# Nuclear $E1$ Peak Splitting

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A schematic calculation is given for the splitting of the $E1$ giant resonance into (two) component peaks, taking into account the large collective energy shift. The observed intensity ratio of about 1:2 between the main peaks of $O^{16}$ is calculated for the first time by assuming the quadrupole-quadrupole component of effective two-body forces in the nucleus to be enhanced by almost a factor 2 over the value given by a local potential of Yukawa shape. Similar calculations show this assumption to be compatible with the additional salient features of $C^{12}$ and $C^{13}$ photodisintegration, such as the "pygmy" resonance. These three nuclei are then simply generalized to serve as typical examples of the closed shell, closed shell minus a few particles, and closed shell plus a few particles. Detailed consideration of highly deformed nuclei by this method looks prohibitively laborious in detail but should generally yield the hydrodynamic intensity ratio of 1:2. The approach emphasizes that the peak splitting for such nuclei is a function of the $E1$ excited state rather than of the ground state.

## I. INTRODUCTION

THE "giant resonance" of $E1$ photonuclear absorption is not generally expected to be a smooth, simple peak but to show splitting into a number of distinct components; such components have already been resolved in a number of experiments. Recently, it has also become apparent that the giant resonance energy contains a substantial upwards shift because of the collective nature of the excitation[1]; the empirical magnitude[2] of the average shift (neglecting resolution into components) is $\Delta \approx 7$–$8$ MeV, roughly independent of mass number $A$. Since $\Delta$ of this magnitude represents some 30–60% of the total resonance energy $W_1$, it is important to consider the role of the collective shift in peak splitting. The present paper is an attempt in that direction, based on procedures previously used[2] for discussing $\Delta$.

Three classes of $E1$ peak structure may be distinguished: satellite peaks, relatively weak and well separated from the main peak; splitting of the main peak into a number of comparable components; division of the main peak into just two principal components with a 1:2 intensity ratio, as predicted by the classical hydrodynamic treatment of spheroidally deformed nuclei.[3,4] The most favorable targets for observing the last effect have naturally been the heavy nuclei; the weak satellites have been known for some time[5–7] as "pygmy resonances" or low-energy shoulders in nuclei like $C^{13}$, $N^{14}$, and $F^{19}$; and resolution of multiple components in the main peak has so far been feasible only for light nuclei.[8]

Specific cases considered below are $O^{16}$, $C^{12}$, and $C^{13}$, for which it is concluded that:

(i) The energy but *not* intensity distribution among the various components can be naively interpreted in terms of nuclear subshells.

(ii) The effective two-nucleon potential in the nucleus must have an exceptionally strong quadrupole-quadrupole term.

(iii) In the excited state, subshells of large $l$ must be relatively lowered in energy.

The naive interpretation of $E1$ excitation in terms of individual subshells is valid only under neglect of two-body forces in the nucleus; the net effect of these forces is mainly to alter the naive intensity ratios while making only a secondary contribution to the energy values. The quadrupole-quadrupole potential strength may be specified as follows: The two-body interaction is taken as a Yukawa form, expanded into multipoles, and a factor of order 1.9 used to multiply the $Y_2(\theta_{ij})$ term. The result is, of course, a nonlocal potential, which reflects the influence of surrounding nuclear matter on the two-particle interaction.

An effort is made to generalize from these examples to heavy nuclei. The enhanced quadrupole-quadrupole interaction is qualitatively responsible for the deformations required in the nuclear collective model,[9] so that some indirect connection is established between the present considerations and the quasi-classical treatment of peak splitting in terms of spheroidal deformation. Unfortunately, it does not seem feasible to make the correlation more immediate by means of the present approach, which would require explicit construction of highly deformed states for heavy nuclei

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[1] R. A. Ferrell, Bull. Am. Phys. Soc. 3, 49 (1958); G. E. Brown, and M. Bolsterli, Phys. Rev. Letters 3, 462 (1959).
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[3] K. Okamoto, Progr. Theoret. Phys. (Kyoto) 15, 75 (1956); Phys. Rev. 110, 143 (1958).
[4] M. Danos, Bull. Am. Phys. Soc. 1, 135 (1956); Nucl. Phys. 5, 23 (1958).
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[6] H. E. Johns, R. J. Horsley, R. N. H. Haslam, and A. Quinton, Phys. Rev. 84, 856 (1951).
[7] R. J. Horsley, R. N. H. Haslam, and H. E. Johns, Phys. Rev. 87, 756 (1952); J. V. G. Taylor, L. B. Robinson, and R. N. H. Haslam, Can. J. Phys. 32, 238 (1954).
[8] Surveyed, for example, in N. W. Tanner, Gordon Conference, 1961 (unpublished).
[9] J. P. Elliott, Proc. Roy. Soc. (London) 245, 128, 562 (1958); V. Bargmann and M. Moshinsky, Nucl. Phys. 18, 697 (1960); R. S. Willey, Phys. Rev. 126, 1127 (1962).

from a complete set of single-particle orbitals. One can only emphasize the qualitative conclusion that even for spheroidal nuclei the peak splitting is a function of the *excited* state and not of the ground state, although, of course, the quadrupole deformations may be extremely similar for both states.

## II. PROCEDURE

In reference 2 the giant resonance peak energy was estimated from the sum-rule formula for the harmonic mean energy of $E1$ absorption, written in a form equivalent to

$$\begin{aligned} W_1 &= \langle Q|\hbar\omega+\mathcal{V}|Q\rangle, \\ Q &= (a^2A)^{-1/2}\sum_{m=\{n\}}\psi_m(1)\psi_m^*(2)(z_1-z_2), \\ \mathcal{V} &= -4\{\mathfrak{M}(12)+\mathcal{H}(12)P_{12}^\sigma\}. \end{aligned} \quad (1)$$

Here $A$ is the nuclear mass number. The nuclear wave function is taken from ideal harmonic oscillator (i.h.o.) model with range parameter $a\approx 1.0\times A^{1/6}$ F, and energy unit $\hbar\omega\approx 40A^{-1/3}$ MeV, both values being empirical. The function $Q$ represents the collective excited state or hole-particle combination produced by $E1$ absorption; the set of states $\{n\}$ includes only the last occupied states in the i.h.o., on which the operator $z$ can induce a transition without violating the exclusion principle. The normalization is such that $\langle Q|Q\rangle=1$. The indices $m$ include orbital and spin states, but isotopic spin has already been eliminated. The quantities $\mathfrak{M}$ and $\mathcal{H}$ are the charge exchange terms in the two-nucleon potential, written as

$$\sum_{i<j}\mathcal{W}(ij)+\mathcal{B}(ij)P_{ij}^\sigma-\mathcal{H}(ij)P_{ij}^\tau-\mathfrak{M}_{(ij)}P_{ij}^\tau P_{ij}^\sigma,$$

with $P_{ij}^\sigma$ and $P_{ij}^\tau$ the respective spin and isotopic spin exchange operators. For the i.h.o. without spin-orbit coupling $\langle P_{ij}^\sigma\rangle=\frac{1}{2}$.

Consider the simplest case where $W_1$ splits into just two components. To do this, one must separate $Q$ into two orthogonal parts that remain so under $(\hbar\omega+\mathcal{V})$. In practice, it is simplest to find components $Q_+$ and $Q_-$ that have different eigenvalues $\hbar\omega_+>\hbar\omega_-$; these are the energy differences of corresponding single-particle states in the shell model, which are sometimes known experimentally. The matrix elements of $\mathcal{V}$ are then computed and the combination $(\hbar\omega+\mathcal{V})$ diagonalized. To define the quantities involved, let

$$Q=\cos\alpha Q_+ + \sin\alpha Q_-,\quad \langle Q_\lambda|Q_\mu\rangle=\delta_{\lambda\mu}, \quad (2)$$

and on this basis

$$\begin{aligned} (\hbar\omega+\mathcal{V}) &= \begin{pmatrix} E_+ & V \\ V & E_- \end{pmatrix}, \\ E_\pm &= \hbar\omega_\pm+\langle Q_\pm|\mathcal{V}|Q_\pm\rangle, \\ V &= \langle Q_\pm|\mathcal{V}|Q_\mp\rangle. \end{aligned} \quad (3)$$

Diagonalization yields the two peak energies

$$\begin{aligned} W_\pm &= E\pm D, & D &= (D_0^2+V^2)^{1/2}, \\ E &= \tfrac{1}{2}(E_++E_-), & D_0 &= \tfrac{1}{2}|E_+-E_-|, \end{aligned} \quad (4)$$

The intensity ratio between components in the naive decomposition of Eq. (2) was

$$r_0=I_-/I_+=\tan^2\alpha. \quad (5a)$$

For the corresponding ratio between peaks at $W_-$ and $W_+$

$$\begin{aligned} r &= [1-\cos(2\alpha-\beta)]/[1+\cos(2\alpha-\beta)], \\ -\pi/2 &\leq \beta=\tan^{-1}V/D_0\leq\pi/2. \end{aligned} \quad (5b)$$

Here "intensity" is measured as the harmonic integral of $E1$ absorption under a peak.

The diagonalization procedure above is quite conventional and could, in principle, be extended to an $n$-fold decomposition. Such algebraic complication is scarcely warranted by present data, and we approach more complex peak structure as a succession of twofold decompositions.

## III. $O^{16}$

The main peak at $W_1\sim 24$ MeV seems definitely to be split into components at about 22 and 25 MeV. There are probably at least three more weak $E1$ peaks in the region 16–20 MeV, but the present discussion considers only the two main components. A summary of $(\gamma,p)$ reactions[10] indicates an intensity ratio $r\approx 0.5$ for the 22- and 25-MeV components, and $(\gamma,n)$ measurements also indicate that the upper component is the stronger.[11] Theoretical treatments[12,13] have generally given $r\gtrsim 2$.

The three main $E1$ transitions in $O^{16}$ are $1p_{3/2}\to 2s_{1/2}$, $1p_{3/2}\to 1d_{5/2}$, $1p_{1/2}\to 1d_{3/2}$. We shall ignore the two other possible transitions ($1p_{3/2}\to 1d_{3/2}$, $1p_{1/2}\to 2s_{1/2}$), for these have $\Delta j=0$, and such $E1$ transitions are known[12,14] to be much less intense than those with $\Delta j=\pm 1$. From the measured binding energies of $N^{15}$, $N^{15*}$ and $O^{17}$, $O^{17*}$ relative to $O^{16}$ one can estimate[12] $\hbar\omega_0=18.5$, 17.6, and 16.5 MeV for the three respective transitions. One should, however, allow for dilation of the $O^{16*}$ dipole state at 24 MeV relative to the $O^{16}$ core in $N^{15}$ or $O^{17}$. The most obvious consequence is to mitigate the spin-orbit splitting. Consideration of $E1$ overtones[15] suggests a factor of order 1.3 for the expansion of $O^{16}$; if the spin-orbit force goes as $(1/r)(dV/dr)$ with $dV/dr$ a constant, the $d_{5/2}-d_{3/2}$ splitting is reduced by $5[(1/1.3)-1]\approx 1.1$ MeV, ren-

[10] E. G. Fuller and E. Hayward, Karlsruhe Photonuclear Conference, 1960 (unpublished), paper No. P9.
[11] C. Tzara (private communication).
[12] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **242**, 57 (1957); G. E. Brown, L. Castillejo, and J. A. Evans, Nucl. Phys. **22**, 1 (1961).
[13] V. Gillet, thesis, Université de Paris, 1962 (unpublished).
[14] E. D. Courant, Phys. Rev. **82**, 703 (1951).
[15] J. H. Carver, D. C. Peaslee, and R. B. Taylor, Phys. Rev. **127**, 2198 (1962).

dering the last two transitions degenerate at $\hbar\omega_0=16.9$ MeV. The average energy of these transitions is still 17.2 MeV, which exceeds the i.h.o. value of $40A^{-1/3}=16$ MeV for this case. One must also expect that dilation of the excited state lowers the orbitals of high $l$ value relative to the others; if we reduce the $p-d$ transition energy to give a weighted average of 16 MeV, the values assigned for the three transitions are $\hbar\omega=18.5$, 15.5, and 15.5 MeV.

Under these approximations it happens that the separation of $\hbar\omega$ into two values for Eq. (3) is just according to the $l$ value ($s$ or $d$) of the final orbital. This provides the basis

$$Q_+=(1/\sqrt{2})[\psi_{1p}(1)\psi_{2s}^*(2)-(1\leftrightarrow 2)],$$
$$Q_-=(1/\sqrt{2})[\psi_{1p}(1)\psi_{1d}^*(2)-(1\leftrightarrow 2)], \quad (6)$$
$$\tan\alpha=\sqrt{5},$$

where repeated neglect of $\Delta j=0$ transitions is the justification for ignoring spin-orbit distinctions entirely in Eq. (6). This simplicity is fortuitous for $O^{16}$ and does not occur in general. For example, in $Ca^{40}$ there are five major $E1$ transitions instead of three; and since expansion effects should not, in general, produce accidental collapse of spin-orbit distinctions, the pattern of the main peak splitting should be relatively complex and weakly differentiated. This is, in fact, observed to be the case.[8,11]

The potential integrals for $O^{16}$ are[16]

$$V_+=\langle Q_+|\mathcal{V}|Q_+\rangle=F^0(1p2s,1p2s)-(1/3)F^1(1p2s,2s1p),$$
$$V\ =\langle Q_\pm|\mathcal{V}|Q_\mp\rangle=-(\sqrt{2}/5)F^2(1p1d,1p2s)+(\sqrt{2}/3)F^1(1p1d,2s1p), \quad (7)$$
$$V_-=\langle Q_-|\mathcal{V}|Q_-\rangle=F^0(1p1d,1p1d)+(1/5)F^2(1p1d,1p1d)-(1/15)F^1(1p1d,1d1p)-(9/35)F^3(1p1d,1d1p).$$

Here the $F^k$ are Slater integrals with $k$ denoting the orbital momentum transfer between two particles in the orbits specified. The first and third (second and fourth) entries refer to the same particle. If we take $\mathcal{V}=V_0(\kappa r)^{-1}\exp(-\kappa r)$, $\kappa^{-1}=1.4$ F, then

$$V_+\approx V_-\approx 0.082V_0, \quad V\approx 0.0008V_0. \quad (8)$$

It would, thus, appear that the original decomposition in Eq. (6) is practically diagonal in $\mathcal{V}$ also, which implies that

$$r\approx r_0=5, \quad (9)$$

in disagreement with observation. The total shift $\Delta$ is known to be about 7.5 MeV and must be set equal to $V_+\approx V_-$, yielding peak energies of 23 and 26 MeV. Each of these peak energies is almost 1.5 MeV too high, although the splitting (3 MeV) and average value (23.5 MeV) are about right. This discrepancy is directly related to the incorrect intensity ratio in Eq. (9).

A known physical feature of nuclei not yet encompassed in the calculation is the exceptional strength of quadrupole-quadrupole interactions, which presumably is the main source of collective rotational states.[9] Assuming that this is also a feature of the charge exchange interaction, it can be approximated by adjoining an enhancement factor $\chi>1$ to the $F^2$ integrals only in Eq. (7). One then obtains

$$V_+=0.082V_0, \quad V_-=[0.082+0.039(\chi-1)]V_0, \quad V=[0.0008+0.026(\chi-1)]V_0, \quad (10)$$

For $\chi=1.65$, $V_0=65$ MeV, the result is

$$W_+=24.5 \text{ MeV}, \quad W_-=21.8 \text{ MeV}, \quad r=0.5. \quad (11)$$

A reversal of the intensity ratio has now occurred, in agreement with experiment. It should be remarked that this result is rather sensitive to the value chosen for $(\hbar\omega_+-\hbar\omega_-)$.

One should attempt to compare this result with previous calculations.[12,13,17] They all use essentially similar forces and procedures and arrive at similar conclusions, with one exception[17]: The dipole strength concentrates in the upper two of five levels, with $r\approx 2$; the strongest peak at about 22 MeV is mostly $1p_{3/2}\rightarrow 1d_{5/2}$, while that at 25 MeV is even more predominantly $1p_{3/2}\rightarrow 1d_{3/2}$. Since this was an unfavored transition to begin with, its increase to $\frac{1}{3}$ of the total intensity is remarkable and must be close to a maximal value; that is, such an approach can scarcely be expected to obtain $r\lesssim 2$. The emphasis on the $1p_{3/2}\rightarrow 1d_{3/2}$ transition is related to the fact that the average shift $\Delta$ computed by these methods is of order 3.5–4.0 MeV, or just half the value inferred from an empirical survey of all nuclei[2]; correspondingly the calculation must emphasize transitions with highest $\hbar\omega$, of which $1p_{3/2}\rightarrow 1d_{3/2}$ is foremost.

There is actually rather little mixing of the simple one-particle transitions in these calculations; the final eigenstates average about 90% "pure" with respect to a single transition, in spite of various choices for the interaction potential. This is because the interactions chosen have all had the feature of being essentially $s$-wave potentials (which do not mix the $s$ and $d$ orbitals) with only moderate spin dependence.

From this discussion it seems clear that only by introducing non-$s$-wave potentials can the ratio $r$ be brought down to the neighborhood of 0.5; the most obvious choice is a quadrupole-quadrupole interaction, which the calculations above suggest is adequate for the purpose. If we had included the $1p_{3/2}-1d_{3/2}$ transition, the appropriate value of $\chi$ might have been different; in fact, subsequent sections favor a somewhat larger $\chi$.

One might, at first, hope to distinguish between the

[16] For these expressions and for values of some of the integrals, the author is indebted to Dr. F. C. Barker.

[17] J. Sawicki and T. Soda, Nucl. Phys. 28, 270 (1961). In this reference an unexplained ratio of $r\approx 1$ was obtained from the same assumptions (Table 3) as gave $r\approx 2$ for all other authors.

present wave functions and those of the conventional calculation[12,13] by angular distribution measurements. If the emitted particles in each peak come mostly from direct interaction and not through compound nucleus formation, their angular distributions correspond directly to the wave functions involved. For example, the distribution from the $1p_{1/2}-1d_{3/2}$ transition is

$$1-\tfrac{3}{5}\cos^2\theta, \qquad (12a)$$

where $\theta$ is the angle between emitted particle and incident beam. Because of threshold effects only the $p_{1/2}$ ground state at $A=15$ is available for direct interaction in the lower peak at $W_-\approx 22$ MeV; and the present model agrees with the conventional ones in assigning negligible $1p_{1/2}-2s_{1/2}$ transition strength to this peak, so that both models predict an angular distribution like Eq. (12a). For the upper peak at $W_+\approx 25$ MeV, the conventional models give about 90% $1p_{3/2}-1d_{3/2}$ transition with 7% $1p_{1/2}-1d_{3/2}$ as the principal mixture. The direct interactions leaving the residual nucleus in its first excited $p_{3/2}$ level thus have the unique angular distribution

$$1+\tfrac{3}{4}\cos^2\theta. \qquad (12b)$$

The present model has a mixture of $1p_{3/2}-1d_{5/2}$ and $1p_{3/2}-2s_{1/2}$ contributions and yields instead of Eq. (12b) the form

$$(\tan\gamma-3/\sqrt{5})^2+(3/\sqrt{5})[(\tfrac{3}{2})\tan\gamma -3/\sqrt{5}](1+\cos^2\theta), \qquad (12c)$$

where $\tan\gamma$ is the ratio of $2s_{1/2}$ to $1d_{5/2}$ amplitudes. If this is cast in the form $1+\lambda\cos^2\theta$, it is clear that $\lambda>0$ for $\tan\gamma>2/\sqrt{5}=0.9$, reaching $\lambda_{\max}=1$ for $\tan\gamma=3/\sqrt{5}=1.34$ and declining to $\lambda=0$ as $\tan\gamma-\infty$. On the present model the amplitude ratios in the upper peak are approximately $5\sqrt{3}:\sqrt{3}:2$ for $1p_{3/2}\rightarrow 2s_{1/2}$, $1p_{3/2}\rightarrow 1d_{5/2}$, and $1p_{1/2}\rightarrow 1d_{3/2}$, so that $\tan\gamma\approx\sqrt{5}$. The angular distribution (12c) then becomes

$$1+0.77\cos^2\theta. \qquad (12d)$$

The models are not accurate enough to render the difference between Eqs. (12b) and (12d) of any significance, quite aside from experimental uncertainties.

Both models also predict rather similar values for the ground to first excited state ratio of the residual nucleus from the $W_+$ peak. The percentage of $p_{1/2}$ in the eigenfunction is about 7% for the conventional models, 10% for the present one; when multiplied by velocity factors to yield reaction ratios, these numbers become something like 10–15% and 15–20%, respectively. Experimentally, the ratio seems[10] to be of order 20%, but this can scarcely be claimed to distinguish between the models. The accidental degeneracy of Eqs. (12b) and (12d) could perhaps be resolved by polarization measurements, but these would involve polarization correlation between two of the three quantities incident photon, emitted particle and residual nuclear state.

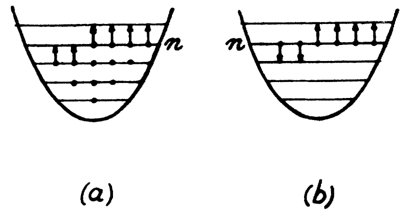


FIG. 1. Diagrams for $E1$ excitation of the i.h.o. with partly filled $n$th shell: (a) actual transitions; (b) equivalent transitions all originating in the last major shell.

## IV. $C^{12}$

The main peak at 23 MeV has always appeared quite narrow, with indications[18] of a prominent "tail" on the high-energy side to compensate somewhat for the relatively small cross section in the main peak. Recent experiment[19] has suggested that a substantial part of this high-energy tail consists of a distinct peak centered at $\sim$29 MeV. A preliminary value for the intensity ratio is $(I_{23}/I_{29})=r\sim 2$. With the wisdom of hindsight one can then perceive that previous measurements are all compatible with this interpretation. Electron bombardment[18] would suggest an intensity ratio $r\sim 2$, although the upper peak is not at all resolved; more detailed betatron measurements[5] on $C^{12}$ $(\gamma,n)$ actually show a slight dip between the peaks and an intensity ratio more like $r\approx 3$. The inelastic scattering of 185 MeV protons from C should have a prominent contribution from virtual photons at angles $\lesssim 10°$; the main $E1$ peak appears above background at an excitation of 22–23 MeV, again with a distinct high-energy tail above background in the 25–30 MeV region.[20] The integrals under the inelastic peaks, after the subtraction of a large smooth background, indicate $r\sim 4$. It, thus, seems reasonable to infer the existence of peak splitting in $C^{12}$ with a spacing of about 6 MeV and $r\approx 3\pm 1$. The corresponding unsplit peak energy is $W_1\approx 25$ MeV, in excellent agreement with the general formula $(40A^{-1/3}+7.5)$ MeV. The existence of a double peak in this energy region is even more clearly apparent[5] in $C^{13}$, although with a somewhat narrower spacing and more nearly equal intensity ratio.

For $C^{12}$ the main $E1$ transitions are $1p\rightarrow(2s,1d)$, much as in $O^{16}$; and also some transitions $1s\rightarrow 1p$, which arise because the $1p$ shell is not entirely filled. It is, perhaps, more convenient to regard these latter as $1p\rightarrow 1s$ transitions, with amplitudes prefixed by a minus sign. The equivalence of these two forms is indicated pictorially in Fig. 1. The procedure can obviously be extended to any higher shell in exactly the same way; for any partly filled shell

$$Q_+=Q_\downarrow, \quad Q_-=Q_\uparrow, \quad \tan\alpha<0, \qquad (13)$$

[18] W. C. Barber, W. D. George, and D. D. Reagan, Phys. Rev. **98**, 73 (1955); V. J. Vanhuyse, and W. C. Barber, Nucl. Phys. **26**, 233 (1961).
[19] B. Ziegler, Nucl. Phys. **17**, 238 (1960).
[20] H. Tyrén and Th. A. J. Maris, Nucl. Phys. **3**, 52 (1957).

where $Q_\uparrow(Q_\downarrow)$ includes all transitions from the last, partly filled shell to the next higher (lower) shell. Tighter binding of particles in interior orbits would make $\hbar\omega_\downarrow > \hbar\omega_\uparrow$, which is the basis of the assignment in Eq. (13).

Comparison among binding energies of $C^{12}$ and appropriate states of $C^{12}$ plus or minus one nucleon indicates $\hbar\omega \approx 16.6$, 17.7, 24.5 MeV for $1p_{3/2} \rightarrow 2s_{1/2}$, $1p_{3/2} \rightarrow 1d_{5/2}$, $1s_{1/2} \rightarrow 1p_{1/2}$. We again neglect the transition $1p_{3/2} \rightarrow 1d_{3/2}$ and assume that the $C^{12}$ ground state has all $p_{3/2}$ orbitals filled and all $p_{1/2}$ orbitals empty, so that no $1p_{1/2} \rightarrow (2s_{1/2}, 1d_{3/2})$ transitions are present. We repeat the prescription followed in $O^{16}$ and drop the energy of the second transition by 2.1 MeV relative to the first, to allow for expansion of the nuclear wave function in excited states. It also seems likely that the $p_{1/2}$ binding energy at $A = 13$ suffers exceptionally from the exclusion principle because of subshell closing at $A = 12$; we accordingly lower the energy of the third transition by a similar (rather arbitrary) amount. The corrected energies are then $\hbar\omega = 16.6$, 15.6, 22.4 MeV. With or without these corrections it is clear that the natural grouping into two peaks separated by about 6 MeV is associated with the basis in Eq. (13); specifically,

$$Q_+ = P_{1/2} q_{1s}, \quad \tan\alpha = -\sqrt{8}, \qquad Q_- = P_{3/2}[(5/6)^{1/2} q_{1d} + (1/6)^{1/2} q_{2s}]. \tag{14}$$

Here $q_{1d}$ and $q_{2s}$ are just the functions in Eq. (6), and $q_{1s}$ is the same with $2s \rightarrow 1s$; the quantities $P_{1/2}$ and $P_{3/2}$ are projection operators on $j$ of the $1p$ orbitals.

This is a case where a complete decomposition would involve three components in the $E1$ peak; but we consider only the major splitting indicated by Eq. (14), lumping the two parts of $Q_-$ together as one peak. Detailed calculation as for $O^{16}$ shows that the splitting between components in $Q_-$ will be $\sim 1$ MeV, for which some experimental evidence has recently appeared.[11,21] The potential matrix elements are

$$\begin{aligned}
V_+ &= F^0(1p1s,1p1s) - \tfrac{1}{9}(1+2R)F^1(1p1s,1s1p),\\
V &= (\tfrac{5}{6})^{1/2}[-(\chi/5)F^2(1p1s,1p1d)\\
&\quad + \tfrac{1}{9}(2+R)F^1(1p1s,1d1p)]\\
&\quad - [(1/6)(2/9)]^{1/2}(1-R)F^1(1p1s,2s1p),\\
V_- &= \tfrac{5}{6}[F^0(1p1d,1p1d) + (\chi/10)F^2(1p1d,1p1d)\\
&\quad - (2/45)(1-R)F^1(1p1d,1d1p)\\
&\quad - (3/70)(4+R)F^3(1p1d,1d1p)]\\
&\quad + (2\sqrt{5}/6)[-(\sqrt{2}/10)\chi F^2(1p1d,1p2s)\\
&\quad + (\sqrt{2}/6)(4-R)F^1(1p1d,2s1p)]\\
&\quad + \tfrac{1}{6}[F^0(1p2s,1p2s) - \tfrac{1}{9}(2+R)F^1(1p2s,2s1p)].
\end{aligned} \tag{15}$$

Here $R = \mathcal{K}/(2\mathcal{M}+\mathcal{K})$ is a ratio introduced by the spin-orbit projections; $R \approx 0.4$ appears a suitable numerical value.[22] After insertion of numerical values for the $F^k$, it appears that $\chi$ must be somewhat larger than for $O^{16}$: with $\chi = 2.0$, $V_0 \approx 60$ MeV,

$$V_+ \approx V_- \approx 0.12 V_0 \approx 7.2 \text{ MeV}, \qquad V = -0.019 V_0 = -1.1 \text{ MeV}. \tag{16}$$

We finally obtain

$$W_- = 22.8 \text{ MeV}, \quad W_+ = 29.8 \text{ MeV}, \quad r = 3.4. \tag{17}$$

The splitting is perhaps a little large; but the intensity ratio is satisfactorily reduced from $r_0 = 8$, which would probably have rendered the upper peak unobservable. Changes in $\chi$ or $V_0$ designed to make $r$ smaller would increase the energy difference $(W_+ - W_-)$ still further; this makes it seem difficult to obtain a calculated $r$ much lower than in Eq. (17), which lies within the experimental range.

The wave functions corresponding to Eq. (17) are over 95% pure $Q_+$ and $Q_-$ and, hence, quite similar to those of the conventional calculation,[13,23] where the main transitions are $1p_{3/2} \rightarrow 1d_{5/2}$ and $1s_{1/2} \rightarrow 1p_{1/2}$ with $W_\mp \approx 22$, 34 MeV and $r \approx 4$.

## V. $C^{13}$

The notable feature of $E1$ absorption by $C^{13}$ is the presence of a "pygmy" resonance at about 13.5 MeV. The intensity (harmonic cross section) ratio $r \approx 0.3 \pm 0.1$ relative to the main peak at 25.5 MeV[5] is about a factor 3 too large to be interpreted by a simple model comprising one neutron outside an undisturbed $C^{12}$ core. To judge from the previous cases of $O^{16}$ and $C^{12}$, the present method of calculation shows the energy of the pygmy resonance to be almost entirely determined by the $1p_{1/2} \rightarrow 1d_{3/2}$ transition of the last nucleon, while the relative intensity of the pygmy resonance is strongly affected by the interaction energy $\mathcal{V}$ and may bear little resemblance to that of the naive model. The main peak in $C^{13}$ is also split in a way resembling $C^{12}$, as one should expect, with[5] $W_-$, $W_+ \approx 24$, 28 MeV and $r \sim 2$. The latter value is rather uncertain, but both $r$ and the peak splitting appear distinctly smaller for $C^{13}$ than for $C^{12}$. The present treatment implies a direct connection between these two changes.[24]

The wave function now contains three terms:

$$Q = (8/3)^{1/2} P_{3/2}[(\tfrac{5}{6})^{1/2} q_{1d} + (\tfrac{1}{6})^{1/2} q_{2s}] - (\tfrac{1}{4})^{1/2} P_{1/2} q_{1s} + (\tfrac{1}{3})^{1/2} P_{1/2}[(\tfrac{5}{6})^{1/2} q_{1d} + (\tfrac{1}{6})^{1/2} q_{2s}]. \tag{18}$$

[21] F. K. W. Kirk, K. H. Lokan, and E. M. Bowey (to be published).

[22] D. C. Peaslee, Phys. Rev. **124**, 839 (1961).

[23] N. Vinh-Mau and G. E. Brown, Nucl. Phys. **28**, 89 (1962).

[24] It is also of interest to compare total intensities for $C^{12}$ and $C^{13}$. According to reference 5 the harmonic cross section for $C^{13}$ is $\sigma(13) \approx 7.5$ mb up to 30 MeV, which includes both components of the main peak. The simple $A^{4/3}$ law implies a corresponding harmonic integral for $C^{12}$ of $\sigma(12) \approx 6.8$ mb. The actual cross section observed[5] for the 23-MeV peak is about $\sigma_-(12) \approx 5.0$ mb, a substantial discrepancy again suggesting a higher energy component in the $C^{12}$ peak. One, thus, obtains an independent estimate for the intensity ratio in $C^{12}$: $r \approx 5.0/(6.8-5.0) \approx 3$.

The first two are the primary constituents of the main peak and resemble those for $C^{12}$, except that one $p_{1/2}$ hole has been removed; the third term is the contribution of the corresponding $p_{1/2}$ particle. To consider the main peak, we simply repeat the considerations for $C^{12}$, neglecting the perturbation of the pygmy resonance. The naive intensity ratio $r_0=32/3\approx 11$ is even larger than for $C^{12}$. The potential matrix elements $V_+\approx V_-\approx 7.2$ MeV are taken over directly from $C^{12}$, as well as the cross term $V=-1.1$ MeV. The observed value of the splitting gives $D\approx 2$ MeV, so that $\sin\beta \approx -0.55$, and the calculated $r=2.3$, in qualitative accord with observation. The corresponding values of $E_-$, $E_+$ are 24.3, 27.7 MeV; subtracting $V_+\approx V_-\approx 7.2$ MeV leaves $\hbar\omega\approx 17.1$, 20.5 MeV for the $1p_{3/2}\rightarrow(2s_{1/2}1d_{5/2})$ and $1s_{1/2}\rightarrow 1p_{1/2}$ transitions in $C^{13}$. The net change from $C^{12}$ is that these single-particle transition energies move closer together in $C^{13}$, which seems reasonable if their exceptional separation in $C^{12}$ arises from subshell closing.

Lumping the two components of the main peak together yields $\bar{W}=25.0$ MeV. We use this as $E_+$ for the upper half of a double peak consisting of the main and pygmy resonances. For the pygmy resonance,

$$\begin{aligned}V_-=&\tfrac{5}{6}[F^0(1p1d,1p1d)-(1/45)(1-4R)F^1(1p1d,1d1p)\\&-(3/35)(1+R)F^3(1p1d,1d1p)]\\&+(2\sqrt{5}/6)(\sqrt{2}/9)(1-R)F^1(1p1d,2s1p)\\&+\tfrac{1}{6}[F^0(1p2s,1p2s)-\tfrac{1}{9}(1+2R)F^1(1p2s,2s1p)]\\&=0.088V_0\approx 5.3\text{ MeV},\end{aligned}\tag{19}$$

using the Slater integrals from $C^{12}$ and $V_0\approx 60$ MeV. The single-particle energy difference $(d_{5/2}-p_{1/2})$ is about 3.7 MeV for $A=13$; and the spin-orbit splitting for $O^{16}$ is $d_{3/2}-d_{5/2}\approx 5.0$ MeV. In the discussion of $C^{12}$ it appeared that for the excited state *both* $p_{1/2}$ and $d$ levels should be lowered by about the same amount; accordingly, for $p_{1/2}\rightarrow d_{3/2}$ we take $\hbar\omega\approx 8.7$ MeV, and $E_-=8.7+5.3=14.0$ MeV.

The interaction potential matrix element is

$$\begin{aligned}V=&(32/35)^{1/2}\langle[(\tfrac{5}{6})^{1/2}q_{1d}+(\tfrac{1}{6})^{1/2}q_{2s}]P_{3/2}|\mathcal{V}|P_{1/2}\\&\times[(\tfrac{5}{6})^{1/2}q_{1d}+(\tfrac{1}{6})^{1/2}q_{2s}]\rangle-(3/35)^{1/2}\langle[(\tfrac{5}{6})^{1/2}q_{1d}\\&+(\tfrac{1}{6})^{1/2}q_{2s}]P_{3/2}|\mathcal{V}|P_{1/2}q_{1s}\rangle.\end{aligned}\tag{20}$$

The second term of Eq. (20) is already available from $C^{12}$, and the first is calculated in a similar way. For $\chi=2.0$, one obtains

$$V=0.027V_0\approx 1.6\text{ MeV}.\tag{21}$$

The original separation is $D_0=5.5$ MeV, whence $\tan\beta\approx 0.3$, and the original intensity ratio is $r_0=4/35\approx 0.1$. From these values one obtains $D=6.0$ MeV, or $W_-$, $W_+=13.5$, 25.5 MeV and $r=0.27$, in good agreement with observation.[25]

## VI. DISCUSSION

The i.h.o. model used above implies that the entire $E1$ sum rule is contained in the giant resonance. This is not true, especially for light nuclei; some of the cross section is pushed up to higher energies by quasi-deuteron effects (two-particle correlations) and by the influence of overtones[15] or 3 quantum jumps. There is no reason to expect these modifications to act differently for different peak components, however, so that present considerations should on the average be valid for giant resonances of real nuclei, as long as only intensity ratios are considered and not absolute intensities. For nuclei as light as $C^{12}$ to $O^{16}$ the first overtone occurs at $W_3\sim 50$ MeV, so there is no danger of confusion with the peak splitting examined here.

The absolute potential magnitude $V_0$ determined here seems to agree with the formulas of reference 22: taking $w=1.7\pm 0.2$,

$$-4(\mathfrak{M}+\tfrac{1}{2}\mathfrak{H})_0\approx 80\pm 5\text{ MeV}.\tag{22}$$

These formulas assumed $\chi=1$, whereas we have found $V_0$ $(\chi=2.0)=60$ MeV, $V_0$ $(\chi=1.65)\approx 65$ MeV: By linear extrapolation $V_0$ $(\chi=1.0)\approx 75$ MeV. It is not clear to what extent the factor $\chi>1$ is intrinsic to the two-nucleon interaction and to what extent it reflects the collective influence of surrounding nucleon matter. The latter effect must certainly be present and will cause fluctuations in $\chi$ regarded as a phenomenological parameter.

*Note added in proof.* The appropriate value of $\chi$ may be somewhat smaller than indicated in the text, say, $\chi\approx 1.7\pm 0.2$. Recent measurements on $O^{16}$, L. N. Bolen and W. D. Whitehead [Phys. Rev. Letters **9**, 458 (1962)] indicate $r\approx 1$, for which $\chi\approx 1.4$; and for $C^{12}$ the high-energy component may occur at a separation[18,21] of only 3 MeV above the main peak, corresponding to $\chi\approx 1.8$.

The foregoing calculations appear to have some general features:

(i) The energy splitting of the $E1$ peaks mainly corresponds to energy differences of single-particle levels, when due allowance is made for reduction of orbital differences in the excited state.

(ii) Relative peak intensities bear little relation to this naive model and are strongly affected by the two-body interaction $\mathcal{V}$, which generally tends to equalize component intensities.

(iii) Collective deformation of the nuclear ground state plays no direct role in peak splitting. The enhancement factor $\chi>1$ is physically significant for (ii), but such enhancement is a cause rather than a result of nuclear ground state deformation.

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[25] It is of interest to note that a recent calculation [F. C. Barker, Nucl. Phys. **28**, 96 (1961)] obtains agreement with the pygmy resonance in $C^{13}$ by a procedure equivalent to the mixing of core $(p_{3/2})$ and external $(p_{1/2})$ particle excitations. Such mixing also occurs in the deformed-model calculations of V. G. Neudachin and V. N. Orlin [Nucl. Phys. **31**, 338 (1962)] for $C^{12}$, $C^{13}$, as well as $Li^7$, $Mg^{24}$, and $Mg^{25}$, with neglect of nondiagonal matrix elements like $V$.

Statement (i) is that the differences between $\hbar\omega_\pm$ are of predominant significance for determining $W_\pm$, while $V_+ \approx V_-$ is relatively constant at $\sim 7$ MeV, and the interaction energy $V \sim 1$–2 MeV is quite small. Statement (ii) is the well-known remark that wave functions are much more sensitive to potential parameters than energy eigenvalues. Finally, statement (iii) runs counter to certain current beliefs and is discussed further below.

## VII. HEAVIER NUCLEI

The labor of these calculations increases as a fairly high power of $A$. We, thus, try to extrapolate to heavier nuclei from the cases considered, regarding them as respective prototypes for the closed shell ($O^{16}$), the closed shell minus a few particles ($C^{12}$), and the closed shell plus one or several nucleons ($C^{13}$).

(a) The closed shell. The number of major components in the peak is determined by the $l$ subshells of the excited state and is equal to $N$, the i.h.o. number for the ground state. In the limit of large $A$, $N \approx (3A/2)^{1/3}$. In the i.h.o. or any other single-particle model the energy scale varies as $\hbar\omega \sim A^{-1/3}$; thus the total width of the pattern varies as $N(\hbar\omega) \sim$const, although the pattern itself becomes increasingly complex. For even medium weight nuclei resolution will become impossible, and this structure will constitute the "intrinsic width" of an $E1$ peak. To judge from the $O^{16}$ case this intrinsic width should be of order $\Gamma_0 \sim 3$ MeV, which is not in disagreement with the data.

(b) The hole states in a closed shell have higher $E1$ excitation energies than the particles; thus a few holes in a closed shell give rise to a minor peak above the main one.

(c) Conversely, a spare particle or two outside a closed core has an especially low binding and, therefore, excitation energy and gives rise to a low-energy satellite. The energy splitting in both (b) and (c) can be several times $\Gamma_0$ and, hence, may be resolvable even with present techniques of measurement.

Some evidence for satellite peaks of type (c) may perhaps be discerned around[26,27] $A = 90$ and[28] $A = 209$. Because of experimental difficulties and the interference of the $W_3$ overtone for heavy nuclei,[15] good examples of (b) are hard to obtain except in light nuclei.

These remarks apply to heavy nuclei in the immediate neighborhood of closed shells. What do the present considerations suggest about intermediate regions containing "highly deformed" nuclei? We consider only nuclei with $J_0 = 0^+$ in the ground state, for experiment shows[29] that adjacent nuclei in the Ho region have the same $E1$ peak pattern for $J_0 = 0$ and $J_0 = \frac{7}{2}$. This seems reasonable, since the giant resonance must be a function of many nucleons acting collectively, to which the contribution of the last odd nucleon should be relatively slight. The higher excited collective states of a deformed nucleus will include odd parity, $K = 0$ bands and odd parity, $K = 1$ bands. Let the $J = 1$ states of two such bands concentrate most of the $E1$ matrix element from the ground state; these two states then provide a suitable basis for the decomposition of Eq. (2) by $Q_{K=0}$ and $Q_{K=1}$. Because $K \neq 0$ bands are of double weight relative to $K = 0$ bands, $r_0 = 0.5$ for $E_0 < E_1$ ("prolate") or $r_0 = 2$ for $E_1 < E_0$ ("oblate"). There is no obvious reason for much deviation of $V_+$ and $V_-$ from the constant value of $\Delta \approx 7.5$ MeV, so that the naive peak separation $D_0$ is just that between $\hbar\omega_0$ and $\hbar\omega_1$. Furthermore, the $K = 0$ and $K = 1$ bands are eigenstates of the nuclear forces, and the states absorbing most of the $E1$ matrix elements will be eigenstates of the charge exchange forces, in particular; thus, to first approximation, the off-diagonal potential element $V = (Q_\pm | \mathcal{V} | Q_\mp) = 0$. The results then agree with the naive model:

$$r = r_0 = 0.5 \text{ or } 2, \quad |D| = |D_0| = \hbar|\omega_0 - \omega_1|. \tag{23}$$

The magnitude of $|D|$ is presumably a measure of the "deformability" of the nucleus, although its exact equivalence to measures based on low-lying states is not obvious.

The conclusions in Eq. (23) do not add anything to those obtained from the classical hydrodynamic model. It is, perhaps, nonetheless worthwhile to remark the difference in point of view: Here the peak splitting is regarded as a property of the excited state, which divides between $K = 0$ and $K = 1$ collective bands in the deformed nucleus. To do the calculation explicitly would require an operator for generating these $K = 0$ and $K = 1$ states from an independent particle model; but for heavy nuclei this procedure is not yet possible even for the low-lying $0^+$, $2^+$, $\cdots$ collective states.[30] Without knowing the details of such explicit operators, one cannot insist on a perfect correspondence between collective properties in the excited $E1$ states and those in the ground and low-lying states—although, of course, one expects great similarity.

This emphasis upon the collective properties of the $E1$ excited state rather than those of the ground state provides relief from a couple of minor embarrassments. If $E1$ peak splitting were a function ground-state deformation alone, peak splitting for $J_0 = 0$ nuclei would not occur *in principle*, in contradiction with experiment; for a basic postulate of quantum mechanics is that for a system with $J < 1$ no tensor quantity (deformation) can have any meaning nor lead to any observable consequences. Again, the existence of tensor contributions to $\gamma - \gamma$ scattering by nuclei[29] depends primarily on the tensor relations between the low-lying

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initial and final states of the nucleus and does not directly reflect the properties of the summed-over intermediate states that constitute the $E1$ giant resonance. The literal connections among all these phenomena arising in a deformed i.h.o. model simply indicate that the model is too simplified (it expresses deformation by one parameter only) for realistic application to nuclei.

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# Positron Decay of $Y^{88}$†*

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The weak positron spectrum in the decay of $Y^{88}$ has been investigated using a $4\pi$-positron-scintillation spectrometer. Two measurements of the spectrum yield experimental shape factors which are consistent above 250 keV with that expected for a unique once-forbidden transition. The average end-point energy obtained in these two measurements is 761±9 keV. The positron branching was measured and found to be 0.20±0.01%. The log$ft$ is 9.4, and log$f_1t$ is 8.7. On the basis of recent measurements of the gamma-ray intensities in the $Y^{88}$ decay the electron capture branching to the 1840-keV level is determined to be 5.8±0.7%, which yields an electron capture to positron ratio of 29±4 for this transition.

## I. INTRODUCTION

ACCORDING to the presently accepted decay scheme[1] (See Fig. 1.), $Y^{88}$ decays primarily by an electron-capture transition to the 2740-keV second excited state of $Sr^{88}$. The 900-, 1840-keV gamma cascade arising in the de-excitation of this level is the most prominent feature in the decay. The existence, also, of a very weak positron group populating the 1840-keV first excited state of $Sr^{88}$ has been known since at least as early as 1948. The results of magnetic spectrometer measurements by Peacock and Jones,[2] reported at that time, indicate that the intensity of the positron group is 0.19±0.04% of the total decay and that the end-point energy is 830±20 keV. Though the Fermi-Kurie plot constructed from their experimental positron distribution appears, roughly, to be linear above about 200 keV, they made no specific assertion about the shape. A positron distribution arising from internal-pair de-excitation of the 1840-keV state in $Sr^{88}$ (populated in more than 90% of the decays) can be expected in the energy interval up to about 820 keV. Peacock and Jones were aware of this possibility but asserted that the effect did not cause appreciable distortion in their measurements. Later, Stirling and Goldberg[3] remeasured the $Y^{88}$ positron spectrum using a double-focusing spectrometer. They reported that their gross experimental positron distribution was not a pure positron group, and concluded that it could be reasonably interpreted as a superposition of a positron spectrum having an end-point energy around 580 keV and a positron distribution from the internal-pair de-excitation of the 1840-keV level in $Sr^{88}$. More recently, Ramaswamy and Jastram[4] have measured the $Y^{88}$ positron spectrum using a $4\pi$ scintillation spectrometer, gating on the an-

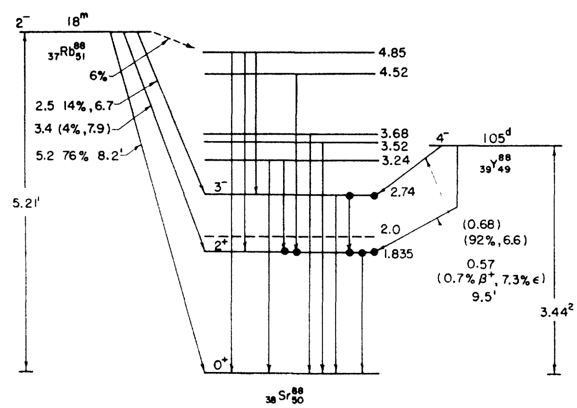


FIG. 1. $Y^{88}$ decay scheme as given in the Nuclear Data Sheets, 1960. The values for the log $f_1t$ and branching percentage of the first-excited-state transition are inconsistent.

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