gives

$$A_2(\nu_F,0)=\frac{1}{\pi}\int_0^\infty d\nu'\,\mathrm{Im}\tilde{A}_2\left(\nu',1+2\frac{\nu'+1}{\nu_F}\right)\frac{2}{\nu'-\nu_F}. \quad \text{(A3)}$$

We can now insert the $P$- and $D$-wave resonances into this expression through Eqs. (10) and (6) with the parameters $\nu_R\Gamma_1{}^1=4.6$, $\nu_R=5.0$ and $\nu_R{}^2\Gamma_2{}^0=4.4$, $\nu_R=9.2$, which were obtained in the coupled $P$-$D$ calculation. It is found, however, that the $I=0$, $l=0$ state itself cannot be neglected in Eq. (A3). To take it into account, $\lambda$—and hence $A_{(0)0}(\nu_F)$—was varied until a calculation of the type described above gave a resonance, which, when added to the $P$ and $D$ resonances in Eq. (A3), gave back the same value of $\lambda$ through Eq. (A1). Such a self-consistent calculation gives $\lambda=-0.13$.[17] The corresponding values of $F_{(0)0}{}^1+A_{(0)0}(-\omega_1)$ and $F_{(0)0}{}^2$ are $-1.50$ and 19.0, respectively. These give a scattering length of 3.4, which agrees with the experimental value deduced by Desai.[16]

In the foregoing calculation of $A_2(\nu_F,0)$, the delta-function approximation given by Eqs. (8), (9), and (10) was used for the $I=0$ $S$-wave resonance. In general, with a large scattering length, such an approximation may be dubious. However, on the basis of some rough estimates, it appears that the approximations (7) and (10) are both reasonable ones in this particular case.

The $I=2$ $S$-state can be calculated just as any other state, since Eq. (5) does converge here. However, it is somewhat simpler to follow, instead, the same procedure as for $I=0$, but with $A_{(0)2}(\nu_F)\simeq-2\lambda$, and $A_{(0)2}'(\nu_F)\simeq-3\nu_F{}^{-1}A_{(1)1}(\nu_F)$ instead of Eqs. (A1) and (A2). With $\lambda=-0.13$, this gives $F_{(0)2}{}^1+A_{(0)2}(-\omega_1)=2.08$, and $F_{(0)2}{}^2=-16.2$. The corresponding scattering length is 0.06.[18] At higher energies, the phase shift becomes negative but remains comparatively small.

[17] The fact that the coupling constant $\lambda$ is not a fundamental constant, but can be calculated through the $I=2$ amplitude was first pointed out by G. F. Chew (private communication). This should be contrasted with the situation in conventional Lagrangian field theory, where $\lambda$ has to be specified in advance.

[18] This value falls within the experimental limits obtained by J. Kirz, J. Schwartz, and R. D. Tripp, Phys. Rev. 126, 763 (1962).

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# Elastic $\mu^-$ Scattering in Nuclear Emulsion*†

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An emulsion stack was exposed to a separated beam of $\sim 2\times10^7$ $\mu^-$ of energy $52\pm8$ MeV at the CERN synchrocyclotron. Muons with endings in a region near the incoming edge of the plates were traced back to scatters, thus discriminating in favor of events with momentum transfers from 100–160 MeV/$c$. The 78 events found give evidence that the muon behaves merely as a heavy electron, in contradiction to the anomalous muon-nucleus scattering reported in several cosmic-ray experiments. Our data indicate, however, a possibility of a small amount of scattering in excess of that predicted, particularly for momentum transfers $>130$ MeV/$c$. This may be ascribed either to unresolvable inelastic scattering, to inaccuracies in the parameters of the nuclear charge distribution, or to the breakdown in the representation of the many-body nucleus by a smoothed-out potential. Of the 78 events, one was an elastic scatter by hydrogen, which is consistent with the Mott scattering formula.

## I. INTRODUCTION

In recent years numerous cosmic-ray experiments have indicated an anomalously large nuclear scattering of muons, often giving results consistent with scattering against point nuclei.[1] Difficulties with energy determination of both incoming and scattered muons, pion background, and multiple scattering corrections have indicated a need for similar experiments with accelerator beams of known composition and momentum. Several such experiments have already been performed,[2–7] none of which have given evidence of

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[1] For a summary of experiments since 1958, see G. H. Rawitscher, Phys. Rev. 124, 1978 (1961). For experiments before 1958, see G. N. Fowler and A. W. Wolfendale, *Progress in Elementary Particles and Cosmic-Ray Physics* (North-Holland Publishing Company, Amsterdam, 1958), Vol. 4, p. 123.

[2] B. Chidley, G. Hinman, P. Goldstein, R. Summers, and R. Adler, Can J. Phys. 36, 801 (1958).

[3] G. E. Masek, L. D. Heggie, Y. B. Kim, and R. W. Williams, Phys. Rev. 122, 937 (1961).

[4] C. Y. Kim, S. Kaneko, Y. B. Kim, G. E. Masek, and R. W. Williams, Phys. Rev. 122, 1641 (1961).

[5] M. Bardon, P. Franzini, and J. Lee, Phys. Rev. Letters 7, 23 (1961).

anomalous scattering. However, of these only two measured predominantly elastic scattering in the region of momentum transfer $>120$ MeV/$c$, which is the region of interest in the present experiment.

One, by Citron *et al.*,[6] measured the scattering rate of negative muons ($p=180$ and 240 MeV/$c$) by carbon for momentum transfers up to 250 MeV/$c$. The expected rate was calculated using the Mott formula and carbon form factors derived from electron scattering measurements.

The present experiment is part of a program for studying the elastic scattering of muons by Ag and Br nuclei in nuclear emulsion. The first experiment in this program, reported earlier by Connolly *et al.*,[7] measured $\mu^+$ scattering for momentum transfers of 100–160 MeV/$c$. The results were compared to the theoretical calculations of Rawitscher[8] who solved the Dirac equation for point muons scattering from extended nuclei (with charge distribution given by electron scattering measurements).[9] No anomalous effects were found.

The present paper reports the final results of the corresponding $\mu^-$ experiment. These results also indicate no anomalous effects of the type reported in cosmic-ray experiments,[1] but there may be a small discrepancy between Rawitscher's calculations and experiment for momentum transfers $>130$ MeV/$c$. (See Fig. 1.) Rather than being interpreted as a nonelectromagnetic muon-nucleus interaction, which would be inconsistent with accelerator experiments,[2–7] the possible discrepancy could be attributed to unresolved inelastic scattering, incorrect choice of nuclear charge distribution parameters in the calculations, or breakdown in the representation of the nuclear interaction by a potential. Inelastic scattering is discussed in Secs. II and IV, and appears to be of minor importance to this experiment both on theoretical and experimental grounds.

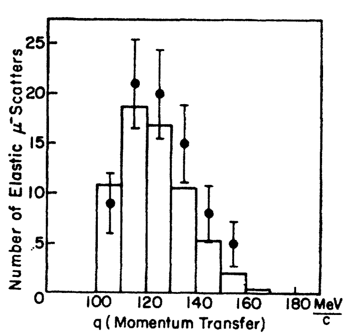


FIG. 1. Histogram of observed number of events compared to predicted number of events using Rawitscher's cross sections. Only rms statistical errors are shown.

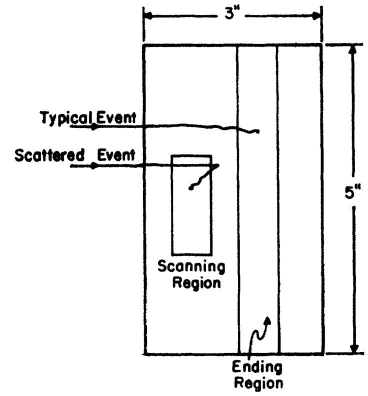


FIG. 2. Geometry of emulsion stack.

## II. EXPERIMENTAL PROCEDURE

A stack of 600-$\mu$ Ilford G-5 pellicles (7.2 cm$\times$12.5 cm $\times$6.6 cm thick) was exposed to $2.1\times10^7$ negative muons from the separated, magnetically analyzed muon beam at the CERN synchrocyclotron. Only those incoming tracks were accepted which had ranges between 38 and 54 mm from the incoming edge, the median range being 45.4 mm (corresponding to $T_\mu=52$ MeV). This restriction on range kept us far from the pion ending region and resulted in a ratio of muon endings to pion endings of at least 1000 to 1 in the region of interest.

The plates were scanned for muon endings in the region from 11.5 to 27.5 mm from the incoming edge. (See Fig. 2.) This region should contain a large percentage of the endings from high momentum transfer scatters.

All endings in the scanning region were traced back 2 mm. If at this point the track has a projected angle $<30°$ to the incoming beam and a projected length $>1$ mm per plate (i.e., a space angle $<46°$), it was dropped. Otherwise it was followed further until it satisfied the above-mentioned criteria or until it scattered.

Using the above procedures, about 1.7 miles of track were effectively scanned for events with high momentum transfer, and, before corrections, 79 events were found. These events then were checked for pion contamination and inelasticity.

To determine the pion contamination, scanners traced back along all tracks giving stars in the scanning region. Five tracks ended in a star, had no star at the scatter, and also had a momentum transfer $q>100$ MeV/$c$ (i.e., appeared to be perfectly good scatters except for a star at the ending). Grain counts, before and after the scatter, then indicated that of the five, three were certainly inelastic events. The endings of the remaining two events (both of which had $130<q<140$ MeV/c) indicated that the events were caused by pions. Since 30% of all negative pion endings in emulsion have zero prongs,[10] there should, therefore, be about one event in our data which appears to be due to a muon,

[6] A. Citron, C. Delorme, D. Fries, L. Goldzahl, J. Heintze, E. G. Michaelis, C. Richard, and H. Øverås, Phys. Letters **1**, 175 (1962).

[7] P. L. Connolly, J. G. McEwen, and J. Orear, Phys. Rev. Letters **6**, 554 (1961).

[8] G. H. Rawitscher, Phys. Rev. **112**, 1274 (1958); **124**, 1978 (1961).

[9] R. Herman and R. Hofstadter, ***High-Energy Electron Scattering Tables*** (Stanford University Press, Stanford, California, 1960).

[10] For a summary of experiments see G. Brown and I. S. Hughes, Phil. Mag. **21**, 779 (1957).

but is, in fact, due to a pion with a zero prong ending. This was subtracted in quoting our final result. No muon stars were found, although according to the work of Moringa and Fry[11] we would have expected two or three.

As regards inelasticity, one might expect that in this experiment inelastic events could result from excitation of the giant resonance. For intermediate nuclei the giant resonance is about 20 MeV above the ground state,[12] hence its excitation would result in an increase in grain density at the scatter of $>15\%$ for all events. Grain counts were made before and after every scatter. No evidence of giant resonance excitation was found. Since, however, the steepness of some of the scattered tracks made their grain counts somewhat unreliable, an indirect grain density comparison was also made, using the range after scatter in combination with the grain count at scatter of the relatively flat incoming track and the grain count of its neighboring tracks. Again the results were negative.

Also all events were examined for internal conversion electrons at the scatter. These would always occur if excited Ag nuclei were to cascade to the ground state through their 90-keV isomeric levels.[13] No such electrons were found.

Finally, it should be noted that of our 78 events, one was a $\mu$-$p$ scatter with $q=131$ MeV/$c$, as evidenced by a 424-$\mu$-long recoil proton and by a check of the kinematics. This is consistent with the prediction of the Mott scattering formula for muons on protons.

### III. THEORETICAL PREDICTIONS

The cross sections for scattering from Ag and Br nuclei were taken from the work of Rawitscher,[8] who solved the Dirac equation for point muons scattering from nuclei having the Wood-Saxon charge distribution as determined by electron scattering measurements.[9] Since these differential cross sections were given for only three values of the energy, our interpolations can introduce errors of up to an estimated 10%.

The average differential cross section for the emulsion $d\bar{\sigma}/d\Omega$ as a function of $R$ (range after scatter) and $\theta$ (scattering angle) was obtained by averaging the Ag and Br values and multiplying by a factor 1.025 to account for scattering off the lighter C, N, O, and H nuclei. For the regions of momentum transfer studied, $d\bar{\sigma}/d\Omega$ varied from 3.5 to 0.01 mb/sr, a typical value being 0.197 mb/sr for a range after scatter $R=19.5$ mm and $\theta=94.3°$ (momentum transfer $q=125$ MeV/$c$).

Knowing $d\bar{\sigma}/d\Omega$ one can predict the average number of events $\bar{N}_{ij}$ for the $i$th interval of $R$ and the $j$th interval of $\cos\theta$ (called bin $ij$) from the relation

$$\bar{N}_{ij}=N_{\mathrm{AgBr}}N_{\mu}f_{ij}\Delta R_i(d\bar{\sigma}/d\Omega)_{ij}2\pi\Delta(\cos\theta_j)\epsilon_{ij}e,$$

where

$N_{\mathrm{AgBr}}=2.039\times10^{22}$ cm$^{-3}$ = number of Ag and Br nuclei/cm$^3$,

$N_{\mu}=(1.78\pm0.008)\times10^6$ = number of muons of range 38–54 mm entering plates scanned,

$\Delta R_i=1$ mm for all $R$ (i.e., $7\leq R\leq 51$ mm),

$\Delta(\cos\theta_j)=0.05$ for all $\theta$ (i.e., $66.4°\leq\theta\leq180$),

$f_{ij}$ = fraction of incoming muons of range 38–54 mm which could contribute to scattering in bin $ij$,

$\epsilon_{ij}$ = total geometrical loss factor (top, bottom, and side losses),

$e$ = scanning efficiency.

The factor $f_{ij}$ was included because our restriction on muon range and scanning region did not permit detection of all scatters into bin $ij$. To calculate $f_{ij}$, a determination was made from the geometry of the scanning region of the largest and smallest total range for tracks which could end in the scanning region and fall into bin $ij$. Then by looking at the integral range curve for muon endings, the fraction of the incoming tracks that fell between these ranges was determined. Values of $f_{ij}$ ranged from 0.994 to 0.026, with a value of 0.719 for the typical event $R=19.5$ mm, $\theta=94.3°$ ($q=125$ MeV/$c$).

The total geometrical loss factor $\epsilon_{ij}$ accounts for the decrease in events near the boundaries of the scanning region due to finite size of the emulsion stack. This correction was not large except for bins with large $R\sin\theta(>30$ mm). For the typical event mentioned above ($R=19.5$ mm, $\theta=94.3°$), $\epsilon_{ij}=0.962$.

The scanning efficiency $e$ was determined by looking for discrepancies between independent determinations of the number of muon endings in some given region. For a total of 560 endings examined, the over-all scanning efficiency was found to be 98%.

Once all the above factors were determined, $\bar{N}_{ij}$ was calculated. Then using the relation $q=2p_0\sin\theta/2$, the average momentum transfer corresponding to each bin was determined. Finally, the predicted number of events in our stack for a given range of $q$ was determined by adding the $\bar{N}_{ij}$'s within that range of $q$. The results, plotted as a function of $q$, are shown in Fig. 1.

### IV. DISCUSSION

As seen in Fig. 1, our results are consistent with Rawitscher's calculations,[8] but there is an indication of a possible excess of events for the larger $q$ values. For the 6 experimental points in Fig. 1, a $\chi^2$ test gives a probability of 51% for finding a least-squares sum greater than that of our experiment. However, the fact

[11] H. Moringa and W. F. Fry, Phys. Rev. 87, 182 (1952).

[12] J. M. Blatt and V. F. Weiskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 656.

[13] H. H. Landolt and R. Bornstein, in *Numerical Data in Science and Technology*, edited by A. M. and K. H. Hellwege (Springer-Verlag, Berlin, 1961), Group I, Vol. 1; M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958).

that all but one of the experimental points are too high does not show up in the results of this test. If the total of 78 observed scatters are compared to the 64.4 expected, the probability of a larger $\chi^2$ is only 12%. For the total number of events with $q > 120$ MeV/$c$, this number becomes 5.7%. Assuming the maximum systematic error of 10% for every predicted value in Fig. 1, the probability of finding a larger least-squares sum for the total number of events with $q > 120$ MeV/$c$ increases to 14%.

Any possible excess scattering found here, however, is quite inconsistent with the anomalous point nucleus scattering reported in cosmic-ray experiments.[1] Our high momentum transfer points are a factor of $\sim 50$ below those for a point nucleus. (See Fig. 3.) Furthermore, the possibility of a discrepancy due to a finite muon radius or to a nonelectromagnetic muon-nucleus interaction would be inconsistent with other recent experiments.[6,7,14] Possible explanations for an excess of events are:

(1) Presence of inelastic events which could not be distinguished from elastic ones. In particular, it is quite possible that despite the absence of giant resonance excitations and excitations of the first excited states at 90-keV there may have been some excitation of $>300$ keV levels, which our emulsion techniques could not have resolved. However, excitations of lower levels might not cause any great difficulty since as Breit has pointed out[15] although new channels are opened up with each excitation, there is a compensating effect on the original "coherent" channel caused by the second order term arising from the coupling of the new channels with the old. The compensation is exact in the classical limit. However, for higher excitations, no such statement can be made.

(2) Slight inaccuracies in the two parameters (nuclear radius and skin depth) which were used in Rawitscher's calculations. In his more recent paper,[8] Rawitscher indicates that a 10% change in skin depth will change the cross section by as much as 15% for $\theta = 150°$, but gives no data pertaining to changes in the nuclear radius. Of course, any change in the two parameters must still give consistency with $\mu^+$, $e^+$, and $e^-$ results.

(3) Distortion of the nucleus by the muon. As Rawitscher[8] and Breit[15] point out, the effect has not been investigated in this case, and might not be negligible.

[14] G. Charpak, F. J. M. Farley, R. L. Garwin, T. Mueller, J. C. Sens, V. L. Telegdi, and A. Zichichi, Phys. Rev. Letters **6**, 128 (1961).

[15] G. Breit (private communication).

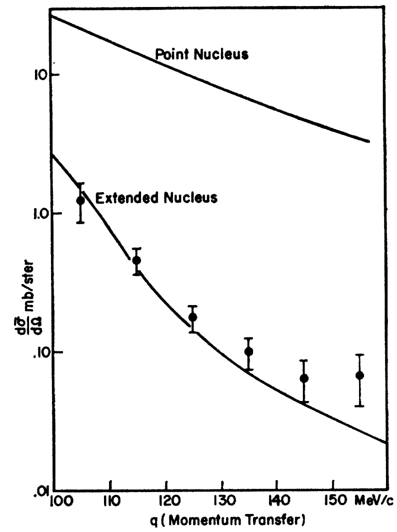


FIG. 3. One-half the sum of the differential cross sections for elastic scattering of 26-MeV $\mu^-$ on Ag and Br as calculated by Rawitscher plotted against momentum transfer $q$. Experimental points are normalized to Rawitscher's curve by making use of the ratios of observed to predicted numbers of events.

(4) Breakdown of Rawitscher's assumption that a smoothed-out potential can replace the effect of a many-body nucleus. Certainly for large enough $q$ the potential model must break down since the muon will tend to interact with individual nucleons rather than the nucleus as a whole.

We conclude that both our $\mu^-$ and $\mu^+$ experiments give further evidence that the muon behaves merely as a heavy electron and that the calculations of Rawitscher[8] are reasonably good even in regions where the nuclear form factor reduces the cross section by a factor of 100 below that of the point charge.

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