finally a complete loss of the sense of direction.

In attempting to explain the mechanism by which the phase difference determines the position of the image, two general theories have been advanced. One of these, which we may term the direct perception theory, assumes that the listener is able to take cognizance of a difference in phase, as such, and by virtue of his past experience, or that of his ancestors, is able to associate any particular phase difference with the direction from which he has been accustomed to receive sounds having that phase difference.

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[1] G. W. Stewart, PHYS. REV., June, 1917, p. 502.

Lord Rayleigh[1] has attempted to explain in a general way the observed facts on the basis of this theory. He pointed out that for a fork of 128 cycles, moving around the head, the phase difference would never exceed about one eighth of a period, and commented on the fact that in the laboratory experiment the location effect persisted for phase differences right up to half a period. He noted also that at a frequency around 512, the phase difference with the source opposite one ear should be 180° and concluded there could be no discrimination at that frequency for sounds directly right or left. For somewhat higher frequencies he pointed out that the same phase difference could be produced by a sound on either the right or the left, and considered it fortunate that our sense of direction fails us at the point where it would begin to give misleading results. It should be noted that in arriving at the phase difference for an actual source at one side he made use of the very rough assumption that the difference in the paths to the two ears is one foot. He also drew other conclusions from the unjustifiable assumption that the sense of lateralness is a maximum when the phase difference is 90°. On the whole, his results were not particularly conclusive.

Against this theory certain psychologists advanced the argument that it involved a violation of the general principle that the sensation resulting from the stimulation of a sensory nerve is independent of the nature of the stimulus. In other words, they held that the characteristic phase of the sound stimulus at each ear could not be preserved in the nervous impulses reaching the brain as would be necessary for a perception of difference in phase. Apparently, however, this general principle, has been deduced from rather limited experimental evidence, and it is not considered by psychologists generally to be very definitely established. This objection, therefore, is not sufficient to warrant a rejection of the theory, provided it can be shown to be satisfactory in other respects.

Nevertheless, it was probably this argument which led to attempts on the part of physicists to show how differences in phase could give rise to differences in the intensity of the stimuli, which latter would cause a location on the side of the stronger stimulus. This resulted in various theories being advanced which we may designate as cross-conduction theories, since they are based on the assumption that an appreciable fraction of the sound entering each ear is conducted through the head to the other ear and there combines with that entering directly.

Myers and Wilson[2] present such a theory. In order to make the directions fit the experiments they find it necessary to introduce an

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[1] Rayleigh, Phil. Mag., 13, 1907, pp. 214 and 316.

[2] C. S. Myers and H. A. Wilson, Proc. Roy. Soc., 80, 1908, p. 260.

arbitrary phase reversal in passing through the head. Even then their theory explains only the most general features of the experiments, since it makes the displacement of the image proportional to the sine of the phase difference. More recently Stewart[1] has embodied the same idea in a more elaborate theory and uses it to account for the location when the phase difference is between 90° and 270°. For the rest of the cycle he assumes direct perception of phase.

Against this type of theory numerous objections have been raised. Probably the most important of these is the experimental evidence showing that the amount of sound actually reaching the ear by cross-conduction is far too small to produce the effects ascribed to it. A good summary of this evidence is given by Peterson.[2] Further, if the location effect due to phase difference were determined purely by the resulting intensity relation it should be very strongly affected by changes in the relative intensity of the sounds coming to the two ears. Under certain conditions, however, it is found to be practically unaffected by very considerable changes of this sort. In order to give even a qualitative explanation of the phenomena it is necessary to introduce a purely arbitrary assumption of phase reversal within the head. To make the agreement at all quantitative even over a limited range of frequency, it has been necessary to set up a very complicated theory involving a rather large number of assumptions.

To put the situation briefly then, the cross-conduction theory, while it does give something of an explanation, is open to very serious objections. The direct perception theory, on the other hand, while it has nothing very serious against it, has so far had no very positive evidence advanced in its favor, such as would be furnished by a quantitative explanation of the experimental facts. The most obvious line of attack then is to find out whether or not any such explanation is possible on the basis of direct perception.

Purpose.

It is the present purpose to attack this question by attempting to trace a correlation between the phase differences produced at the ears by actual sources in various positions and the positions of the images which result when sounds of equal intensity and predetermined phase differences are applied experimentally to the two ears.

Data and Discussion.

For this the first essential is a knowledge of the phase at the ear as a function of the position of the source for various frequencies. Stokes[3]

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[1] G. W. Stewart, PHYS. REV., June, 1917, p. 514.

[2] Joseph Peterson, Psychol. Rev., Vol. 23, No. 5, p. 333, 1916.

Lord Rayleigh, Theory of Sound, Ch. 17.

has developed a method whereby it is possible to calculate, for any point on a great circle of a rigid sphere, the phase of a sound whose source is in the plane of the great circle. The wave-length of the sound and the distance of the source are expressed in terms of the radius of the sphere. Stewart[1] has applied this method to the calculation of certain special cases. At the time this work was undertaken, T. C. Fry, a colleague in the Western Electric Co., had, independently, applied the method to certain other cases. A study of these two sets of results led to the conclusions described below. Later Fry extended his calculations securing the results from which have been plotted the curves given in Figs. 1 to 5.

Stewart assumes the head to be such a rigid sphere of 60 cm. circumference, the ears being at opposite ends of a diameter. For a single case, corresponding to a source, of frequency about 280 cycles, distant about 480 cm., he plots the difference in phase at the two ears as a function of the direction of the source. The only deduction which he draws from this curve is that, "The phase difference changes the most rapidly when the source of sound is in front or behind the hearer."

In plotting the curves of Figs. 1 to 5, the circumference of the head was assumed to be 55 cm. (radius, $r = 8.75$ cm.). The angular separation of the ears in the horizontal plane was taken as 165° and the velocity of sound as 340 m. per second. The figures represent progressively increasing frequencies. The individual curves of each figure are for sources at various distances R from the center of the head, ranging from $2r$ to infinity. (Unfortunately, the method of calculation was not adaptable to sources at the surface of the head where $R = r$.) Abscissæ represent the angular displacement ϕ of the source from a point straight in front, being taken positive to the right and negative to the left, 180° being at the back. Ordinates represent the phase lead P at the right ear over the left. Negative values then mean that the phase is leading at the left ear. A phase lead $180° + x$ ($x < 180°$) at one ear is equivalent to a lead of $180 - x$ at the other ear. In other words, those parts of the curve where the phase lead exceeds 180° in either direction must have 360° either added or subtracted so as to keep the ordinates between $-180°$ and $+180°$. This has been done in Figs. 3, 4 and 5.

From this theoretical information it is possible to predict, on the basis of the direct perception theory, where a listener should place the image when the phase difference at the ears is given any arbitrary value. For it is only necessary to read from the curve corresponding to the frequency of the sound, the value of ϕ corresponding to the given value of P.

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[1] G. W. Stewart, Phys. Rev., 1914, 4, p. 252.

As the curves give only those values of ϕ lying between $-90°$ and $+90°$, they give only the images located in front of the observer. If extended to $\phi = 180°$ in each direction the curves would be nearly symmetrical with respect to $\phi = \pm 90°$. (The departure from symmetry is due to taking the ears as 165° apart rather than 180°.) This means that any particular phase difference could be produced by a source either in front of or behind the observer and hence we should expect the image to be in either one or both of these positions. As already noted some experimenters find it in one place and some in the other. Inasmuch as the majority seems to find the image in front, the discussion will be limited to those quadrants with the understanding that the same general conclusions hold for images at the back.

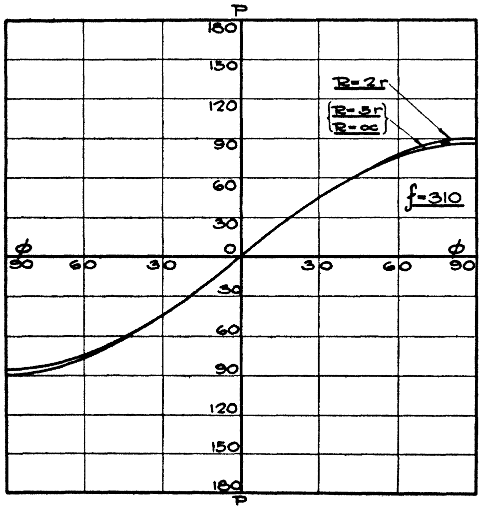

Fig. 1.

Consider first the case of the low frequency note of 310 cycles, shown in Fig. 1. For values of phase difference less than about 86°, we may for each ordinate choose an abscissa lying on any one of an infinite number of curves distributed between those of $R = r$ and $R = \infty$. This means that so far as phase alone is concerned there is in this region nothing to fix the distance of the image, while its angular position is determined, within limits, by this distance. The observed fact that the image is generally at a considerable distance is probably to be explained by the equality of intensity. Generally speaking, for any given direction the intensities at the ears become more nearly equal the greater the distance of the source. This then would explain why the image in moving around from the front would stay at a considerable distance till it was opposite the ear for a phase difference of 86°. For a further increase in phase there is no longer a corresponding abscissa on the curve for $R = \infty$ so it is necessary to go to the curve for the greatest distance which will still give a point. This means that once the image has reached a position near 90° its direction remains unchanged, but in spite of equal intensity, it moves in toward the ear, reaching it for a phase difference somewhat greater than 90°, being the maximum value of the hypothetical curve for $R = r$. This is in agreement with the observations quoted above for the case of continuously changing phase difference. Beyond this point there is no corresponding position for an actual source and hence the curves tell us nothing as to where an image is to be expected. However, it does not follow that such phase differences are foreign to the experience of the listener. They may very easily be produced by the reflections which occur in an enclosed space. Or they may occur when the source is within the head, or when the latter is in direct contact with the sounding body. Most observers describe the image as being located within the head in this case. Bowlker,[1] however, speaks of the image as being spread over a considerable angle on either side of 90°. In any case the definiteness of the sense of location should be much less than for small phase differences. It is, therefore, not surprising that, as Stewart has observed, the position of the image is affected much more by unequal intensities when the phase difference is near 180° than when it is near zero.

Consider next the effect of increasing the frequency. Up to something over 600 cycles (Figs. 1 and 2) the curves are of the same general form and we should expect the same general behavior of the image, as is found to be the case. The main variation is that the phase difference at which the image reaches its maximum lateral displacement increases with the frequency. Thus for 620 cycles (Fig. 2) the sense of lateralness should be a maximum for about 170°, instead of 90° as Rayleigh assumed. In support of this variation in the value of P for maximum image displacement, may be cited the results obtained by Myers and Wilson[2] using a variable path arrangement. They give curves in which the sensation of lateralness, measured in arbitrary units, is plotted against the path difference, and attempt to show a connection between this apparent displacement and the sine of the phase difference. An inspection of the curves, however, shows that for a frequency of 128, the maximum displacement occurs for a phase difference considerably less than 90°, while above 90° the displacement shows a decided falling off. At 384 cycles, the maximum is more nearly at 90°, while for 512, it is

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[1] T. J. Bowlker, Phil. Mag. (6), 15, p. 318, 1908.

[2] C. S. Myers and H. A. Wilson, *loc. cit.*

on the side toward 180°. What is perhaps a more striking confirmation is a statement made by Bowlker,[1] in connection with some experiments in which various path differences were obtained by placing tubes of different lengths over the two ears and listening to a distant sound. He says "A wave-length of about 36 inches (frequency about 370) . . . gave an image displaced 90°. Sound with longer wave-length gave a displacement of 90° before a phase-difference of half a wave-length was reached, and the sound image then seemed to spread over a continually increasing length of arc on each side of 90°."

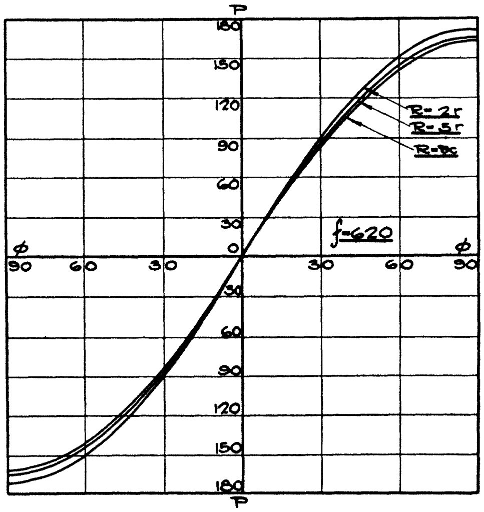

Fig. 2.

As the frequency is still further increased a marked change occurs in the nature of the curves owing to the fact that an actual source may then cause a phase difference greater than 180°. For a frequency of 930 cycles, Fig. 3 shows that for ordinates above 126° there are, on the curve for $R = \infty$, two abscissæ, one positive and one negative. This means two images, one on the right and one on the left. In the range between 126° and 106° there can be no image at infinity on the left, but there can be one at some distance greater than $2r$. (As before, the equality in intensity should tend to keep the image as far away from the head as is consistent with the phase relations.) For P less than the critical value corresponding to $R = r$, there is only one image as at lower frequencies.

Starting then with $P = 0$, the image is in front. As P increases the image moves rather slowly to the right keeping at a considerable distance.

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[1] Bowlker, *loc. cit.*

When P is somewhat less than 106° it has moved out about 20°, when a second image appears at the left ear. This new image moves quite rapidly outward from the ear, reaching a great distance by the time the first has moved to 33° ($P = 126°$). It then swings around in a circle toward the front, at first rather rapidly and then more slowly, arriving at 51° on the left at the same time that the other image reaches the same position on the right ($P = 180°$). To complete the cycle P must be varied from -180 to 0°. The image on the right now moves on around the circle with increasing speed till opposite the ear, when it moves in to the ear and disappears. Meanwhile, the left image moves slowly toward the front reaching there for $P = 0$. That is to say, the two images follow the same path relative to their respective sides but in opposite directions. The one on the side where the phase-lead is increasing begins in front and disappears at the ear, the one on the other doing the reverse. It will be noticed that the paths followed are exactly the same as for the low frequencies discussed above. The difference lies in the magnitude of the variation of P required to cover the path.

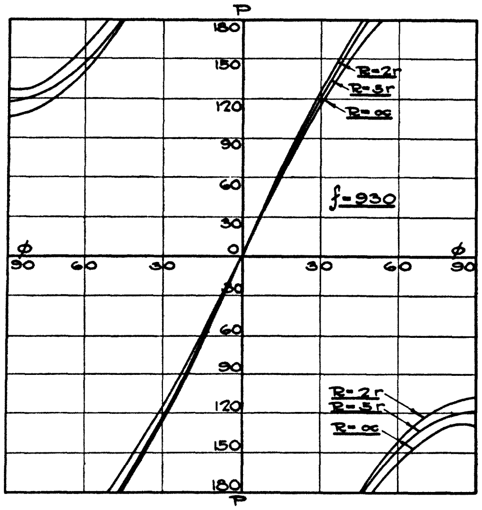

Fig. 3.

For still higher frequencies the range on either side of 180° in which two images are present increases progressively (see Fig. 4) until the whole cycle is covered. For higher frequencies still, where the possible phase difference exceeds 360°, there should be a third image for values of P close to zero, being on the right for small positive values of P and on the left for negative. Fig. 5 shows a case where this third image is present for most of the cycle. For still further increasing frequencies it follows from the above that the greatest number of images in the field during any part of a cycle increases by one each time the maximum possible phase difference increases by 180°.

Before the foregoing picture of the sound images for high frequencies can be used to predict the experimental results to be obtained at those frequencies, there must be something known or assumed as to the ability of the listener to perceive and localize a plurality of simultaneous images, giving to each the location assigned to it by the phase relations. It is quite conceivable that the listener would be unable to do this, in which case he might either have the sensation of a single image in some sort of average position or he might have no sense of direction at all. In either case his judgment would be much less certain than for a single image. Under these circumstances he would probably be influenced considerably by the equality of intensity, which would tend to cause him to locate the image in the median plane, particularly since at high frequencies the variation of intensity ratio with the position of the source is more marked than at low. This assumption appears to be in agreement with the results of most observers, who find that from about 600 cycles upward the sense of location by phase difference becomes progressively less trustworthy. That the change should appear to be gradual follows from the fact that the region of two images is at first confined to a small part of the cycle and gradually extends over the remainder. So far then, the theoretical deductions are not inconsistent with the experiments, on the assumption that more than one image can not be located separately.

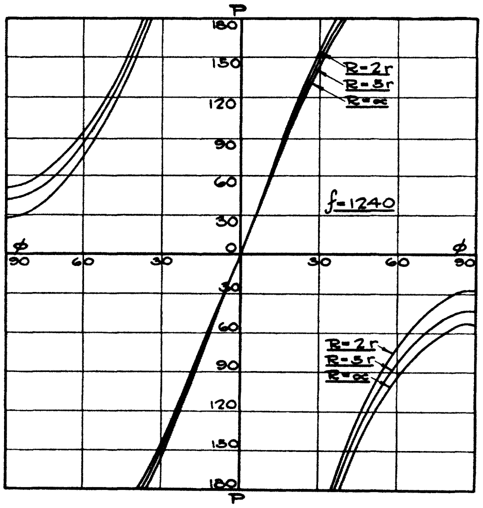

Fig. 4.

That this is not universally true, however, is shown by a series of experiments by Bowlker,[1] in which he was able to recognize two and even three separate images, in the field at once. Whether this was due to some peculiarity of his apparatus or of his individual sense of hearing is not known, but in either case the experiments are of special interest as being probably the only ones available for testing the above theory regarding multiple images.

The method used was an improvement of that referred to above in which tubes of unequal length were used. In order to eliminate possible inequalities of intensity due to the unequal lengths of the tubes, or to sound shadows cast by the head, he used exactly similar tubes on the two ears, bending them upward so as to make the openings horizontal in a plane well above the top of the head. The difference in path was then secured by facing in various directions relative to the source. The latter consisted of an organ pipe about 30 feet away. The distance L between the openings was 31 inches. The path difference was given by L into the sine of the angle turned through from the position facing the source. With this arrangement he was able to locate the direction of the sound image with considerable exactness, especially near the front. Due to some asymmetry either of the tubes or the ears, the image, when facing the source, was not always exactly in front. It was necessary for some frequencies to turn through as much as 4° to bring it in front. This position was then used as the corrected zero. At moderately low

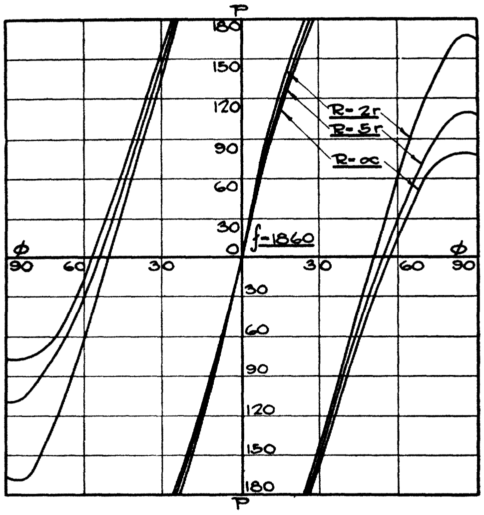

Fig. 5.

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[1] Bowlker, *loc. cit.*

frequencies the image moved around toward the side of leading phase, until at a point corresponding to a path difference of half a wave-length ($P = 180°$), it jumped to a similar position on the left. Continuing the rotation till this new image was straight in front the angle was generally found to agree quite closely with a path difference of a whole wave-length. The position of the image just before and after it crosses to the other side he calls the crossover angle. At frequencies of 263 and 335 he gives this as "90° (wide image)," which agrees with his results for unequal tubes. At higher frequencies it decreases progressively, having the following values; for 485 cycles, 40° to 50°; 690 cycles, 36°; 970 cycles, 18°, and 1290 cycles, 15°. He speaks of this cross-over angle as being "near the maximum displacement that phase will produce with the particular wave-length under observation."

At this point he reduced the separation of the openings to 15 inches so as to avoid the necessity of measuring very small angles. Repeating the experiment with 1,290 cycles he found for the cross-over-angle, 20°. He also observed that for all positions corresponding to phase differences between 180° and about 100° both images were present in the field at once. At 1,675 cycles he says: "Two images were evident during nearly the whole range—there were practically always two and sometimes three images evident, though I had some doubts whether the central image was always real or a result of attention to the two side images." In place of the cross-over angle he gives the maximum displacement as 12°.

He then reduced the apertures of the tubes from 2 inches to 0.6 inches. For a note of 2,090 cycles he says, "now three images could always apparently be heard together when facing one of them." The maximum displacement is here given as about 9°. At 2,310 cycles, "the image further to the left seemed stronger and tended to draw off one's attention." At 3,050 cycles, "it was very difficult to determine even approximately the position of an image, the one to the left of the two or three in the field of view seeming the loudest as a rule. I only felt sure that phase was still playing a part in fixing the maxima and minima which gave rise to the centers of the sound images."

The general agreement of these experiments with the theoretical results is obvious. The order of the changes in the images with increasing frequency is in striking agreement. First at low frequencies there is the progressive increase in the value of P to give maximum lateral displacement, accompanied by a narrowing of the region in which the image appears diffuse. This represents the range of frequency in which the maximum of the $P - \phi$ curve is less than 180°. Next comes a range in which the displacement of the image for $P = 180°$ is less than 90° and decreases progressively with increasing frequency. This corresponds to the diminishing value of ϕ for $P = 180°$, over the frequency range in which the values of maximum phase difference exceed 180°. Along with this comes the appearance of a second image on the opposite side, the part of the cycle for which it is present increasing progressively. When it has covered the whole cycle a third image appears, corresponding to the condition where the maximum value of P exceeds 360°.

The quantitative agreement is, however, not exact. The greatest angular displacement in the case of multiple images is always given as quite small and never reaches 90° as the theory would indicate. Also the frequencies at which the additional images appear are rather higher than the theoretical values. The second of these discrepancies is probably a result of the first. Suppose, for example, that in Fig. 4 the observer could not detect the second image when its displacement exceeded say 45°. The second image ($R = \infty$) would then appear when P reached 170° instead of 50° as it otherwise would. In order to have it appear at 50° under the assumed conditions it would be necessary to go to a higher frequency. That the second image should not have been detected under certain conditions is not surprising, when it is considered that practically all the other experimenters have failed to observe it at all. Furthermore, the assumptions underlying the theoretical calculations are such that we should not expect a very close agreement with experiment. The head is not a sphere nor is its circumference likely to be exactly 55 cm. The ears do not lie on a great circle and in most cases are not separated by exactly 165°. In fact, the differences, both physical and psychological, among the different observers are so great that there should be wide variations in the experimental results themselves. It should be strongly emphasized, therefore, that although fairly definite quantitative values have been given, it has been done chiefly for the sake of clearness in exposition, and there is no justification for using these values to predict with equal definiteness the results of any given experiment.

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