Scattering of 8-GeV μ Mesons on Electrons*

G. BACKENSTOSS, B. D. HYAMS, G. KNOP,† P. C. MARIN,‡ AND U. STIERLIN
CERN, Geneva, Switzerland
(Received 25 October 1962)

The scattering of polarized 8-GeV μ^- mesons on electrons in magnetized iron has been measured. The spin dependence of the cross section gives the helicity for μ^- mesons from π^- decay. The cross section for producing knock-on electrons up to 3.4 GeV is compared with the value calculated in first order, corrected for the emission of hard photons. Measurement and calculation of the cross section agree within the experimental error, which is $\pm 7\%$.

I. INTRODUCTION

IN the collision of a μ meson with energy $E_\mu \gg m_\mu$ with an electron initially at rest, the maximum transferable energy to the electron is

$$E_{\max} \approx E_\mu \left(\frac{2E_\mu m_e}{m_\mu^2 c^2 + 2m_e E_\mu} \right).$$

The four-momentum transfer to the electron is $(2m_e E_{\max})^{1/2}$. It is, thus, necessary to have μ mesons with energies in the multi-GeV range to observe momentum transfers corresponding to distances of the order of one fermi.

The CERN proton synchrotron may be used to produce an intense, pure beam of high-energy μ mesons. We have used it to measure muon-electron scattering of 8-GeV μ mesons.

The cross section for the production of a knock-on electron of energy E_e by a μ meson is given to first order by[1]

$$\sigma(E_e, \mathbf{P}_e \cdot \mathbf{P}_\mu) dE = \frac{2\pi r_0^2 m_0 c^2}{\beta_\mu^2 E^2} \left[1 - \beta_\mu^2 \left(\frac{E}{E_m} \right) + \frac{1}{2} \left(\frac{E}{E_\mu} \right)^2 - \mathbf{P}_e \cdot \mathbf{P}_\mu \frac{E}{E_\mu} \left(1 - \frac{E}{E_m} + \frac{E}{2E_\mu} \right) \right] dE, \quad (1)$$

where $\mathbf{P}_e$, $\mathbf{P}_\mu$ are the electron and μ-meson polarization vectors.

We have carried out an experiment designed primarily to measure the spin dependence of this cross section, to determine the helicity of the μ mesons from π-meson decay. The sign of the helicity thus determined has been briefly reported elsewhere[2]; a more detailed account of some aspects of that experiment is given here.

We have also measured, with the same arrangement, the differential cross section for μ-e scattering for knock-on electrons in the range 1.0–3.4 GeV ($E_{\max} = 3.4$ GeV), thus observing four momentum transfers up to 60 MeV/c. At this momentum transfer the scattering cross section might be expected to deviate by about 7% from point charge scattering for a deviation from quantum electrodynamics at 0.7 F.[3] Our determination of the cross section is limited by the precision with which the electron energy could be measured, and amounts to about $\Delta\sigma/\sigma = 7\%$ for 3-GeV electrons.

Radiative corrections to the cross section given by Eq. 1 including hard photons have been calculated by Rollnik *et al.*[4] They do not include the radiation

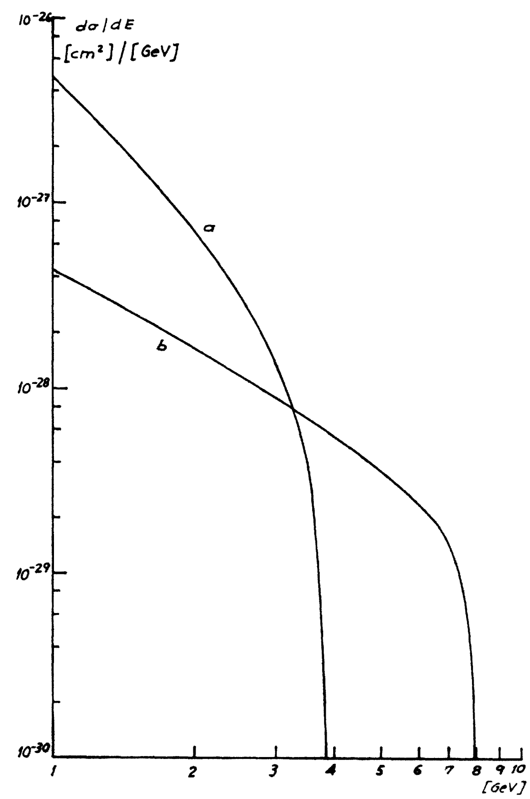

FIG. 1. Calculated cross sections as a function of the transferred energy. (a) $\sigma(\mu,e)$ from Eq. (1); (b) $\sigma(\mu,\gamma)$ from Eq. (2).

* Preliminary results have been reported in the *Proceedings* of the *Aix-en-Provence International Conference on Elementary Particles* (C. E. N. Saclay, France, 1962), Vol. I, p. 147.

† Present address: Physikalisches Institut der Universität, Bonn, Germany.

‡ Present address: Laboratoire des Hautes Energies, Orsay, Seine et Oise, France.

[1] A. M. Bincer, Phys. Rev. **107**, 1434 (1957). A. E. Alikhanov and V. A. Lyubimov, Zh. Eksperim. i Teor. Fiz. **36**, 1334 (1959) [translation: Soviet Phys.—JETP **9**, 946 (1959)].

[2] G. Backenstoss, B. D. Hyams, G. Knop, P. C. Marin, and U. Stierlin, Phys. Rev. Letters **6**, 415 (1961).

[3] S. D. Drell, Ann. Phys. (New York) **4**, 75 (1958).

[4] V. Gorgé, M. Locher and H. Rollnik (to be published).

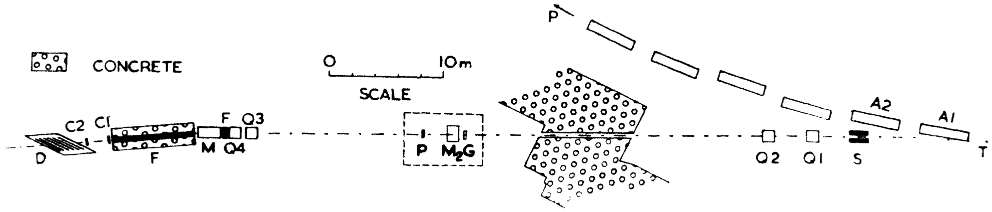

FIG. 2. Experimental arrangement. (a) π-μ beam. P: circulating proton beam; A: synchrotron magnets; S: Pb collimator; Q: quadrupole lenses; M: dispersing magnets; F: graphite filter; C: counters; D: target and detector. (b) electron beam (the elements added for these measurements are shown inside the dotted line). G: graphite; P: lead converter.

from the μ mesons; however, this is estimated to be negligible compared with that from the electron. Figure 1 shows the theoretical cross section without radiative corrections.

In the course of the experiment, the bremsstrahlung cross section for the radiation of quanta in the range 4.0–7.0 GeV by μ mesons on iron has been determined. These measurements have about the same accuracy as existing published calculations on μ-meson bremsstrahlung,[5] which give a cross section (Fig. 1)

$$\sigma(E_\gamma,E_\mu)dE_\gamma=\alpha\frac{N}{A}Z^2r_e^2\left(\frac{m_e}{m}\right)^2\frac{dE_\gamma}{E_\gamma}F(E_\mu,E_\gamma,r_n) \quad (2)$$

for the radiation of a quantum of energy E_γ by a μ meson of energy E_μ, where $F(E_\mu,E_\gamma,r_n)$ is a slowly varying function of E_γ, and depends on the nuclear radius r_n. For this process only small four-momentum transfers play a significant role, so it is not as ensitive test for "structure" of the μ meson.

II. EXPERIMENTAL ARRANGEMENT

1. Outline of Arrangement

The general layout of the apparatus is shown in Fig. 2. A collimated beam of negative π mesons is produced, and the high-energy μ mesons arising from the π decays in the forward direction are scattered on electrons in a target D. The knock-on electron energy is measured by using the target material as a total absorption detector of electron-photon cascades, and recording the light signal generated in sheets of plastic scintillator interleaved with sheets of magnetized iron. This arrangement is unable to distinguish between the showers produced by knock-on electrons and the showers produced by bremsstrahlung quanta. However, as shown in Fig. 1, Eqs. (1) and (2) predict that there is a range of shower energies above the maximum transferable energy to an electron to which only the bremsstrahlung quanta can contribute. At lower energies, the bremsstrahlung contribution is very small.

The following experimental conditions had to be satisfied in order to compare the experiment with the predictions of Eqs. (1) and (2). A beam of μ mesons was required having less than 10^{-4} π-meson contamination. It was essential that only the μ meson initiating a shower was present in the time during which the shower signal was recorded, since a second μ meson would add to the signal recorded. The μ-meson energy had to be known, and a precise calibration had to be made for the electron shower energy, since the scattering cross section is so rapidly varying a function of energy. The efficiency of the counter for recording showers of a given energy had to be determined. The signal recorded for a knock-on electron of a fixed energy had to be very insensitive to reversal of the sense of magnetization of the iron to avoid spurious effects simulating a spin dependence of the scattering cross section. The solution of these problems is elaborated below.

2. Beam Layout for μ Mesons

To produce a μ-meson beam (Fig. 2), a beam of negative particles was focused on the collimator slot S. It was emitted at 0° from the target T, a 0.5-mm Al rod placed in the circulating proton beam, and had a mean energy of 12.1 ± 0.1 GeV/c and a spread of 0.3 GeV/c. The beam emerging from S was collimated by quadrupoles Q_1Q_2; it contained about 8×10^5 particles per pulse. These particles, mostly π mesons, had a flight path of about 42 m in air. The beam of μ mesons decaying in the forward direction was made convergent by quadrupoles Q_3 and Q_4, after which it was dispersed by a magnet M to select the required momentum band of μ mesons which it deflected by 84 mrad. A filter column F of 7.2 m of graphite of 30×30 cm² cross section, embedded in baryte concrete blocks, absorbed out the π mesons and other strongly interacting particles in the beam, leaving a purified beam of μ mesons (see Sec. III.2).

Counters C_1 and C_2, both effectively 7×7 cm² in area, defined the μ-meson beam, which then traversed the combined target and detector D. Counters C_1 and C_2 gave a coincidence rate of about 2000 counts per machine pulse for 2×10^{11} circulating protons in the synchrotron.

The μ mesons emerging from the graphite filter and accepted by the counter had a relatively wide angular divergence. For this reason, a direct measurement of

[5] B. Rossi, *High-Energy Particles* (Prentice Hall, Inc., Englewood Cliffs, New Jersey, 1956), p. 61.

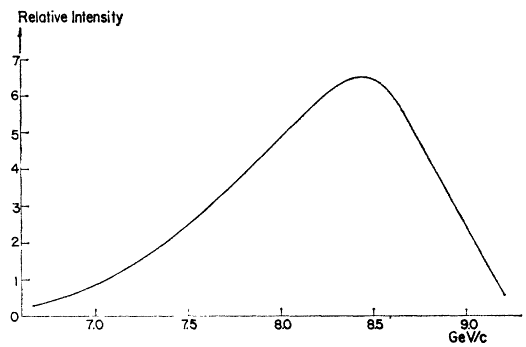

FIG. 3. Momentum spectrum of the muons hitting the detector system, calculated with a Monte Carlo program.

their energy spectrum would have been a difficult task. Therefore we have calculated their energy distribution with the aid of a computer program, taking into account the kinematics of the π decay, the decay path, the acceptance of the particles by the quadrupoles and magnet, the scattering and energy loss in the graphite filter, and the geometry of the counters C_1C_2. This calculated energy spectrum is shown in Fig. 3.

To ensure that only one μ meson was traversing the counter while a shower was being recorded, an anticoincidence counter A, 50 cm high and 80 cm wide, was placed in front of the detector, centered on counter C_2. Counter A had an aperture 7×7 cm^2 at its center so that beam particles were not vetoed. To ensure that only one beam particle entered during the time of recording, counter C_2 was used to provide an anticoincidence signal if its signal exceeded 1.8 times the most probable signal for a single particle.

3. Beam Layout for Calibration Electrons

A beam of electrons of known energy was used for calibrating the detector's output response to high-energy knock-on electrons from μ mesons. This was produced from the 12-GeV negative meson beam in the following way (Fig. 2). Twenty cm of graphite was placed in the beam at G, a $0.3\times5\times5$ cm^3 sheet of lead was placed at P, and the magnet M_2 was energized. Q_1 and Q_2 were energized so as to image the π-meson beam on the lead P. π^0 mesons produced in G gave γ rays which struck P and gave electron-positron pairs. The graphite filter F was removed. Quadrupoles Q_3 and Q_4 produced an image of P on the counters C_1C_2 for the electrons of each required energy. The magnet M_1 was energized so as to deflect these electrons through 84 mrad, the constant angle which the counter made to the negative meson beam. Magnet M_2 was necessary to sweep the noninteracting part of the charged beam and all the charged secondary particles produced in G away from the electron beam. This arrangement gave around 2 electrons per machine pulse in a 10% momentum band. There was a negligible contamination of other particles in the beam. Although this system gave a very much smaller electron flux than we obtained by other methods, it had the advantage that it used the identical electronics selection system for calibration as for the experiment, and magnet M_1 was used for both the μ-meson and electron-energy determination in the same geometry.

4. Detector

The detector consists of 20 sheets of 1-cm-thick iron, which formed part of a magnetic circuit, and which could be magnetized to 2×10^4 G. They are interleaved with 20 sheets of 1-cm-thick plastic scintillator D_1–D_{20}. The sheets are 30×40 cm^2 in area, and are inclined at an angle of 30° with respect to the beam, presenting a surface 30 cm high and 20 cm wide normal to the beam. The photomultiplier voltages were measured and adjusted to 0.1%, and the gain of the counters was equalized to within 1% by calibration with the μ-meson beam. The electronic circuitry was so arranged that only showers initiated in the first 10 counters D_1–D_{10} were recorded, so that all showers recorded were totally absorbed in the counter.

5. Electronics

A somewhat simplified block diagram of the electronics is shown in Fig. 4. The μ-meson flux accepted is recorded on a scaler which registers the coincidences C_1,C_2(providing C_2 is not too big), $\bar{A}$ referred to as Coinc. 1. The signals from the first 10 scintillation counters D_1–D_{10} were added and sent through discriminator 3, which was set to accept showers of more than 0.35 GeV and responded to the passage of about 2% of the μ mesons traversing the counter. The signals from all 20 counters were also added together and sent to a 200-channel pulse-height analyzer (PHA) via a fast linear "gate" circuit. The pulses were recorded by opening the fast gate when, and only when, a coincidence pulse, Coinc. 1, was in coincidence with a large pulse from the first 10 counters. The linear "gate" transistor circuit was developed for this experiment and is linear to within 1% over a range of more than 10 to 1.[6] The gate was fully open for some 50 nsec but passed a measurable signal over an interval of about 150 nsec.

The anticoincidence channels had to meet rather special requirements. The signal from C_2 was required to give an anticoincidence if either one large pulse arrived, or if two normal pulses arrived within 75 nsec of each other, the time for which the gate was not fully closed. This was achieved with a transistor circuit[6] measuring the charge arriving at gate 1 within a time interval of ±75 nsec around the arrival time of each beam particle. The channel $\bar{A}$ presented the problem that a flux of some 2×10^4 μ mesons, particles

[6] C. Carter, B. Hyams, and U. Stierlin, CERN Internal Report NP 61–18 (unpublished).

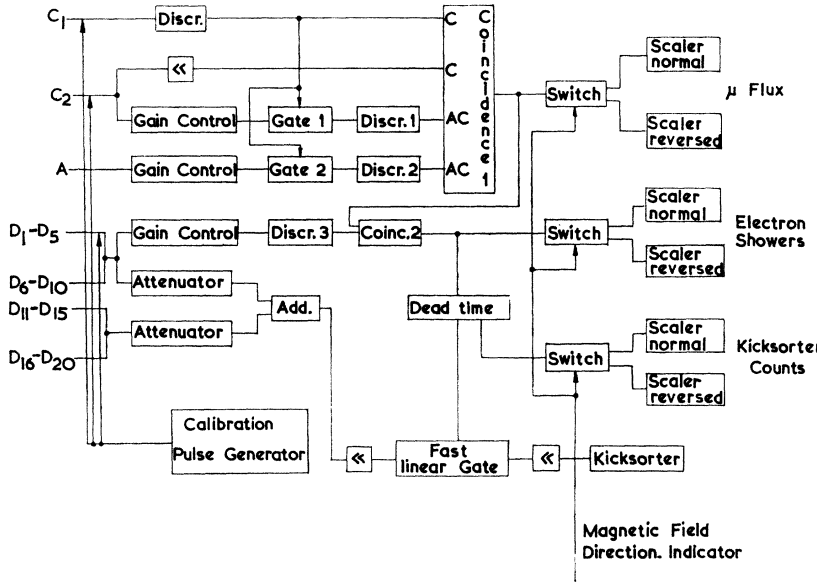

FIG. 4. Block diagram, simplified. C: defining counters; A: beam surrounding anticoincidence shield; D_1-D_{20}: scintillation counters of the detector.

with unwanted momenta, traversed these counters each machine pulse, which lasted for some 30 msec. Thus, the duty cycle for the arrival of anticoincidence pulses reached to the order of 30% during part of the machine pulse. To achieve efficient operation a circuit arrangement, gate 2,[6] was devised, which passed anticoincidence pulses when, and only when, a coincidence pulse was present. This reduced the fraction of time during which anticoincidence pulses reached the coincidence circuit from 30% to a maximum of about 1%.

The "dead-time" circuit[6] ensured that independently of the rate of arrival of pulses at the fast gate after a pulse was accepted, a minimum waiting time, just exceeding the PHA recovery time, elapsed before the gate would transmit the next signal.

The loss of counts due to the inoperative time of the PHA was measured and the observed cross section was corrected accordingly. In order to monitor the stability of the gain of the electronic system throughout the run, calibration pulses with stabilized amplitude were injected at the outputs of counters 1, 2, and D_2 throughout the run. About 10 such pulses were recorded in channel 90 of the PHA per machine pulse. Thus all the scattering data were recorded simultaneously with a calibration signal which made it possible to ensure that the electronic gain of the system was constant within $\pm 0.5\%$.

6. Measures Taken to Minimize Systematic Asymmetries

The counter assembly was designed to minimize systematic asymmetries capable of simulating a real spin dependence of the scattering. The magnetized iron plates formed part of a closed magnetic circuit to reduce the stray field. The photomultipliers were well shielded against magnetic field and oriented in groups canceling systematic magnetic asymmetries.

The μ mesons and a calculable part of the electron shower they initiate are deflected systematically up and down with magnetic field reversal. The response of the counters to particles traversing them at different points along a vertical axis was measured. From this measurement it was calculated that the asymmetry so introduced was negligible compared with the effect sought.

The electronic system recorded the knock-on energy spectrum for "normal" and "reversed" sense of magnetization on two halves of a split-memory PHA, and the μ-meson fluxes on separate scalers. The magnetization was reversed every minute by a clock in order to reduce the effect of drifts in sensitivity. The connection of the contacts of the relay measuring the direction of magnetization of the iron, and addressing the scaler and PHA pulses, was reversed after each hour's data was recorded. Thus, any appreciable purely electronic asymmetry could be readily measured and was averaged to zero during data collection.

III. PERFORMANCE OF THE APPARATUS

1. Sandwich Counter Performance

The response of the sandwich counter to incident electrons of selected energy is shown in Fig. 5. It is seen that the mean voltage output signal is linearly proportional to the electron energy within 1% over the energy range 1 GeV to 6 GeV.[7] A nonlinearity above 6 GeV is accounted for by nonlinearity of the photomultipliers at these signal levels.

The response of the counter to the μ-meson beam is shown in Fig. 6. This was measured by accepting all μ mesons in the beam, by setting the bias on discriminator

[7] A more detailed description of the performance of the counter will be published in Nucl. Instr. Methods.

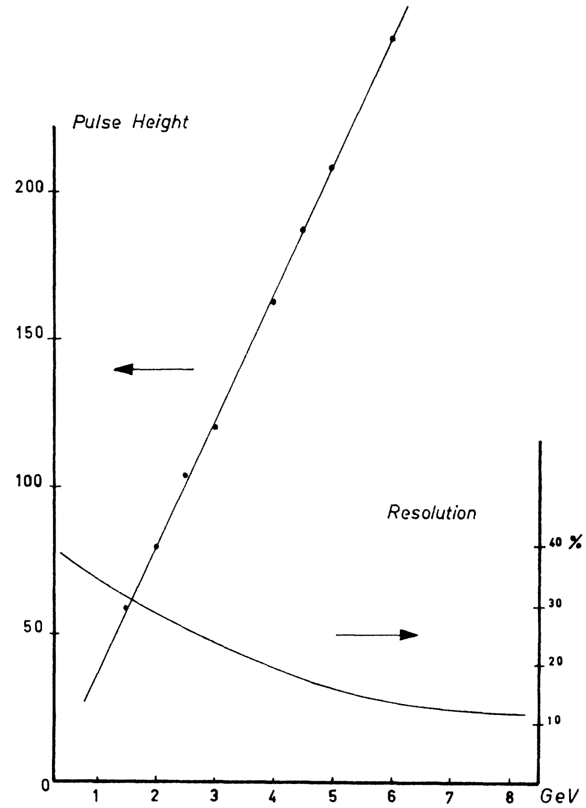

FIG. 5. Energy response and resolution of the total absorption scintillation detector as a function of the incoming electron energy. The resolution shown is the full width at half-height.

3 to zero. The width of the peak is mainly due to energy loss fluctuations[8] which are larger than the instrumental width from statistical fluctuations in the number of photoelectrons counted. The mean energy loss of a μ meson in traversing the counter is calculated to be 600 MeV. The data plotted in Fig. 6 show that an electron of 600 MeV would give only 80% of the voltage signal produced by a μ-meson traversal of the counter. This may be accounted for qualitatively by the different mechanism for the energy transfer. For μ mesons, e.g., the ratio of track length in scintillator to track length in iron is in the ratio of their respective thicknesses. However, electron-positron tracks in a cascade shower essentially all originate in the iron, and thus the ratio of track length in scintillator to that in iron is less than for μ mesons.

The effective thickness of the counter as a target is dependent on the energy bias imposed by the discriminator 3 and the rate of growth of an electron shower. The rate of growth of electron-photon cascades initiated by electrons of known energies was measured in the counter. This was done by recording the amplitude distribution of signals with only the first counter, and then the first two counters, etc., switched on. From these data, and the known discriminator setting 3, we calculated the efficiencies of detection of electrons of a given energy from μ mesons as a function of the plate of origin of the electron, and the variation of effective target thickness with the energy of the electron. It was found that for knock-on electron energies in the range 1 to 8 GeV the target efficiency changed by only 4%.

The variation of the counter response with reversal of the maximum magnetic field to an unbiased sample of μ mesons was recorded periodically throughout the run. The shift of the μ-meson peak was measured to be $(0.05\pm0.13)\%$. This encouraged us to believe that there was no serious over-all asymmetry in the apparatus.

2. Purity and Energy of the μ-Meson Beam

The response of the counter to an incident beam of 7-GeV π mesons is shown in Fig. 6. The mean signal corresponds to an energy loss of about 3.5 GeV with a half-width of the order of 2 GeV. Figure 6 shows that the abundance of π mesons present in a "μ beam" may be measured from inspection of the shape of the detector energy response curve. To have a measure of the attenuation of the π mesons by the graphite absorber, a set of observations was made of the counter response with varying absorber thicknesses in the beam. By extrapolating these data to 760 cm of graphite, we conclude that there were not more than 2×10^{-5} π mesons per μ meson in the beam. This implies that π-meson interactions contributed less than 3% of the high-energy showers at any energy.

IV. RESULTS CONCERNING MUON HELICITY

Data were collected with two different settings of the beam parameters. One run of 9 h used 7.1-GeV/c muons

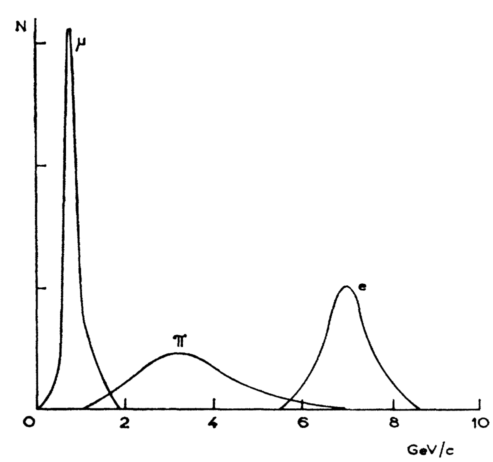

FIG. 6. Pulse-height spectrum produced by μ mesons, π mesons, and electrons of 7 GeV in the total absorption scintillation detector. Abscissa: energy scale as calibrated with electrons. Areas under the peaks are normalized to correspond to equal numbers of μ, π, and e.

[8] K. R. Symon, Harvard University, thesis 1948 (unpublished), quoted in reference 5, p. 32.

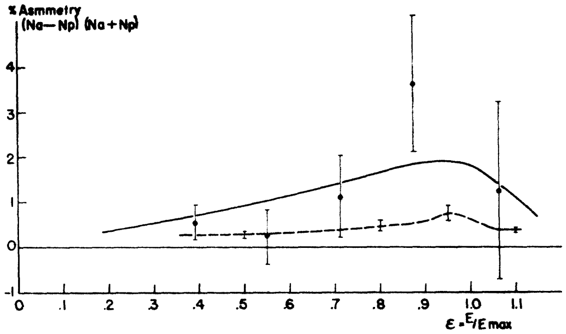

FIG. 7. Asymmetry of the cross section for knock-on electrons produced by muons in Fe, with N_p magnetization parallel, N_a antiparallel to the muon spin. The abscissa is the ratio of the energy transferred to the maximum transferable energy. The errors of the points show the standard deviation. The points are corrected for the instrumental asymmetry, the measurement of which is shown by the dashed line. The full line shows the calculated curve for a muon helicity of +1.

($E_{\max}=2.9$ GeV/c) with a calculated polarization of 0.62, and another run of 17 h used 8.0-GeV/c muons ($E_{\max}=3.43$ GeV/c) with a calculated polarization of 0.85.

These data were combined and the quantity

$$A=(N_a-N_p)/(N_a+N_p)-\alpha$$

is plotted in Fig. 7 as a function of the variable $E_{\rm sh}/E_{\max}$ where N_a, N_p denote a magnetic field direction parallel and antiparallel to the muon velocity, $E_{\rm sh}$ is the energy of the shower recorded and α a small correction described below. The solid curve shows the theoretical prediction for the muon helicity +1 taking into account the spectral distribution of the incoming muons, their polarization, and the resolution of the detector.

The experimental observations have been corrected for an instrumental asymmetry, α. As mentioned above, the over-all asymmetry was measured to be negligible. However, there could still be asymmetries in some groups of counters which averaged out to zero for the whole counter. Thus muons would show no asymmetry, but showers giving up their energy predominantly in the middle of the counter would be sensitive to this instrumental asymmetry. To investigate this effect the variation of the response of small groups of adjacent counters with field reversal was measured. Small but measurable changes of gain were observed. The asymmetry, α, due to these changes in gain, of the order of 2×10^{-3}, has been evaluated and is plotted, with the errors in its determination, as the dashed curve in Fig. 7.

Our final measurement yields for the helicity

$$H(\mu^-)=+0.80\pm0.31,$$

in agreement with the theoretical prediction of +1, and our preliminary result.[2] The measurement differs by 5.8 standard deviations from a helicity −1.

V. RESULTS CONCERNING MUON-ELECTRON SCATTERING, AND MUON BREMSSTRAHLUNG

We recorded showers produced by 8.0-GeV μ mesons for 10 h for the measurement of μ-electron scattering. The observed rate of signals in the PHA has been expressed in Fig. 8, as dN/dE, the rate of energy transfer per GeV, using the electron calibration of Fig. 5.

However, we cannot compare these values of dN/dE directly with the cross sections of the two elementary processes of Fig. 1, because we have recorded the energy transfer distribution from μ mesons traversing our detector which is a thick target. Therefore, we must take into account multiple processes which also include the ionization loss of the μ meson. Since we are only interested in the high-energy tail of the distribution, we could use an approximation method for the calculation, considering first transfers of energy above a cutoff energy E_c, chosen to be 100 MeV. We calculated the distribution function arising from multiple collisions with energy transfer less than E_c. The latter distribution can be represented by a Gaussian. The distribution obtained from the Gaussian, all single and double collisions producing knock-on electrons and bremsstrahl quanta above E_c was calculated. Higher order collisions above E_c were estimated to be negligible. The resulting distribution was then folded with the μ-meson energy spectrum Fig. 3, the counter resolution of Fig. 5, and the small correction for energy bias in the recording system.

The whole program of calculation was carried out with the CERN mercury computer. The result is shown in Fig. 8. Curve b includes radiative corrections, curve a does not. Radiative corrections to the μ bremsstrahlung have not been included. It should be emphasized

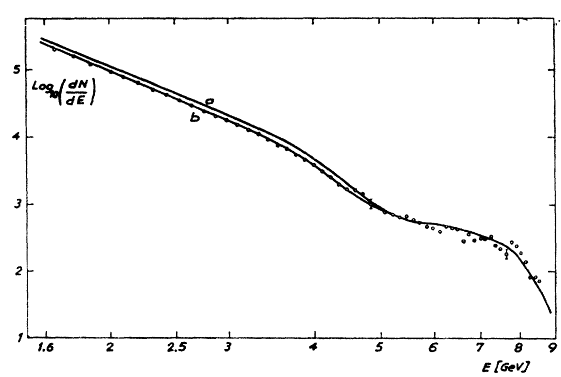

FIG. 8. Measured distribution dN/dE, the rate of knock-on electrons and bremsstrahlung-quanta produced by 8-GeV μ mesons per GeV. Abscissa: total energy loss including energy loss of the μ meson. For comparison the calculated distribution is shown (a) without and (b) with radiative corrections.

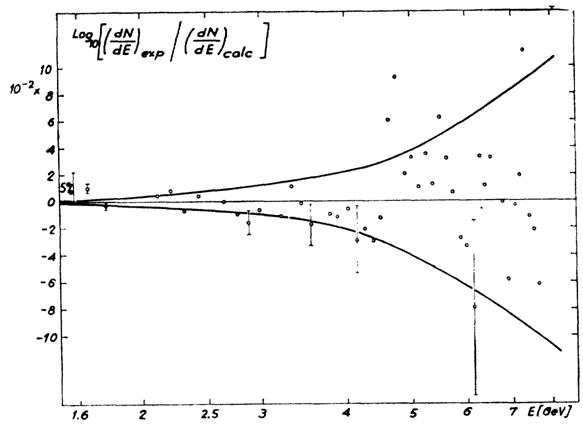

FIG. 9. Logarithm of the ratio of the experimental points to the computed curve including radiative corrections (Fig. 8). Abscissa: total energy loss including energy loss of the μ meson. The lines show a statistical error of 1 standard deviation.

that the energy scale gives the total energy loss in the counter, including that due to the μ meson.

Figure 9 shows the logarithm of the ratio of the experimental points to the computed curve (with radiative corrections) shown in Fig. 8. This is done to display the results on an expanded scale.

The main error in the result of the experiment comes from the indeterminacy of the energy calibration of the shower detector. The calibration accuracy was about 1% and there were three known causes of random drifts each of about 0.5%. We estimate, therefore, that there was a total uncertainty of about 2% in the energy scale. This has been calculated to introduce an error in the cross section of 6.8%, 8.2%, 11.8%, for 1, 2, 3 GeV knock-on electrons, respectively.

The statistical errors are shown on a few points of Fig. 9, being 0.7%, and 5% for 1- and 3-GeV knock-on electrons, respectively. The absolute cross section is uncertain to about $\pm 2\%$ due to uncertainty in the effective target thickness.

The most sensitive comparison of our data with calculations is made by normalizing the absolute cross section to fit the calculated curve exactly at 1.6 GeV. Then experiment and calculation at 4.0 GeV agree to within 3%. This agreement is within our estimated systematic errors of $\pm 5.0\%$, arising from the errors in energy calibration.

From this limit on a deviation, we can infer an upper limit on a cutoff parameter Λ describing a (small) deviation from the predictions of quantum electrodynamics.[3] We conclude that $1/\Lambda \leq 0.64$ F.

This result is not compatible with the tentative conclusion that $\Lambda^{-1} \gg 1$ F drawn from a cosmic-ray μ-e scattering experiment.[9]

ACKNOWLEDGMENTS

We are indebted to the CERN proton synchrotron machine division for their work and willing assistance throughout the experiment.

We are grateful to Dr. H. Rollnik for illuminating discussions and for his calculation of the radiative corrections for our experiment.

---

[9] R. F. Deery and S. H. Neddermeyer, Phys. Rev. **121**, 1803 (1960).