Determination of the Deuteron Form Factor from Coherent π^0 Photoproduction*

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Coherent photoproduction of π^0 mesons from deuterium has been used to measure the deuteron form factor. Measurements of the production cross section were carried out for photon energies from 470 to 514 MeV by detecting momentum-analyzed recoil deuterons. These measurements were analyzed in terms of the impulse approximation calculation of Hadjioannou. The deuteron form factor has been determined in the range of momentum transfers from 1.74 to 2.74 F^{-1}. The results are consistent with the predictions of a repulsive core potential; however, there is some ambiguity in the interpretation due to the uncertainty in the magnitude of the multiple scattering corrections to the impulse approximation. Measurements of the cross section were also made as a function of photon energy for two constant momentum transfers, 1.76 and 1.96 F^{-1}. The range of energies covered was 300 to 500 MeV. The results indicate that there is a large reduction of the cross section with respect to the impulse approximation prediction near the (3,3) resonance. This is in qualitative agreement with calculated corrections arising from multiple scattering of the meson in the deuteron. The measurements show that this deviation decreases as the photon energy increases in the region above resonance.

I. INTRODUCTION

DETAILED knowledge of the structure of the deuteron provides an important test for a meson theory of the nucleon-nucleon interaction. Although nucleon-nucleon scattering experiments and studies of photodisintegration of the deuteron allow the construction of semiphenomenological potentials describing the neutron-proton interaction there has been, up to now, only one class of experiments that are highly sensitive to the short-range structure of the interaction when the total energy of the n-p system is very low. These experiments involve the scattering of electrons from deuterium either elastically or inelastically or other processes which are dependent on the form factor of the deuteron. The information that can be extracted directly from this type of experiment can otherwise only be obtained from potentials established by using data from experiments in which the neutron and proton have high relative energies. The low-energy, short-range structure of the interactions must then be determined by assuming the established potential is valid for all lower relative energies of the particles. The purpose of the present experiment was to study the structure of the deuteron using the determinations of the deuteron's form factor from coherent photoproduction of π^0 mesons from the deuteron: $\gamma+d\rightarrow\pi^0+d$.

The results of electron scattering determinations[1,2] of the deuteron's elastic form factor, F_d^2, are consistent with the predictions made from a neutron-proton interaction having a repulsive core. In a similar manner, determinations of the deuteron's form factor from coherent π^0 photoproduction can be used to extract information about the neutron-proton interaction. Hadjioannou[3] has recently completed a theoretical study of this process in which various deuteron models are considered. (Hereafter this reference is referred to as H.)

Previous studies[4,5] of coherent π^0 photoproduction from the deuteron were not intended to investigate the structure of the deuteron and had limited results for values of momentum transfer at which the finite structure of the deuteron affects the cross-sections appreciably. All acceptable models of the deuteron give form factor curves which converge at low momentum transfers. It is at high momentum transfers that the predictions from various models diverge and that the coherent π^0 photoproduction results can most usefully be compared with those from electron scattering.

In the present series of measurements we have studied the deuteron form factor at momentum transfers, q, in the range from 1.74 to 2.74 F^{-1}. The pholon energies employed were in the range from 470 to 515 MeV.

The measured cross sections for the reaction $\gamma+d\rightarrow\pi^0+d$ have been related to F_d^2 for the deuteron by using the calculation of H. This calculation was carried out in the impulse approximation and used dispersion relation formulas to determine the single nucleon photoproduction amplitudes. The amplitudes are corrected for kinematic effects arising from the internal momentum distribution of the nucleons in the deuteron. In the derivation of his results he used both Yukawa type and repulsive core deuteron models, including the D state contributions.

In addition, measurements of the cross-section were made as a function of photon energy for two constant momentum transfers, 1.76 and 1.96 F^{-1}. The range of photon energies employed was from about 300 to 500

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[1] J. A. McIntyre and G. R. Burleson, Phys. Rev. **112**, 2077 (1958).

[2] J. I. Friedman, H. W. Kendall, and P. A. M. Gram, Phys. Rev. **120**, 992 (1960).

[3] F. T. Hadjioannou, Phys. Rev. **125**, 1414 (1962) (referred to in the text as H).

[4] B. Wolfe, A. Silverman, and J. W. DeWire, Phys. Rev. **99**, 268 (1955).

[5] H. L. Davis and D. R. Corson, Phys. Rev. **99**, 273 (1955).

MeV. The purpose of this set of measurements was to determine whether this process has the energy dependence predicted from the impulse approximation description.

II. EXPERIMENTAL METHOD

The experiment was carried out with a 600-MeV electron beam from the Stanford Mark III linear accelerator. The major part of the data was taken with the 36-in. 180° double-focusing magnetic spectrometer, used to momentum analyze the recoil deuterons from coherent π^0 photoproduction (Run I). The experimental apparatus used has been previously described.[6] Two of the data points were later measured with completely new instrumentation (Run II).

The target assembly[7] used had a provision for moving either a liquid hydrogen target or a liquid deuterium target into the beam line. The liquid targets were 8.0 in. long and 1 in. in diameter with 0.001 in. stainless steel walls. In Run I, a Cu radiator 0.03 of a radiation length was placed 3 in. from the edge of the liquid target in the beam line. The deuterons were detected in a plastic scintillation counter, 0.120 in. thick, viewed by a 6810 photomultiplier tube. The counter defined a momentum interval in the focal plane of the spectrometer about 1.5% wide. In order to separate the recoil deuterons from protons resulting from photodisintegration of the deuteron the output pulses of the scintillation counter were analyzed with a 256-channel RCL pulse height analyzer. The deuteron and proton peaks in the pulse height distributions could be separated unambiguously. However, the deuteron peaks rested on a continuum and the major part of the errors quoted for most of the measured points resulted from the estimates of the background beneath the deuteron peaks.

The background continua beneath the deuteron peaks in the pulse height spectra were in the range from 10 to 30% of the heights of the deuteron peaks. Although the deuteron yields from different runs under similar conditions were reproducible within statistical fluctuations the background continua were occasionally quite different, both as to height and shape. A substantial fraction of this background came from neutrons produced both in the target and in the primary beam collimating and ditching apparatus which were able to penetrate the concrete counter shielding and give either proton recoils in the plastic scintillators or neutron-capture gamma rays from capture in the shielding. The uncertainties in the measured deuteron yields from the background were largely a consequence of our inability to determine precisely the shapes of the spectra under the peaks. The shapes were not entirely reproducible.

In Run II we made use of the newly installed 72-in. spectrometer and a more complex deuteron-detecting system. Two thicknesses of copper radiator (0.03 and 0.05 radiation length) could be inserted in the beam line. One-inch tungsten slits 11 in. from the target defined the portion of the target observed by the spectrometer and prevented spurious radiation from the radiator and from the end windows of the target cell from reaching the entrance slits of the spectrometer. Internal baffles at the 90° position in the spectrometer served to define the acceptance aperture. A three-element scintillation counter telescope was used to separate momentum analyzed protons and deuterons. The telescope defined a 0.7% momentum interval in the focal plane of the spectrometer. The outputs from the first two elements of the telescope were put in coincidence and integral discriminators in each channel were adjusted using coincidence-gated pulse height analysis so that protons could trigger neither channel. A variety of copper absorbers were introduced between the second and the third counter. The absorber thickness was chosen to stop deuterons of the selected momentum. Coincidence-gated pulse height data from either of the first two counters was stored in different portions of the memory of the pulse height analyzer depending on whether or not there had been a triple coincidence. In this manner we achieved a substantial reduction in the background under the deuteron peaks.

III. CORRECTIONS TO DATA

In both Run I and Run II the electron beam was allowed to pass through the target after leaving the copper radiator. As a consequence some of the pion photoproduction (about 25%) resulted from interactions between the deuterons and the virtual photons from the electron's field. Since there is little difference in the production resulting from these and from real photons for the kinematic conditions of this experiment the uncertainties introduced by this effect are considerably smaller than the errors in the measurements. Nearly all the virtual photons come from electrons which are scattered by less than 5° and are thus not far off the energy shell.[8] The average four-momentum transfer of the electrons contributing to pion production is thus small; the magnitude of the longitudinal part of the virtual photon field is thus also small. In addition, pion photoproduction at the energies of this experiment is, to a large extent, a consequence of the magnetic dipole part of the electromagnetic field. The magnetic dipole part supports no longitudinal component and the small longitudinal contributions from the electron's field can only affect the other amplitudes. The value of the equivalent radiation length X_e of the virtual photon field used in the analysis of this experiment was taken from the measurements of Panofsky *et al.*[9] These

[6] R. Hofstadter, Rev. Mod. Phys. **28**, 214 (1956).

[7] S. Sobottka, Ph. D. thesis, Stanford University, 1960 (unpublished).

[8] R. H. Dalitz and D. R. Yennie, Phys. Rev. **105**, 1598 (1957).

[9] W. H. K. Panofsky, W. M. Woodward, and G. B. Yodh, Phys. Rev. **102**, 1392 (1956).

measurements yielded a value of $X_e=0.0178\pm0.0023$ for the electroproduction of pions with kinematics similar to those of the present experiment.

Because of the short duty cycle of the linear accelerator it was not feasible to detect the recoil deuterons in coincidence with a decay gamma ray from the produced π^0. Thus, a few sources of background enter into these measurements.

(1) There are deuteron recoils from Compton scattering. This process is estimated to produce a background of about 1%. This estimate is made from an extrapolation of the ratio of cross-sections for Compton scattering[10,11] and π^0 photoproduction[12,13] from the proton in conjunction with the calculation of Capps[14] which relates Compton scattering from the proton to that from the deuteron. Measurements of Compton scattering from the deuteron at photon energies around 100 MeV by Hyman *et al.*[15] are a factor 1.6 larger than that predicted by an impulse approximation calculation. Because of the uncertainty in this estimate an error of 2% is introduced into the results.

(2) A second source of background is recoil deuterons from electron-deuteron scattering. Because of the kinematic difference between elastic electron scattering and pion photoproduction,[16,17] background resulted only from electrons which had radiated in the radiator or target before scattering or from a scattering in which a photon was emitted in the scattering process. This effect made the following contributions to the observed deuteron counting rates. At a momentum transfer of $q\simeq1.74$ F^{-1} the contributions were 14 and 8% at the angles 62 and 58.5°, respectively; at $q\simeq1.94$ F^{-1} the contribution at 58.5° was 12%. At all other points the increase in the deuteron-counting rate was 4% or less. The data were corrected for this effect.

(3) Another source of recoil deuterons is coherent double π photoproduction, $\gamma+d\rightarrow 2\pi+d$. At all points of the form factor measurements (470 MeV$<E_\gamma<$514 MeV) at which this process could contribute background, the thresholds were only slightly below the end point energy of the bremmstrahlung spectrum. Rough estimates of this background based on cross sections[18] of π pair photoproduction from the proton indicate that it was small compared to the errors in the measurements and consequently was neglected.

The horizontal entrance slits of the spectrometer in Run I were sufficiently small ($\frac{3}{4}$ in. separation) so that interactions of the beam with the entrance and exit windows could not produce background deuterons. In Run II 1 in. tungsten slits excluded this background. However, it was necessary to determine that the beam of electrons and photons leaving the radiator did not interact with the side walls of the target. Such interactions would produce deuterons copiously from pick-up processes occurring in photonuclear disintegrations. This was investigated by attempting to detect deuterons with the liquid hydrogen target in the beam line. No deuterons were detected in these measurements.

IV. MEASUREMENTS

As these measurements were to be analyzed in terms of an impulse approximation calculation, this suggested that the experiment be carried out at photon energies at which the corrections to the theory are reduced. The form factor measurements were made with an average photon energy of 485 MeV. At this energy the photoproduced pion is considerably above the (3,3) resonance with respect to the nucleons in the deuteron. The probability that the pion scatters in the deuteron after being produced is considerably smaller than when the energy of the pion is near resonance. It is thus expected that multiple scattering corrections to the impulse approximation decrease as the photon energy increases above the resonance. This is confirmed by the results of the present experiment and is discussed in Sec. VI. It was, thus, advantageous for this reason and for simplicity of interpretation to separate the q dependence and the energy dependence of the cross section. These dependences have different origins and are of different physical interest. The q dependence is related mainly to the probability of the deuteron staying intact in the production process and gives information about the form factor of the deuteron. The energy dependence comes mainly from the energy dependence of the amplitude for photoproduction from the nucleon. Two types of measurements were made. The first was a determination of the momentum transfer dependence of the cross section for a nearly constant photon energy. The momentum setting and the angle of the spectrometer were both varied for each value of q in order to select a constant photon energy. When the angular dependence of the energy loss of the deuterons in the target was taken into account, it was found that the photon energies had varied from 470 to 514 MeV. The second set of measurements consisted of determinations of the energy dependence of the cross section at two values of momentum transfer, $q\simeq1.76$ F^{-1} and $q\simeq1.96$ F^{-1}. In this procedure the momentum setting of the spectrometer was kept constant as the spectrometer was rotated in angle to accept recoil deuterons from photoproduction at varying photon energies.

Absolute cross sections were determined by compar-

[10] L. Auerbach, G. Bernardini, I. Filosofo, A. Hanson, A. Odian, T. Yamagata, in *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. 2, p. 291.
[11] G. Pugh, R. Gomez. D. Frisch, and G. Janes, Phys. Rev. **105**, 982 (1957).
[12] Y. Goldschmidt-Clermont, L. Osborne, and M. Scott, Phys. Rev. **97**, 188 (1955).
[13] D. C. Oakley and R. L. Walker, Phys. Rev. **97**, 1283 (1955).
[14] R. H. Capps, Phys. Rev. **106**, 1031; **108**, 1032 (1957).
[15] L. Hyman, R. Ely, D. Frisch, and M. Wahlig, Phys. Rev. Letters **3**, 93 (1959).
[16] W. K. H. Panofsky and E. A. Alton, Phys. Rev. **110**, 1155 (1958).
[17] J. I. Friedman, Phys. Rev. **116**, 1257 (1959).
[18] R. M. Friedman and K. M. Crowe, Phys. Rev. **105**, 1369 (1957).

ing the deuteron counting rates to counting rates of recoil protons from elastic electron-proton scattering. Absolute cross-sections for elastic electron proton scattering have been previously measured.[19] The value of the electron–proton cross section used to normalize the deuteron counting rates of Run I was $(9.70\pm0.068)\times10^{-31}$ cm²/sr for detecting the recoil proton. In Run II the $q=2.16$ F^{-1} deuteron point was normalized using proton recoils detected at $\theta=59°.4$; the value of the proton cross section employed was $(5.65\pm0.4)\times10^{-31}$ cm²/sr. The $q=2.74$ F^{-1} point was normalized using proton recoils at $\theta=48°.5$; the value of the proton cross section employed was $(1.64\pm0.11)\times10^{-31}$ cm²/sr. An examination of the pulse height distributions resulting from protons and deuterons indicated, for both runs, that the detection efficiency was essentially unity for both types of particles to within an error of a few percent.

V. RESULTS OF FORM FACTOR MEASUREMENTS

The measured cross sections are given in Table I. The errors listed were compounded from statistical errors, the error in the proton cross section used to normalize the data, and the errors in estimating the continua beneath the deuteron peaks in the deuteron pulse height distributions. The errors from the continuum subtractions were estimated to be about 10%. The errors given in Table I represent standard deviations.

TABLE I. Numerical results of the present measurements of the square of the structure form factor of the deuteron, F_{eff}^2, defined in the text. The laboratory angle of deuteron recoil, the gamma ray energy, and the momentum transfer to the deuteron (in reciprocal F) is given as well as the measured cross section. The listed uncertainties include not only those contributions from statistical fluctuations but, in addition, a 7% uncertainty in our knowledge of the electron-proton scattering cross sections[a] and a 10% contribution from uncertainties in determinations of the backgrounds under the deuteron peaks. The data of Run II have uncertainties, from all effects combined, of 10%. Some of the data, as noted, is included in Table II for comparison. The percentage spread of gamma-ray energies that contributes to each point is approximately 5% from variation of the kinematics over the acceptance angle of the spectrometer, and, from all effects combined, from 6 to 6.3%. These numbers represent the full width at half maximum of the relevant gamma-ray spectrum (cf. also Fig. 1).

θ_d lab	ν (MeV)	q (F^{-1})	F_{eff}^2	$(d\sigma/d\Omega)_d$ (μb/sr) center of mass
62.0	516	1.74	0.084±0.017	1.07±0.22
58.5	508	1.94	0.044±0.010	0.62±0.14
58.5	508	1.94	0.044±0.008	0.63±0.12
56.0	468	1.94	0.049±0.010	1.16±0.24
54.4	491	2.16	0.051±0.0053	0.99±0.10[b]
50.4	482	2.34	0.031±0.0043	0.66±0.092
46.2	473	2.56	0.023±0.0032	0.55±0.077
42.4	470	2.74	0.020±0.0028	0.47±0.066
42.4	470	2.74	0.019±0.0019	0.45±0.045[b]

[a] See C. de Vries, R. Hofstadter, and R. Herman, Phys. Rev. Letters **8**, 381 (1962); and reference 19.
[b] Run II.

[19] F. Bumiller, M. Croissiaux, E. Dally, and R. Hofstadter, Phys. Rev. **124**, 1623 (1961).

TABLE II. Numerical results of the present measurements at two essentially constant values of q. The table gives the quotient of the measured value of F_{eff}^2, and the square of the form factor, $F_{\text{R.C.}}^2$, predicted for the deuteron, assuming a repulsive core neutron-proton potential. The great deviations between theory and experiment are discussed in the text. Other quantities, including the errors, are discussed in the caption to Table I. Some of the data from that table, as noted, are included here for comparison. Definitions of F_{eff}^2 and $F_{\text{R.C.}}^2$ and discussion of the results are included in the text (cf. also Fig. 2).

Degrees lab	ν (MeV)	q (F^{-1})	$(F_{\text{eff}}^2/F_{\text{R.C.}}^2)$	F_{eff}^2	$(d\sigma/d\Omega)_d$ (μb/sr) center of mass
40.0	302	1.82	0.21±0.027	0.019±0.0025	2.34±0.30
42.5	313	1.81	0.28±0.036	0.025±0.0032	3.04±0.39
45.0	328	1.80	0.32±0.042	0.029±0.0038	3.15±0.41
47.5	344	1.78	0.32±0.044	0.030±0.0040	2.86±0.39
50.0	362	1.77	0.35±0.047	0.034±0.0046	2.78±0.37
52.5	383	1.76	0.39±0.057	0.039±0.0057	2.26±0.33
55.0	408	1.76	0.46±0.071	0.045±0.0070	2.00±0.31
58.5	456	1.75	0.68±0.12	0.068±0.0150	1.58±0.29
62.0	516	1.74	0.81±0.16	0.084±0.0170	1.07±0.22[a]
40.0	330	2.00	0.25±0.030	0.015±0.0018	1.67±0.20
42.0	339	1.99	0.28±0.039	0.017±0.0025	1.82±0.27
44.0	351	1.98	0.31±0.045	0.019±0.0028	1.80±0.26
45.0	358	1.97	0.33±0.040	0.021±0.0025	1.83±0.22
46.0	366	1.97	0.39±0.057	0.025±0.0037	2.02±0.30
47.5	378	1.97	0.35±0.043	0.023±0.0028	1.60±0.20
48.0	380	1.97	0.33±0.055	0.021±0.0035	1.42±0.24
50.0	397	1.96	0.44±0.055	0.029±0.0037	1.51±0.19
53.0	430	1.95	0.53±0.096	0.034±0.0062	1.24±0.23
56.0	468	1.95	0.74±0.14	0.049±0.0095	1.16±0.24[a]
58.5	508	1.94	0.64±0.13	0.044±0.0086	0.62±0.14[a]
58.5	508	1.94	0.64±0.10	0.044±0.0070	0.63±0.11[a]

[a] Data appears also in Table I.

In Fig. 1 the points

$$F_{\text{eff}}^2=\frac{(d\sigma/d\Omega)_{\text{exp}}}{\frac{2}{3}|\mathbf{K}^{(+)}|^2+|L^+|^2}$$

are plotted for various momentum transfers, where $(d\sigma/d\Omega)_{\text{exp}}$ represents the experimental cross sections for $\gamma+d\rightarrow\pi^0+d$ and the values for $\frac{2}{3}|\mathbf{K}^{(+)}|^2+|L^{(+)}|^2$ are taken from reference 3. The quantity F_{eff}^2 can be regarded as the square of the effective form factor for the process and can be related to the form factors of

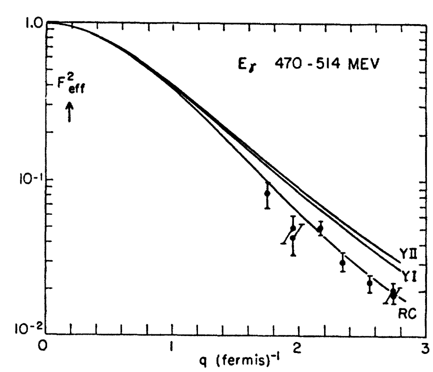

FIG. 1. Comparison of the measured values of F_{eff}^2 for the deuteron with the predictions based on a repulsive core (R.C.) and two Yukawa-type neutron-proton potentials. The data are from Table I and a discussion of the results displayed here is given in the caption to that table.

the deuteron. It is shown in reference 3 that

$$F_{\text{eff}}^2 \simeq F_0^2 + 4\epsilon^2 G^2_{uw} - \frac{[2|\mathbf{K}^{(+)}\cdot\hat{q}|^2 - \frac{2}{3}|\mathbf{K}^{(+)}|^2]}{\frac{2}{3}|\mathbf{K}^{(+)}|^2 + |L^+|^2} \times[\sqrt{2}\epsilon(1-\tfrac{1}{4}\sqrt{2})F_{uu}G_{uw} + \epsilon^2 G_{uw}^2] - \tfrac{4}{3}\sqrt{2}\epsilon^3|K^{(+)}|^2 G_{uw}^2.$$

To define the quantities in the foregoing expression it is useful to introduce the expression for the deuteron wave function. The wave function is given by

$$\psi_d(\mathbf{r}) = \frac{1}{(4\pi)^{1/2}}\left[(1-\epsilon^2)^{1/2}\frac{u(r)}{r} + \epsilon\frac{w(r)}{r}S_{12}\right]\chi_t,$$

where $(1-\epsilon^2)^{1/2}[u(r)/r]\chi_t$ is the S wave part; $\epsilon[w(r)/r] \times S_{12}\chi_t$ the D-wave part; $S_{12}=3(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$, the tensor operator; χ_t is the spin function representing the triplet spin state; and ϵ^2 is the fraction of D state. The relevant deuteron from factors are defined as follows:

$$F_0 = \int_0^\infty j_0(qr/2)[(1-\epsilon^2)u^2(r) + \epsilon^2 w^2(r)]dr,$$

$$F_{uu} = \int_0^\infty j_0(qr/2)(1-\epsilon^2)u^2(r)dr,$$

$$G_{uw} = \int_0^\infty j_2(qr/2)\epsilon(1-\epsilon^2)^{1/2}u(r)w(r)dr.$$

The quantity $\mathbf{q}$ is the momentum transfer in the process and is equal to the recoil momentum of the deuteron in the laboratory system. $K^{(+)}$ and $L^{(+)}$ are the matrix elements for spin-flip and non-spin-flip transitions, respectively. It should be noted that $|K^{(+)}|^2$ and $|L^{(+)}|^2$ implicitly contain the appropriate kinematic factors which make them proportional to cross sections.

The three curves in Fig. 1 represent the predictions of F_{eff}^2 for three deuteron models calculated by H. Two of these models, the Yukawa I and II, use Yukawa-type deuteron potentials. The third model utilizes the repulsive core calculated by Gartenhaus.[20] The properties of these models have been previously discussed.[21]

VI. ENERGY DEPENDENCE OF THE CROSS SECTION AT CONSTANT MOMENTUM TRANSFERS

In this set of runs the cross section was measured for photon energies from about 300 to 500 MeV at nearly constant momentum transfers of 1.76 and 1.96 F^{-1}. The purpose of these runs was to investigate whether the energy dependence of this process is consistent with that predicted by the impulse approximation. Because most of these measurements were made with photon energies relatively close to the (3,3) resonance a somewhat simpler analysis was performed on all data taken at energies of 460 MeV or less. From an earlier impulse approximation calculation by Chew and Lewis[22] the cross section is given by

$$(d\sigma/d\Omega)_{\gamma+d} = \{\tfrac{2}{3}|K_1^+ + K_2^+|^2 + |L_1^+ + L_2^+|^2\}W_d F_0^2.$$

K and L are the matrix elements for spin-flip and non-spin-flip transitions and the subscripts 1 and 2 refer to the two nucleons in the deuteron; the superscript + indicates that only the isotopic scalar part of the photoproduction matrix element contributes to this process because the deuteron is an isotopic singlet; W_d is the density-of-states factor. The quantity F_0^2 appears in the expression for $(d\sigma/d\Omega)_{\gamma+d}$ instead of F_{eff}^2 because the D-state contribution to the photoproduction was not included in this calculation. At the momentum transfers of this set of runs, 1.76 and 1.96 F^{-1}, F_0^2 and F_{eff}^2 differ at most by about 1 and 5%, respectively. In the following discussion F_0^2 will be replaced by F_{eff}^2. The cross section for π^0 photoproduction from the proton is similarly given by

$$(d\sigma/d\Omega)_{\gamma+p} = \{|K_p|^2 + |L_p|^2\}W_p.$$

The dispersion relation description of the matrix elements shows that $K_1^+ = K_2^+$ and $L_1^+ = L_2^+$ and that to a very good approximation $K_1^+ = K_p$ and $L_1^+ = L_p$. With the omission of the S-wave phase shifts and the small P-wave phase shifts in the evaluation of the matrix elements, the two cross sections evaluated in the center-of-mass system can be written as follows:

$$(d\bar{\sigma}/d\Omega)_{\gamma+d} = (8/3)|M(\nu)|^2[2+5\sin^2\theta](\bar{W}_d/v_d')F_{\text{eff}}^2,$$
$$(d\bar{\sigma}/d\Omega)_{\gamma+p} = |M(\nu)|^2[2+3\sin^2\theta](\bar{W}_p/v_p'),$$

where θ is the angle in the center-of-mass system; $\bar{W}_d = \bar{P}_d^2/(\bar{v}_d + \bar{v}_{\pi^0})$, where $\bar{P}_d$ is the deuteron momentum in the final state; $\bar{v}_d$ and $\bar{v}_{\pi^0}$ are the deuteron and pion velocities in the final state, all evaluated in the center-of-mass system; and $v_d' = P_d'/M_d + c$, where P_d' is the initial momentum of the deuteron in the center-of-mass system. $|M(\nu)|^2$ represents the energy dependence of the square of the matrix element, for a constant momentum transfer. The energy dependence of $(d\bar{\sigma}/d\Omega)_{\gamma+d}$ can thus be predicted in terms of that of $(d\bar{\sigma}/d\Omega)_{\gamma+p}$. In order to determine whether the former has the correct energy dependence F_{eff}^2 can be evaluated as a function of photon energy ν. The quantity F_{eff}^2, which is related to the form factors of the deuteron, should mainly be a function of momentum transfer and have a dependence on energy of only a few percent. Any energy dependence greater than this would indicate corrections to the impulse approximation.

In the photoproduction from the deuteron the major energy dependence is a function of the energy in the center-of-mass of the photon and the nucleon from which production takes place. Because of the internal momentum of the nucleon in the deuteron there is a spread in this energy for a given incident photon energy. This is an important effect in the resonance region. H has pointed out, however, that the matrix element is sharply peaked for internal momenta $P=0$ and $P=-q/2$, and that the average of the matrix elements for these two internal momenta is a reasonable approxi-

[20] S. Gartenhaus, Phys. Rev. **100**, 900 (1955).
[21] J. A. McIntyre and S. Dhar, Phys. Rev., **106**, 1074 (1957).
[22] G. F. Chew and H. W. Lewis, Phys. Rev. **84**, 779 (1951).

mation. Following this point of view,

$$F_{\text{eff}}^2=\frac{d\bar{\sigma}}{d\Omega}(\nu,\theta)\big|_{\gamma+d}\frac{3}{8}\left[(|M|^2)_{\text{Av}}(2+5\sin^2\theta)\frac{\bar{W}_d}{v_d'}\right]^{-1},$$

$$(|M|)_{\text{Av}}=\frac{1}{2}\left[\left\{\frac{v_p(\nu_1)}{\bar{W}_p(\nu_1)}\frac{(d\bar{\sigma}/d\Omega)(\theta,\nu_1)|_{\gamma+p}}{(2+3\sin^2\theta)}\right\}^{1/2}+\left\{\frac{v_p(\nu_2)}{\bar{W}_p(\nu_2)}\frac{(d\bar{\sigma}/d\Omega)(\theta,\nu_2)|_{\gamma+p}}{(2+3\sin^2\theta)}\right\}^{1/2}\right],$$

where $\nu_1=\nu$ and $\nu_2=\nu+\mathbf{K}\cdot\mathbf{q}/2M_p-q^2/8M_p$; ν and K are the laboratory energy and momentum of the photon and M_p is the nucleon mass. The cross sections for π^0 photoproduction from the proton used to evaluate F_{eff}^2 are taken from least-squares fits[23] to a number of different measurements. The cross sections at 90° were used since these are least sensitive to the small amount of S wave—P wave interference in the production. This method of analysis has been compared with the results of the H calculation in the energy region where the simplifying approximations are least valid. At a photon energy of 460 MeV the calculated cross sections differ from the dispersion relation results by less than 5%. Inasmuch as the dispersion relation calculation has ambiguities which are greater than this discrepancy the above method is certainly satisfactory for the analysis of the constant q measurements for photons energies of about 450 MeV or less.

Since the elastic electron-deuteron scattering results are consistent with a repulsive core model of the deuteron, it is instructive to compare the experimental values of F_{eff}^2 with those predicted from this model. In Fig 2 values of (F_{eff}^2) exp$/F_{\text{R.C.}}^2$ are plotted as a function of photon energy for constant momentum transfers of about 1.76 and 1.96 F^{-1}. The results indicate that the experimental cross section is considerably smaller near resonance than that predicted by the impulse approximation. It should be noted that this conclusion is independent of the deuteron model chosen since the results would indicate even greater deviations for deuteron potentials without a repulsive core. A previous measurement[4] made with photon energies in the interval of 250 to 300 MeV yielded a cross section considerably smaller than that predicted by theory. The present experiment, though spanning a different energy region, is in qualitative agreement with the earlier result. Chappelear[24] has considered multiple scattering corrections to the impulse approximation as applied to this process. His calculations indicate sizable reductions in the cross section, but not as large as those observed in this experiment.

The present experiment also shows that the deviations from the impulse approximation result decrease as a function of photon energy above the resonance. Both the 1.76 and 1.96 F^{-1} points show this effect in Fig. 2.

[23] K. Berkleman and J. A. Waggoner, Phys. Rev. **117**, 1364 (1960).
[24] J. Chappelear, Phys. Rev. **99**, 254 (1955).

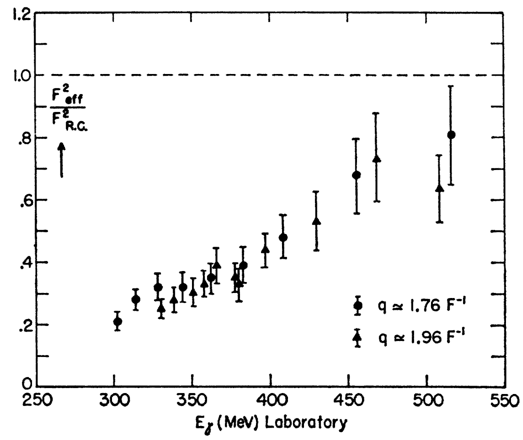

FIG. 2. The quotient of the measured F_{eff}^2 for the deuteron and the square of the form factor, $F_{\text{R.C.}}^2$, predicted for the deuteron assuming a repulsive core neutron proton potential. The data are from Table II and a discussion of the results is included in the captions to that table and to Table I and, in more detail, in the text.

DISCUSSION OF FORM FACTOR MEASUREMENTS

The results of F_{eff}^2 measured at photon energies between 514 and 470 MeV (Fig. 1) are consistent with the predictions for a repulsive core model of the deuteron. However, there is some ambiguity in the interpretation of the experimental results which is introduced by the lack of knowledge of the multiple scattering corrections at these energies. Chappalear's calculations, carried out at lower energies, give corrections that reduce the impulse approximation result. Thus, on this basis, experimental values of F_{eff}^2 greater than those predicted from a repulsive core model would contradict the electron scattering results. However, experimental results which fall along the repulsive core predictions or somewhat below them cannot be satisfactorily interpreted until reliable estimates are made of the multiple scattering corrections. The present measurements, on the average, lie somewhat below this curve; because of this and because of the experimental uncertainties in the measured values of F_{eff}^2, multiple scattering corrections as large as 20% would permit consistency with the electron scattering results.

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