## MAGNETO-STRICTION.

## MAGNETO-STRICTION WITH SPECIAL REFERENCE TO PURE COBALT.

## BY HOWARD A. PIDGEON.

## PART I. THE WIEDEMANN EFFECT.<sup>1</sup>

HE inter-relations between magnetization and mechanical strain in the ferro-magnetic metals are widely varied in their nature and some of them are extremely complex in character. Their study has formed such a wide field of investigation that no attempt will be made here to more than briefly outline some of the more salient features of the extensive literature on the subject.<sup>2</sup>

## THE JOULE EFFECT.

In 1847 Joule' found that the length of a soft-iron bar was slightly increased when magnetized, and at about the same time Matteucci<sup>4</sup> discovered that the longitudinal magnetism in an iron rod was increased by the application of a longitudinal pull. It was later discovered by Vallari<sup>5</sup> that this effect is reversed in sufficiently strong magnetic fields, and is commonly known as the Vallari reversal. The reciprocal relations between magnetization and mechanical strain discovered by Joule and Matteucci were shown by the work of later investigators to extend throughout the field of magneto-striction and have been dealt with from theoretical considerations by J. J. Thomson,<sup>6</sup> Kirchhoff,<sup>7</sup> Heydweiller,<sup>8</sup> Gans,<sup>9</sup> Houstoun<sup>10</sup> and others, who have derived mathematical relations

<sup>1</sup> This is the first of a series of articles in preparation under the general title given above.

 $^{\rm 2}$  For a fuller discussion of these effects the reader is referred to the following: Poynting and Thomson, Electricity and Magnetism; Ewing, Magnetic Induction in Iron and Other Metals; Maxwell, Electricity and Magnetism, Vol. II., 3d edition, pp. 90—94; Wledemann, Electricitat. Vol. 3, p. 519. A fairly comprehensive bibliography of the subject up to that date is given at the end of an article by H. G. Dorsey, PHYS. REV., Vol. 30, p. 718, 1910.

<sup>&</sup>lt;sup>8</sup> Phil. Mag., Vol. 30, pp. 76, 225, 1847.

Comptes Rendus, I847.

<sup>&</sup>lt;sup>5</sup> Pogg. Ann., 1868.

Thomson, Applications of Dynamics to Physics and Chemistry.

<sup>~</sup> Wied. Annalen, x88\$, Vol. a4, p. 52.

<sup>&</sup>lt;sup>8</sup> A. Heydweiller, Ann. d. Phys., Vol. 11, p. 602, 1903.

<sup>&</sup>lt;sup>9</sup> R. Gans, Ann. d. Phys., Vol. 13, p. 634, 1904.

<sup>10</sup> Phil. Mag., Vol. 21, p.78,1911.

SECOND SERIES.

for these reciprocal relations, which, however, have been only very roughly verified by data,<sup>1</sup> due no doubt largely at least to magnetic and elastic hysteresis which play a very important role in all magneto-elastic phenomena.

The first really comprehensive view of the Joule effect, as the change in length due to magnetization is called, is due to the extensive work of Shelford Bidwell<sup>2</sup> who worked with specimens of iron, nickel and cobalt, and also studied the change in the effect produced by the application of longitudinal stress. As a result of his work and also that of more recent investigators, especially of Honda and his co-laborers,<sup>3</sup> the genera relations between longitudinal magnetization and strain in iron and nickel have been quite definitely established, and with considerably less certainty in cobalt.

The principal features of the work done may be summarized as follows: soft iron elongates when subjected to a magnetic field of weak or moderate strength, retracts in larger fields and in strong fields becomes shorter than its original length. The effect of pull is to increase the intensity of magnetization in weak or moderate fields, and to decrease it in strong fields. The results for steel are similar, but the initial magnetic elongation is less in general.

In the case of nickel, longitudinal magnetization is always accompanied by a decrease in length which approaches an asymptotic value in strong fields, while the effect of longitudinal tension is to diminish the magnetization for all values of the magnetic field.

In the case of cobalt the results have been far less conclusive. Bidwel14 experimenting with a short rod of cast cobalt found a decrease in length with increasing magnetic field, followed by a retraction reaching the original length at about 75o gauss, and in still stronger fields the specimen became longer than its original length. Thus we see that the effect in cobalt was found to be the exact reverse of that in iron. This result was later confirmed by Nagaoka and Honda' for cast cobalt, but a similar annealed specimen was found to decrease in length gradually and did not reach a maximum shortening even in a magnetic field of 2,000 gauss. It was found as expected that the effect of longitudinal tension upon the cast cobalt was to diminish the magnetization in fields of moderate strength and to increase it in very strong fields, while longi-

<sup>1</sup> Honda and Tereda, Phil. Mag., Vol. 14, 1907, pp. 65-107.

'Proc. Roy. Soc., I886, Vol. 4o, pp. xo9, 257; Phil. Trans. , x888, Vol, I49, p. 205; Proc. Roy. Soc., x89o, Vol. 4y, p. 469.

\* Phil. Mag., 1898, Vol. 46, p. 261; 1900, Vol. 49, p. 329; 1902, S. 6, Vol. 4, pp. 45, 338, 459, 537; 1903, S. 6, Vol. 6, p. 392; 1905, S. 6, Vol. 10, p. 548, 642.

<sup>4</sup> Phil. Trans., vol. 149.

<sup>5</sup> Phil. Mag., Vol. 4, p. 51, 1902.

Vol. XIII.]<br>No. 3.

tudinal pull always diminished the intensity of magnetization in annealed cobalt.

Now all of these specimens were undoubtedly of very impure cobalt, so there has remained some doubt as to what the character of the effect would be in specimens of pure cobalt, since it is known that a marked change may be produced by relatively small quantities of impurities. Bidwell gives no analysis of his specimen but makes the statement that it was soft and easily worked which indicates a considerable amount of impurity since pure cobalt is hard and brittle and not easily worked without special heat treatment. Honda,<sup>1</sup> however, gives an analysis of his specimens which contained from four to five per cent. of nickel, more than one per cent. of iron, and a very high carbon content of from I.38 to I.64 per cent.

It was shown by the work of Kelvin,<sup>2</sup> Ewing<sup>3</sup> and others that the effect of longitudinal compression upon magnetization is just the reverse of that produced by longitudinal pull, and that the effect of transverse stress upon longitudinal magnetization is just the reverse of that produced by longitudinal stress.

The work of Nagoaka and Honda' has shown that magnetization is also attended by a change in volume which is, in general, of a much smaller order than the change in dimensions.

## THE WIEDEMANN EFFECT.

Some years previous to Joule's discovery Wiedemann' experimenting with iron wires made the interesting discovery that when the specimen was suspended in a very small vertical magnetic field, while at the same time an electric current flowed through the specimen, its free end was observed to twist in such a direction that to an observer looking along the specimen in the direction of the flow of current and also in the direction of the magnetic lines of force, the lines of twist were in the direction of a right-handed screw, that is the twist was positive in direction. The direction of twist was reversed upon the reversal of either the current or the longitudinal held. If the current in the specimen remained constant and the longitudinal field was gradually increased, the twist reached a maximum in fields of from 15 to 36 gauss, gradually decreased in stronger fields until in very strong fields the direction of twist changed and became negative or left handed. This is commonly known as the Wiedemann effect.

<sup>&</sup>lt;sup>1</sup> Phil. Mag., S. 6, Vol. 4, p. 48.

<sup>~</sup> Kelvin, Reprint of Papers, Vol. II., pp. 332—407.

 $*$  Loc. cit., p. 202.

<sup>~</sup> Phil, Mag. , S. S, Vol. 46, p. 26'; S. g, Vol. 49, p. 3&9.

<sup>&</sup>lt;sup>5</sup> Electricität, Bd. 3.

SECOND SERIES.

Wiedemann also found that the reciprocal relations already referred to hold here too, for if a wire in which a current was flowing was twisted it was found that longitudinal magnetization was developed, or if a specimen was subjected to a longitudinal field and then twisted circular magnetization resulted.

Investigation by Knott,<sup>1</sup> and Nagoaka and Honda<sup>2</sup> with nickel wires showed that the twist was always negative in direction and much larger than in iron.

The inability to draw cobalt into the form of wires until recently proved an almost insurmountable obstacle to the study of the Wiedemann effect in that metal. However, Honda and Shimizu' did succeed in making some measurements on two cobalt rods 2x cm. in length and approximately one centimeter in diameter. They found that for cast cobalt the twist in small magnetic fields was in the same direction as for nickel. It reached a maximum in a somewhat higher field and then decreased, finally reversing its direction in very strong fields. The magnitude of maximum twist was found to be approximately the same as that in similar specimens of iron. In the case of annealed cobalt the twist was very small and did not reach a maximum until a field of about 200 gauss was reached; it then gradually decreased but did not reverse even in very strong fields.

Beside the uncertainty due to the use of impure cobalt, there is still an additional uncertainty here because it was found that the curves obtained for the twist in similar rods of iron and nickel varied considerably from those obtained when wires approximately one millimeter in diameter were used. It would therefore seem especially desirable in this case to obtain data using specimens of as nearly pure cobalt as possible. Although there are many other interesting magneto-elastic effects we shall not discuss them here since the Joule and Wiedemann effects are the fundamental ones and the only ones studied in this work.

As the result of a detailed study of his own work and that of others, Kelvin was able to explain many of the magneto-elastic effects on the basis of magnetic *a* olotropy produced by stress. In the case of iron in moderate fields he showed that the effect of a simple pulling stress is to produce a greater permeability along than across the lines of strain, while a compressional stress has just the opposite effect. Above the Vallari reversal point the effect is reversed. Kelvin further showed that the idea of magnetic *a* olotropy may be employed to explain the Wiedemann effect and its reciprocal relations. In this case by the super-position of

<sup>&</sup>lt;sup>1</sup> Trans. Roy. Soc. Edin., Vol. 32, p. 193, 1883.

<sup>&</sup>lt;sup>2</sup> Phil. Mag., S. 6, Vol. 4, p. 61.

<sup>~</sup> Phil. Mag. , S. 6, Vol. g, p. 65o.

Vol.  $XIII.$ ]<br>No. 3.

#### MAGNETO-STRICTION. 213

the circular field due to the current in the specimen, upon the longitudinal field, the direction of the resultant field in any given element is a diagonal lying in a plane tangent to the element and making an angle with the transverse plane, which varies from zero at the center to a maximum at the circumference. According to Kelvin's theory this must result (for moderate fields} in an elongation in the direction of the resultant field and a shortening in a direction at right angles to it. Since the change in volume is of a smaller order, there must result a shearing strain in a plane making an angle of 45 degrees with the resultant field and, consequently, this strain wi11 have a component in a direction tangent to the element and in a plane perpendicular to the axis of the wire. Such a strain must result in a twist of the specimen. Applying similar reasoning one can predict the nature of the effect produced by stress upon circular or longitudinal magnetization.

Although the conception of magnetic *a* eolotropy is sufficient to explain many of the magneto-elastic effects qualitatively at least, after making due allowance for hysteresis effects, it does not afford a satisfactory explanation of many of the experimental facts.' For example, according to this theory the Wiedemann effect may be regarded as only a special case of the Joule effect, and one would accordingly expect to find a comparatively simple relation between the two. However, this is not the case, as has been pointed out by S. R. Williams.<sup>2</sup>

#### OBJECT OF THIS WORK.

The object of this work is two-fold: first, to study the Joule and Wiedemann effects in specimens of pure cobalt wire and if possible to establish a definite relation between the two effects; second, to make a comparative study of these effects in specimens of cobalt, iron and nickel wire previously subjected to exactly the same heat treatment. As has already been indicated, previous investigation of the Wiedemann effect in cobalt has been of such a meager character and all work on cobalt with such impure specimens that it seemed highly desirable to repeat the work with pure specimens now available.

Moreover, as has been pointed out by S. R. Williams,<sup>3</sup> there has beer a great lack of coordination in the work done in this field. Much of the experimentation has been upon specimens of whose chemical composition or previous history but little or nothing is known, and since both have a very important influence upon the magneto-elastic effects, much of the data taken has a questionable value so far as making a comparative study

<sup>&#</sup>x27; See Eming's Magnetic Induction in Iron and Other Metals, pp. 244 and 246.

<sup>~</sup> PHYS. REv., Vol. 32, p. 295.

<sup>&</sup>lt;sup>8</sup> PHYS. REV., Vol. 34, p. 258.

HOWARD A. PIDGEON.

Seconi<br>Ser<mark>ies</mark>.

is concerned. It is evidently highly desirable that as many as possible of these effects be studied in the same specimens which have previously been subjected to the same heat treatment. It was with this object in view that a considerable portion of this work was undertaken. This paper will deal with the Wiedemann effect in cobalt and for comparison, also in iron and nickel; while a later paper will deal with the Joule effect and a comparative study of these and other magnetic phenomena.

#### SPECIMENS.

The specimens of cobalt used were obtained from the laboratory of the School of Mining in Queen's University, through the generosity of Eugene Haanel, director of the Mines Branch, under whose direction H. T. Kalmus and others made an extensive study of cobalt' and succeeded not only in producing very pure metal on a commercial basis but also by special heat treatment, in drawing it into wires suitable for many tests otherwise almost impossible. As will be seen from the following analysis given by the School of Mining, the specimens were comparatively pure.







The nickel specimens were from wire labelled "pure nickel" and the iron from the core of an old induction coil, so although no chemical analysis has thus far been made they are undoubtedly of fairly pure material. All of the specimens tested were approximately 25 centimeters in length and varied from 0.85 mm. to 1.00 mm. in diameter. The exact dimensions are given in Table I.

<sup>1</sup> See Bulletin entitled, The Physical Properties of the Metal Cobalt, Canadian Department of Mines, Part II., by H. T. Kalmus and C. Harper.

214

Vol., XIII.]<br>No. 3. MAGNETO-STRICTION. 2 IS

Specimens A, C, E and H were annealed at 800 degrees centigrade for three hours, after which the temperature was very slowly reduced to that of the room, the process occupying several hours. Oxidation was prevented by keeping a stream of hydrogen flowing through the furnace. After this treatment the iron and nickel were extremely soft and flexible but the cobalt was still hard and showed a decidedly crystalline structure. Cobalt specimen 8 has the same composition as C but was left in the original condition. Nickel specimen F is the same as E except that it was not annealed. Data was also obtained from specimens of iron and nickel annealed by alternating current, but the results are not given as they did not differ materially from those for specimens E and H.

#### APPARATUS.

The arrangement of apparatus is shown diagrammatically in Fig. I. The solenoid, S, was attached to the plank,  $P_1$ , supported firmly between two massive brick pillars about three feet apart. The specimen,  $W$ , was



soldered to two brass rod extensions and the whole suspended by means of the knife-edge support, O, from the plank,  $P_2$ , which was supported by the brick piers entirely independent of the plank,  $P_1$ . This was to guard against any possible disturbance due to mechanical displacement produced by the field in the solenoid.

The earth's field was determined and the vertical component compensated for by an additional winding on the outside of the solenoid. As it was found in some cases that the rise in temperature of the specimen due to the heating of the solenoid produced quite an appreciable effect

HOWARD A. PIDGEON.

upon the readings, a double-walled water jacket, J, made of brass tubing was placed inside the solenoid and water from a hydrant kept flowing through it during the experiment. Electrical connection with the specimen was made by means of the mercury cups,  $V_1$  and  $V_2$ , into which dipped copper contacts attached to the specimen. By means of a special construction illustrated in the figure, a small weight,  $F$ , could be suspended from the specimen without interfering with its freedom of motion. The weight was made just large enough to keep the wire accurately vertical in the central line of the solenoid, and to prevent the weight from vibrating it was suspended in oil at the end of a long flexible cord. The entire suspended system was enclosed to prevent the disturbing influence of currents of air in the room.

As it was desired to measure angles varying from zero to considerably more than one degree in magnitude with an accuracy of at least one second for the smaller angles, a special device was necessary for the purpose. After trying several methods, the very simple one illustrated in Fig. I was chosen. Lengthwise in the fairly narrow slit of the collimator, C, was mounted a fine glass fiber. A Nernst glower,  $N$ , brilliantly illuminated the slit, rays from which after being rendered parallel by the collimator lens fell upon the plane mirror,  $M$ , mounted upon the brass rod attached to the lower end of the specimen. The reflected parallel rays were focussed by an achromatic lens,  $L$ , forming an image at  $I$ , which was viewed by the traveling microscope,  $T$ . This image when viewed through the microscope showed a very well-defined diffraction pattern produced by the double slit, and consisted of a large number of alternate light and dark parallel bands. By special adjustment two or three of these lines could be made to appear predominantly clear and well defined, and upon these very good settings could be made by means of double cross-hairs in the eye-piece of the microscope.

The system was calibrated by replacing the specimen and mirror,  $M$ , with another mirror mounted in the axis of a lever approximately 65 cm. in length moved by a micrometer screw. It was found that one division of the micrometer screw of the microscope corresponded to approximately one second of twist of the specimen. By taking an average of two or three settings it was possible to set within one half division, corresponding to 0.5" of arc or 0.02" per centimeter length of specimen, for small angles. With larger deflections the image became less distinct so that the accuracy of setting was not quite as great but in all cases amply sufhcient. Considerable difficulty in obtaining measurements was experienced due to vibrations in the building or produced by passing street cars, so that it was found necessary to take data at night when traffic was reduced.

 $\begin{bmatrix} \text{Vol. } \text{XIII.} \\ \text{No. } 3. \end{bmatrix}$ 

#### MAGNETO-STRICTION. 217

A diagram of the electrical connections is shown in Fig. r. A special form of rheostat,  $R_1$ , made of carbon lamps was selected, which although it did not have the ease of manipulation nor advantage of continuous change of resistance possessed by a liquid or slide-mire rheostat, had the very decided advantage of certainty of setting permitting readings to be easily repeated, which was very desirable in this work. The rheostat was so constructed that all the lamps could easily be placed in series, in parallel, or in various combinations of lamps in parallel with a group of lamps. For example, with switches  $a, b$  and  $c$  closed, lamp  $I$  was connected directly across the line in parallel with  $3$ ,  $4$  and  $5$  in series. This arrangement made it possible to increase the current from zero to a maximum by as few or as many steps as desired. The adjustable rheostat,  $R_2$ , was connected in parallel with  $R_1$  to obtain the higher values of current.

Current through the specimen was controlled by means of two field rheostats,  $R_3$  and  $R_4$ , in parallel with a slide wire rheostat,  $R_5$ . The latter provided for fine adjustment necessary to keep the current constant during a run.

The current was read by means of Weston milli-voltmeters provided with one and five ampere shunts. They were frequently calibrated by means of a potentiometer, standard cell and standard resistances.

The solenoid was 38 cm. in length and hence the field was not quite uniform throughout the 25 cm. length of the specimen. From approximate dimensions the variation from the maximum, of the average field over the length of the specimen was computed. As no accurate data for the solenoid were obtainable it was necessary to determine its constant experimentally. This was done by means of a ballistic galvanometer and a ballistic coil whose dimensions were accurately known. The galvanometer was calibrated by means of a standard condenser. The constant after applying the correction indicated above was found to be 2zo.5 gauss per square cm. per ampere. A later determination using a mutual inductance instead of the condenser gave a result differing from that given above by only a small fraction of one per cent.

#### METHOD OF OBSERVATION.

The specimen was thoroughly demagnetized before each run by gradually decreasing an alternating current flowing through the solenoid, which process was repeated several times with decreasing voltages applied. Demagnetization by less frequent reversals made by hand and also by a mechanical commutator constructed for the purpose, was also tried and as no difference was observed it was concluded that the demagnetization by A.C. was complete in most cases. (Certain exceptions will be noted later.)

Keeping the circular 6eld constant four runs were made as follows: both longitudinal and circular 6elds direct; both reversed; circular field direct, longitudinal field reversed; circular field reversed, longitudinal field direct. In many cases one or more repetitions of these runs were made. When both fields were direct the relation of circular and longitudinal fields and of twist for iron, was right handed or positive according to the convention previously mentioned.

#### DATA AND RESULTS.

Examination of the plotted results showed that these four runs gave two distinct curves, one when both fields were direct or reversed, the other when either one was reversed. Which one of the curves was obtained evidently depended upon the direction of twist in the specimen since the agreement between the curves of each pair seems to eliminate other possibilities. This lack of agreement has been noticed by other investigators and ascribed to various causes, consequently a study of this effect was made to determine its origin.

Curves r and z, Fig. 2, show typical results for cobalt specimen A, when both fields are direct and when one is reversed respectively. Curve 3 is the average of curves I and 2.

Curves  $5$ , 6 and 7 show the same thing for iron specimen H; and curves  $9$ , 10 and 11 for nickel specimen E.

It was found in the case of every specimen that after careful demagnetization the application of either the longitudinal or circular field alone produced a small twist, whose value varied with different specimens and the magnitude of the magnetizing field. The initial twist due to the circular 6eld was, however, extremely small for most of the specimens. The maximum value of the twist produced by the longitudinal field varied from 0.18" to 4.33" per cm. in different specimens and, although its value varied somewhat with different runs on the same specimen, it was always in the same direction and seemed to be practically independent of the previous condition of magnetization or of demagnetization so long as the latter was reasonably thorough. In general the twist was greater in unannealed than in annealed specimens of the same material. Curves 4, 8 and  $12$  show average results for these correction curves for specimens A, H and E respectively.

In the case of nickel the initial twist due to the circular field varied quite erratically with different runs both in magnitude and direction. However, if the twist, as measured from the original zero when neither

218

VOL. XIII.]<br>No. 3. MAGNETO-STRICTION.

field was operating, be plotted from the correction curve obtained when the longitudinal field alone was operating, as a new zero axis of twist the resulting corrected curves almost coincide. An example of the result of this operation is shown in Fig. 2 by the double row of dots, obtained



by applying curve  $I2$  as a correction to curves  $\rho$  and  $I0$ . It is seen the dots almost coincide with curve  $II$  which is the average of curves  $g$  and 10. In some cases there was some divergence near the peaks of the curves but the agreement was always good for higher values of the longitudinal field.

Treating the data for iron in the same manner gave even better results, an example of which is shown by the dots almost coinciding with curve 7, obtained by applying curve  $\delta$  as a correction to curves  $\zeta$  and  $\delta$ .

In the case of cobalt, however, the best agreement was obtained by measuring the twist not from the initial zero but from that obtained after the application of the circular field.

An analysis of these rather complex results indicates that the lack of symmetry in twist is apparently due to various combinations of the three

Second<br>S<mark>eries.</mark>

following factors: imperfect demagnetization, change in temperature of the specimen due to the heating effect of the current in it, and *a* olotropic structure of the specimen.

Imperfect demagnetization was in evidence in the annealed nickel specimens only, which apparently accounts for the initial twist of erratic character due to circular field. This is not at all surprising in this case since for very small values of the longitudinal field, not only is the susceptibility relatively high making perfect demagnetization difficult, but the twist is also very large, thus greatly magnifying the effect of even an extremely small amount of residual magnetism. Indeed, it may be added that this method affords an extremely delicate means of detecting residual magnetism in annealed nickel. No such effect is observed in hard drawn nickel since both the initial susceptibility and the twist are very low for small values of the longitudinal field.

The fact that this initial twist in cobalt and iron due to the circular field alone, was always in the same direction and of about the same magnitude for any given specimen seems to indicate that if residual magnetism were present it was so small that its effect could not be observed. The observed twist was evidently due to heating and a olotropy in these cases.

Twist caused by either of these two effects alone could be distinguished by the fact that *a*eolotropy produces a true magnetic twist for which correction is necessary as in the case of iron; while the effect of heating is merely to change the zero of twist by producing a shift in the equilibrium configuration of the specimen allowing a partial adjustment of residual strains. Such was the case with cobalt. However, if both of the above effects be present, the net twist due to circular field is the algebraic sum of the two components, correction for only one of which should be made. Consequently, the application of the corrections in the manner previously described results, in general, in two parallel curves, the intercept between which is twice the variation of the correction actually made for the circular field (it was zero in the case of cobalt) from the component due to *a* olotropy alone.

Since the corrected curves for iron and cobalt almost coincide as shown by the curves, we conclude that the twist due to circular field in the case of the former is due almost wholly to æolotropy, and to rise of temperature in the case of the latter.

That this initial effect in cobalt is due primarily to change in temperature is further supported by the following facts. Not only did the maximum twist show a very marked increase with rise of temperature, but in one case after making a run at approximately 6o degrees centiVol.  $XIII.$ ]<br>No. 3.

grade and demagnetizing, <sup>a</sup> twist of approximately 4.o" per cm. was observed on reducing the temperature to approximately 15 degrees.

Furthermore, the initial twist in cobalt seemed to increase approximately as a parabolic function of the current which one would expect if due to the heating effect.

On the other hand the eftect of temperature change in iron and nickel was so small as to be almost completely masked by the other factors.

The twist produced in all specimens by the application of the longitudinal field alone is apparently due almost wholly to *æolotropy* caused perhaps by permanent residual strains sustained in drawing, by subsequent coiling of the wire, etc. As we have seen, such an effect may give a circular component to longitudinal magnetism.

This view is supported by the fact that annealing reduced the effect, and by the following experiment. One end of an iron specimen which

	$I_c = 0.1229.$		$I_c = 0.2051.$		$I_c = 0.615.$	$I_c = 1.230.$	
Н.	θ.	Н.	θ.	Н.	θ.	Н.	θ.
1.50	0.87	1.50	2.50	1.39	8.11	2.93	26.90
2.14	2.32	2.14	6.02	2.14	14.41	4.71	31.39
3.21	3.28	3.21	8.21	3.16	18.12	6.64	30.61
4.28	3.72	4.28	9.16	4.28	19.52	9.33	27.43
5.57	3.74	5.57	9.18	5.56	19.29	12.74	23.49
7.28	3.57	7.28	8.71	7.33	18.01	17.08	19.34
9.21	3.29	9.31	7.96	9.26	16.45	21.41	16.08
11.25	3.01	11.30	7.30	11.30	14.90	25.9	13.61
14.35	2.66	14.35	6.41	14.35	12.94	32.3	10.89
18.42	2.26	18.41	5.43	18.40	10.82	53.3	6.01
23.88	1.83	23.88	4.42	23.92	8.65	77.2	3.49
36.4	1.18	30.4	3.41	30.4	6.76	102.7	2.16
49.5	0.79	49.7	1.89	36.4	5.57	124.5	1.51
69.3	0.50	59.3	1.45	49.7	3.75	146.3	1.06
84.6	0.30	69.4	1.14	59.3	2.92	179.3	0.61
104.8	0.23	84.7	0.81	69.4	2.30	204.1	0.38
133.8	0.15	104.9	0.54	84.7	1.71	275.5	0.00
168.7	0.08	133.9	0.30	104.9	1.19	347	$-0.20$
196.6	0.07	169	0.16	134.0	0.76	401	$-0.30$
246.5	0.02	197	0.10	169.1	0.48	450	$-0.34$
332	0.00	223	0.05	197.0	0.33	501	$-0.35$
362.5	$-.003$	275	$-0.02$	223	0.23	552	$-0.34$
450.2	$-0.05$	334	$-0.06$	274	0.03	608	$-0.33$
		392	$-0.10$	334	$-0.03$	659	$-0.31$
		452	$-0.13$	391	$-0.10$	708	$-0.30$
				451	$-0.13$	800	$-0.25$
						854	$-0.22$

TABLE II.

Annealed Iron Specimen H. Current, Ic, in Specimen in Amperes per Square MilLimelers.

HOWARD A. PIDGEON.

	$I_c = 2.459.$		$I_c = 3.554.$		$I_0 = 5.331$	
Н.	θ.	Н.	θ.	Н.	θ.	
2.89	24.35	2.5	18.37	2.3	12.04	
4.6	33.98	4.1	27.71	4.1	20.32	
6.6	37.68	5.6	34.55	5.7	28.31	
9.4	37.12	7.7	38.85	7.8	35.42	
12.8	33.80	10.4	39.62	10.5	39.50	
17.1	29.07	14.0	37.27	14.1	40.52	
21.5	24.82	17.4	33.92	17.8	39.23	
26.1	21.40	21.1	30.25	21.5	36.17	
32.6	17.42	26.5	25.80	27.1	31.95	
42.1	13.41	34.6	20.62	35.1	26.35	
53.4	10.11	44.1	16.01	44.9	20.87	
77.7	6.08	54.2	12.61	66.0	13.53	
103.0	3.86	65.7	9.95	88.1	8.98	
125.3	2.67	73.0	8.59	106.3	6.76	
147.4	1.87	83.0	6.39	126.1	4.86	
179.7	1.11	106.2	4.68	154.0	3.01	
205.7	0.71	126.0	3.36	190.9	1.55	
282	$-0.05$	153.9	2.10	242.5	0.25	
406	$-0.59$	190.7	1.09	300	$-0.60$	
508	$-0.68$	242	0.24	353.5	$-1.05$	
616	$-0.59$	329	$-0.56$	400	$-1.33$	
715	$-0.53$	397	$-0.92$	445	$-1.45$	
864	$-0.49$	490	$-1.05$	492	$-1.45$	
		591	$-0.99$	544	$-1.42$	
		685	$-0.90$	593	$-1.37$	
		778	$-0.80$	637	$-1.35$	
				700	$-1.28$	
				779	$-1.19$	

had been tested previously, was twisted through g6o degrees, leaving a permanent twist of r8o degrees. Subsequent tests showed that the average maximum twist was reduced and in strong fields the twist mas increased in one direction and decreased in the other. The twist due to either circular or longitudinal field alone was greatly increased, the latter being almost doubled as shown by curves  $I_3$  and  $I_4$ , Fig. 2. Data for curve  $13$  were taken before and for curve  $14$  after twisting.

One peculiarity still remains unexplained. In some cases the corrected curves shomed a considerable divergence in the region of maximum twist. It is perhaps due to a difference in the modulus of rigidity for direct and reversed twist, as magnetic twist must always be accompanied by actual mechanical strain since the tangential component of shear due to the magnetic fields does not vary uniformly from the center to the circumference of the wire.

At any rate it seems that one may safely conclude that the average curve for any given specimen gives very closely at least, the single curve

SECOND<br>SERIES.

Vol. XIII. $\begin{bmatrix} \text{Vol.} & \text{XIII.} \\ \text{No. } & \text{3.} \end{bmatrix}$ 

that would be obtained if experimental conditions could be better controlled and the specimen freed from accidental eccentricities.

In this work the data and curves shown are the result of selecting data from runs giving nearly average results for twist in the two directions and averaging these results. Where runs could not be found very closely approximating the average result an actual average of the different runs is given.

The results for iron are shown in Table II. and in Fig. <sup>3</sup> for several values of current as indicated. In general characteristics the results



agree quite well with those obtained by other observers; in fact they show a striking similarity to results obtained for steel tubes by S.R. Williams. ' It will be observed that the values of the maximum twist increase roughly proportional to the current in the specimen at first then seemingly approach a maximum. This is what one would expect if the Wiedemann effect is a special case of the Joule effect, since the elongation in iron reaches a maximum in fairly low fields, and consequently with large circular fields the magnitude of the resultant field and its angle with the axis of the wire do not combine to form a condition so favorable for a large twist.

The maxima which gradually move into higher longitudinal fields with increase of circular field, are not only very sharp in this specimen but also occur at somewhat lower fields than observed by others. This is  $1$  Loc. cit.

#### HOWARD A. PIDGEON.

doubtless due to the extremely soft, well-annealed condition of the iron, since in another specimen evidently not so well annealed by alternating current, the maxima were broader and came in stronger fields.

The curves show rather striking characteristics in stronger fields. Not only do they all reverse their direction of twist at approximately the same value of longitudinal field,—<sup>a</sup> fact the reason for which is not apparent,—but in still higher fields they reach a maximum negative twist and at the highest values of field obtained, seem to be very gradually approaching the axis of zero twist. So far as the author knows this characteristic has never been previously observed. Apparently in only one other instance' has the Wiedemann effect been studied with values of longitudinal field above 4oo or 5oo gauss, and that was with rods about one centimeter in diameter whose characteristics were quite different. <sup>A</sup> further study of this feature, —for which the Joule effect offers no apparent explanation,—will be necessary to determine its cause and whether it is characteristic of all well annealed iron specimens.

The results for annealed nickel specimen E are given in Table III.

Annealed Nickel Specimen E. Current, Ic, in Specimen in Amperes per Square Millimeters.								
	$I_c = 0.306.$		$I_c = 0.630.$		$I_c = 1.277.$			
Н.	θ,	Н.	$\theta.$	Н.	θ.			
2.9	16.57	2.9	39.2	2.8	56.8			
4.5	27.17	4.5	55.0	4.6	79.0			
6.5	32.97	6.5	63.8	6.6	93.7			
9.2	33.30	9.2	64.6	9.4	99.8			
12.7	28.88	12.7	56.5	12.8	94.5			
16.86	23.17	17.1	45.8	17.1	81.5			
21.3	18.64	21.4	37.4	21.6	69.0			
25.7	15.61	25.8	31.5	26.1	59.4			
32.1	12.68	32.2	25.6	32.7	49.0			
41.7	9.90	41.8	20.1	42.4	39.0			
52.9	7.86	53.2	16.07	66.8	25.85			
77.0	5.80	77.4	11.39	78.3	22.43			
102.5	4.36	102.8	8.81	103.8	17.28			
124.3	3.64	124.8	7.34	126.4	14.32			
146.0	3.16	146.6	6.23	148.3	12.18			
179.7	2.64	180.2	5.12	182.3	9.83			
223.2	2.10	224	4.13	216.4	8.40			
279	1.80	280	3.33	289.5	6.30			
349	1.38	406	2.32	392	4.69			
405	1.18	508	1.86	503	3.66			
507	1.00	619	1.54	620	3.03			
619	0.84	721	1.36	720	2.59			
718	0.80	867	1.15	866	2.19			
867	0.66							

TABLE III.

Annealed Nickel Specimen E. Current, Ic, in Specimen in Amperes per Square Millimeter.

<sup>1</sup> Honda and Shimizu, Phil. Mag., Vol. 5, S. 6, p. 650.

224

Seconi<br>Series

Vol. XIII.]<br>No. 3.

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# [Second<br>[Series.

## TABLE IV.

Unannealed Nickel Specimen F. Current, Ic, in Specimen in Amperes per Square Millimeters.

	$I_c = 0.641.$		$I_c = 1.281.$		$I_c = 2.563.$	$I_e = 3.704.$	
Н.	θ.	Н.	θ.	Н.	θ.	Н.	$\theta$ .
2.7	0.02	2.6	0.02	2.5	0.09	2.7	0.14
4.5	0.05	4.3	0.06	4.3	0.17	4.5	0.235
6.4	0.065	6.3	0.09	6.2	0.24	6.4	0.36
12.6	0.14	8.6	0.15	8.6	0.36	9.1	0.51
21.4	0.32	11.8	0.21	11.8	0.49	16.9	1.10
32.2	0.73	15.7	0.34	19.9	1.06	25.8	2.74
41.8	1.08	19.9	0.50	30.1	2.66	41.7	6.54
53.2	1.44	29.9	1.18	38.9	4.19	53.3	8.30
66.2	1.74	38.7	1.98	49.4	5.50	65.9	9.66
77.3	1.92	49.2	2.68	61.4	6.54	77.2	10.62
88.2	2.03	61.2	3.23	71.9	7.21	102.5	11.63
102.6	2.12	82.0	3.83	82.3	7.68	117.6	11.96
115.8	2.16	95.2	4.06	95.9	8.08	124.2	12.04
124.5	2.17	101.5	4.14	101.7	8.24	136.4	12.07
136.9	2.19	109.0	4.22	109.3	8.37	146.0	12.06
146.5	2.19	115.0	4.26	115.9	8.45	163.9	11.96
164.2	2.17	126.4	4.29	127.4	8.54	179.2	11.78
179.9	2.13	135.1	4.30	136.0	8.56	204.0	11.46
204.5	2.06	151.0	4.28	152.0	8.52	222.2	11.16
223	2.01	165.1	4.22	166.4	8.44	278	10.26
256	1.92	187.6	4.14	188.6	8.26	347	9.18
348	1.65	203.5	4.06	205.5	8.08	400	8.49
404	1.52	232	3.90	237	7.72	450	7.91
505	1.32	252	3.80	257.5	7.48	499	7.40
613	1.16	285	3.60	295	7.07	555	6.90
711	1.04	314	3.42	321	6.80	603	6.51
785	0.98	363	3.18	373	6.28	655	6.12
851	0.92	407	2.96	417	5.88	695	5.84
		455	2.78	464	5.50	760	5.44
		498	2.60	508	5.18	825	5.06
		550	2.44	565	4.82		
		595	2.31	609	4.56		
		640	2.19	660	4.28		
		682	2.08	695	4.12		
		682	2.08	777	3.78		
		765	1.90				

and Fig. 4. They agree remarkably well with results obtained by other observers. It will be observed that for nickel the twist is opposite in direction to that in iron, does not reverse its direction and the maximum value is more than four and one half times as great as in iron. The maximum twist in iron for the highest value of current used is approximately 40.5" per cm., while for a slightly higher value of current in nickel it is x9o" per cm. As in iron the peaks advance at first slowly, then more



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Annealed Cobalt Specimen A. Current, Ic, in Specimen in Amperes per Square Millimeters.



HOWARD A. PIDGEON.

$I_c = 3.279$ .			$I_c = 4.734.$	$I_c = 7.102.$	
Н.	θ.	Н.	θ.	Н.	θ.
2.7	0.10	2.5	0.24	2.5	0.72
4.6	0.20	4.3	0.52	4.3	1.48
6.5	0.43	6.4	0.98	6.4	2.66
9.1	0.95	8.9	1.97	12.5	7.77
12.7	2.03	12.5	3.79	20.1	15.45
16.9	3.92	16.9	6.41	25.5	18.99
21.2	5.80	21.2	9.07	31.8	22.58
25.6	7.25	25.6	11.27	41.2	25.83
32.1	9.06	32.0	13.77	52.3	27.48
41.6	10.63	41.6	15.96	64.8	27.69
52.9	11.55	53.0	17.10	75.8	27.17
65.8	11.83	65.7	17.38	86.5	26.40
77.6	11.70	76.6	17.14	100.5	25.19
87.1	11.46	87.4	16.70	122.1	23.25
95.5	11.19	101.7	16.00	143.8	21.45
101.5	11.00	123.4	14.84	176.4	18.98
123.3	10.20	145.0	13.67	200.5	17.47
144.5	9.45	178.1	12.18	275	13.94
177.5	8.48	202.8	11.26	341	11.76
219.8	7.40	276	9.04	393	10.45
276	6.36	344	7.66	489	8.61
342	5.37	399	6.82	590	7.30
396	4.77	498	5.66	680	6.37
494	4.00	603	4.83	807	5.42
596	3.40	695	4.24		
687	3.00	827	3.57	8.9	4.78
817	2.55			16.8	11.93

rapidly into higher longitudinal fields with increasing circular field. However, the peaks of the curves seem to approach a maximum value of twist much less rapidly than in the case of iron. Here again the sharply defined maxima occurring in comparatively low fields are an indication of good annealing. A specimen of the same wire not so well annealed by alternating current gave curves (not shown) having flatter maxima of less magnitude and occurring in higher longitudinal fields as in the case of iron.

This is very strikingly illustrated by the results for specimen F, shown in Table IV. and Fig. 5, which was a hard drawn nickel wire exactly the same as specimen E except that it was not annealed. Here the broad flat maxima, not only occur in very much higher fields, but have only about one thirteenth the magnitude of the former. It clearly shows the importance in work on magneto-striction of knowing the previous history of the specimen, especially the heat treatment.

In Table V. and Fig. 6 are shown the results for cobalt specimen, A, and in Table VI. and Fig. <sup>7</sup> for the less pure specimen, C. <sup>A</sup> comparison





Annealed Cobalt Specimen C. Current, Ic, in Specimen in Amperes per Square Millimeters.



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	$I_c = 3.083.$		$I_c = 4.45I$		$I_c = 6.677.$
Н.	θ.	Н.	θ.	Н.	θ.
2.7	0.12	2.7	0.27	2.7	1.20
4.5	0.23	4.5	0.53	4.5	2.52
6.4	0.42	6.4	0.94	6.4	4.66
9.1	0.80	9.1	1.83	9.08	9.00
12.6	1.78	12.6	4.23	13.5	16.32
16.9	5.00	16.96	10.20	16.9	26.55
21.2	9.33	21.2	16.58	21.2	35.60
25.6	12.93	25.6	21.40	25.5	42.25
31.9	16.68	32.0	25.85	31.85	48.42
415	18.53	41.6	28.69	41.3	51.88
52.7	19.07	52.9	29.13	52.4	51.68
65.5	18.57	65.6	28.08	65.1	49.10
76.5	17.72	76.5	26.67	76.3	46.10
97.3	16.81	87.2	25.25	87.0	43.18
101.6	15.62	101.7	23.32	101.1	39.40
123.2	13.93	123.3	20.73	122.9	34.49
144.7	12.46	144.8	18.41	144.4	30.15
177.8	10.64	177.9	15.65	177.1	25.23
210.8	9.27	202.2	14.01	201.6	22.34
276	7.25	276.1	10.51	275.8	16.28
341	5.89	344	8.49	343	12.83
397	5.06	398	7.29	398	10.92
497	4.01	496	5.78	498	8.46
602	3.31	601	4.73	604	6.78
695	2.82	694	4.01	696	5.71
832	2.32	828	3.28	833	4.60



230

SECOND<br>Series.

Fig  $7$ .

of the curves shows that the maximum twist for the purer specimen is much less than for the other and occurs at higher values of the longitudinal 6eld. This is not surprising when we consider that the purer cobalt is more highly crystalline and consequently exhibits a greater magnetic hardness and a correspondingly lower susceptibility; properties which as we have seen in the case of nickel, produce exactly this sort of an effect.

These results for pure cobalt differ very materially from those obtained



by Honda,<sup>1</sup> for not only is there no reversal of twist in strong fields such as he found for a cast cobalt rod, but the curves show very little resemblance to those he obtained for an annealed cobalt rod. The difference is doubtless due to the difference in purity of cobalt.

Examination of the curves also shows that there is a very marked similarity between the results for cobalt and nickel, making due allowance for the difference in the magnitude of twist in the two cases. In fact, there is actually less difference between the curves for annealed specimens of cobalt and nickel than between those for annealed and unannealed nickel. (It will be shown in a later publication that this similarity holds also for the Joule effect.) This suggests the possibility that, if a more satisfactory method of annealing cobalt could be found reducing it to a less marked crystalline condition or at least to a finer crystalline texture,

<sup>&</sup>lt;sup>1</sup> Honda and Shimiza, Phil. Mag., Vol. 5, S. 6, p. 650.

HOWARD A. PIDGEON.

the results might compare even more favorably with those for nickel. Such a suggestion receives some support from the results for cobalt specimen 8, given in Table VII. and Fig. 8. This specimen differs from specimen C, only in not being subjected to further annealing.

Here again we see the marked effect of annealing and that the result corresponds exactly with that found in iron and nickel. Indeed it may be stated as a general characteristic of all three metals that, the more thorough the annealing the greater is the magnitude of the maximum twist, the sharper the peaks of the curves, and the lower the values of the longitudinal field in which they occur. Now this is exactly the sort of effect that annealing has upon the susceptibility. So, qualitatively

TABLE VII. Unannealed Cobalt Specimen B. Current, Ic, in Specimen in Amperes per Square Millimeters

	$I_c = 0.308.$		$I_c = 0.769.$		$I_e = 1.539.$	
Н.	θ.	Н.	θ.	Н.	θ.	
4.3	0.008	2.6.	0.01	2.6	0.02	
9.0	0.02	4.4	0.03	4.5	0.04	
16.6	0.04	6.4	0.04	6.4	0.06	
20.9	0.07	9.0	0.05	9.0	0.08	
25.4	0.15	12.4	0.06	12.5	0.13	
31.7	0.34	16.7	0.11	16.7	0.21	
40.8	0.58	21.0	0.20	21.1	0.40	
52.0	0.80	25.4	0.40	25.5	0.82	
64.5	0.94	31.8	0.84	31.8	1.74	
75.7	1.02	40.97	1.49	40.9	3.05	
86.2	1.03	52.1	2.04	52.2	4.07	
94.5	1.04	64.7	237	64.7	4.71	
100.2	1.02	75.9	2.52	76.1	5.01	
106.8	1.02	86.5	2.57	86.6	5.09	
121.7	0.98	94.8	2.57	95.0	5.08	
133.7	0.95	100.5	2.55	100.6	5.05	
$143.0 \cdot$	0.92	107.1	2.52	107.1	5.00	
160.4	0.86	122.1	2.44	122.2	4.84	
175.5	0.81	134.1	2.34	134.1	4.64	
199.7	0.74	143.7	2.26	145.6	4.49	
216.4	0.69	161.2	2.11	161.0	4.20	
272.5	0.56	176.2	1.98	176.5	3.95	
340	0.45	200.6	1.81	200.6	3.58	
393	0.39	218.7	1.69	217.9	3.34	
493	0.28	274.5	1.36	275	2.69	
599	0.24	343	1.10	342	2.12	
695	0.20	397	0.93	395	1.82	
839	0.16	498	0.73	497	1.41	
		607	0.58	606	1.12	
		704	0.47	704	0.93	
		853	0.38	849	0.78	

232

Second<br>S<mark>erie</mark>s

 $\begin{bmatrix} \text{Vol. } & \text{XIII.} \\ \text{No. } & \text{3.} \end{bmatrix}$ 



at least, there seems to be a close connection between the susceptibility and the Wiedemann effect. However, the latter is so complex that the relation is far from being a simple one. It is hoped that further study may bring out more definite relations.

The results for iron specimen H, nickel specimen E, and cobalt speci-



men C are presented in a somewhat different way in Figs. 9, 10 and 11 respectively, where for several values of the longitudinal field, values of current in the specimen are plotted as abscissæ and the corresponding twist as ordinates. In the case of nickel and cobalt it is seen that for values of the longitudinal field very large compared with the circular

field, the twist is approximately proportional to the latter. This is exactly what would be expected if the Wiedemann effect be a modification of the Joule effect. For, let  $P$ , Fig. 12, be any particle in a cylin-



drical element of the wire. The approximate net effect of combined longitudinal and circular magnetization is to displace the particle through the distance  $\lambda$ , in the direction  $PP'$  of the resultant magnetic field, making



the angle  $\theta$  with the length of the element in the direction of the axis of the wire. The twist,  $\theta$ , is given by,

$$
\theta = \frac{\lambda}{r} \sin \theta
$$

$$
= \frac{\lambda}{r} \frac{H_e}{\sqrt{H^2 + H_e^2}}
$$

Vol. XIII.]<br>No. 3. MAGNETO-STRICTION. 235

where  $H<sub>c</sub>$  is the circular field, H the longitudinal field and r the distance of the particle from the axis. If H is very large compared to  $H<sub>c</sub>$  then

$$
\theta = \frac{\lambda}{rH} H_c.
$$

Previous study of the Joule effect has shown that  $\lambda$  in nickel, reaches practically a constant value in moderately strong magnetic fields. From the equation it is seen that for constant values of  $H$ ,  $\theta$  is very nearly a linear function of  $H<sub>c</sub>$ . This result, derived only for high values of magnetic field in a cylindrical element, at least gives an indication of what may be expected in a wire. For any given value of  $H<sub>c</sub>$  the twist varies inversely as  $H$ , if  $H$  is very large, and consequently approaches zero as a



limit as  $H$  is increased indefinitely. The curves indicate that this is perhaps the case for nickel and cobalt.

These results correspond quite well with what one might predict from the molecular theory of magnetism; for in strong longitudinal fields which magnetize the material almost to saturation, the molecular magnets are almost all aligned in the direction of the field so that the only effect of the circular field is to produce a deflection of these molecular magnets proportional to the deHecting field with a corresponding deformation of the magnetized material.

Cobalt becomes saturated only in much stronger fields than nickel and consequently the agreement is not so good.

In this case as in that of all the magneto-elastic effects in iron, the results are very complex and any attempt to explain them without further study is fruitless.

Any analysis of magneto-elastic effects is made much more difficult by

the marked hysteresis occurring in all of them. In Figs. 13, 14 and 15 are shown hysteresis curves for iron, nickel and cobalt respectively, obtained by keeping the circular field constant and cyclically varying the longitudinal 6eld. The comparatively large area inclosed in the hys-



teresis loop for cobalt indicates its rather extreme magnetic hardness and low susceptibility, which has been previously mentioned.

Some of the features of the Wiedemann effect will be still further dis-



cussed in a later publication in connection with the Joule effect and other magnetic phenomena.

Vol. XIII.]<br>No. 3.

MAGNETO-STRICTION.

## SUMMARY.

Experimental curves are shown for the Wiedemann effect in specimens of pure cobalt wire, and for the purpose of comparison, results are also shown for specimens of iron and nickel subjected to the same heat treatment. A comparison of the results is made and certain features of the Wiedemann effect discussed.

The results for pure cobalt differ materially from those obtained by



other observers working with impure specimens, and show a close similarity to the results for nickel, the twist, however being much less.

Certain eccentricities of twist observed in all specimens are studied and an explanation given which seems to account for nearly all of the observed facts.

Hysteresis curves for iron, nickel and cobalt are shown.

Experimental results for the Joule effect and other magnetic phenomena wi11 be presented in a later publication.

The writer wishes to express his thanks to various members of the department staff for helpful suggestions given, and especially to Professor E. L. Nichols through whom the specimens were obtained and under whose direction the work was done.

PHYSICAL LABORATORY, CORNELL UNIVERSITY, September, 1918.