ON THE CHARACTERISTICS OF ELECTRICALLY OPERATED TUNING FORKS.

BY H. M. DADOURIAN.

SYNOPSIS.—A series of experiments was performed to determine the conditions affecting the period of an electrically operated tuning fork with the following results: (1) The more massive the base of the fork and the table upon which it is placed the smaller is the period. This effect, however, is not greater than one part in 10,000. (2) A change in the constants of the electrical circuit containing the electromagnet affects the period. The effect is less than one part in 10,000 for moderate changes necessary for keeping the amplitude of vibration constant. (3) The period increases linearly with the increase in the length of the gaps between the contact springs and contact points. For the forks used this increase was about one part in 500 for a change of 0.1 mm. in the length of the gaps. (4) For a given fork there is an amplitude at which the period has a stationary value. This may be a maximum or a minimum depending upon the arrangement of the mounting of the contact springs. At small amplitudes the change in the period due to a variation in the amplitude may be considerable. (5) The temperature effect increases from 1.04×10^{-4} at $-25°$ C. to 1.43×10^{-4} at $56°$ C. The values corresponding to temperatures above $0°$ C. are from 20 to 40 per cent. greater than those obtained by other observers. There are no other published data giving the temperature effect below $0°$ C. (6) By keeping the temperature, the length of the gap and the amplitude constant, a well-made fork can be relied upon to give a constancy of rate accurate to one part in 50,000. (7) The theoretical expression for the period of vibration of a bar holds good for a tuning fork. (8) The velocity of sound in the steel of which the forks used were made is 5.49×10^5 cm. per sec. (9) The coefficient of modulus of elasticity of the steel is 19.10×10^{11}. (10) The temperature coefficient of the modulus of elasticity increases from -2.2×10^{-4} at $-25°$ C. to -2.96×10^{-4} at $56°$ C. These values were computed from the relation $\epsilon = -(2\theta + \alpha)$, where ϵ is the temperature coefficient of modulus of elasticity, θ the temperature effect upon the period of the fork, and α the coefficient of linear expansion of steel.

I. INTRODUCTION.

THE determination of the frequency of tuning forks and the study of the causes affecting the frequency have been the subjects of investigation by a large number of physicists during the last hundred years or more. Consequently the author would not have thought of carrying out the researches described in the following pages had not circumstances led him to them. While working on problems of sound ranging with the Engineer Detachment of the United States Army in Princeton, it became part of the author's work to adjust and to determine the periods of vibration of a number of tuning forks. As these forks were slightly different in shape and in details of mounting from those used by previous investigators it was found desirable to know the extent to which the results obtained by them held good for these forks. A brief investigation was therefore planned with this end in view. But as the work progressed, interesting and important results were obtained which led the author to widen the scope of inquiry until the problem had been carefully studied in all its phases.

II. Description of the Forks.

Most of the experiments described in the following pages were carried out with forks made by the Western Electric Company, of the type shown in Fig. 1. The fork proper is milled out of a solid piece of soft machine steel and is electro-galvanized to prevent rusting. The prongs of the fork are about 39.5 cm. long, 0.95 cm. thick, 1.90 cm. deep, and 2.90 cm. apart. The outer curves at the shoulders as well as the inner curve are circular. Consequently the prongs are thinner at the shoulders than the prongs of standard Koenig forks of the same general dimensions, and therefore vibrate about their fixed ends as axes more like rigid bars than do the prongs of the Koenig type of fork.

The fork is rigidly attached to the back of a brass casting which has the form of a rectangular trough. The electromagnet is provided with movable pole-pieces placed on the outside of the prongs. This feature had been introduced by the Western Electric Company to satisfy the special conditions for which the fork was originally designed and is not at all necessary for its general serviceability. In fact the common type of electromagnet with pole-pieces between the prongs is preferable for a fork for which constancy of frequency is the principal desideratum.

A steel contact spring, 0.28 mm. thick and 3.92 mm. wide, is attached to each of the prongs by means of a steel clamp screwed to the prong at a point about midway between its ends. The contact spring is reënforced with two shorter springs placed one on each side and clamped together. When the fork is adjusted the free end of each contact spring comes midway between two platinum contact points soldered to the ends of two brass screws. The latter are set in brass bars provided with set screws for the purpose of making the contact screws fast after adjustment. These bars are screwed to a piece of micanite which forms the top piece of the bridge over the prongs. In order to secure good electrical contact a circular piece of platinum foil is soldered to each side of the free end of each contact spring.

The scheme of electrical connections is shown in Fig. 2, where the arrowheads marked A, B, C and D indicate the four contact points at the ends of the horizontal brass screws shown in Fig. 1. The condensers

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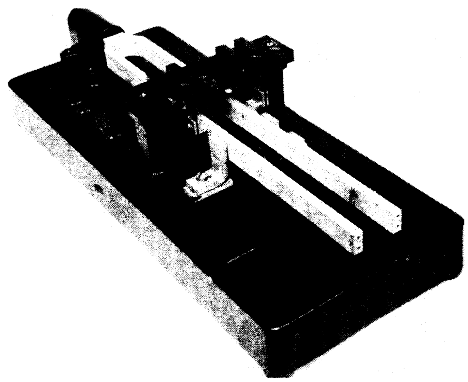

Fig. 1.

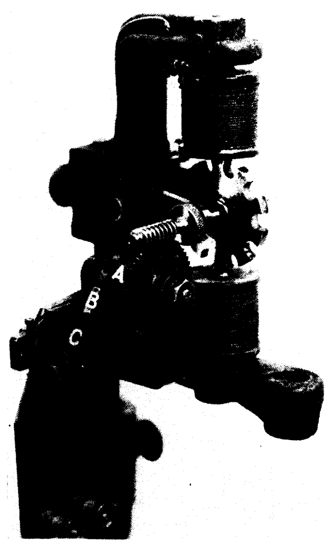

Fig. 4.

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marked *c* have each a capacity of 0.5 M.F., the resistance coils marked *r* have each a resistance of 150 ohms, and the field coil, *R*, of the electromagnet has a resistance of 225 ohms and about 6,300 turns of wire. The condensers and the resistance coils are attached to the under side of the brass base. The contact points *C* and *D* do not form parts of the electrical circuits, but they are necessary for the symmetry of the mechanical action of the contact points upon the contact springs. The contact point *D* is provided with the necessary connections, so that it can be made use of electrically in case it is desired to energize the electromagnet of the phonic wheel every half period instead of every full period,

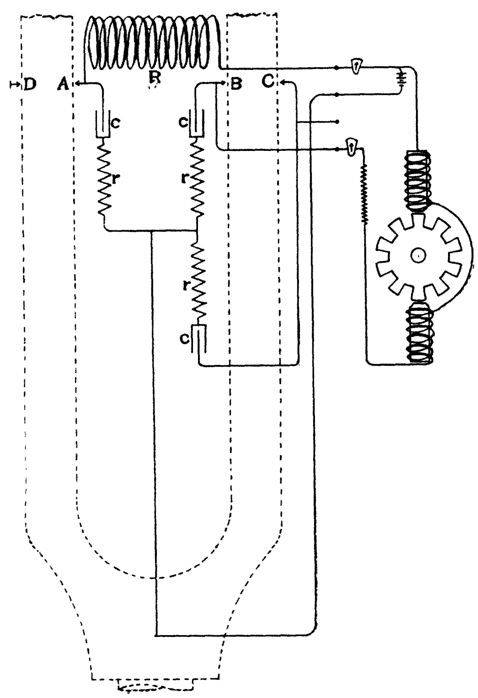

Fig. 2.

for instance. It may be stated here parenthetically that the wheel runs with greater smoothness between two than between four pole-pieces as the fork functions under normal running conditions remarkably free of sparking at the contact points, so that the pieces of platinum foil soldered at the ends of the contact springs last indefinitely.

In addition to the W. E. (Western Electric) forks three others were used. One of these was a Leeds and Northrup fork kindly loaned by the Company. Another was a fork made by Pirard and Coeurdevache of

Paris. The third was an old 50 V. D. Koenig fork mounted upon the base of one of the W.E. forks. In order to adapt the Koenig fork to the W.E. base, contact springs were clamped to the sides of the prongs at points about one-fourth of the length from the free ends. The way in which this was done is indicated in Fig. 3, where A is a piece of steel channeled to a depth slightly less than the total thickness of the three springs between it and the prong. This piece was attached to the prong by means of two screws which pass through holes made in the springs. The arrangement is an improvement over that of the original W.E. forks, first because it is simpler and second because it does not unnecessarily modify the uniformity of the effective thickness of the prong in the neighborhood of the clamp, as the spring in contact with the prong has only one-third the width of the clamp in contact with the prong in the W.E. fork.

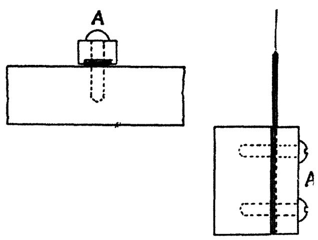

Fig. 3.

III. Method of Determination of the Period.

The period of vibration of the fork was determined by a method devised by Captain H. B. Williams. Chronographic records of every fiftieth complete vibration were compared with the records of the mean-time clock of the Princeton University Observatory. In order to record the vibration the phonic wheel and contact making device shown in Fig. 4 were used as intermediary between the fork and the chronograph. When the fork is set in vibration the circuit, of which the field coils of the phonic wheel form a part, is made and broken at the contact point A, Fig. 2, thus periodically exciting the electromagnet of the wheel. If the armature of the wheel is given a motion of rotation so that one of its teeth comes nearly in front of each of the pole-pieces of the magnet it falls in step with the magnetization of the electromagnet and rotates synchronously with the vibration of the fork. The rotation of the armature causes the toothed wheel A, Fig. 4, to rotate and to push against the rod B every time one of its teeth passes by the end of the latter. The rod B then comes into contact with the rod C, thus closing the circuit which operates the pen of the chronograph. The gearing intervening between the phonic wheel and the contact maker is such that this occurs once in every fifty complete vibrations of the fork. The short flat springs at the lower ends of the rods B and C hold them in position and connect them to the binding post at the lower left hand corner of Fig. 4 to which are attached the leads of the chronographic circuit. The time of contact may be made as short as desired by adjusting the position of the contact maker relative to the wheel A, and of the rod C relative to the rod B. The clock records were obtained by the intermediary of a quick contact making device, attached to the scope wheel of the observatory clock, of which the contact maker shown in Fig. 4 is a simplified copy.

The phonic wheel was invented by La Cour and Rayleigh, independently. They used it, however, to compare the frequencies of two forks by the stroposcopic method and not to obtain chronographic records of a fork. The method described in the preceding paragraph which may, properly, be called the *chronographic method*, is more convenient and is capable of yielding a greater accuracy than any other method hitherto used for determining the period of a tuning fork, as will be observed from the following statement.

First, in the chronographic method a record of two hours can be obtained without any more care on the part of the experimenter than that of winding up the clock-work of the chronograph. The record obtained can be read with great precision, and being permanent can be read at one's convenience and kept for future reference. The clock does not have to be adapted to the fork nor the fork to the clock; consequently the frequency of any fork can be obtained by comparing its record with that of the clock, the rate of which is practically constant and known very accurately. The rate of the mean time clock used in these experiments was known to one part in 200,000.

Second, in this method the recording arrangement does not affect the motion of the fork as in the case of the vibrographic method in which the variable pressure of the drum against the recording style must have an appreciable effect upon the period of the fork. In this connection it may be stated that an experiment was performed in which the period of W.E. fork No. 496 was observed while the mean current through the field coils of the phonic wheel was changed from 60 to 180 milliamperes, but no appreciable change in the period was observed.

Third, the method gives a direct comparison of the fork with a primary standard like a clock, and not with a secondary standard in the form of another fork.

The precision of the chronographic method depends upon (*a*) the accuracy with which the rate of the clock is known, (*b*) the accuracy of the positions of the fork signals relative to the positions of the clock signals on the chronographic record, (*c*) the length of the record, and (*d*) the precision with which it is read. The error due to the rate of the clock may be made negligible by using a clock the rate of which is accurately determined. The error due to the irregularities of the positions of the signals need not be more than .01 sec. in a well-constructed instrument. The chronographic record can be read with great accuracy by means of a differential scale often used by astronomers. So that even if an error of .05 sec. is made, due to the imperfections of the recording apparatus, the effect of this comparatively large error can be made negligible by taking a fairly long record. If the record is an hour long, for instance, an error of .05 sec. due to the end signals will introduce in the determination an error of one part in 72,000 only.

IV. Effect of Base.

It follows from theoretical considerations that the base upon which the fork is mounted should have no mass at all or it should have an infinite mass in order that it absorb no energy from the fork. It is to be expected therefore that the base of a fork will have some effect upon the period. In order to study this effect the following experiments were performed.

The W.E. fork No. 496 was placed upon a brick pier with a marble top and its period determined. A lead weight of 10 kg. was then placed upon the standard by means of which the fork proper is attached to its base and the period was again determined. The result of increasing the mass of the base by this means was to decrease the period by one part in 30,000.

A similar experiment was made with the Pirard and Coeurdevache fork and a decrease of one part in 1,000 observed. The reason for the comparatively large change in the period of this fork can be accounted for by the fact that the P. & C. fork weighs about 4 kg. while the W.E. forks weigh about 12 kg.

Next the effect of the table upon which the W.E. fork was placed was determined by observing its period while it was (*a*) upon the pier, (*b*) upon a wooden table and (*c*) upon a shaky stool. The result was a progressive increase in the period, the period on the stool being one part in 10,000 greater than on the pier.

The relatively large weight of the base and mounting of the W.E. forks is therefore a desirable feature, which makes the effect on the period of the mass of the table upon which it is placed relatively small.

V. Effect of Changes in the Constant of the Electrical Circuit.

In Fig. 5 let oo' indicate the position of rest of the contact spring which is in series with the field coil of the electromagnet, bb' and cc' indicate the positions of extreme displacement, and aa' its position at the instant it comes into contact with A, Fig. 2, and closes the circuit of the electromagnet. On account of the self induction of the circuit the driving current does not attain its maximum value instantaneously but increases according to the well-known exponential law during the excursion of the spring from aa' to bb' and back to aa'. At aa' the spring leaves the contact point thus opening the circuit and permitting the current to die down. The values of the current at different parts of a complete vibration are roughly indicated by the curve $ab'a'ca$. The motion of the prongs is opposed by the magnetic field, while the spring moves from aa' to bb', is helped while moving from bb' to cc' and is opposed again while moving from cc' to aa'. Thus the motion of the prongs is opposed during the time the prongs come together and is helped the rest of the time. Therefore a periodic but non-harmonic force is impressed upon the prongs in addition to the forces of restitution and of damping. When a steady state of vibration is reached the energy supplied per cycle by this force equals the energy dissipated by the damping forces. This energy is a function of the area enclosed by the broken curve $ab'a'ca$. The area as well as the position of its center can be altered while the maximum value of the current is kept constant by changing the constants of the exponential curves and the position of the line aa'. In other words the energy applied per cycle and the effective phase of its application may be altered by changing the time constant of the circuit and the length of the gap between the spring and the contact point.

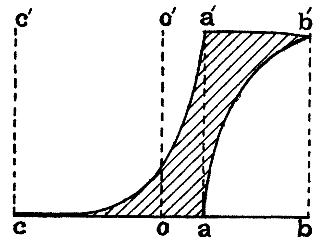

Fig. 5.

It will be seen in Part VII. that for constancy of frequency it is necessary to keep the amplitude constant. But to do this it may be necessary to regulate the driving current and hence it is of importance to know to what extent, if at all, a change in the time constant due to a change in the resistance of the circuit affects the period while the amplitude is kept constant. This point was investigated with the W.E. fork No. 488 in two experiments. In one of these the effect of an additional non-inductive resistance was observed, and in the other the effect of adding a self induction to the circuit was determined, the amplitude being kept constant in both cases. The results of the first experiment are shown by the curves of Fig. 6, where the abscissas denote the external non-inductive resistance added to the circuit of the electromagnet, and the ordinates denote the percentage increase in the period of the fork in ten-thousandths parts of the period corresponding to zero external resistance.

Curve I. was obtained with the length of the gaps between the contact springs and the contact points equal to .05 mm., while curve *II.* was obtained with the length equal to 0.3 mm. It will be observed that the effect is greater for the longer gap. But the effect is negligible for both, so long as the change in the resistance is small. If a change of 100 ohms is made in the resistance of the circuit to keep the amplitude constant,

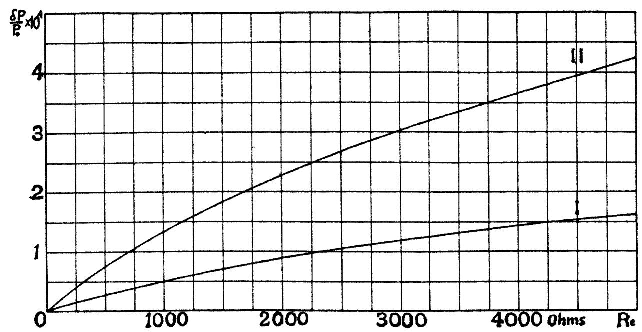

Fig. 6.

the change in the period is under one part in 100,000 for curve I. and under two parts for curve *II.*

The effect of an increase in the self induction of the circuit was next observed by placing alternately an inductive resistance and a non-inductive resistance of equal value in series with the coil of the electromagnet and the change in the period was found to be less than one part in 10,000.

VI. Effect of the Length of the Gap.

It is evident from Fig. 5 that a change in the position of the line *aa'* would result in a change in the area of the cycle *ab'a'ca* and the position of the center of this area. Consequently changing the gap between the contact spring and the contact point introduces a change in the amount of energy applied to the fork per period and in the phase of application of this energy. Furthermore the circumstances of the mechanical action of the contact points upon the springs, and through them upon the prongs, is changed when the gaps are changed. Therefore an alteration in the length of the gaps is bound to affect the period of vibration of the fork.

This effect was studied by determining the period of vibration for gaps of different lengths. The results of an experiment with the W.E. fork No. 488 are shown by curve *I* of Fig. 7, where the abscissas denote the lengths of the gaps between the springs and the contact points in fractions of one millimeter, while the ordinates denote percentage increases in the period in ten-thousandths of the period corresponding to zero gap.

All four of the gaps were made of the same length, changing them from .04 mm. to .25 mm. in ten steps. It will be observed that the effect is very nearly linear and that it is comparatively large, being one fifth of one per cent. per one tenth of a millimeter change in the length of the gaps.

Curve *II* was obtained with the Koenig fork. In spite of the fact that the springs were less stiff than those of fork No. 488 the gap-effect of the Koenig fork is greater than that of No. 488. This can be accounted for

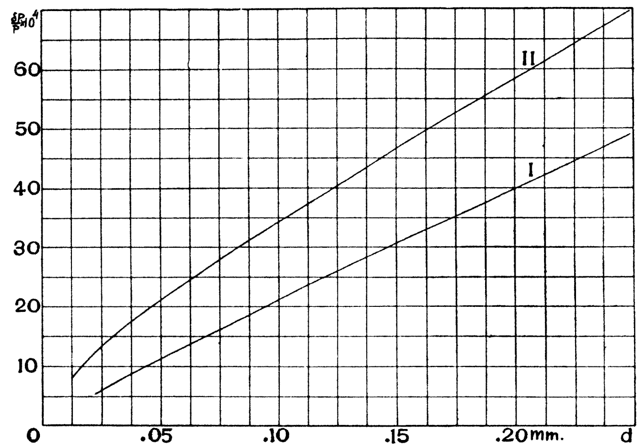

Fig. 7.

by the fact that the springs were nearer the free ends of the prongs in the case of the Koenig fork and consequently the moment of the forces acting upon the springs was greater in proportion, and that the forks were relatively thinner and consequently their motion was more easily affected by this moment.

These results show that the gap-effect is considerable and that the gaps must be kept constant in length if constancy of frequency is desired. This fact does not seem to have been recognized by students and makers of tuning forks, as is evident from the form of contact springs of standard types of forks. Most of these forks have twisted platinum wires for contact springs which change their shape under the action of the contact points, making the length of the gaps variable. The springs of the W.E. forks can be relied upon to keep their shape and position so that the length of the gaps do not vary more than .01 mm.

It was found that the positions of the springs relative to the contact points were slightly changed when the position of the fork was changed. This was due to a redistribution of the weight of the fork among the four rubber feet of the base and the consequent strain in the frame of the base. In order to avoid possible errors from this source, the gaps were examined whenever the forks had to be moved and one of the feet raised by placing a few thicknesses of paper, thereby bringing the springs to positions midway between the contact points. This precaution can be made unnecessary by making the frame of the base more rigid and providing it with three instead of four feet.

The effect upon the period of fork No. 488 of eliminating the outer contact points was determined by observing the period alternately with the outer contact points at 0.1 mm. from the springs and at distances beyond the reach of the springs. The average results of four separate determinations was to change the time of 50 periods from 1.00096 sec. with the outer contact points at 0.1 mm. to 1.00369 sec. with the outer contact points beyond the reach of the springs, the inner contact points being kept at 0.1 mm. from the springs. In other words the mechanical action of the outer contact points upon the springs decreases the period by a fraction of about 27 parts in 10,000.

VII. Effects of Changes in Amplitude.

Mercadier[1] and Ettinghausen[2] have observed that the frequency of a fork increases with decreasing amplitude and approaches assymptotically the frequency corresponding to zero amplitude. Poske[3] and Heerwagen[4] on the other hand have found that the frequency increases linearly with decreasing amplitude. Heerwagen has expressed his results by the equation

$$n = n_0 - p\alpha$$

where n and n_0 denote the frequency, α the amplitude, and p a constant. Hartmann-Kempf[5] has observed, however, that the frequency increases more rapidly than Heerwagen's linear equation would imply and has expressed his results by the empirical relation

$$n = n_0 - (p + \Delta\alpha)\alpha.$$

The differences in the conclusions attained by these investigators can be accounted for in the light of the results obtained from the following experiments upon the effect of changes of amplitude upon the period. Throughout these experiments the temperature and the spark gaps were kept constant. The amplitude was measured by means of a traveling

[1] Mercadier, C. R., 83, p. 800, 1876; Journ. de Phys., 5, p. 201, 1876.

[2] A. Ettinghausen, Pogg. Ann., 156, p. 337, 1875.

[3] H. Poske, Pogg. Ann., 152, p. 448, 1874.

[4] F. Heerwagen, Diss. Dorpat, 1890.

[5] R. Hartmann-Kempf, Ann. d. Phys., 13, p. 124, 1904.

microscope provided with a vernier to read twentieths of one millimeter. The microscope was focused upon a bright spot at the end of one of the prongs and the double amplitude measured; the double amplitude of the other prong was then measured and one half of the average of the two observations taken as the amplitude of the fork.

Curves *I* and *II* in Fig. 8 show the results obtained with the W.E.

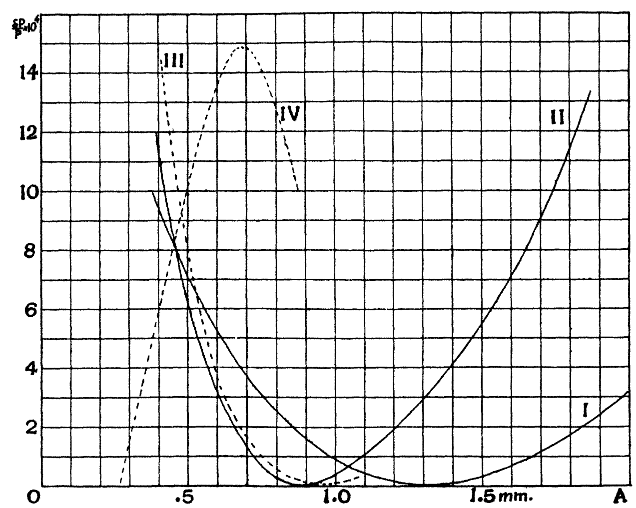

Fig. 8.

fork No. 496. The abscissas denote single amplitudes in millimeters and the ordinates denote percentage increases in the period in ten-thousandths of the minimum period observed. The changes of amplitude of curve *I* were made by increasing the current in the field coils of the electromagnet while the pole-pieces were kept fixed. On the other hand, the changes in the amplitude of curve *II* were obtained by moving the pole-pieces relative to the prongs while the current was kept constant. The curves represent the averages of two sets of curves, one obtained by increasing the amplitude and the other by decreasing it. The two component curves did not quite coincide; each formed, however, a very smooth curve representing the experimental data.

These curves bring out the interesting fact that there is an amplitude for which the period has a minimum value. In other words there is an amplitude in the neighborhood of which the period has a stationary value and consequently is not affected by small changes of amplitude. The value of this amplitude is different for the two curves because of the difference in the manner in which the changes were broüght about. We shall see later that the amplitude corresponding to the stationary value of the period depends also upon the stiffness of the contact springs and upon the length of the gaps.

Curve *III* was obtained with the Leeds & Northrup fork. It will be observed that it has the general shape of curves *I* and *II*. This was to be expected because the Leeds & Northrup contact arrangements are somewhat similar to those of the W.E. forks. Curve *III* and curve *IV* could not be extended further to the right because the amplitude could not be increased beyond certain limits on account of the proximity of the prongs to the pole-pieces of the electromagnet.

Curve *IV* was obtained with a Pirard & Coeurdevache fork. In this case the period has a maximum instead of a minimum value because in the P. & C. fork the contact springs as well as the contact points are attached to the base and consequently the prongs are not in contact with the springs during a fraction of the period which forms a greater and greater portion of the period as the amplitude is increased. This explanation is sustained by the experiment described in the last paragraph of Part VI. where it was shown that relieving the prongs from the mechanical action of the outer contact points resulted in a considerable increase in the period.

Amplitude-period curves were also obtained for the Koenig fork and

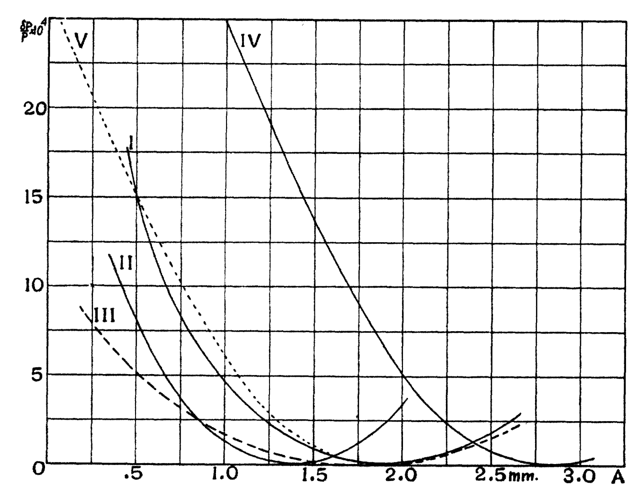

Fig. 9.

for the W.E. fork No. 488. Curve *III* of Fig. 9 was obtained with the Koenig fork, the changes in the amplitude being obtained by changing the driving current, while curves *I* and *II* were obtained with the W.E. fork No. 488. In the experiments corresponding to curve *I* the pole-pieces of the magnet were moved; in the experiment corresponding to curve *II* the driving current was changed. It will be observed that as in the case of fork No. 496 the amplitude corresponding to the minimum period is smaller for curve *I*.

Curve *IV* was obtained with the 488 fork under the same conditions as curve *I* except that the gaps were changed from .05 mm. in the case of curve *I*, to 0.30 mm. in the case of curve *IV*. In order to compare these two curves, curve *V* similar to curve *IV* was drawn so that its minimum point coincides with that of curve *I*. It will be observed that the rate of decrease of the period is greater for *I* than for *V*. This can be accounted for on the ground that for a given magnitude of the mechanical action of the contact points upon the springs, the energy of the prongs was greater in the experiments corresponding to curve *V*. This explanation is in accord with the fact that the curves are steeper to the left of the minimum point, where the amplitude is smaller and consequently the energy of the prongs is smaller.

From the foregoing considerations it will be expected that for a given length of gap and at a given amplitude less than the amplitude which corresponds to the stationary value of the period, the rate of decrease

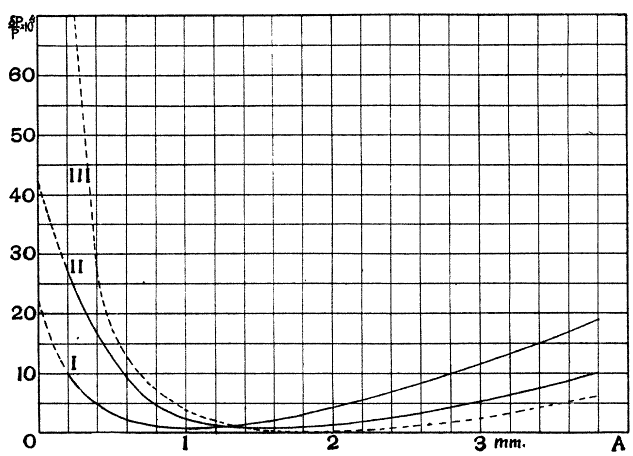

Fig. 10.

of the period will be smaller the smaller the stiffness of the springs. Certain experiments, the results of which are indicated by the curves of Fig. 10, sustain this conclusion. All three of these curves were obtained with fork No. 488 with gaps equal to .05 mm., the amplitude being altered by changing the driving current. In the experiment represented by curve *I* the fork had the same contact springs as in the experiment of curve *I* of Fig. 9, namely, springs with cross-sections of .41 mm. × 4.64 mm.

In the experiments represented by curves *II* and *III* the fork had springs with cross sections 0.28 mm. × 3.92 mm., and .48 mm. × 7.86 mm., respectively. A glance at these curves shows that the stiffer the springs the steeper are the curves on both sides of the minimum point, and the greater is the amplitude corresponding to the minimum period.

The results of other experimenters can be made to agree among themselves and with the results obtained by the author if it is supposed that the former have to do with that part of the period-amplitude curve which lies to the right of the minimum point and that the region investigated by Mercadier and Ettinghauser comes nearer the point of minimum period than the region studied by Poske, Heerwagen and Hartmann-Kempf.

The more important results of the experiments described in Part VIII. may be summarized in the following terms.

First, the period-amplitude curve passes through a stationary value and consequently there is an amplitude at which the fork can be operated with maximum constancy of period.

Second, the amplitude which corresponds to the stationary value of the period depends upon the stiffness of the contact springs and the length of the gaps.

From the results of the experiments described in Parts V. and VI. certain inferences may be drawn with regard to the best position for the contact springs. For a given contact pressure the action of the contact points upon the springs, and through them upon the prongs, is greater the nearer the springs are placed to the free ends of the prongs. Furthermore, for a given change in the amplitude the change in the contact pressure is greater the nearer the springs are placed to the free ends. On the other hand there is a limit to the extent to which the position of the springs can be approached to the shoulders of the fork. Hence there must be a position which is more favorable than any other. The determination of this position did not form a part of the investigations described here, but considerations based upon the mechanical action of the contact points upon the prongs, the position of center of percussion of the prongs, and the nodal points of the first harmonic of the vibration have led the author to the conclusion that the most favorable position for the contact springs must lie between the middle of the prong and one third of the distance from the free ends.

VIII. Temperature Effect.

A score or more physicists have determined the effect of temperature upon the frequency of tuning forks and have obtained concordant results.

These men, however, do not appear to have appreciated the importance of controlling certain factors which we have found to affect the frequency. It was deemed worth while, therefore, to redetermine the temperature effect under the favorable conditions which could be secured in the light of the experiments described in the preceding pages. Another reason which induced the author to carry his investigations into this phase of the tuning fork problem was the fact that he had at his disposal facilities to extend the investigation on the temperature effect to temperatures below zero degree centigrade, to which region the older investigators had not carried their work.

The work was carried on in the constant temperature rooms of the Palmer Physical Laboratory. The larger of the two rooms was provided with a thermostat which responded readily to changes of temperature as small as 0°.1. The smaller room had no temperature regulating device but its temperature could be lowered still further. Two separate determinations were made. The first, made in the larger room, covered the range of temperatures between – 3°.6 C. and 56°.8 C. The second, made in the smaller room covered the range of temperatures between – 25°.8 C. and 21°.2 C.

The W.E. fork No. 488 and the Koenig fork were placed side by side on a table in the middle of the larger room and a box with open ends was built over them. A microscope with a scale in the ocular was focused upon a bright spot at the end of one of the prongs of each fork, in order to observe and adjust the amplitude of vibration. Two thermometers made by R. Fuess with ranges of – 30° C. to 100° C. and scales divided into tenths of one degree were suspended so that the bulb of one was placed between the prongs and near the stem of one of the forks. A Callander recorder was set up to observe the temperature without going into the constant temperature room in use, but it was found to be not quite sensitive enough to give the temperature to the desired degree of precision; therefore it was used only to keep a rough record of the changes. The following general method of procedure was used during the first experiment: About one half hour after the temperature of the room had reached a desired value the room was entered and the gaps of the forks were examined to see if there had been any changes in the lengths of the gaps from the value .05 mm. to which they had been adjusted. Each fork was then set into vibration and the amplitude adjusted, if necessary; fork No. 488 was left vibrating and the open ends of the box were covered with pieces of woolen cloth. After about ten minutes the room was entered again, the thermometers read (this could be done without opening the box), and the chronograph started. Twenty minutes later the room was entered again, the thermometers were read, the Koenig fork was started and its record also taken for twenty minutes, after which the thermometers were read again. In this manner a large number of records were obtained which covered the range of temperatures between $-3°.6$ C. and $56°.8$ C. and extended over several days.

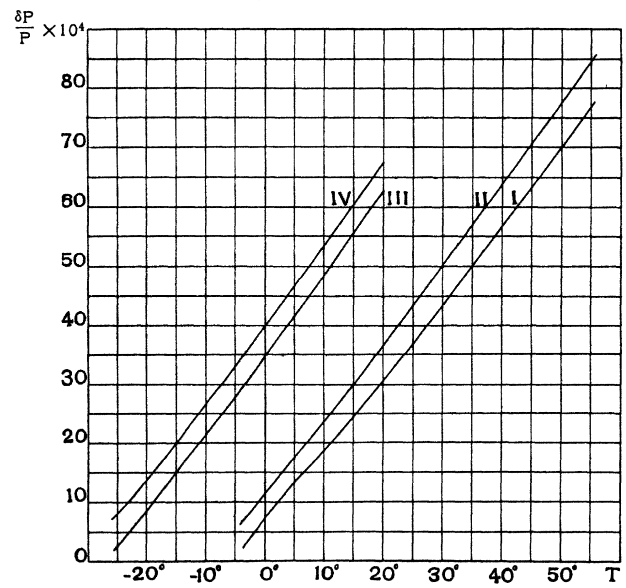

Fig. 11.

The results obtained with fork No. 488 were plotted on a large scale upon a millimeter cross-section paper with the temperatures as abscissas and the corresponding values of 50 complete periods as ordinates and a smooth curve was obtained with a continuously increasing slope. From this curve *II* of Fig. 11 was drawn, where the ordinates give the percentage increases in the period in ten-thousandths of the period at 0° C.

The period-temperature curve of the Koenig fork had an inflection point at about 11° and another at about 35°. There is no doubt that this behavior of the Koenig fork was due to slight variations in the spark gaps observed during inspections of the gaps made before taking records. The variation in the gaps must have been due to the expansion and contraction of the vulcanite piece which carried the contact points. In the regular W.E. forks this piece was of micanite which has a much smaller coefficient of expansion. Curve *I* which represents the results obtained by the Koenig fork is therefore of interest because it shows the importance of having spark gaps of constant length as well as because it supports the general conclusions derived from the other three curves of Fig. 11.

In the second experiment the Koenig fork was replaced by the W.E. fork No. 313 and the apparatus was set up in the smaller room. This room did not have a thermostat and consequently its temperature could not be kept constant; therefore the following method of procedure was adopted. The room was cooled several degrees below the temperature in the neighborhood of which it was desired to take a record; then the refrigerating machine was stopped and the temperature was allowed to rise for several hours and then records of the two forks were taken in the same manner as in the first experiment. This was repeated until ten pairs of records were taken while the temperature was lowered in the manner just described from 21°.2 C. to − 27°.C. The temperature of the room was then allowed to rise from − 27° to the normal room temperature and 15 more pairs of records were taken. The experiment took eight days and the rate of change of temperature was so slow during the intervals when records were taken that the temperature could be considered as constant. The results of this experiment are indicated by

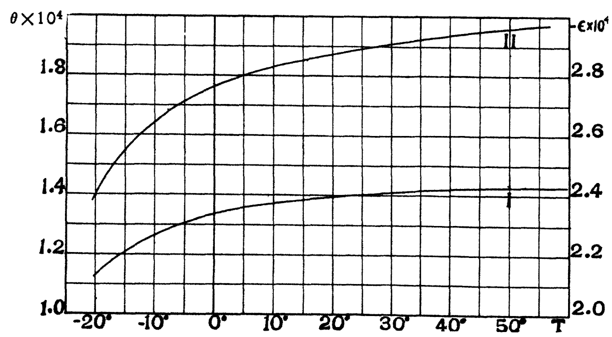

Fig. 12.

curves *III* and *IV* of Fig. 11, curve *III* corresponding to No. 488 and curve *IV* to No. 313.

The results indicated by the curves *II*, *III* and *IV* were plotted accurately on a large scale and the different rates of increase in the period at different temperatures were determined for each curve. The average values for the three curves are given in Table I., where the numbers in the first column represent the temperatures and those in the second column the corresponding values of the temperature coefficient θ defined by

$$\theta = \frac{1}{P_0}\frac{\partial P}{\partial t},$$

or

$$P = P_0(1 + \theta t).$$

TABLE I.

t.	θ.	$-\epsilon$.	t.	θ.	$-\epsilon$.
– 20°	1.14 × 10^{-4}	2.39 × 10^{-4}	20°	1.38 × 10^{-4}	2.87 × 10^{-4}
– 10°	1.27	2.65	30°	1.40	2.91
0°	1.33	2.77	40	1.42	2.95
10°	1.36	2.83	50°	1.43	2.97
			– 20 to 50	1.34	2.79

These results are represented graphically by curve *I* of Fig. 12. The values of θ given in Table I. are from 10 to 40 per cent. higher than the average value of the results obtained by former observers as will be seen from Table II.

TABLE II.

Observer.	Range of Temp.	Temp. Coef.	Observer.	Range of Temp.	Temp. Coef.
Mercadier[1]......	3°–26°	0.96×10^{-4}	Michelson[5]......	12°–24°	1.00×10^{-4}
Kayser[2].........	0°–26°	0.98	Lang[6]..........	14°–19°	1.11
Koenig[3].........	3°–26°	1.11	Pierpaoli[7].......	0°–30°	1.05
Koenig[3].........	26°–56°	1.07	Woodruff[8].......	20°–200°	1.08
McLeod & Clarke[4]	15°–26°	1.10	Dadourian......	– 26°–57°	1.34

It will be shown in Part XII. that the effect of temperature upon the period of a fork is mainly due to its effect upon the elasticity of the fork and that only about 5 per cent. of the former effect is due to the increase in the dimensions of the fork resulting from a rise of temperature.

IX. CONSTANCY OF PERIOD.

The constancy of the performance of the W.E. fork No. 488 was tested under the following conditions: The pole-pieces of the electromagnet were set at 1.37 mm. from the prongs. The contact points were adjusted so that the length of the gaps was .05 mm. A potential of 24 volts was applied directly to the binding posts on the fork. Under these conditions an amplitude of 1.58 mm. was obtained which corresponds to the stationary period of the fork under the given conditions. The fork was operated continuously from 8.43 a.m. until 12.45 M., and chronographic records were taken. The period was then determined for different parts

[1] Mercadier, Journ de Phys., 5, p. 201, 1876; C. R., 83, p. 822, 1876.
[2] Kayser, Wied. Ann., 8, p. 444, 1879.
[3] Koenig, Wied. Ann., 9, p. 394, 1880.
[4] McLeod & Clarke, Phil. Trans. Roy. Soc., 171, part I., p. 1, 1880.
[5] Michelson, Am. Journ. Sci., 25, p. 61, 1883.
[6] Lang, Wied. Ann., 29, p. 132, 1886.
[7] Pierpaoli, Rend Linc., 4 (1), p. 714, 1888; 5 (2), p. 265, 1889.
[8] Woodruff, PHYS. REV., 16, p. 325, 1903.

of the run from careful measurements of the record, taking intervals of 20 minutes. The results of these determinations are given in Table III. where the numbers in the first column give the temperatures at the middle of the different 20-minute intervals, those in the second column are the corresponding observed values of 50 complete periods, the numbers of the third column are these values corrected to 20°.50 C. and those of the last column are the percentage deviations of the values of the periods thus determined from their mean value.

Table III.

Temperature.	50 P.	50 P_c.	$\delta P/P$.
20°.30	1.000885 sec.	1.000913 sec.	-8.4×10^{-6}
20°.34	1.000903	1.000925	3.6
20°.38	1.000910	1.000927	5.6
20°.46	1.000925	1.000931	9.4
20°.50	1.000336	1.000934	13.6
20°.56	1.000934	1.000926	4.6
20°.66	1.000923	1.000901	− 20.4
20°.69	1.000940	1.000913	− 8.4
20°.72	1.000954	1.000923	1.6
		1.0009214	± 8.4

Another set of determinations was made the next day with the spark gaps equal to 0.3 mm. the pole-pieces at 1.0 mm. from the prongs, and an amplitude of 1.75 mm. The fork was operated from 12.55 until 4.19 P.M. and a continuous record taken. The values of 50 periods were then determined as in the preceding experiment and the results given in Table IV. were obtained.

Table IV.

Temperature.	50 P.	50 P_c.	$\delta P/P$.
20°.77	1.004153 sec.	1.004134 sec.	-10×10^{-6}
20°.74	1.004168	1.004155	11
20°.72	1.004186	1.004175	31
20°.70	1.004181	1.004170	26
20°.67	1.004152	1.004148	4
20°.65	1.004153	1.004151	7
20°.63	1.004136	1.004137	− 7
20°.62	1.004126	1.004136	− 8
20°.61	1.004115	1.004122	− 22
20°.60	1.004114	1.004121	− 23
		1.0041443	± 14.9

A glance at the last columns of Tables III. and IV. shows the remarkable constancy of performance of the fork. In the first run the maximum deviation from the mean for a run of four hours is only one part in 50,000 while the average deviation is less than one part in 100,000. In the second run the deviation from the mean is larger. But in this case the deviation is systematic indicating that the conditions affecting the period were not kept as constant as during the first run. The great constancy of performance shown by the fork is all the more remarkable because it is a commercial article in the manufacture of which no great effort had been made to secure such important requirements as a high degree of uniformity of thickness of prongs and symmetry relative to the axis of the fork. This investigation shows conclusively that a proper control of the factors affecting the period is of prime importance.

X. Expression for Period of Vibration.

It has been shown by Mercadier[1] that the expression for the period of transverse vibration of a bar obtained from the theory of elasticity holds good for a tuning fork provided the projection of the median line of the prongs upon the geometric axis of the fork is taken for the length of the prongs and a small correction term is added to this. The expression for the period of vibration of a bar is

$$P = \frac{4\pi\sqrt{3}}{r^2 v}\frac{l^2}{a} \qquad (1)$$

where l is the length and a the thickness of the bar, v the velocity of sound in the bar, and r the root of the equation

$$(e^r + e^{-r}) \sin 2r + 2 = 0 \qquad (2)$$

and equals 1.87011.[2] For rods of the same material v is constant, therefore we can write

$$P = K\frac{l^2}{a} \qquad (3)$$

Mercadier's modified expression for tuning forks is

$$P = K\frac{(l + \lambda)^2}{a} \qquad (3')$$

where λ is a correction term introduced in order to satisfy Mercadier's experimental results and equals $.012l$. Evidently the value of λ will depend upon the shape of the fork near the shoulders of the prongs. Since the W.E. forks had a slightly different shape from those used by Mercadier it was thought desirable to determine the value of λ for these forks. Using Mercadier's experimental value of K, namely, 1/81,827

[1] E. Mercadier, C. R., 79, pp. 1001, 1069, 1874; Journ. de Phys., V., p. 201, 1876.

[2] Poisson, Traité Mecanique, II., p. 390.

sec./cm., the values given in the first column of Table V. were obtained for ten different forks.

Table V.

λ.	λ/l.	E.	$\delta E/E$.
+ 0.16 cm.	+ 0.0040	18.80×10^{11}	− 0.016
+ 0.07 "	+ 0.0018	18.92	− 0.009
+ 0.06 "	+ 0.0015	18.98	− 0.006
+ 0.06 "	+ 0.0015	19.03	− 0.004
+ 0.04 "	+ 0.0010	19.07	− 0.002
− 0.03 "	− 0.0007	19.16	− 0.003
− 0.05 "	− 0.0012	19.02	+ 0.004
− 0.09 "	− 0.0022	19.30	+ 0.010
− 0.10 "	− 0.0025	19.30	+ 0.011
− 0.17 "	− 0.0042	19.42	+ 0.017
− 0.005 "	− 0.0001	19.10	.0092

It will be observed (*a*) that the highest value of λ obtained is only one third of 0.48 cm. required for the W.E. forks by Mercadier's empirical relation $\lambda = 0.012\, l$, (*b*) that the values are fairly evenly distributed on the two sides of zero, and (*c*) that their algebraic sum differs from zero by an amount which falls within the experimental errors of the measurement of the lengths of the prongs. Therefore the unmodified equation (3) gives closer results for the W.E. forks than Mercadier's equation (3′).

XI. The Modulus of Elasticity and Velocity of Sound in Steel.

Replacing v in equation (1) by its expression in terms of the modulus of elasticity E and the density ρ we have

$$P = \frac{4\pi\sqrt{3}}{r^2}\frac{l^2}{a}\sqrt{\frac{\rho}{E}}. \qquad (4)$$

Putting in the last equation 1.87011 for r, 7.363 for ρ and the values of P, l, and a obtained from the ten W.E. forks the values of E given in the third column of Table V. were obtained. The value 7.363 for ρ was obtained from a careful determination of the density of four pieces which were cut off the ends of the prongs of different forks in adjusting their periods. Using this value of ρ and the average value of E given at the bottom of the third column, the value 5.093×10^5 cm. per sec. was obtained for the velocity of sound in steel.

The deviations of the values of λ and of E from their mean values are small and undoubtedly due to the facts that (*a*) the prongs were not as uniform as might have been reasonably expected, and that (*b*) some of the forks were cut in more at the shoulders than others.

XII. TEMPERATURE COEFFICIENT OF MODULUS OF ELASTICITY OF STEEL.

Let P_0, a_0, l_0, ρ_0 and E_0 denote the values at 0° C. of the quantities involved in equation (4), and P, a, l, ρ *and* E denote their values at any other temperature t, then we can write

$$\frac{P}{P_0} = \frac{a_0}{a} \cdot \frac{l}{l_0} \cdot \left(\frac{\rho}{\rho_0}\right)^{1/2} \left(\frac{E}{E_0}\right)^{1/2}. \qquad (5)$$

Introducing in the last equation the temperature coefficients α, θ, and ϵ defined by

$$l = l_0(1 + \alpha t),$$
$$P = P_0(1 + \theta t),$$
$$E = E_0(1 + \epsilon t),$$

we obtain, to a first order of approximation, the relation

$$\theta = -\frac{\alpha + \epsilon}{2},$$

or

$$\epsilon = -(2\theta + \alpha). \qquad (6)$$

Since α and θ are both positive magnitudes ϵ, the temperature coefficient of the modulus of elasticity must be negative. In other words the modulus of elasticity decreases with increasing temperature. Taking the coefficient of linear expansion of steel to be 1.1×10^{-5} and substituting in equation (6) the values of θ given in the second column of Table I. we obtain the figures of the third column for the values of ϵ corresponding to the temperatures given in the first column. The variation of ϵ with the temperature is shown graphically by curve *II* of Fig. 12. The unmistakeable increase of the absolute value of ϵ with the increase of temperature indicated by Table I. and curve *II* is in agreement with results obtained by Pizati[3] and Dodge.[10]

The relatively large differences among the values of ϵ obtained by different observers must be due to the variety of the specimens of steel used and also due to experimental errors. It may be stated that the indirect method used in this paper is more accurate than the more direct methods by which the other observers have determined the temperature coefficient of elasticity.

In conclusion the author wishes to acknowledge his indebtedness and express his thanks to Professor W. F. Magie for placing the facilities of the Palmer Physical Laboratory at his disposal; to Professor E. H. Loomis for helping him place the constant temperature plant in working order; to Professor H. N. Russell for the use of the observatory chrono-

TABLE VI.

Observer.	$-\epsilon$.	Observer.	$-\epsilon$.
Wertheim[1]	2.8×10^{-4}	Gray, etc.[7]	2.47×10^{-4}
Kohlrausch & Loomis[2]	4.6	Wassmuth[8]	2.64
Pizati[3]	1.95	Walker[9]	2.5
Mayer[4]	2.21	Dodge[10]	1.9
Miss Noyes[5]	3.4	Dadourian	2.79
Shakespeare[6]	3.87		

graph; and to Captain H. B. Williams, who had charge of the sound ranging work of which the present investigation formed a part, for his interest in the work.

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1 Wertheim, Pogg. Ann., 2, p. 1, 1848.
2 Kohlrausch & Loomis, Pogg. Ann., 141, p. 481, 1870.
3 Pizati, Nuo. Cim., 1, p. 181; 2, p. 137, 1877; 3, p. 152, 1879; 5, pp. 34, 135, 145, 1879.
4 Mayer, Brit. Assoc. Rep., p. 573, 1894.
5 Miss Noyes, PHYS. REV., 3, p. 433, 1896.
6 Shakespeare, Phil. Mag., 47, p. 539, 1899.
7 Gray, Blyth and Dunlop, Proc. Roy. Soc., 67, p. 180, 1900.
8 Wassmuth, Phys. Zeits., 6, p. 755, 1906.
9 Walker, Proc. Roy. Soc. Edin., 28, p. 562, 1907.
10 Dodge, PHYS. REV., 5, p. 373, 1915.

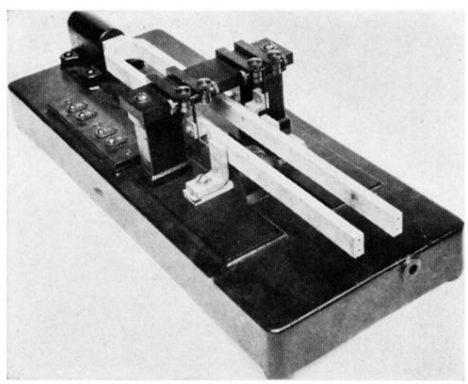

Fig. 1.

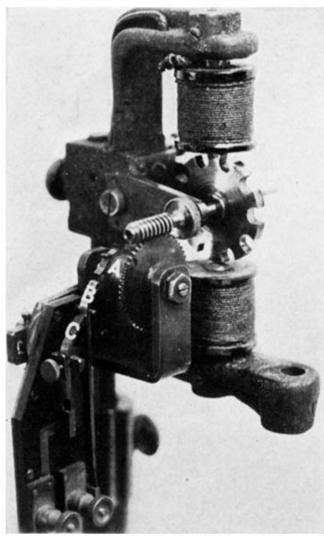

Fig. 4.