PHYSICAL REVIEW VOLUME 129, NUMBER 2 5 JANUARY 1963

Microwave Emission from Non-Maxwellian Plasmas*

H. Fields, G. Bekefi, and Sanborn C. Brown
Research Laboratory of Electronics and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 22 August 1962)

Microwave measurements of the radiation from a weakly ionized low-energy plasma immersed in a magnetic field show significant departures from the Kirchhoff-Planck law. The departures can be explained by taking account of the non-Maxwellian distribution of the radiating electrons. By comparing the measured and calculated radiation temperatures we estimate the distribution of electron velocities and their mean energy.

I. INTRODUCTION

THE radiation spectra emitted by plasmas at radio and microwave frequencies can be grouped into three broad classes. The first class includes spectra that can be interpreted on the basis of the classical theory of thermal radiation. The radiation originates predominantly from the motions of the free electrons in the field of ions and atoms (bremsstrahlung), and from the orbital motion of free electrons in externally applied magnetic fields (cyclotron radiation). The electrons are assumed to have a Maxwellian distribution of velocities and the classical radiation laws and concepts are accepted as valid. The radiation intensity cannot exceed that of a black body with a temperature equal to that of the free electrons.

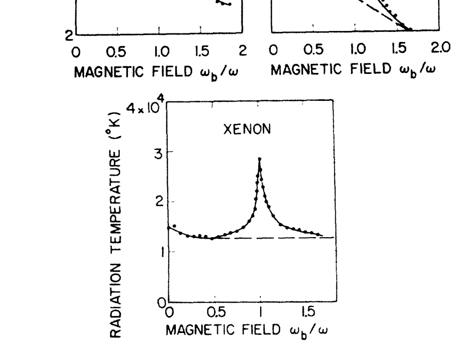

FIG. 1. The radiation temperature as a function of magnetic field measured in neon and argon (discharge current = 10 mA, gas pressure = 0.28 mm-Hg), and xenon (discharge current = 20 mA, gas pressure = 5.9 mm-Hg).

* This work was supported in part by the U. S. Army, the Air Force Office of Scientific Research, and the Office of Naval Research; and in part by the U. S. Atomic Energy Commission [AT(30-1)-1842]; and in part by the Air Force Command and Control Development Division under Contract AF 19(604)-5992.

The second class of radiation is thought to originate from cooperative motions of the free charges induced by perturbations of the plasma (plasma oscillations). The radiation intensity often exceeds blackbody intensity by many orders of magnitude. The interpretation of these emission spectra is at best semiquantitative.

In the third class we include radiation not due to cooperative motions, but where nonthermal spectra arise from departures of the velocity distribution of the radiating electrons from a Maxwellian distribution. Here, too, the radiation can differ greatly from that which would be observed in a Maxwellian plasma with electrons of the same average energy. The concept of blackbody radiation is not applicable. In this paper, we are concerned only with radiation that we have grouped in this class.

On the basis of the classical theory of thermal radiation, the emission of electromagnetic waves from the plasma is characterized by the temperature T of the free electrons that participate in the emission and absorption processes. The essence of the theory is the Kirchhoff-Planck law which can be stated as follows: In a given volume element of plasma, the ratio of the emission coefficient j_ω to the absorption coefficient α_ω, for radiation in the angular frequency interval between ω and $\omega+d\omega$, is

$$j_\omega/\alpha_\omega = B(\omega,T,n). \tag{1}$$

The quantity B represents the intensity of equilibrium, blackbody radiation in the volume element considered and is only a function of the frequency ω, the local temperature T, and the refractive index n of the plasma. A measurement of $B(\omega,T,n)$ is a means of finding the electron temperature.

A cylindrical positive column of a glow discharge was subjected to an axial magnetic field and the apparent electron temperature was deduced from microwave radiation measurements. Figure 1 illustrates the results that were obtained in neon, argon, and xenon glow discharges. The apparent temperature is plotted as a function of the magnetic field, at a fixed frequency of observation ω equal to $6\pi\times10^9$ rad/sec; ($\omega_b = eB/m$ is the electron cyclotron frequency, where B is the strength of the magnetic field). We note the large peaks in the apparent temperature at the electron cyclotron frequency, $\omega=\omega_b$. These peaks must not be confused with

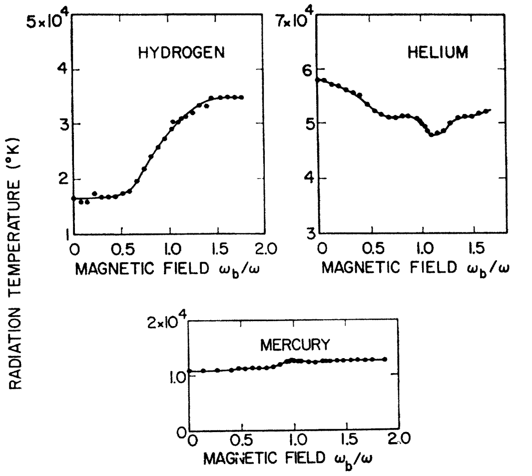

FIG. 2. The radiation temperature as a function of magnetic field measured in hydrogen and helium (discharge current = 10 mA, gas pressure = 0.28 mm-Hg), and mercury (discharge current = 60 mA, gas pressure = 0.088 mm-Hg).

the enhanced emission, j_ω, caused by cyclotron radiation, which in these experiments would be exactly balanced by the enhanced cyclotron absorption, α_ω, in a plasma with a Maxwellian distribution of electron velocities. In contrast to Fig. 1, Fig. 2 shows the same measurements in discharges produced in hydrogen, helium, and mercury. We now observe the absence of peaks in the apparent temperature.

The foregoing observations are consistent with calculations of j_ω/α_ω for a lowly ionized, low-energy plasma with a non-Maxwellian distribution of electron velocities. The difference in the observations shown in Figs. 1 and 2 is explained on the basis of the energy dependence of the electron-neutral collision cross sections.

II. THEORY

A. Calculations of the Emission and Absorption Coefficients

Our calculations and experiments are concerned with plasmas whose refractive index is close to or equal to unity, so that the interpretation of the measurements is not subject to difficulties caused by reflections from plasma boundaries. Also, rather simple concepts of the quantum-mechanical theory of radiation can be used in deriving the emission and absorption coefficients.

In the volume element of plasma surrounding point $\mathbf{r}$, we choose a group of electrons of density $dN(\mathbf{v}')$ that have velocities between $\mathbf{v}'$ and $\mathbf{v}'+d\mathbf{v}'$; and a second group of electrons $dN(\mathbf{v})$, with velocities between $\mathbf{v}$ and $\mathbf{v}+d\mathbf{v}$. The energy of the electrons labeled with a prime is greater than that of the unlabeled electrons.

As a result of a radiative interaction of an electron, a photon of frequency ω is either emitted or absorbed. The frequency of the photon is given by the Bohr frequency condition,

$$\hbar\omega=\tfrac{1}{2}mv'^2-\tfrac{1}{2}mv^2, \tag{2}$$

where $2\pi\hbar$ is Planck's constant h. The form of the Bohr condition implies that a free electron remains in a free state after completion of the emission or absorption process. Transitions of this kind are the main sources of radio and microwave emission from plasmas.

Three fundamental processes can take place when an electron in one state ($\mathbf{v}'$ or $\mathbf{v}$) goes to state ($\mathbf{v}$ or $\mathbf{v}'$). These processes are: spontaneous emission, absorption, and stimulated emission. The net absorption α_ω is the difference between the absorption and the stimulated emission.

Spontaneous emission goes on at a rate that is independent of the presence or absence of the radiation field. Let $\eta_\omega(\mathbf{v}',\mathbf{r},\mathbf{s})$ be the rate at which energy is emitted spontaneously per unit solid angle, in the direction $\mathbf{s}$ by one electron with a velocity in the range dv' (which then decays to the state v), thus giving rise to radiation in a unit frequency interval. The rate of emission by $dN(\mathbf{v}')$ electrons per unit volume of plasma is

$$\eta_\omega(\mathbf{v}',\mathbf{r},\mathbf{s})dN(\mathbf{v}'). \tag{3}$$

Writing Eq. (3) in terms of the steady-state distribution function of electron velocities f, we obtain for the rate of spontaneous emission,

$$\eta_\omega(\mathbf{v}',\mathbf{r},\mathbf{s})f(\mathbf{v}',\mathbf{r})d^3v', \tag{4}$$

where d^3v' is the volume element in velocity space.

Stimulated emission is proportional to the intensity of radiation I_ω at the point of the medium in question. Let $\eta_{\omega S}(\mathbf{v}',\mathbf{r},\mathbf{s})$ be the rate of emission by an electron $\mathbf{v}'$ per unit intensity of radiation present per unit solid angle per unit frequency interval. The total rate by $dN(\mathbf{v}')$ electrons per unit volume is

$$\eta_{\omega S}(\mathbf{v}',\mathbf{r},\mathbf{s})I_\omega(\mathbf{r},\mathbf{s})f(\mathbf{v}',\mathbf{r})d^3v'. \tag{5}$$

An electron in the lower state $\mathbf{v}$ reaches state $\mathbf{v}'$ after absorption of a photon of energy $\hbar\omega$. Let $\eta_{\omega A}(\mathbf{v},\mathbf{r},\mathbf{s})$ be the rate of absorption by this electron per unit intensity of radiation present. The rate of absorption by $dN(\mathbf{v})$ of these electrons is

$$\eta_{\omega A}(\mathbf{v},\mathbf{r},\mathbf{s})I_\omega(\mathbf{r},\mathbf{s})f(\mathbf{v},\mathbf{r})d^3v. \tag{6}$$

We can compute the emission and absorption coefficients, $j_\omega(\mathbf{r},\mathbf{s})$ and $\alpha_\omega(\mathbf{r},\mathbf{s})$, at a given point $\mathbf{r}$ of the plasma and in a given direction $\mathbf{s}$ by integrating Eqs. (4), (5), and (6) over all electron velocities and noting that α_ω is defined by the relation $dI_\omega=-\alpha_\omega I_\omega ds$. Omitting, for simplicity, the vectors $\mathbf{r}$, $\mathbf{s}$ from our notation, we obtain

$$j_\omega=\int\eta_\omega(\mathbf{v}')f(\mathbf{v}')d^3v', \tag{7}$$

and

$$\alpha_\omega=\int\eta_{\omega A}(\mathbf{v})f(\mathbf{v})d^3v-\int\eta_{\omega S}(\mathbf{v}')f(\mathbf{v}')d^3v', \tag{8}$$

where the magnitudes of $\mathbf{v}'$ and $\mathbf{v}$ are related through Eq. (2). Note that $d^3v'\neq d^3v$. In the special case of an isotropic distribution it follows from Eq. (2) that $d^3v'/d^3v = v'^2dv'/v^2dv = v'/v$.

The three emission and absorption rates η_ω, $\eta_{\omega S}$, and $\eta_{\omega A}$ bear definite relationships to one another. These relationships are calculated by assuming that the rate at which an electron emits or absorbs is independent of the state of the plasma; that is, they do not depend either on the intensity of radiation field, or on the distribution of electron velocities. This is a valid assumption in tenuous plasmas.

For the purpose of establishing relationships between the three rates, we may choose any state for the plasma. We enclose the whole medium in a large adiabatic shield of constant temperature T, and allow everything to come to thermodynamic equilibrium with the container. When the plasma is in this state, the following properties are true:

(a) The electrons have a Maxwellian distribution of velocities, given by

$$f(v) = N(m/2\pi kT)^{3/2}\exp(-mv^2/2kT), \tag{9}$$

where N is the electron concentration.

(b) The intensity of the radiation field is everywhere the same within the container and is given by the Planck radiation formula (for one polarization)

$$B(\omega,T) = (\hbar\omega^3/8\pi^3c^2)[\exp(\hbar\omega/kT)-1]^{-1}, \tag{10}$$

where c is the velocity of light in free space.

(c) For every pair of states, v' and v, in which the electrons find themselves, the rate of "emitting" transitions, $v' \rightarrow v$, is exactly balanced by the rate of "absorbing" transitions, $v \rightarrow v'$. This is the principle of detailed balance.

Using statements (a), (b), and (c), in conjunction with Eqs. (4), (5), and (6), we obtain

$$\eta_\omega(\mathbf{v}')e^{-mv'^2/2kT}d^3v' = B(\omega,T)[\eta_{\omega A}(\mathbf{v})e^{-mv^2/2kT}d^3v - \eta_{\omega S}(\mathbf{v}')e^{-mv'^2/2kT}d^3v']. \tag{11}$$

After substituting Eq. (2) in (11) and rearranging terms, we find that

$$B(\omega,T) = \frac{[\eta_\omega(\mathbf{v}')/\eta_{\omega S}(\mathbf{v}')]}{[\eta_{\omega A}(\mathbf{v})d^3(v)/\eta_{\omega S}(\mathbf{v}')d^3(v')]e^{\hbar\omega/kT}-1}. \tag{12}$$

Equations (10) and (12) must hold simultaneously at all frequencies. For this to be true, the following identities must be satisfied:

$$\eta_\omega(\mathbf{v}') = (\hbar\omega^3/8\pi^3c^2)\eta_{\omega S}(\mathbf{v}'), \tag{13}$$

and

$$\eta_{\omega A}(\mathbf{v})d^3v = \eta_{\omega S}(\mathbf{v}')d^3v'. \tag{14}$$

Equations (13) and (14) are equivalent to the well-known relations between the Einstein A and B coefficients,[1] but written in a form more appropriate in the calculation of the emission and absorption rates by free electrons.

The emission and absorption coefficients given by Eqs. (7) and (8) may be simplified, since any two of the three parameters η_ω, $\eta_{\omega S}$, $\eta_{\omega A}$ can be eliminated, with the aid of Eqs. (13) and (14). In terms of the rate of spontaneous emission,

$$j_\omega = \int \eta_\omega(\mathbf{v}')f(\mathbf{v}')d^3v', \tag{15}$$

and

$$\alpha_\omega = (8\pi^3c^2/\hbar\omega^3)\int \eta_\omega(\mathbf{v}')[f(\mathbf{v})-f(\mathbf{v}')]d^3v'. \tag{16}$$

It is useful to denote the ratio of the emission coefficient to the absorption coefficient, j_ω/α_ω, by a quantity S_ω called the source function. From Eqs. (15) and (16), we obtain

$$S_\omega = \frac{\hbar\omega^3}{8\pi^3c^2}\frac{\int \eta_\omega(\mathbf{v}')f(\mathbf{v}')d^3v'}{\int \eta_\omega(\mathbf{v}')[f(\mathbf{v})-f(\mathbf{v}')]d^3v'}. \tag{17}$$

When the distribution of electron velocities is Maxwellian, it can be shown that the source function S_ω equals the blackbody intensity $B(\omega,T)$, given by Eq. (10). Thus, in agreement with Eq. (1),

$$S_\omega \equiv j_\omega/\alpha_\omega = B(\omega,T). \tag{18}$$

Equation (18) is generally proved for systems that are in exact thermodynamic equilibrium. Here, we derived it under much less restrictive conditions. Since the parameters j_ω and α_ω are properties of the radiating electrons only, Eq. (18) should be true whenever the electrons have a Maxwellian distribution, and no other criteria need be imposed. Since Eq. (17) refers to an elementary volume element, it remains valid even if $f(v)$ and $\eta_\omega(v)$ vary from point to point in the plasma.

We now calculate the emission coefficient, the absorption coefficient, and the source function for radiation at low frequencies by taking Eqs. (15), (16), and (17) to the limit, $\hbar\omega \rightarrow 0$. We consider only isotropic distributions of electron velocities since in our plasmas the drift velocities of the electrons in the applied dc electric field are very small compared to their random velocities.

As the frequency decreases, the speed v' of the upper state approaches more and more closely the speed v of the lower state. We write that

$$v' = v + \Delta v, \tag{19}$$

where Δv, obtained from Eq. (2), is given by

$$\Delta v \approx \hbar\omega/mv. \tag{20}$$

We expand the distribution function in a Taylor series,

[1] A. Einstein, Physik. Z. **18**, 121 (1917); G. Cillie, Monthly Notices Ray Astron. Soc. **92**, 820 (1932); L. Oster, Laboratory of Marine Physics, Yale University, Technical Memorandum No. 73, April, 1961 (unpublished).

with the result that

$$f(v')=f(v)+\frac{\partial f}{\partial v}\Delta v+\cdots. \qquad (21)$$

Inserting these results in Eqs. (15), (16), and (17) gives

$$j_\omega=\int \eta_\omega(v)f(v)4\pi v^2dv, \qquad (22)$$

$$\alpha_\omega=-(8\pi^3c^2/\omega^2)\int \eta_\omega(v)\frac{\partial f(v)}{\partial U}4\pi v^2dv, \qquad (23)$$

and

$$S_\omega=\frac{\omega^2}{8\pi^3c^2}\left[-\frac{\int \eta_\omega(v)f(v)v^2dv}{\int \eta_\omega(v)[\partial f(v)/\partial U]v^2dv}\right]. \qquad (24)$$

Here $U=\frac{1}{2}mv^2$ is the kinetic energy of the electron.

The source function for each of the two polarizations of the radiation, given by Eq. (24), depends on the detailed behavior of the radiating processes taking place inside the plasma and on the direction of propagation of the radiation. The one exception arises for a plasma with a Maxwellian distribution, where Eq. (24) reduces to the simple result,

$$S_\omega\equiv B(\omega,T)=(\omega^2/8\pi^3c^2)(kT), \qquad (25)$$

which is the Rayleigh-Jeans limit, $\hbar\omega\ll kT$, of the Kirchhoff-Planck equations (10) and (18).

We may write the source function of Eq. (24) symbolically as

$$S_\omega=(\omega^2/8\pi^3c^2)(kT_r), \qquad (26)$$

and call T_r the radiation temperature. This is the quantity to which we refer in our measurements shown in Figs. 1 and 2. Unless the distribution function is Maxwellian, T_r is a fictitious temperature, and is merely a convenient way of describing the radiation field. Generally, T_r is a function of frequency ω, the radiation process or processes η_ω, the distribution function f, and the direction of propagation of the radiation.

The source function given by Eq. (26) and the radiation temperature are closely related to the intensity of radiation I_ω that emanates from a plasma. If we assume that T_r is everywhere the same within the plasma, and sum the emissions and absorptions at every point of a ray, we obtain

$$I_\omega=(j_\omega/\alpha_\omega)[1-\exp(-\tau)]. \qquad (27)$$

Here τ is the optical thickness given by

$$\tau=\int_0^L \alpha_\omega ds, \qquad (28)$$

where ds is an element of length along the path of the ray and L is the total length that the ray traversed in the plasma. Substituting Eq. (26) in Eq. (27), we obtain

$$I_\omega=(\omega^2/8\pi^3c^2)kT_r[1-\exp(-\tau)]. \qquad (29)$$

When the distribution function is non-Maxwellian, the intensity I_ω can greatly exceed the blackbody radiation $B(\omega,T)$ of a plasma with electrons of the same mean energy. Indeed, calculations[2] suggest that, for certain emission processes and for certain distribution functions, the absorption coefficient α_ω can be negative (stimulated emission exceeds absorption), in which case the radiation amplifies in passing through the plasma. In our experiments a negative absorption has not occurred; however, an increase in the stimulated emission in excess of what it would have been in a Maxwellian plasma has been observed. This is basically the origin of the peaks shown in Fig. 1.

A classical approach to the calculation of the radiation temperature, through the use of Maxwell's equations and the Boltzmann equation, leads to the same result as given above.[3] An alternate treatment of radiation phenomena to that discussed in this paper is one in which the noise generated by the plasma is ascribed to electron current fluctuations. This concept leads to a Nyquist theorem for plasmas[4] and relates the time-averaged mean-square fluctuations to the plasma conductivity and the electron temperature. By extending the Nyquist theorem to plasmas with non-Maxwellian distributions, Plantinga[5] derived the same expression for the radiation temperature as we do here, except that he restricted his calculations to bremsstrahlung in the absence of magnetic fields. Bunkin[6] derives a Nyquist theorem valid for dense plasmas both in the presence and absence of magnetic fields. His result for the radiation temperature reduces to ours in the limit of a plasma with a refractive index close to unity. Calculations of the tensor conductivity elements of a magnetized plasma, from which the absorption coefficient can be calculated, are given by Harris,[7] Weibel[8] and Sagdeev and Shafranov.[9] These calculations reduce to our absorption coefficient given by Eq. (23), in the limit of tenuous plasmas, with isotropic distributions.

[2] R. Q. Twiss, Australian J. Phys. **11**, 564 (1958); G. Bekefi, J. L. Hirshfield, and S. C. Brown, Phys. Fluids **4**, 173 (1961); Phys. Rev. **122**, 1037 (1961); J. Schneider, Z. Naturforsch. **15a**, 484 (1960); Phys. Rev. Letters **2**, 504 (1959).

[3] S. C. Brown and G. Bekefi, "Conference on Plasma Physics and Controlled Nuclear Fusion Research, Salzburg, September 1961," Paper No. CN-10/171 [J. Nucl. Fusion (to be published)].

[4] P. Parzen and L. Goldstein, Phys. Rev. **82**, 724 (1951); S. M. Rytov, *Theory of Electrical Fluctuations and Thermal Radiation* (U.S.S.R. Academy of Sciences, Moscow, 1953) [translation: Air Force Cambridge Research Center, Bedford, Massachusetts, Report. No. AFCRC-TR-59-162, July, 1959 (unpublished)]; H. A. Haus, J. Appl. Phys. **32**, 493 (1961).

[5] G. H. Plantinga, Philips Research Repts. **16**, 462 (1961).

[6] F. V. Bunkin, Soviet Phys.—JETP **14**, 206 (1962).

[7] E. G. Harris, Phys. Rev. Letters **2**, 34 (1959); also Oak Ridge National Laboratory Report, Oak Ridge, Tennessee and University of Tennessee, Knoxville, Tennessee (unpublished).

[8] E. S. Weibel, Phys. Rev. Letters **2**, 83 (1959).

[9] R. Z. Sagdeev and V. D. Shafranov, Soviet Phys.—JETP **12**, 130 (1961).

B. Derivation of the Radiation Temperature for Cyclotron Emission

We now consider the special case when cyclotron emission is the major mechanism for radiation. The rate of energy radiation by a nonrelativistic electron, orbiting in a magnetic field of intensity B, is given by,[10]

$$\eta(v_\perp)=\frac{e^2\omega_b{}^2v_\perp{}^2(1+\cos^2\theta)}{32\pi^2\epsilon_0c^3}. \tag{30}$$

The radiation appears at a frequency ω equal to the electron cyclotron frequency ω_b; $v_\perp$ is the speed of the electron in a direction perpendicular to the direction of the magnetic field; ϵ_0, the free-space permittivity, and θ is the angle between the direction of propagation of the radiation and the magnetic field.

When collisions of electrons with other particles are the only mechanism for broadening the cyclotron emission line (we neglect possible broadening due to the thermal motion of the electron) we obtain for frequencies at and close to ω_b

$$\eta_\omega(v_\perp)d\omega=\frac{e^2\omega_b{}^2v_\perp{}^2(1+\cos^2\theta)}{32\pi^3\epsilon_0c^3}\frac{\nu(v)d\omega}{\nu^2(v)+(\omega-\omega_b)^2}, \tag{31}$$

where $\nu(v)$ is the rate of collisions of electrons with molecules and is generally a function of the electron speed. Calculations, employing Boltzmann's equation and Maxwell's equations, show[3] that $\nu(v)$ is the collision frequency for momentum transfer. Note that when ν is independent of v, the line broadening shown in Eq. (31) is that originally given by Lorentz.

Making use of Eqs. (24), (26), and (31), we obtain the radiation temperature for our plasma

$$kT_r=-\frac{\displaystyle\int\frac{\nu(v)}{\nu^2(v)+(\omega-\omega_b)^2}f(v)v^4dv}{\displaystyle\int\frac{\nu(v)}{\nu^2(v)+(\omega-\omega_b)^2}\frac{\partial f(v)}{\partial U}v^4dv}. \tag{32}$$

The radiation temperature is seen to be independent of the electron density and density gradients. This is of particular value in our experiments where radial gradients in density do exist. It is noteworthy that, in the determination of T_r, the absolute value of $\nu(v)$ is not required, only its variation with velocity need be known.

The angular dependence of the emission rate does not appear in Eq. (32). This and the invariance with electron density come from our assumption of a tenuous plasma. For radiation from a dense medium $\eta_\omega(v)$ has a more complicated dependence on direction and, in this case, T_r would be a function of the direction of observation, relative to the magnetic field and of the electron density.

The radiation temperature given by Eq. (32) is for the polarization that would generally be called the extraordinary ray,[11] which is characterized by a resonant denominator in the expression for $\eta_\omega(v)$. The other polarization does not exhibit a resonant denominator, and therefore this ray does not contribute appreciably to the total radiation temperature in the neighborhood of $\omega=\omega_b$.

Equation (32) is also applicable[3] to radiation in the absence of a magnetic field ($\omega_b=0$), when the emission η_ω results from electron-neutral collisions. The radiation temperature takes on the same values as in the presence of a magnetic field but it does so at a different frequency of observation. For example: In the presence of a magnetic field the value of T_r at $\omega=\omega_b$ is the same as the value of T_r at $\omega\to 0$, in the absence of a magnetic field. In the presence of a magnetic field any significant variations in T_r that may occur in the narrow frequency range between $\omega=\omega_b$ and $|\omega-\omega_b|\approx\nu$, would in the absence of a magnetic field be spread out over a much larger frequency range from $\omega\ll\nu$ to $\omega\gg\nu$. The large variations shown in Fig. 1 in the presence of a magnetic field would not be seen in its absence, unless measurements were made over a very large frequency range.

C. Evaluations of T_r and Comparison with Measurements

The evaluation of the radiation temperature T_r of Eq. (32) presupposes a knowledge of the distribution function $f(v)$ and the collision frequency $\nu(v)$. In calculating T_r we made use of the known results of the collision cross sections for neon,[12] argon,[13] hydrogen,[14] helium,[15] and mercury,[16] from which $\nu(v)$ is derived.

Since the distribution functions in the various discharges we have studied were not known, we chose an arbitrary distribution function with variable parameters. In the quantitative comparisons between experiments and theory, given later, these parameters are adjusted for a best fit between calculations and measurements, enabling us to estimate the distribution function in the discharges studied.

We chose a distribution function of the form,

$$f(v)\propto\exp(-bv^l), \tag{33}$$

[10] H. Rosner, Rept. Republic Aviation Corporation Report, AFSWC-TR-58-47, Farmingdale, Long Island, New York, 1958 (unpublished).

[11] V. L. Ginzburg, *Propagation of Electromagnetic Waves in Plasma* (Gordon and Breach Science Publishers, New York, 1962).

[12] R. B. Brode, Rev. Mod. Phys. **5**, 257 (1933). A. L. Gilardini and S. C. Brown, Phys. Rev. **105**, 31 (1957).

[13] C. Ramsauer and R. Kollath, Ann. Physik **12**, 529 (1932); J. C. Bowe, Phys. Rev. **117**, 1416 (1960); also R. Brode, reference 12.

[14] R. W. Crompton and D. J. Sutton, Proc. Roy. Soc. (London) **A215**, 467 (1952); G. Bekefi and S. C. Brown, Phys. Rev. **112**, 159 (1958); also R. Brode, reference 12.

[15] L. Gould and S. C. Brown, Phys. Rev. **95**, 897 (1954); also R. Brode, reference 12.

[16] H. Margenau and F. P. Adler, Phys. Rev. **79**, 970 (1950); C. W. McCutchen, *ibid.* **112**, 1848 (1958); also R. Brode, reference 12.

where b and l are arbitrary positive parameters. The distribution function is normalized such that

$$\int_0^\infty f(v)4\pi v^2 dv = N. \tag{34}$$

The mean energy $\bar{U}$ is given by

$$\bar{U} = N^{-1}\int_0^\infty \tfrac{1}{2}mv^2 f(v)4\pi v^2 dv. \tag{35}$$

When $l=2$, f is a Maxwellian distribution; when $l=4$, f is a Druyvesteyn distribution.

If the applied magnetic field is parallel to the applied electric field and if inelastic collisions could be neglected, the first order, spherically symmetric distribution function of the form given above is obtained.[17] We assume $f(v)$ independent of magnetic field over a range of magnetic fields around $\omega=\omega_b$, where the peaks in the apparent electron temperature have been observed. Since, in the discharges studied, inelastic collisions cannot be neglected, the choice of the distribution function given by Eq. (33) is at best a crude trial function.

The collision frequency has a complicated energy dependence, and the expression for the radiation temperature [Eq. (32)] was evaluated on a digital computer for various combinations of the distribution parameter l and the mean energy $\bar{U}$. Figure 3(a) shows the results of calculations for hydrogen, for one value of the distribution parameter, $l=4$, and for various electron energies $\bar{U}$. We observe that for electrons of low energy ($\bar{U}=1$ eV) the radiation temperature is a maximum at the electron cyclotron frequency, $\Delta=0$, on the abscissa of

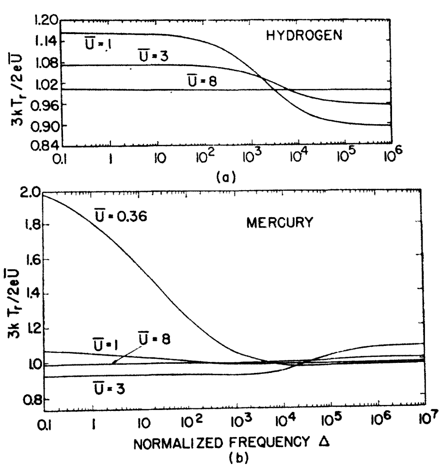

FIG. 3. The radiation temperature calculated for (a) hydrogen and (b) mercury as a function of the normalized frequency, Δ, defined by Eq. (36). The distribution parameter l defined by Eq. (33) equals 4.

[17] W. P. Allis, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 21, pp. 383–444.

Fig. 3. The parameter Δ is defined as

$$\begin{aligned}\Delta &= |\omega-\omega_b|^2/[10^2 p_0(2e/m)^{1/2}]^2 \\ &= 2.84\times10^{-16}|(\omega-\omega_b)/p_0|^2,\end{aligned} \tag{36}$$

where p_0 is the gas pressure in mm Hg normalized to zero degrees centigrade. For values of $|\omega-\omega_b|\gg 0\,(\Delta\gg 0)$, the radiation temperature drops off and reaches an asymptotic value. We show only one-half of the spectral distribution of T_r since the curve is symmetrical about $\omega=\omega_b$. Thus, T_r exhibits a peak at a frequency equal to the electron cyclotron frequency.

As the mean electron energy increases, the peak becomes smaller and vanishes for an electron energy $\bar{U}=8$ eV. A peak occurs whenever the collision frequency is an increasing function of velocity, in the range where the distribution function contributes most to the integrands of Eq. (32). This contribution is predominant in the neighborhood of electron energies equal to the mean electron energy. On the other hand, no peaks occur when the mean electron energy falls in the energy range where the collision frequency is essentially independent of velocity.

Calculations of T_r in mercury are shown in Fig. 3(b). At low electron energies the collision frequency is a very steeply rising function of velocity and for that reason the peak in T_r greatly exceeds that computed in hydrogen. We compute a dip in T_r for an electron energy of 3 eV. In this energy range the collision frequency is a steeply decreasing function of velocity and in such cases dips rather than peaks can arise.

Calculations for other gases show similar characteristics. If most electrons of a plasma are situated in an energy range in which the collision frequency is an increasing function of velocity, the radiation temperature exhibits a peak; if most electrons are in an energy range where $\nu(v)$ is a decreasing function of velocity, a dip can be expected. When the collision frequency is essentially independent of velocity, in the energy range in question, T_r shows no frequency dependence, which can be deduced directly from Eq. (32) by setting $\nu(v)$ equal to a constant.

The calculations for argon are shown in Figs. 4(a) and 4(b). Figure 4(a) shows the radiation temperature at $\omega=\omega_b$ as a function of the mean electron energy $\bar{U}$ for different values of the distribution parameter l. When the distribution function is Maxwellian ($l=2$) the radiation temperature is equal to the electron temperature defined by $T=2e\bar{U}/3k$. When $l=5$ the radiation temperature exceeds the value $2e\bar{U}/3k$ and is less than this value when $l=1$. ($l>2$ implies that there is an excess of slow particles and a deficiency of fast particles, as compared with a Maxwellian distribution, and vice versa for $l<2$.)

Figure 4(b) shows the ratio of the peak value of the radiation temperature in argon at $\omega=\omega_b$ to its asymptotic value at $\omega\to\infty$, as a function of the mean electron energy $\bar{U}$, for different values of the distribution

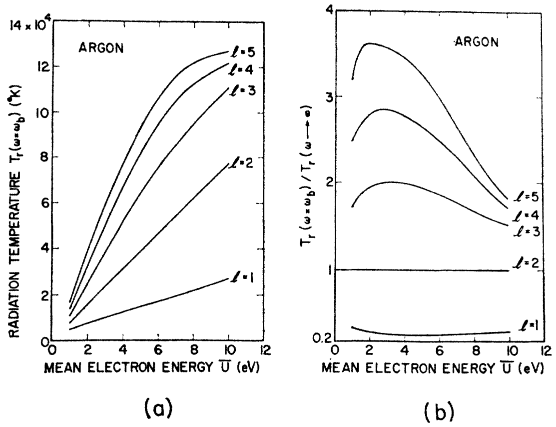

FIG. 4. (a) The radiation temperature at $\omega=\omega_b$, calculated as a function of the mean electron energy for various values of the distribution parameter l. (b) The ratio of the radiation temperature at $\omega=\omega_b$ to the radiation temperature at $\omega\to\infty$, calculated as a function of the mean electron energy, for various values of the distribution parameter l.

parameter l. This graph illustrates the magnitudes of the peaks above the background value of T_r, or of the dips below this background. We see that even for small departures from a Maxwellian distribution (e.g., $l=3$) the radiation temperature can be as large as twice that of a plasma with a Maxwellian distribution containing the same number of electrons with the same mean energy.

On the basis of these calculations, the experimental results shown in Figs. 1 and 2 can now be understood. The large peaks in the radiation temperature at cyclotron resonance in neon, argon, and xenon (Fig. 1) are due to non-Maxwellian electron distributions in gases with strongly increasing collision frequencies $\nu(v)$ at the operating energies of these plasmas (2 to 6 ev). The absence of peaks in T_r in hydrogen, helium, and mercury, as shown in Fig. 2, results from one of two effects. Either these plasmas maintain themselves at energies where the collision frequency is approximately invariant with energy [as noted before, theory shows that when $\nu(v)$ is constant, T_r is independent of the frequency or magnetic field, whatever the distribution function may be] or the absence of peaks is due to an electron distribution close to a Maxwellian. We cannot distinguish between one or the other possibility.

A physical explanation of the variations in T_r is the following. Consider the helical electron motion in the dc magnetic field. The motion is interrupted at random intervals by collisions with atoms. Between collisions, the electron motion is either in such a phase with the radiation field that it absorbs energy from it (absorption), or the phase of the motion is such that it relinquishes some of its energy to the field (stimulated emission). If the gas is one in which the electron-neutral collision frequency ν increases with energy, an electron losing energy will acquire a lower probability of collision. On the other hand, an electron gaining energy acquires a larger probability. Those electrons which find themselves in phase for stimulated emission remain in phase for this process longer (because of the lower ν) than those electrons which find themselves in phase for absorption. Since initially the probability that an electron be in phase for either process is the same, it is possible to increase stimulated emission relative to the absorption from electrons in a given velocity interval. The reverse is true if ν decreases with increasing speed.

The net absorption is the sum of all contributions from all velocity intervals of the distribution of electron speeds. A general comparison of the amount of absorption in a low-energy interval (where electrons can absorb relatively large amounts of energy due to their low ν, but have no energy to relinquish) with the amount of stimulated emission in a high-energy interval shows that for most distribution functions net absorption occurs even though ν increases with energy. Nevertheless, radiation intensities in excess of those from plasmas in thermal equilibrium can be obtained as confirmed by our experiments and calculations.

So far, we have only discussed the radiation temperature observed close to the cyclotron frequency. The general trends of a decreasing radiation temperature with increasing magnetic field, which forms the background for the peaks seen in the experimental curves of Fig. 1, are not explained on the basis of the present theory. They are, however, explained by the theory of the positive column, which states that the reduction of radial diffusion in the positive column with increasing magnetic field, enables the plasma to maintain the same current at a lower axial voltage. This leads to a decrease of the mean electron energy with increasing magnetic field. The rate of decrease of $\bar{U}$ with increasing magnetic field is smaller the higher the pressure and the higher the ionic mass. Thus, the slope of T_r, shown in Fig. 1, decreases as the mass of the ion increases.

The general trend of the radiation temperature described above is violated in hydrogen and helium, as seen in Fig. 2. Here we note that T_r increases with magnetic field over part of the range. The increase in T_r with magnetic field occurs at precisely the magnetic field at which the instability of Lehnert,[18] Kadomtsev and Nedospasov[19] sets in. This instability is characterized by an increase in the axial voltage at a critical magnetic field, which we observed to be accompanied by the rise in the radiation temperature. [In the particular measurements shown in Fig. 2 the onset of the instability occurred at 650 G for hydrogen and at approximately 1200 G for helium. In the remaining gases of Figs. 1 and 2 no instability was observed because it occurs outside the range of the available magnetic fields (2000 G).]

Various tests showed that the presence or absence of peaks was not associated with the above instability. For

[18] F. C. Hoh and B. Lehnert, Phys. Fluids 3, 600 (1960).
[19] B. B. Kadomtsev and A. V. Nedospasov, J. Nucl. Energy C1, 230 (1960).

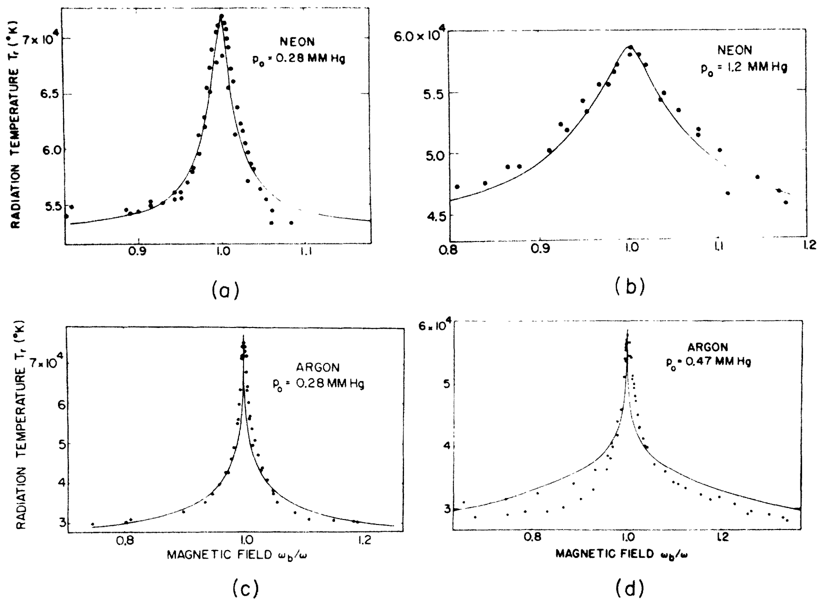

FIG. 5. The radiation temperature as a function of magnetic field. The points are experimental values for a current of 10 mA. The solid curves were computed for the following values of the distribution parameter l and the mean energy $\bar{U}$: curve (a): $l=3.4$; $\bar{U}=7.75$ eV; curve (b): $l=3.3$; $\bar{U}=6.35$ eV; curve (c): $l=4.13$; $\bar{U}=4.55$ eV; curve (d): $l=3.02$; $\bar{U}=4.52$ eV.

example, in helium, no peak was observed at $\omega=\omega_b$, whether the critical field for the onset of the instability was adjusted to fall at, below, or above $\omega=\omega_b$.

D. Determination of the Mean Energy and Velocity Distribution of Electrons in Neon and Argon

By assuming that the distribution functions $f(v)$ in the positive column of the dc discharges in neon and argon are of the form given by Eq. (33), we estimate $f(v)$ and $\bar{U}$ by the following procedure. We take the experimental value of the radiation temperature T_r at $\omega=\omega_b$ and T_r at $\omega\rightarrow\infty$ [obtained from the value of T_r at the dashed line directly below the resonance (see Fig. 1)] and transfer these values to Figs. 4(a) and 4(b). Thus, Fig. 4(a) gives us a set of values of l and $\bar{U}$. We get another such set from Fig. 4(b). By plotting $\bar{U}$ versus l for both sets of points, we obtain two curves that have one point of intersection. This point of intersection gives us the sought for values of $\bar{U}$ and l. Whether the distribution function given by Eq. (33), with the values of $\bar{U}$ and l obtained by the above method, is appropriate to the particular discharges studied, can now be determined by computing the complete spectrum of T_r and comparing it with experiment.

Figures 5(a) and 5(b) compare the calculated and measured values of the radiation temperature in neon. The results are given for two values of gas pressure, for magnetic fields in the immediate vicinity of $\omega_b=\omega$. Figures 5(c) and 5(d) present similar comparisons in argon discharges.

The experiments are seen to be in satisfactory agreement with the calculated values. We found that a small change in $\bar{U}$ and l from the values that give the best fit results in large disagreement between the calculated and measured values of the radiation temperature. This indicates that the method by which we estimate $\bar{U}$ and l is quite accurate. However, this does not mean that other forms of the distribution function may not give equally good fits to the experimental results, and that the distribution function that we chose is unique. We have not tried other distribution functions.

Figure 6(a) presents the values of $\bar{U}$ and l in neon for various gas pressures ranging from approximately 0.1 to 3 mm Hg, for a positive column immersed in a dc magnetic field equal to 1000 G. We note that over this range of pressure the distribution parameter l remains constant. The mean energy decreases with increasing

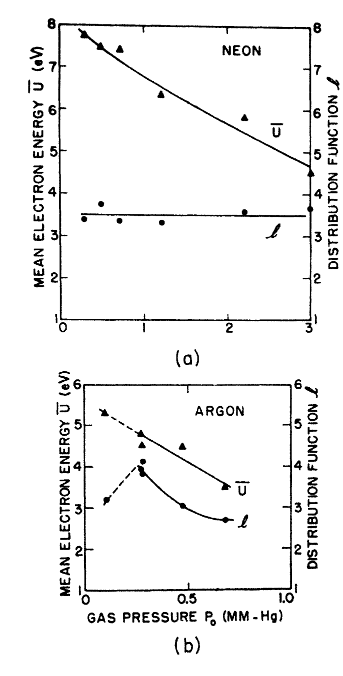

FIG. 6. Variation of the mean electron energy $\bar{U}$ and of the distribution parameter l with pressure, in neon and argon.

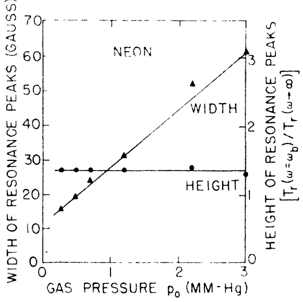

FIG. 7. The width and the height of the resonance peaks as a function of pressure.

pressure in agreement with the theory of the positive column.

Figure 6(b) illustrates how these quantities ($\bar{U}$ and l) behave in argon, over a range of gas pressure from about 0.1 to 1 mm Hg. The distribution parameter l varies considerably with gas pressure. The departure from a Maxwellian distribution appears to be greater at the low than at the high pressures. The difference in the behavior of l as a function of pressure in neon and argon may be due to the very marked difference in the velocity dependence of the respective collision cross sections for electron-neutral encounters in the two gases. Whereas in neon the cross section is sensibly constant with energy, in argon it is very energy dependent and exhibits a deep Ramsauer minimum.

In referring to the formula in Eq. (32) for the radiation temperature, it can be shown that if the distribution function is independent of the gas pressure and of the electron density, T_r exhibits the following properties:

(a) The width of the peak or dip in T_r is proportional to the collision frequency and thus to the gas pressure, and is independent of the electron density.

(b) The height of the peak or dip normalized to T_r at $\omega \to \infty$ is independent of both the gas pressure and the electron density.

These properties of the radiation temperature lend themselves to experimental check in the case of neon (where l was found to be independent of pressure). This is shown in Fig. 7. We observe that the width of the peak is proportional to the gas pressure and that the height of the peak normalized to T_r at $\omega \to \infty$ is constant with pressure. In the limit of zero pressure the apparent width of the peak tends to a value of 10 G due to the inhomogeneity of the magnetic field over the 50-cm-long section of the positive column from which the microwave radiation was received. Measurements of the width and the height of the peak as a function of electron density are not shown. However, we found[20] that the normalized height and the width remained constant for discharge currents from 2 to 200 mA.

All our experiments were made at such low electron densities that the plasma frequency ω_p was less than ω. As the current and thus the electron density are increased to a value where ω_p is approximately equal to or greater than ω, the peak in the radiation temperature at $\omega=\omega_b$ broadens out and disappears with increasing current; simultaneously peaks in the emission, j_ω, in the absorption, α_ω, and in T_r are observed[21] at the higher harmonics $\omega=n\omega_b$, where $n=2, 3, 4\cdots$. These effects are not understood and fall outside the range of this investigation that deals exclusively with tenuous plasmas. The peaks at the higher harmonics are not associated with the phenomena discussed in this paper. They occur in all gases studied (helium, argon, mercury) and their presence is, therefore, not associated with the energy dependence of the electron-atom collision cross section.

III. EXPERIMENTAL TECHNIQUE

Equation (29) suggests that the radiation temperature can be obtained directly from the intensity, in the limit, when the optical depth τ is greater than unity. Since in our tenuous plasmas τ was generally less than unity, we adopted a technique wherein the measurements of the radiation temperature were independent of the magnitude of the optical depth.[22]

We illuminate the plasma with radiation incident on the side away from the observer. The total intensity that leaves the plasma is then given by

$$I_\omega = I_\omega(\text{inc})\exp(-\tau) + (\omega^2/8\pi^3c^2)kT_r[1-\exp(-\tau)], \quad (37)$$

where $I_\omega(\text{inc})$ is the intensity of radiation incident on the back side of the plasma. The intensity I_ω was then compared periodically with radiation $I_\omega(\text{inc})$ that has not traversed the plasma. We adjust $I_\omega(\text{inc})$ until the radiation along the two channels is the same ($I_\omega(\text{inc}) = I_\omega$). It then follows from Eq. (37) that

$$I_\omega(\text{inc}) = (\omega^2/8\pi^3c^2)kT_r, \quad (38)$$

independent of the optical depth τ.

The assumed constancy of the radiation temperature implicit in Eqs. (37) and (38) may not be true in the discharges studied. We varied the optical depth τ from approximately 0.1 to 2 and measured the radiation temperature with and without the use of the above substitution method. All these methods yielded identical values of T_r as a function of magnetic field. From these measurements we conclude that the radiation temperature must have been sensibly constant over the volume of plasma studied and that the substitution method used of finding T_r in a tenuous plasma was free from experimental anomalies.

[20] H. Fields, G. Bekefi, and S. C. Brown, *Proceedings of the Fifth International Conference on Ionization Phenomena in Gases* (North-Holland Publishing Company, Amsterdam, 1961), pp. 367–375.

[21] G. Bekefi, J. D. Coccoli, E. B. Hooper, Jr., and S. J. Buchsbaum, Phys. Rev. Letters **9**, 6 (1962).

[22] G. Bekefi and S. C. Brown, J. Appl. Phys. **32**, 25 (1961). For transient measurements, see D. Formato and A. Gilardini, *Proceedings of the Fourth International Conference on Ionization Phenomena in Gases, Uppsala, Sweden* (North-Holland Publishing Company, Amsterdam, 1960), Vol. 1, pp. 1A99–104.

The radiation temperature was measured in the positive column of a glow discharge, produced in a tube 100 cm long and 1.25 cm in radius. A heated oxide-coated cathode and a water-cooled nickel anode were used for electrodes. The tube was continuously pumped and after activation of the cathode pressures better than 10^{-7} mm Hg could be maintained. Before taking measurements the tube was cleaned by passing strong currents through it and subsequently pumping the gas out. The measurements of the radiation temperature in the mercury vapor discharge were made in a tube whose walls could be heated up to 150°C. A temperature-controlled bath provided a means of varying the pressure in the mercury discharge tube.

The measurements were made with an S-band radiometer that has been described in detail elsewhere.[22] The discharge tube was inserted into the broad face of a section of S-band rectangular waveguide at an angle of 8° with the waveguide axis. A 50-cm-long section of the positive column was thus situated within the waveguide section that comprised one arm of the radiometer. Flanges, designed to be beyond cutoff for the guide, were used to insure that any possible radiation generated at the electrodes could not enter the guide.

The section of waveguide containing the discharge tube was mounted in a solenoid, 120 cm long, in such a way that the tube was situated along the axis of the solenoid. The magnetic field could be varied continuously from 0 to 2000 G, and was uniform to within 1% over the 50-cm-long section of the positive column from which the microwave radiation was received.

Since the experiments were performed in a waveguide structure, the source function for the radiation now differs from that given in free space. For one mode of propagation of the waveguide, one finds that

$$S_\omega(\omega,T_r)=kT_r d\omega/2\pi. \tag{39}$$

Note, however, that the radiation temperature as given by Eq. (32) remains unchanged, due to the fact that the radiation from our tenuous plasmas couples weakly to the waveguide.

ACKNOWLEDGMENTS

The authors wish to acknowledge the technical help given by J. J. McCarthy. The many helpful discussions with P. E. Schweitzer of the Air Force Cambridge Research Center, Bedford, Massachusetts; and with S. Gruber of MIT are greatly appreciated. The machine calculations and the programming were carried out by the R.L.E.-L.N.S. Computing Group at MIT. One of us (HF) wishes to acknowledge the encouragements he received from Microwave Associates, Burlington, Massachusetts, where he was employed during the time of these measurements.