Electromagnetic Interactions of Neutrinos*

LEONARD ROSENBERG
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania
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The cross sections for the process (a) $\nu+$nucleus $\rightarrow \nu+\gamma$ (virtual Coulomb)+nucleus $\rightarrow \nu+\gamma+$nucleus and the related process (b) $\gamma+$nucleus $\rightarrow \gamma+\gamma$ (virtual Coulomb)+nucleus $\rightarrow \nu+\bar{\nu}+$nucleus are calculated to lowest order in the weak and electromagnetic interactions, assuming the existence of a direct electron-neutrino weak interaction. While the relevant Feynman amplitude is given by a formally divergent expression, a finite amplitude is obtained by imposing the condition of gauge invariance. The contribution of process (b) to the neutrino luminosity of stars is worked out and found to be somewhat smaller than that of the processes: $e^-+e^+ \rightarrow \nu+\bar{\nu}$, $\gamma+e^\pm \rightarrow e^\pm+\nu+\bar{\nu}$. The present calculation is motivated by recent suggestions that stellar evolution may be significantly affected by energy lost by the star due to the emission of neutrinos. A very crude estimate is given for the contribution of the process (c) $\gamma+\gamma \rightarrow \nu+\bar{\nu}+\gamma$ to the neutrino luminosity. It is pointed out that a calculation of process (b) which exists in the literature is non-gauge-invariant.

I

IT has recently been suggested[1] that the direct electron-neutrino interaction implied by the current-current theory of weak interactions[2] gives rise to processes which may significantly increase the "neutrino luminosity" of stars, thereby affecting stellar evolution. One such process would be $\gamma+\gamma \rightarrow \nu+\bar{\nu}$. This reaction, however, has been shown to be forbidden if the weak interaction is strictly local.[3] In fact, in the V-A theory,[2] the most general effective local interaction Lagrangian which is bilinear in the neutrino field and in the electromagnetic field is of the form

$$\mathcal{L}_{\text{eff}}(x)=\lambda\bar{\psi}_\nu(x)\gamma^\mu(1+\gamma^5)\psi_\nu(x)G_{\mu\sigma}(x)\partial_\rho F^{\rho\sigma}(x), \quad (1)$$

where $F^{\rho\sigma}(x)$ is the electromagnetic field tensor and $G_{\mu\sigma}(x)\equiv\epsilon_{\mu\sigma\xi\eta}F^{\xi\eta}(x)$. (The form with $G_{\mu\sigma}$ replaced by $F_{\mu\sigma}$ is also allowed but linear combinations of the two would violate PC invariance.) Equation (1) shows that one of the photons must be virtual since $\partial_\rho F^{\rho\sigma}$ vanishes at all x for an electromagnetic field associated with a real photon.

II

To gain an idea of the magnitude of the effective coupling parameter, λ, we have evaluated the Feynman amplitude, F_a, for the process depicted by the diagram in Fig. 1. The primitive Lagrangian giving rise to this process is taken to be

$$\mathcal{L}(x)=e:\bar{\psi}_e(x)\gamma^\kappa A_\kappa(x)\psi_e(x):+e:\bar{\psi}_e(x)\gamma^0 a_0(\mathbf{x})\psi_e(x):$$
$$+\frac{1}{\sqrt{2}}G:\bar{\psi}_e(x)\gamma^\mu(1+\gamma^5)\psi_\nu(x)\bar{\psi}_\nu(x)\gamma^\mu(1+\gamma^5)\psi_e(x):, \quad (2)$$

where $a_0(\mathbf{x})$ is the static Coulomb field of the nucleus.[4]

We find that

$$F_a=e^2G\frac{1}{\sqrt{2}}\frac{1}{(2k_{10})^{1/2}}(2\pi)^{-6}\varphi_0(\mathbf{k}_2)e_{1\sigma}e_{2\rho}J_\mu R^{\sigma\rho\mu}(k_1,k_2), \quad (3)$$

where e_1 and e_2 are the photon polarization vectors ($e_{2\rho}=\delta_{\rho 0}$) and the other quantities are given by

$$J_\mu=\bar{u}_{\nu,s}(p_\nu)\gamma_\mu(1+\gamma^5)u_{\nu,s_0}(p_{\nu 0}), \quad (4)$$

$$\varphi_0(\mathbf{k})=(2\pi)^{-3/2}\int d\mathbf{x}\, e^{-i\mathbf{k}\cdot\mathbf{x}}a_0(\mathbf{x}), \quad (5)$$

and[5]

$$R^{\sigma\rho\mu}(k_1,k_2)=2\int d^4q\,\text{Sp}\left[\frac{m+\gamma^\kappa q_\kappa+\gamma^\kappa k_{1\kappa}}{(q+k_1)^2-m^2+i\epsilon}\gamma^\sigma\right.$$
$$\left.\times\frac{m+\gamma^\kappa q_\kappa}{q^2-m^2+i\epsilon}\gamma^\rho\frac{m+\gamma^\kappa q_\kappa-\gamma^\kappa k_{2\kappa}}{(q-k_2)^2-m^2+i\epsilon}\gamma^\mu\gamma^5\right]. \quad (6)$$

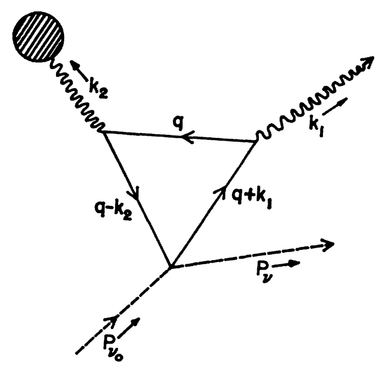

FIG. 1. Feynman diagram for the process $\nu+$nucleus $\rightarrow$ $\nu+\gamma$ (virtual Coulomb)+nucleus $\rightarrow \nu+\gamma+$nucleus. To this must be added the diagram with the momenta of the internal lines reversed.

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[1] H. Y. Chiu and P. Morrison, Phys. Rev. Letters **5**, 573 (1960).

[2] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[3] M. Gell-Mann, Phys. Rev. Letters **6**, 70 (1961).

[4] We use units in which $\hbar=c=1$; $e=(4\pi/137)^{1/2}$ and $G=1.01\times10^{-5}/M^2$, where M is the proton mass.

[5] The addition of the term corresponding to the diagram in which the momenta of the internal electron lines are reversed has the effect that the factor $\gamma^\mu(1+\gamma^5)$ is replaced by $2\gamma^\mu\gamma^5$, as in Eq. (6). Note that the expression $e_{1\sigma}e_{2\rho}R^{\sigma\rho\mu}(k_1,k_2)$ is symmetric under the interchange of the labels 1 and 2, even if one photon is real and the other is virtual.

The integral in Eq. (6) may be cast by parametrization into the form

$$R^{\sigma\rho\mu}=A_1k_{1\tau}\epsilon^{\tau\sigma\rho\mu}+A_2k_{2\tau}\epsilon^{\tau\sigma\rho\mu}+A_3k_1{}^{\rho}k_{1\xi}k_{2\tau}\epsilon^{\xi\tau\sigma\mu}+A_4k_2{}^{\rho}k_{1\xi}k_{2\tau}\epsilon^{\xi\tau\sigma\mu}+A_5k_1{}^{\sigma}k_{1\xi}k_{2\tau}\epsilon^{\xi\tau\rho\mu}+A_6k_2{}^{\sigma}k_{1\xi}k_{2\tau}\epsilon^{\xi\tau\rho\mu},\quad(7)$$

where the A_i for $i\geq 3$, are finite and are given explicitly by

$$A_3(k_1,k_2)=-16\pi^2I_{11}(k_1,k_2)=-A_6(k_1,k_2),$$
$$A_4(k_1,k_2)=16\pi^2[I_{20}(k_1,k_2)-I_{10}(k_1,k_2)]=-A_5(k_2,k_1),\quad(8)$$

with the I_{st} defined by

$$I_{st}(k_1,k_2)=\int_0^1dx\int_0^{1-x}dy\,x^sy^t[y(1-y)k_1{}^2+x(1-x)k_2{}^2+2xy(k_1k_2)-m^2]^{-1}.\quad(9)$$

On the other hand, both A_1 and A_2 are represented by formally divergent integrals and an additional restriction must be invoked to obtain meaningful results for F_a.

Such a restriction is contained in the condition of gauge invariance of F_a, which we impose in the form

$$k_{1\sigma}R^{\sigma\rho\mu}=k_{2\rho}R^{\sigma\rho\mu}=0.\quad(10)$$

Equations (10) and (7) lead to the relations

$$[-A_2+k_1{}^2A_5+(k_1k_2)A_6]k_{1\xi}k_{2\tau}\epsilon^{\xi\tau\rho\mu}=0,\quad(11)$$
$$[-A_1+(k_1k_2)A_3+k_2{}^2A_4]k_{1\xi}k_{2\tau}\epsilon^{\xi\tau\sigma\mu}=0,\quad(12)$$

so that a finite, gauge-invariant amplitude will be obtained if we make the choice

$$A_1=(k_1k_2)A_3+k_2{}^2A_4,\quad(13)$$
$$A_2=k_1{}^2A_5+(k_1k_2)A_6.\quad(14)$$

Equation (3) may be simplified with the aid of the identity

$$(af)|bcde|+(bf)|cdea|+(cf)|deab|+(df)|eabc|+(ef)|abcd|=0,\quad(15)$$

where $|abcd|\equiv a_\sigma b_\tau c_\nu d_\mu\epsilon^{\sigma\tau\nu\mu}$. This identity, along with Eqs. (7), (13), and (14) allow us to write

$$e_{1\sigma}e_{2\rho}J_\mu R^{\sigma\rho\mu}=k_1{}^2(A_3+A_5)|k_2Je_1e_2|+k_2{}^2(A_4+A_6)|k_1Je_1e_2|+[A_3(k_1J)-A_6(k_2J)]|k_1k_2e_1e_2|,\quad(16)$$

where we have used

$$(e_1k_1)=(e_2k_2)=0,\quad(17)$$

whence, also making use of Eqs. (8), the relations $k_1{}^2=0$, and

$$(k_1J)+(k_2J)=(p_{\nu_0}J)-(p_\nu J)=0,\quad(18)$$

we obtain

$$e_{1\sigma}e_{2\rho}J_\mu R^{\sigma\rho\mu}=k_2{}^2\mathfrak{F}(k_1,k_2)|k_1Je_1e_2|,\quad(19)$$

with

$$\mathfrak{F}(k_1,k_2)\equiv16\pi^2\int_0^1dx\int_0^{1-x}dy\times\frac{x(x+y-1)}{[x(1-x)k_2{}^2+2xy(k_1k_2)-m^2]}.\quad(20)$$

Equations (19), (20), and (3) determine the Feynman amplitude, F_a, which takes the form

$$F_a=e^2G\frac{1}{\sqrt{2}}\frac{1}{(2k_{10})^{1/2}}(2\pi)^{-6}\times k_2{}^2\varphi_0(\mathbf{k}_2)\mathfrak{F}(k_1,k_2)\mathbf{J}\cdot(\mathbf{e}_1\times\mathbf{k}_1).\quad(21)$$

The corresponding differential cross section, $d\sigma_a$, is then given by

$$d\sigma_a=(2\pi)^2(p_\nu{}^0)^2\sum|F_a|^2k_{10}{}^2dk_{10}d\mathbf{\Omega}_{k_1}d\mathbf{\Omega}_\nu,\quad(22)$$

where the sum indicates an average over initial spins and a sum over final spins and polarization directions. With the aid of the relations

$$\sum_{s,s_0}J_iJ_j{}^*=\frac{2}{p_{\nu_0}{}^0p_\nu{}^0}p_\nu{}^\sigma p_{\nu_0}{}^\rho[g_{\sigma i}g_{\rho j}+g_{\sigma j}g_{\rho i}-g_{\sigma\rho}g_{ij}+\epsilon_{\sigma i\rho j}],\quad(23)$$

we obtain

$$\tfrac{1}{2}\sum_{s,s_0}\sum_{e_1}|\mathbf{J}\cdot\mathbf{e}_1\times\mathbf{k}_1|^2=\frac{1}{p_{\nu_0}{}^0p_\nu{}^0}\sum_{e_1}\{(p_\nu p_{\nu_0})[\mathbf{e}_1\times\mathbf{k}_1]+2[\mathbf{p}_\nu\cdot\mathbf{e}_1\times\mathbf{k}_1][\mathbf{p}_{\nu_0}\cdot\mathbf{e}_1\times\mathbf{k}_1]\},\quad(24)$$

so that Eq. (22) becomes

$$d\sigma_a=(2\pi)^{-10}(p_\nu{}^0)^2(e^2G)^2[\mathfrak{F}(k_1,k_2)]^2[k_2{}^2\varphi_0(\mathbf{k}_2)]^2\times\tfrac{1}{2}k_{10}{}^3[1-(\hat{p}_{\nu_0}\cdot\hat{k}_1)(\hat{p}_\nu\cdot\hat{k}_1)]dk_{10}d\mathbf{\Omega}_{k_1}d\mathbf{\Omega}_\nu,\quad(25)$$

where the carets denote unit vectors.

It is seen that the $\mathfrak{L}_{\text{eff}}$ of Eq. (1) is, strictly speaking, local only if $\mathfrak{F}(k_1,k_2)$ can be well approximated by $\mathfrak{F}(0,0)=2\pi^2/3m^2$. In this case, comparison with Eq. (21) yields $\lambda=(2\pi)^{-4}e^2(G/\sqrt{2})(2\pi^2/3m^2)$.

To obtain an order of magnitude estimate of σ_a we replace $-k_2{}^2$ and $-2(k_1k_2)$ by $(p_{\nu_0}{}^0)^2=E_{\nu_0}{}^2$ in the denominator of Eq. (20) and neglect the m^2 term. For $E_{\nu_0}\gg m$ this will be grossly inaccurate only in a narrow forward cone where the differential cross section vanishes anyway. Then,

$$\mathfrak{F}(k_1,k_2)\approx16\pi^2/5E_{\nu_0}{}^2,\quad(26)$$

and with

$$-k_2{}^2\varphi_0(\mathbf{k}_2)\approx Ze/(2\pi)^{3/2}\quad(27)$$

the approximate expression for the cross section becomes, for $E_{\nu 0} \gg m$,

$$\sigma_a \approx Z^2 \left(\frac{e^2}{4\pi}\right)^3 \left(\frac{E_{\nu 0}}{M}\right)^2 \times 5 \times 10^{-42} \text{ cm}^2. \quad (28)$$

[Of course, the nuclear form factor, which we have approximated by unity in Eq. (27), will, in fact, prevent the indefinite increase of σ_a with energy.] We therefore conclude that the neutrino bremsstrahlung process considered here may be safely ignored in the analysis of high-energy neutrino experiments in which cross sections of the order of 10^{-38} cm^2 are involved.[6]

III

A perhaps more significant application of the preceding calculation is the evaluation of the contribution of the process $\gamma+\text{nucleus} \rightarrow \gamma+\gamma$ (virtual Coulomb) $+\text{nucleus} \rightarrow \nu+\bar{\nu}+\text{nucleus}$ to the neutrino luminosity of stars. The cross section, σ_b, for this process can be determined with the aid of the substitution rule. We find that

$$\sigma_b \approx Z^2(e^2/4\pi)^3(k_{10}/m)^6 \times 1.25 \times 10^{-49} \text{ cm}^2, \quad (29)$$

where we have now replaced $\mathfrak{F}(k_1,k_2)$ by its upper bound

$$\mathfrak{F}(k_1,k_2) < \mathfrak{F}(0,0) = 2\pi^2/3m^2, \quad (30)$$

and have again employed Eq. (27) for the nuclear form factor. Both approximations will be quite accurate for the photon energies which are relevant, namely $k_{10} \lesssim m$.

It is now a simple matter to calculate the contribution, $\mathcal{E}_b$, to the neutrino luminosity of a star due to the reaction under consideration. $\mathcal{E}_b$ is just the energy lost by the star due to $(\gamma+\text{nucleus} \rightarrow \nu+\bar{\nu}+\text{nucleus})$ reckoned per sec and per cm^3 and is given by (we now include factors of $\hbar$ and c):

$$\mathcal{E}_b = \sum_i \int \frac{2d\mathbf{k}}{(2\pi)^3} [e^{\hbar\omega/\kappa T} - 1]^{-1} \hbar\omega c (N_i Z_i^2) \left(\frac{\hbar\omega}{mc^2}\right)^6 \sigma_0, \quad (31)$$

with $\hbar\omega = \hbar|\mathbf{k}|c$ = photon energy, N_i = number of nuclei of atomic number Z_i per cm^3, and $\sigma_0 = 5.0 \times 10^{-56}$ cm^2. Upon evaluation of the integral we obtain

$$\mathcal{E}_b = 2.3 \times 10^{14} (\kappa T/mc^2)^{10} \text{ erg/sec-cm}^3, \quad (32)$$

where we have inserted some typical value for the factor $\sum_i N_i Z_i^2$.[7] For $T \gtrsim 10^9$ °K this gives contributions to the neutrino luminosity somewhat smaller than those calculated by Chiu and Stabler[8] for the processes

$$e^- + e^+ \rightarrow \nu + \bar{\nu} \quad (33)$$

and

$$\gamma + e^\pm \rightarrow e^\pm + \nu + \bar{\nu}. \quad (34)$$

These latter processes then appear to give the major contributions to the neutrino liminosity of stars, although the cross section, σ_c, for the reaction

$$\gamma + \gamma \rightarrow \nu + \bar{\nu} + \gamma \quad (35)$$

remains to be calculated. In this connection we estimate that $\sigma_c \approx \sigma_b/Z^2$ so that the corresponding contribution to the neutrino luminosity, $\mathcal{E}_c$ is given by

$$\mathcal{E}_c \approx \mathcal{E}_b \times (\text{photon number density}) / \sum_i N_i Z_i^2$$
$$= \mathcal{E}_b [0.2(\kappa T/\hbar c)^3] / \sum_i N_i Z_i^2 \approx \mathcal{E}_b \quad (36)$$

for[7] $\sum_i N_i Z_i^2 \approx 10^{29}$/cm^3 and $T \approx 10^9$ °K.[9]

IV

A previous calculation[10] of the pair production amplitude F_b is fundamentally incorrect since it is manifestly non-gauge-invariant. In fact, the Feynman amplitude [Eq. (1) of reference 10] corresponds to an effective Lagrangian which is obtained from Eq. (1) of the present paper by replacing the gauge-invariant factor $\partial_\rho F^{\rho\sigma}(x)$ by $A^\sigma(x)$. The amplitude will then not vanish for both photons real, in contradiction with Gell-Mann's theorem,[3] and with the result of the present paper. It might be worth emphasing that since the amplitude is given by a formally divergent expression gauge invariance must be explicitly *imposed* to obtain a unique, gauge-invariant, finite result, as is done here through Eqs. (13) and (14). The situation is analogous to the one in standard quantum electrodynamics, which leads to Ward's identity.

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I wish to express my gratitude to Professor H. Primakoff, who suggested this problem to me and with whom I had a number of valuable discussions.

[6] T. D. Lee and C. N. Yang, Phys. Rev. Letters **4**, 307 (1960); G. Danby, J-M. Gaillard, K. Goulianos, L. M. Lederman, N. Mistry, M. Schwartz, and J. Steinberger, *ibid.* **9**, 36 (1962).

[7] One usually writes $\Sigma_i N_i Z_i^2 = 6 \times 10^{23} \rho/\nu$, where ρ is the stellar density and $1/\nu = \Sigma_i Z_i^2 C_i/A_i$; C_i is the weight concentration of the ith constituent. We choose $\rho = 10^5$ g/cm^3 and $1/\nu = 5$ g^{-1}.

[8] H. Y. Chiu and R. Stabler, Phys. Rev. **122**, 1317 (1961).

[9] An estimate of $\mathcal{E}_c$ has also been made by Chiu and Morrison with the result (quoted in reference 1) that $\mathcal{E}_c$ is comparable with the contribution due to the process $e^- + e^+ \rightarrow \nu + \bar{\nu}$.

[10] S. G. Matinyan and N. N. Tsilosani, Zh. Eksperim. i Teor. Fiz. **41**, 1681 (1961) [translation: Soviet Phys.—JETP **14**, 1195 (1962)]. We wish to thank Professor M. Ruderman for calling this paper to our attention.