These two effects tend to compensate each other in cases where the spin of the bombarded nucleus is large.

A complete expression for magnetic scattering by finite-size nuclei with a magnetic momentum was derived by Walecka and Pratt[8]; the connection with Rosenbluth's formula is discussed there.

From Fig. 5 it is seen that for light elements, where magnetic scattering is big and thus our error (due to subtracting the charge cross section) is small, the experimental points follow the theory quite closely. $_5B^{10}$ seems to be the only exception. Since it has a large spin, this might be due to contributions of higher multipoles which are not present in $_5B^{11}$. For heavier nuclei the errors are large, and there seem to be no major conflicts between theory and experiment. The H theoretical point is higher than the experimental because no recoil is assumed in formula (4) which makes an appreciable effect only in H, and removes completely the discrepancy when taken into account.

For the case of Li^7 a calculation of the scattering cross section was made by Willey,[9] using known wave functions for the ground state of Li^7. The calculations were made for a *LS* coupling and for an odd-proton model. The results are listed below for 41.5 MeV:

LS model,	3.70×10^{-32} cm²/sr;
Odd-proton model,	5.71×10^{-32} cm²/sr;
Formula (4),	7.1×10^{-32} cm²/sr;
Experiment,	5.6×10^{-32} cm²/sr.

Although the magnetic moment in Li^7 is in agreement with the predictions of the *LS* model, the old proton model is considerably better than the *LS* model in predicing the scattering results.

The case of He^4 is very special because the absence of magnetic scattering can yield information on an electric dipole moment of the electron. It will be discussed in another publication.[10]

Further work on the lighter nuclei with higher electron energies would be very interesting because of the expected sharp decrease in cross section due to the effect of the magnetic form factors.

We wish to thank Dr. W. C. Barber for constant encouragement and advice, Dr. G. A. Peterson who participated in the initial phases of the work, Prof. R. Hofstadter, Dr. J. Scoffield, Dr. J. D. Walecka, and Dr. R. H. Pratt for stimulating discussions, and J. Carson and E. Wright for generous technical help. One of us (Y. T.) wishes to thank the Ministry of Education of Japan which made possible his stay at Stanford.

[8] J. D. Walecka and R. H. Pratt (private communication).
[9] R. Willey (private communication).
[10] J. Goldemberg and Y. Torizuka (to be published).

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Beta-Gamma Directional Correlation in Re^{188}†

L. D. Wyly, C. H. Braden, and Harry Dulaney
School of Physics, Georgia Institute of Technology, Atlanta, Georgia

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The directional correlation between the 1960-keV beta group and the 155-keV gamma in the decay of Re^{188} has been measured. The coefficient A_2 is found to vary from about 0.17 to about 0.25 over the energy range $W=3$ to $W=4.3$. Limitations on the matrix elements for the sequence $1^-(\beta)2^+(\gamma)0^+$ are discussed. It is found that the directional correlation and a variety of beta spectral shapes may be fitted for values of the parameter $|\xi_1|$ greater than some minimum value which is dependent upon the spectral shape. Information is presented on further limitations of the matrix elements which will result when the shape is known and some discussion is given for the role of a measurement of the beta–circularly polarized gamma correlation.

EXPERIMENTAL PROCEDURE AND RESULTS

THE 17-h Re^{188} was purchased from the Oak Ridge National Laboratory as processed, high specific activity (~20 mC/ml) 91-h Re^{186}. At the time of assay before leaving Oak Ridge the Re^{188} activity was approximately one-third of the total activity. Sources were prepared by evaporation of the nitric acid solution of the $HReO_4$ on 0.25-mil Mylar backing. The sources were transparent. They were grounded by a thin Aquadag line. Data taking for the directional correlation determination began about eight hours subsequent to the assay at Oak Ridge.

The electronic equipment and general experimental procedure has been previously described.[1] The decay scheme of Re^{188} is shown in Fig. 1.[2] A single-channel

† Supported in part by a grant from the National Science Foundation (G6337).

[1] Harry Dulaney, C. H. Braden, E. T. Patronis, Jr., and L. D. Wyly, Phys. Rev. **129**, 283 (1963).
[2] *Nuclear Data Sheets*, Natl. Acad. Sci.–Natl. Res. Council, NRC 59-3-119 (Office of Printing and Publishing, National Research Council–National Academy of Sciences, Washington 25, D. C.).

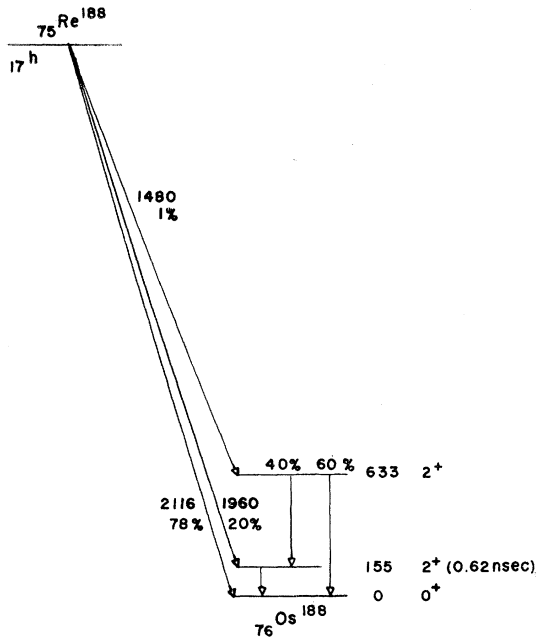

FIG. 1. Principal branches of decay scheme of Re^{188}.

pulse-height analyzer was centered on the 155-keV gamma ray with a window width of approximately 12 keV. Therefore, a portion of the gamma counts was due to the 137-keV gamma present in the decay of the Re^{186}. The beta coincidence spectrum was displayed on a 20-channel analyzer which was gated by the output of a fast-slow coincidence circuit. The beta spectrum between 1060 keV and 1690 keV was divided into approximately 125-keV increments. The mean values of the beta energies are believed accurate to ± 25 keV. At the lower beta energies studied here there will be some interference due to the (1480-keV beta) (155-keV gamma) cascade. However, it is believed that no appreciable error is introduced in the results because of the small branching ratio for the 1480-keV beta decay and the fact that the 155-keV level in Os^{188} decays directly to the ground state in about 60% of the cases.[2] In addition, in the interference energy region the intensity of the 1960-keV beta group is closer to its maximum value than in the case of the 1480-keV beta group.

The relatively short lifetime of this activity necessitated some changes in procedure from that usually employed. No attempt was made to adjust amplifier gains during a data taking cycle and the coincidence counting rate was not normalized to the singles counting rates. Instead, the following procedure was adopted. The beta and gamma channels were calibrated for energy by comparison with the conversion electron lines in Cs^{137} and Bi^{207} and the Bi^{207} x ray and 137-keV gamma in Re^{186} (using an old source in which the Re^{188} had decayed). The rhenium source was placed in the vacuum chamber. After a 10-min pumping period, 15-min coincidence counts were made at angles of 90 deg, 180 deg (two counts), 270 deg, and of the accidental coincidence rate. Data were rejected if either the gamma or beta singles rates departed by more than 3% from the expected values. At the completion of a counting cycle, the beta and gamma channels were recalibrated and the data were rejected if the energy calibration had shifted by more than 3%.

The experimental results are listed in Table I. A preliminary report of the results has been given.[3] The indicated errors are essentially probable errors due to statistical effects. A_2 is the usual coefficient of $P_2(\cos\theta)$ in the expansion of the directional correlation in terms of even-order Legendre polynomials, where A_4 is assumed to be zero. No gamma-gamma coincidences were observed when a 0.5-cm Lucite absorber was inserted between the source and beta detector. Correction for finite angular resolution of the detectors was made according to the method of Rose[4] with the aid of tables of Stanford and Rivers.[5] The increase in the relative concentration of the Re^{186} activity with time mainfested itself by an increase in the accidental coincidence rate with time. Originally the accidental rate was about one-eighth the real rate. The experiment was terminated when satisfactory counting rates required sources which yielded an accidental rate in excess of the real rate. No significant differences in the observed directional correlation were noted due to the relative increase in the Re^{186} activity.

TABLE I. Experimental results for the directional correlation between the 1960-keV beta group and the 155-keV gamma in the decay of Re^{188}. W is the total beta energy in mc^2 units, A_2 is the coefficient of $P_2(\cos\theta)$ in an expansion of the directional correlation in even order Legendre polynomials with $A_4=0$, R is the number of real coincidences at 90–270 deg, and R/C is the ratio of real to accidental coincidences. $A_2{}^*=WA_2/\lambda_2 p^2$.

W	A_2	R	R/C	$A_2{}^*$
3.08	0.189±0.009	19 900	1.7	0.100±0.005
3.33	0.169±0.009	17 600	2.0	0.081±0.004
3.57	0.169±0.008	27 400	2.0	0.074±0.004
3.82	0.213±0.009	19 300	1.9	0.086±0.004
4.06	0.232±0.020	6 200	1.6	0.087±0.007
4.30	0.248±0.024	3 400	1.4	0.086±0.008

INTERPRETATION OF DATA

The procedure used to investigate limitations placed on the beta decay matrix elements has been previously described.[6] The modified beta-gamma directional correlation coefficient, $A_2{}^*=WA_2/\lambda_2 p^2$, is characterized by

[3] L. D. Wyly, Robert E. Wood, and C. H. Braden, Bull. Am. Phys. Soc. **7**, 34 (1962).
[4] M. E. Rose, Phys. Rev. **91**, 60 (1953).
[5] A. L. Stanford and W. K. Rivers, Rev. Sci. Instr. **30**, 719 (1959).
[6] Harry Dulaney, C. H. Braden, and L. D. Wyly, Phys. Rev. **125**, 1620 (1962).

the constraining condition:

$$A_2^*=0.08\pm0.01 \quad \text{for} \quad W=3.571,$$

where W is the total β energy in mc^2 units. When the experimental values corresponding to the two error limits are equated to the theoretical expression given by Kotani[7] and an arbitrary value given to the beta-decay matrix element parameter ζ_1, two conic sections are generated in the x-u plane. The definitions of the matrix element parameters are

$$\zeta_1=-(\xi-W_0/3)u-iC_V\int\alpha/C_A\int B_{ij}-(\xi+W_0/3)x,$$

$$u=i\int\boldsymbol{\sigma}\times\mathbf{r}\Big/\int B_{ij}, \quad x=-C_V\int\mathbf{r}/C_A\int B_{ij}.$$

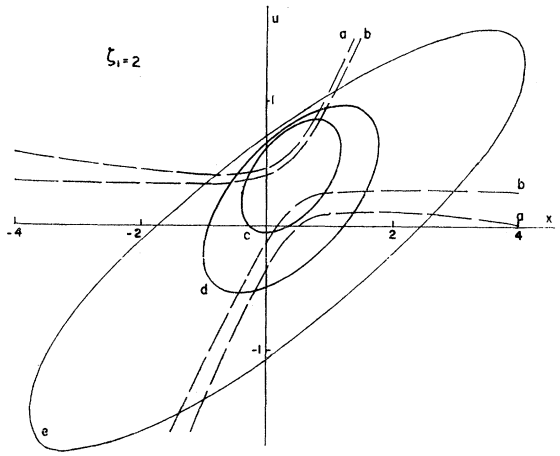

FIG. 2. Conic sections for $\zeta_1=2$, generated by the following constraining conditions: (a) $A_2^*=0.07$; (b) $A_2^*=0.09$; (c) $S=1.2$; (d) $S=1.4$; and (e) $S=1.6$. A_2^* is evaluated at a beta energy of $W=3.571$ and S is the ratio of the beta spectral shape correction at $W=W_0=4.836$ to that at $W=3.327$.

The analysis was carried out only for $|\zeta_1|\leq20$. An example of the results is depicted in Fig. 2 for $\zeta_1=2$.

No precise experimental determination of the beta spectral shape has been reported for the 1960-keV beta group. However, there is some evidence that the shape may be generally similar to that observed for the inner beta spectrum of Re^{186}.[8–10] Accordingly, various assumptions about the spectral shape are incorporated in the analysis. Additional conic sections in the x-u plane are generated by placing on the shape correction the constraining condition:

$$S=C(W=W_0=4.836)/C(W=3.327)=1.2,\ 1.4,\ \text{or}\ 1.6,$$

where $C(W)$ is the beta spectral shape correction factor.[7] These conic sections are also shown in Fig. 2 for $\zeta_1=2$. It will be noticed that the shape and directional correlation conics intersect in several distinct regions. Parameter sets that correspond to the regions of intersection are possible fits to the experimental data but the energy dependence must be checked over the entire experimental energy range. Plots of A_2^* vs the beta energy are given in Fig. 3 for the following parameter sets selected from Fig. 2 as typical of the four regions of intersection for $S=1.4$:

(a) $\zeta_1=2$, $u=-0.53$, $x=-0.4$;
(b) $\zeta_1=2$, $u=0.22$, $x=1.55$;
(c) $\zeta_1=2$, $u=0.93$, $x=0.90$;
(d) $\zeta_1=2$, $u=0.38$, $x=-0.57$.

The experimental values of A_2^*, computed from the results given in Table I, are also shown. The data favor set (a), although not decisively so. Additional data at lower beta energy would be decisive but requires enriched sources with less Re^{186} contaminant, which were not available. In the decay sequence 1^-(934-keV β)2^+ (137-keV γ)0^+ in the similar isotope Re^{186}, an A_2^* that was essentially energy independent, as predicted by parameter set (a) of Fig. 3, was found.[1] Plots of the spectral shape correction factor vs energy are shown in Fig. 4. These curves are generally similar except for parameter set (c) which gives a shape that may not be expected.[8]

The directional correlation data and a spectral shape correction may be fit for conditions that may be characterized approximately as follows: for $S=1.2$, $|\zeta_1|$ must be as large as 2; for $S=1.4$, $|\zeta_1|$ must be as large as 1.5; and for $S=1.6$, $|\zeta_1|$ must be as large as 1. Smaller values of $|\zeta_1|$ do not give simultaneous fits to the directional correlation and shape or, for sufficiently small values of $|\zeta_1|$, no fit to the shape is possible. Values of $|\zeta_1|$ greater than 20 were not investigated but no

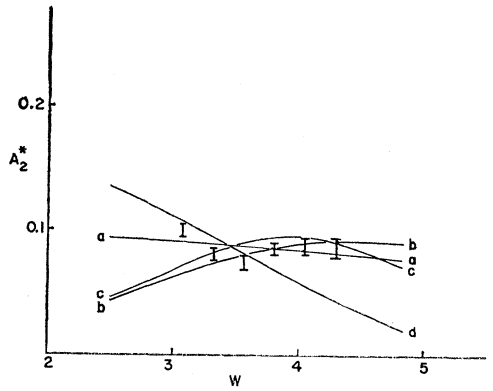

FIG. 3. Plots of A_2^* vs beta energy for the following matrix element parameter sets: (a) $\zeta_1=2$, $u=-0.53$, $x=-0.4$; (b) $\zeta_1=2$, $u=0.22$, $x=1.55$; (c) $\zeta_1=2$, $u=0.93$, $x=0.9$; and (d) $\zeta_1=2$, $u=0.38$, $x=-0.57$. Experimental values of A_2^* are also shown.

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[8] F. T. Porter, M. S. Freedman, T. B. Novey, and F. Wagner, Jr., Phys. Rev. **103**, 921 (1956).
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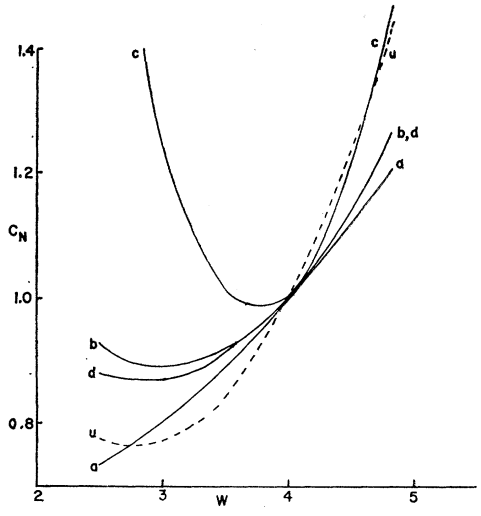

FIG. 4. Plots of the beta spectral shape correction factor normalized to unity at an energy $W=4$, C_N, vs beta energy for the same sets of matrix element parameter as used in Fig. 3. The unique shape (u) for which $\zeta_1=u=x=0$ is also shown.

evidence of poorer fits to the data for the larger values was noticed. In general, the energy dependence of the directional correlation data favor a single one of the regions of intersection of the directional correlation and shape conics. The fits to the experimental data at other

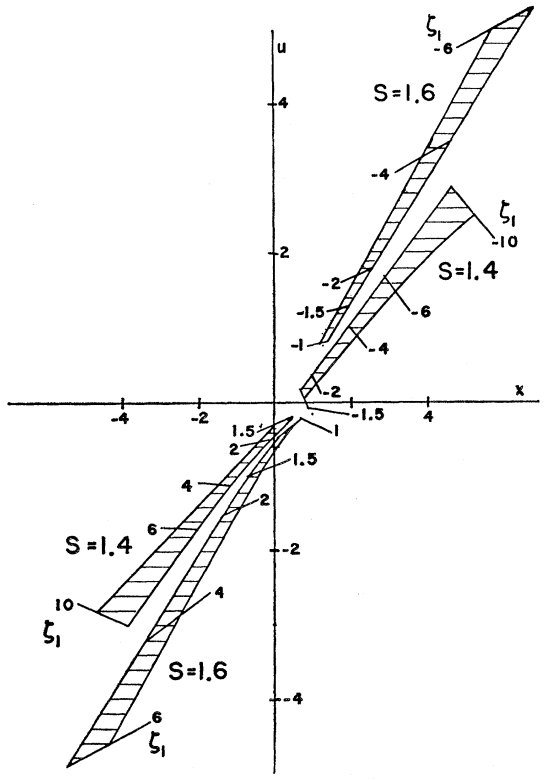

FIG. 5. Loci of points, as a function of ζ_1, corresponding to the most favorable fits to the directional correlation and a spectral shape correction $S=1.4$ or 1.6, in the decay of Re^{188}.

values of the parameter ζ_1 are comparable to that depicted in Figs. 2, 3, and 4. For plots similar to Fig. 2, the most favorable region of intersection is the lower left region for positive ζ_1 and the upper right region for negative ζ_1 (the shape conic for a negative ζ_1 is just the reflection about the origin of the shape conic for the corresponding positive ζ_1).

In order to schematically trace the region at which the most favorable fit to the directional correlation data and a spectral shape may be attained as a function of the parameter ζ_1 we present Figs. 5 and 6. There we plot, in the x-u plane, the locus of points corresponding to the most favorable fit to the directional correlation data and a particular spectral shape characterized by the shape parameter S. In Fig. 5 plots are given for $S=1.4$ and 1.6. In Fig. 6 the plot is given for $S=1.2$. Plots of the modified correlation coefficient, A_2^*, or the normalized shape correction, C_N, vs W for points contained within these loci are generally similar to the curves (a) of Figs. 3 and 4. Also shown in Fig. 6 is the locus of points corresponding to the most favorable fit to the directional correlation and the spectral shape for the very similar decay of Re^{186}.[1,8] The finite extent of the loci of points indicated in Figs. 5 and 6 approximately indicate the uncertainties in the determinations of matrix elements due to errors inherent in the directional correlation measurements. In order to avoid undue confusion on the plot the effect of errors has been suppressed in the case of the locus of points for the Re^{186} decay. It must always be borne in mind that, in fact, the error limits should be considered broader because of limitations in the theoretical formulas and because of experimental difficulties that have not been thoroughly investigated, such as attenuation effects due to perturbations of the intermediate state in correlation experiments. It has been suggested that there may be a limited attenuation of a gamma-gamma directional correlation that proceeds through the 155-keV level in Os^{188}.[11] However, it is not feasible at present to adduce a value for an attenuation coefficient that might be applied to the beta-gamma cascade studied here. Furthermore, it is not evident that the attenuation coefficient applicable to a gamma-gamma cascade should be valid for a beta-gamma cascade proceeding through the same level because of possible differences in the environment to which the intermediate state is subjected. Experimental clarification of this point is desirable and work is proceeding at this laboratory directed towards possible attenuation effects in beta-gamma cascades.

It is of interest to inquire whether some additional types of measurement might better delimit the permissible values of the various matrix element parameters. Some information relative to the beta–circularly polarized gamma correlation is presented in Table II. Theoretical values of the coefficient ω^7 are given for

[11] W. J. King and M. W. Johns, Can. J. Phys. **37**, 755 (1959).

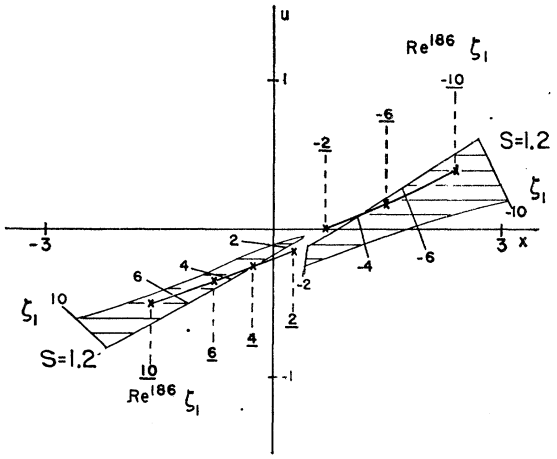

FIG. 6. Loci of points, as a function of ζ_1, corresponding to the most favorable fit to the directional correlation and the spectral shape correction $S=1.2$ in the decay of Re^{188} and the best fit to the directional correlation and the spectral shape in the decay of Re^{186}.

numerous sets of matrix element parameters corresponding to the most favorable fits to the directional correlation data and different spectral shapes characterized by the values of S. The matrix element parameter sets chosen lie, approximately, along the mean of the loci of points shown in Figs. 5 and 6. The polarization coefficient has been computed as a function of the angle between the beta and gamma and for a beta energy $W=4.2$. The tabulated values are for an angle of 180 deg. It will be noted that there is a substantial variation of ω with the matrix element parameter ζ_1, for a particular choice of spectral shape. The parameter ω might also further substantiate the choice of a single intersection region on the conic section plots similar to Fig. 2. For example, in the case of the parameter sets chosen to characterize the four regions of intersection on Fig. 2 for the spectral shape correction corresponding to $S=1.4$ the following values of ω are computed (angle between beta and gamma is 180 deg and beta energy is $W=4.2$): $\omega=0.16$ for set (a), $\omega\sim 1$ for sets (b) and (c), and $\omega\sim -1$ for set (d). It would appear that measurement of the beta–circularly polarized gamma correlation might prove of significant value for this decay.

TABLE II. Theoretical values of the beta-circularly polarized gamma correlation coefficient, ω, in the decay of Re^{188} for various sets of the beta decay matrix element parameters ζ_1, u, and x. The angle between the beta and gamma is 180 deg and the beta energy is $W=4.2$. The beta spectral shape may be characterized by S which is listed for each group of parameter sets.

ζ_1	u	x	ω
	$S=1.2$		
−10	0.3	3.0	0.26
−4	0	1.2	0.46
4	−0.3	−0.7	0.07
10	−0.7	−2.4	0.11
	$S=1.4$		
−10	2.7	5.0	0.26
−4	0.9	2.1	0.36
−1.5	0.12	0.7	0.79
1.5	−0.25	0.25	0.62
4	−1.7	−2.4	0.14
10	−2.9	−4.4	0.16
	$S=1.6$		
−10	8.5	10.2	0.25
−1	0.8	1.25	0.58
1	−0.3	0.5	0.91
10	−8.2	−9.0	0.20

With regard to possible selection rule effects in the beta decay of Re^{188} we encounter a situation similar to that in Re^{186} and Tm^{170}.[1] The beta decay spin sequence $1^-(\beta)2^+$ precludes an obvious argument based on K or j forbiddenness.[7] The large $\log ft$ product for the 1960-keV beta group, $\log ft=8.5$, suggests some attenuation of the "ordinary" first forbidden matrix elements, although not necessarily relative to the B_{ij} matrix element.

We wish to acknowledge the assistance of Nisbet Kendrick and Robert Wood in taking the data.