Decay of Sc^{42m} and the Levels of $Ca^{42\dagger}$

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A 62.0±0.4-sec isomer, Sc^{42m}, has been observed which decays by emission of a positron group of endpoint energy 2.87±0.10 MeV to an excited state in Ca⁴² at 3.191 MeV, followed by a cascade of three γ rays of energies 0.438±0.003, 1.226±0.010, and 1.520±0.010 MeV. The levels of Ca⁴² and their assigned spins are: 1.523 MeV, 2+; 1.836 MeV, 0+; 2.424 MeV, 2+(1+); 2.750 MeV, 4+; and 3.191 MeV, 6+. The spin of Sc^{42m} is 7+. Studies of the reaction $Ca^{40}(He^3,p)$ showed that the first and second excited states of Sc^{42} are at 0.67 ± 0.03 and 1.58 ± 0.05 MeV, respectively, the former being Sc^{42m} . The excitation function for the Ca⁴⁰ (α, pn)Sc^{42m} reaction was also measured. The general trends of levels in even $f_{1/2}$ nuclides are rather well explained by two treatments of the *jj*-coupling model, with exceptions for Ca⁴⁴ and Ca⁴².

I. INTRODUCTION

FOLLOWING the inception of the independentparticle model with the hypothesis of strong spinorbit coupling,¹ there has been considerable theoretical interest in the level structure of nuclides containing several $f_{7/2}$ nucleons outside the Ca⁴⁰ doubly magic core. Several of the theoretical investigations have been based mainly on the independent-particle model,²⁻⁴ whereas others have included collective-model effects.^{5,6} We have obtained further information on coupling of $f_{7/2}$ particles by study of the positron decay of highspin, 62-sec Sc^{42m} (discovered by us⁷ and, independently, by Nelson et al.^{8,9}) which populates levels of Ca⁴² that are not seen in the beta decay^{10,11} of 12.5-h K⁴² (I=2-), or in the positron decay¹² of 0.689-sec Sc⁴² (I=0+).

II. EXPERIMENTS

1. Mass and Charge Assignments

Bombardment of natural calcium targets with He³ ions of 12 to 24 MeV and He⁴ ions of 30 MeV produced a positron and γ -ray activity having a 62-sec half-life.

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- ⁿ J. D. McCullen and J. J. Kraushaar, Phys. Rev. 122, 555 (1961).

¹² J. Jänecke, Z. Naturforsch. 15A, 593 (1960).

Attempts were made to identify the activity with one of the four elements titanium, scandium, calcium, or potassium. In the time available, titanium and scandium could be collected together, separated from calcium and potassium. The separation was made by extracting the titanium and scandium into an 0.5 M solution of thenoyltrifluoroacetone (TTA) in benzene from an 0.1 to 5 M HCl solution of reaction products. The activity appeared in the extracted phase and was, thus, established as titanium or scandium and not calcium or potassium.

Attempts were also made to separate the titanium alone by extracting the tetraphenylarsonium chloride salt of TiCl₆⁼ from HCl solution into chloroform. At no time was more than a few parts per thousand of the initial activity separated by this means, specific for titanium. It is, thus, concluded that the species is an isotope of scandium.

The short-lived activity was shown definitely to be an isotope of scandium by bombardments of K³⁹ (as natural KCl) with 17- to 30-MeV He⁴ ions (producing no Ti isotopes) and subsequent separation of the 62-sec activity by the TTA extraction technique. The bombardments also produced considerable amounts of 7.7min K³⁸ by the $K^{39}(\alpha,\alpha n)$ and $Cl^{35}(\alpha,n)$ reactions and 34.2 min Cl^{34m} by the $Cl^{35}(\alpha,\alpha n)$ reaction. The 62-sec activity was barely distinguishable in the γ -ray spectra of the gross reaction products, but its relative intensity was increased 50 fold and became clearly distinguishable over the K³⁸ and Cl^{34m} activities in the benzene fraction from the TTA separation. Thus, the activity is clearly an isotope of scandium.

The assignment to mass 42 was initially based on the three cross bombardments $Ca^{40}(He^3, p)$, $Ca^{40}(\alpha, pn)$, and $K^{39}(\alpha,n)$ and on the good agreement between the observed γ -ray energies and the energy differences between the levels of Ca42 previously determined by other workers (see below). Also, the yield at 30-MeV He4-ion energy was much too large to be produced by any calcium isotope except Ca40.

The assignment was further checked by measuring the excitation function for production of the activity via He4-ion bombardment of vacuum evaporated CaF2

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 ¹ M. G. Mayer, Phys. Rev. **75**, 1969 (1949); O. Haxel, J. H. D. Jensen, and H. E. Suess, *ibid*. **75**, 1766 (1949).
 ² C. Levinson and K. W. Ford, Phys. Rev. **100**, 13 (1955).

⁸ A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. (London)

A215, 120 (1952).

⁴ I. Talmi, Phys. Rev. 107, 326 (1957)

⁵ B. J. Raz, Phys. Rev. 114, 1116 (1959)

⁶ L. S. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **32**, No. 9 (1960). ⁷ P. C. Rogers, Massachusetts Institute of Technology Labora-

tory for Nuclear Science Progress Report, May, 1961, (unpub-

<sup>bished), p. 54.
⁸ J. W. Nelson, H. S. Plendl, and J. D. Oberholtzer, in</sup> *Proceedings of the Rutherford Jubilee International Conference, Manchester, September, 1961* (Academic Press Inc., New York, 1961), p. 819; see also J. D. Oberholtzer, J. W. Nelson, and H. S. Plendl, Bull. Am. Phys. Soc. 7, 286 (1962).
⁹ J. W. Nelson (private communication).
¹⁰ H. Morinaga, N. Matsuro, and M. Sugawara, Phys. Rev. 114, 1146 (1959).

targets. The 30.72±0.06-MeV He4-ion beam from the MIT cyclotron was degraded by insertion of weighed aluminum foils to give a series of bombarding energies which were determined with the use of a semiconductor particle detector. During each 1- to 3-min bombardment the beam current collected by an electrostatically shielded Faraday cup (which also served as target holder) was monitored by use of a current integrator and pulse-height analyzer operating as a multiscaler.

Following each bombardment, the γ rays from the target were counted directly with a NaI(Tl) crystal. A decay curve of the γ rays of $E_{\gamma} > 0.7$ MeV was measured using the pulse-height analyzer as a multiscaler. The counting arrangement was calibrated against a 3-in. ×3-in. NaI(Tl) crystal (under standard geometrical conditions) from which the absolute disintegration rates were determined. The decay curves obtained were analyzed with the aid of the FRANTIC computer program¹³ to obtain the counting rates of the 62-sec activity at the end of bombardments. The thickness of calcium on the targets was determined by an EDTA titration of the material dissolved from the targets.¹⁴

The excitation function obtained is shown in Fig. 1



FIG. 1. Experimental and predicted excitation functions for production of Sc42m and Sc42 by the He4-ion bombardment of Ca40. Experimental curve represents production of only Sc⁴² ⁿ. Calculated curve is for total production of Sc42 isomers assuming mass value of Sc42m (see text).

where the energy at each point is the He⁴-ion energy at the midpoint of the target. In order to determine if the position and shape were consistent with the assumption of a combination of $Ca^{40}(\alpha, pn)$ and $Ca^{40}(\alpha, d)$ reactions, calculations of the excitation functions predicted by the Monte Carlo method described by Dostrovsky et al.¹⁵ were made. In the calculations the masses from König et al.¹⁶ were used except for those of Ti⁴², Ti⁴¹, and Sc^{42m} . The Ti⁴² and Ti⁴¹ masses were predicted for this work from mirror-nuclide systematics. The mass for Sc⁴² was calculated from that of Ca⁴², the Sc⁴²-Ca⁴² mass difference,¹⁷ and the Sc^{42m}–Sc⁴² mass difference determined in this work (see below). The δ values (used to correct level densities for even-odd effects) were those which Dostrovsky et al.15 had determined for the nuclides around the Z and N=28 closed shells. These values were applied without change to the nuclides around the Z and N = 20 closed shells. The level-density parameter a was assumed to be A/20. Results of the



FIG. 2. Excitation functions for products from He⁴-ion bombardment of Ca⁴⁰ predicted by the Monte Carlo method. In cases where several paths leading to the same final nucleus have been lumped together, the curves are designated by the product nucleus.

calculations are given in Fig. 2. In comparing the magnitude and shape of the calculated and experimental excitation functions, one must recall that the experimental cross sections are those for formation of only the highspin isomer, whereas, the calculated curves are for total production of Sc42 with mass that of Sc42m. The major differences in shapes occur at low energies, probably as a result of the very approximate treatment of the proton and deuteron barrier penetration. In any case, it is clear that the excitation function is that for the (α, pn) and (α,d) reactions and not that for the (α,p) or $(\alpha,p2n)$ reaction. For the latter, the cross sections predicted for He4-ion energies up to 35 MeV were less than 1 mb (threshold = 29.33 MeV).¹⁶

¹³ P. C. Rogers, FRANTIC Program for Analysis of Exponential Growth and Decay Curves, Massachusetts Institute of Technology Laboratory for Nuclear Science, Technical Report No. 76(NYÖ-2303), June 1961 (unpublished).

¹⁴ The authors are indebted to James T. Corless, Massachusetts Institute of Technology for performing this analysis.

¹⁵ I. Dostrovsky, Z. Fraenkel, and G. Friedlander, Phys. Rev. 116, 683 (1959).

 ¹⁶ L. A. König, J. H. E. Mattauch, and A. H. Wapstra, Nucl. Phys. 31, 18 (1962).
 ¹⁷ John Miller (private communication).

0.438

0.511 Me\

1.226 MeV

1.520 MeV

2. Gamma-Ray Spectroscopy

In addition to 0.511-MeV annihilation radiation, γ rays of energies 0.438±0.003, 1.226±0.010, and 1.520 ± 0.010 MeV are observed in the decay of Sc^{42m}. A γ -ray spectrum obtained using a 3-in. \times 3-in. NaI(Tl) crystal and graphite absorber is shown in Fig. 3. The crystal was mounted in a lead house in a geometrical configuration similar to that used by Heath to measure standard γ -ray spectra and to determine peak-to-total ratios.¹⁸ The ratio of intensities of the three γ rays after correction for absorption, peak-to-total ratios, and total counting efficiencies¹⁸ is $1.00 \pm 0.03: 1.00 \pm 0.01: 1.00$ for the 0.44-MeV:1.23-MeV:1.52-MeV γ rays. The upper limit on the intensity of any other γ rays is 1%. Gamma rays corresponding to "crossover" transitions occur in less than 0.3% of the decays.

Gamma-gamma coincidence experiments were performed using two 2-in.×2-in. NaI(Tl) crystals. Pulses from one branch of the system were stored in a 20channel analyzer, previously described by Crut et al.¹⁹ It is possible to obtain pulses from this analyzer when counts are stored in any preselected ranges of channels. Prior to the coincidence experiments, singles spectra were taken using the 20-channel analyzer. The analyzer was then adjusted to give output pulses A or B whenever counts were recorded in the channels corresponding to the photopeaks of one or the other of two of the principal γ rays. When pulses from the two branches of the system were in fast (resolving time 20 nsec) and slow coincidence ($\sim 4\mu$ sec) and the pulse in the first branch fell in the group A or group B channels, the coincidence gate of a 256-channel pulse-height analyzer was opened for analysis of the signal in the second branch. With a group-A pulse in the first branch, the pulse from the second branch was stored in the first 128 channels of the 256-channel memory. If the count in the first branch fell into the B group, a routing pulse from the slow coincidence unit was fed to a programming unit²⁰ which caused the analyzer to store the pulse from the second branch in the second 128 channels. Using this setup, it was possible to obtain simultaneously the spectra coincident with two different γ rays.

In the investigation of the Sc^{42m} decay scheme, we obtained the spectra in coincidence with the 1.23- and 1.52-MeV γ rays. The coincidence spectra indicated that the three γ rays are in coincidence with each other and with annihilation radiation. The relative intensities of the γ rays in the coincident spectra are, within experimental error, the same as those observed in the singles spectra, with the exception of the annihilation radiation whose intensity was 30 to 50% lower than in the singles spectra. The latter observation was the result

Channe ð 10 50 150 200 Channel Number

FIG. 3. Spectrum of γ rays from decay of Sc^{42m} showing the four major peaks and several sum peaks obtained using a 3-in. \times 3-in. NaI(Tl) crystal and graphite absorber.

of good collimation in front of the crystals in the coincidence experiments. The fact that, with the exception of annihilation radiation, the relative intensities of all the γ rays remained the same in the coincident spectra indicates that the lifetimes of the parent states involved are shorter than the 20-nsec resolving time of the fast coincidence circuit. The spin changes in the transitions are thus limited to $\Delta I \leq 2$ (multipolarities E2, M2, E1, or M1, except no M2 for the 0.44-MeV γ ray).

3. Beta-Ray Spectroscopy

Beta-ray spectra were determined using a split plastic scintillator shaped into a 45-deg cone with base diameter of 2.5 in. Analysis of the experimental spectra using the Kurie computer program^{20,21} indicated that the main group of particles had an allowed positron shape with end point energy of 2.87 ± 0.10 MeV. In addition, there were smaller contributions at high energies resulting from γ -plus- β^+ coincidence summing. The only other activity present in appreciable amounts was a few percent of 3.92-h Sc⁴³ from the Ca⁴⁰(α, p) reaction on the calcium target foil. These spectra exhibited no conversion-electron lines of energies between 0.05 and 4 MeV with as much as 2% of the total number of β^+ counts in the spectrum.

4. Half-Life Determination

Decay curves were obtained by use of a 256-channel pulse-height analyzer operating in the multiscaler mode at a channel-advance rate of 10 per min. The decay curves most nearly free of other activities were obtained from samples produced by He4-ion bombardment of calcium and by counting only γ rays of energy greater than 0.55 MeV. The Sc^{42m} decay could be followed for at least seven half-lives before becoming small compared with the activity of long-lived components present.



¹⁸ R. L. Heath, Atomic Energy Commission, Research and Development Report IDO-16408 (unpublished).

 ¹⁹ M. Crut, D. R. Sweetman, and N. S. Wall, Nucl. Phys. 17, 665 (1960).
 ²⁰ Paul C. Rogers, Ph.D. thesis, Massachusetts Institute of

Rogers, Ph.D. thesis, Massachusetts Institute of Technology, September, 1962 (unpublished).

²¹ The Kurie computer program is designed for analysis of experimental β -ray spectra by the least-squares techniques. It also includes a means of correcting for the finite response of scintillation detectors of the types used in these experiments.

Decay data were analyzed using the FRANTIC computer program.¹³ The main problem in determining the half-life accurately arose because of upward gain shifts in the photomultiplier tube resulting from high initial count rates from each Sc^{42m} sample. This condition had the effect on the decay-curve analyses of yielding better fits to the data when a dead-time factor smaller than the experimentally determined value was used. When the measured dead-time factor was used, the analyses indicated the presence of a component of ~ 200 sec half-life in quantities far greater than expected on the basis of reasonable cross-section estimates for reactions involving less abundant calcium isotopes or impurities. The final value selected, 62.0 ± 0.4 sec, was that obtained using an effective dead time per pulse of $11.5 \pm 2.0 \ \mu sec$ (measured value = 18.8 μsec) in analyses of high-activity decay curves. This value of the effective dead time caused the analyses to yield reasonably small amounts of \sim 200-sec activity with a small variance-of-fit to the data. Approximately the same value for the half-life was obtained from analysis of low activity decay curves regardless of which deadtime factor was used. The uncertainty assigned to the quoted half-life value arises mainly from the gain-shift problem.

5. Sc42-Sc42m Energy Difference

The $Sc^{42}-Sc^{42m}$ energy difference was determined by measurement of the energies of proton groups emitted in the $Ca^{40}(He^3,p)Sc^{42}$ reaction with a bombarding energy of 25 MeV. Protons were identified and their energies measured by use of an $E - \Delta E$ counting arrangement employing the electronics system described by Stokes et al.²² A parallel-plate flowing-gas proportional counter served as the ΔE detector and a NaI(Tl) crystal detector was used to measure residual energies. Protons emitted in the $Be^{9}(He^{3}, p)B^{11}$ reaction were used to calibrate the energy scale.

The Q value for formation of the ground state of Sc^{42} is found to be 4.92 ± 0.05 MeV. Proton groups of lower energies are observed, corresponding to formation of excited states of Sc^{42} at 0.67 ± 0.03 and 1.58 ± 0.05 MeV. Although the spectrum of protons was measured at five angles, the angular distributions of the proton groups were not characterized well enough to obtain information on the spins of the levels formed. It is assumed that the first excited state is the 62-sec isomer. As noted below, support for this assumption is obtained from the energy cycle.

III. DISCUSSION

The ground-state isomer of Sc⁴² decays to the 0+ ground state of Ca⁴² with a half-life of 0.689 sec¹² and β^+ end-point energy of 5.389 \pm 0.015 MeV.¹⁷ As determined by our analysis using the Kurie program²¹ for a 256-point numerical integration, the log ft value for

Braams^a Ca⁴²(p,p')(MeV) Buechner and Mazari^b Sc⁴⁵ (p,α) (MeV) This work $Sc^{42m} - \beta^+$ (MeV) 1.523 ± 0.004 1.526 ± 0.006 $1.520{\pm}0.010$ 1.836 ± 0.004 1.836 ± 0.006 Not observed 2.422 ± 0.005 2.425 ± 0.006 Not observed 2.753 ± 0.006 2.750 ± 0.005 2.746 ± 0.014 Not observed 3.191 ± 0.008 3.184 ± 0.015 3.250 ± 0.006 3.297 ± 0.006 3.389 ± 0.006 Others

TABLE I. Excited states of Ca⁴².

^a See reference 25. ^b See reference 24.

this transition, 3.477 ± 0.010 , corresponds to that of a superallowed transition for which $\Delta I = 0$, no parity change, indicating that Sc⁴² has spin 0+. Scandium-42, with one $f_{7/2}$ neutron and one $f_{7/2}$ proton outside the 20Ca2040 core, falls into the class of nuclides covered by the Brennan-Bernstein²³ rule R2, which predicts two rather closely spaced isomers of spins $|J_p - J_n|$ and J_p+J_n , or 0+ and 7+ for Sc⁴². Thus, Sc^{42m} probably has spin 7+, although our upper limits on the intensities of γ rays and conversion electrons resulting from an isomeric transition allow us to say only that $I \ge 5$. We assume that the parity of the level is positive as the result of coupling of two $f_{7/2}$ particles.

The half-life of Sc^{42m} , 62.0 ± 0.4 sec, and positron end point, 2.87 ± 0.10 MeV, give a log ft value of 4.185 ± 0.068 as determined by the Kurie computer program.²¹ This value corresponds to an allowed transition with $\Delta I = 0, \pm 1$, no parity change. The excited state of Ca⁴² populated in this decay deexcites by emission of a cascade of three γ rays. Our data do not alone give the order of emission of the γ rays, but by comparison with the energy levels of Ca42 determined by Buechner and Mazari,²⁴ we find the order of γ rays to be: 0.44, 1.23, and 1.52 MeV. The corresponding levels are compared with those determined by Braams²⁵ and Buechner and Mazari²⁴ in Table I.

In order to test the assignment of the 62-sec activity to the first excited state of Sc42, the Sc42-Ca42 mass difference as determined by three paths is compared. By the first path, the difference is 7.08 ± 0.10 MeV from the sum of the 3.19-MeV level in Ca^{42} and Q_{EC} of the Sc^{42m} β^+ decay from this work. By the second path the difference is 7.08 ± 0.04 MeV from the sum of $Q_{\rm EC}$ for the Sc⁴² β^+ decay¹⁷ and Sc^{42m}-Sc⁴² difference from our work. For the final path, the value is 7.08 ± 0.05 MeV from the Q value for formation of the first excited state of Sc⁴² via the Ca⁴⁰ (He³, p) reaction and Ca⁴⁰ and Ca⁴² masses from König et al.¹⁶ This assignment is in agreement with

²² R. H. Stokes, J. A. Northrop, and K. Boyer, Rev. Sci. Instr. 29, 61 (1958).

²³ M. H. Brennan and A. M. Bernstein, Phys. Rev. 120, 927

^{(1960).} ²⁴ W. W. Buechner and M. Mazari, Rev. Mex. Fis. 7, 117

^{(1958).} ²⁵ C. M. Braams, thesis, Utrecht, 1956 (unpublished). Data given in reference 24.

the measurements by Nelson and co-workers^{8,9} which showed directly that the 62-sec isomer lies above the 0.69-sec Sc⁴² isomer. Their measurements of the thresholds for production of the various states by the $K^{39}(\alpha, n)$ reaction indicate the Sc^{42m} - Sc^{42} energy difference to be $0.526{\pm}0.04~{\rm MeV}$ with a second excited state at 1.34 ± 0.06 MeV. The difference between these energies and those of the same levels as determined in this work $(0.67 \pm 0.03 \text{ MeV} \text{ and } 1.58 \pm 0.05 \text{ MeV}, \text{ respectively})$ are not understood.

Our proposed decay scheme for Sc42m and other information on the levels of Ca42 from nuclear-reaction studies and decay of K42 are given in Fig. 4. Arguments for the assignments of spins and parities to the various levels are given in the following paragraphs.

The first three excited states of Ca⁴² are known to be 2+, 0+, and 2+(1+), respectively, with the 2+favored from systematics,^{10,11} for the third. The fourth excited state, at 2.75 MeV, is concluded by Morinaga et al.¹⁰ to be 0+ or 4+ and by McCullen and Kraushaar¹¹ to be 0+ or ≥ 4 . The fifth excited state, at 3.19 MeV, can be limited to $\geq 4+$ from the allowed Sc^{42m} β^+ decay and observed lower limit $(\geq 5+)$ on the spin of Sc^{42m} . Of the possibilities discussed above for the spin of the 2.75-MeV level we can rule out 0+, as the level is fed from the 3.19-MeV state of spin $\geq 4+$ and the transition multipolarities are limited to E2, E1, or M1. By the same reasoning, the transition from the 2.75-MeV level to the 2+ level at 1.52 MeV requires the spin of the former to ≤ 4 . Thus, we conclude that the 2.75-MeV level has spin 4+.

With the spin of the 2.75-MeV level assigned 4+, only three combinations for the spins of the Sc42m isomer



FIG. 4. Level structure of Ca⁴² and proposed decay scheme of Sc^{42m} with data from the K⁴² decay (references 10, 11).



FIG. 5. Experimental and predicted level structures of even-A, single-closed-shell nuclides in the $f_{7/2}$ region. Ca⁴², this work; Ti⁵⁰ and Fe⁵⁴, reference 28; Ca⁴⁴, reference 29; Cr⁵², reference 30.

and Ca42 level at 3.19 MeV are possible in view of the allowed β decay with no branching and the limitation of multipolarities in the γ transitions as noted above. The three combinations are: (7+, 6+), (6+, 6+), and (6+, 6+)5+) for the isomer and Ca⁴² state, respectively. From measurements of the Ca⁴⁰(α ,d) reaction, Harvey et al.²⁶ have found that the spin of Sc^{42m} is definitely not 6. Therefore, we conclude that the spins of Sc^{42m} and the 3.19-MeV level of Ca⁴² are 7+ and 6+, respectively.

IV. INTERPRETATION

The existence of the Sc^{42m} isomer gives further support to the Brennan-Bernstein²³ hypothesis that nuclides falling in class R2 (containing two odd "particles" or "holes," as distinguished from "particle-hole" combinations, both having j=l+1/2 or j=l-1/2) have two close-lying levels of spins $|J_p - J_n|$ and $J_p + J_n$. As Jänecke²⁷ has pointed out, Cl³⁴ and Sc⁴² are exceptions to the general observation that, in self-conjugate nuclides, the level having isotopic spin, T, of zero lies below the one having T=1.

In Fig. 5, are shown the low-lying levels in the singleclosed-shell (s.c.s.) nuclides having even numbers of $f_{7/2}$ nucleons $\lceil Ca^{42}$ (this work,) Ti⁵⁰ and Fe⁵⁴, ²⁸ Ca⁴⁴, ²⁹ and Cr^{52 30}] and the positions of the levels as calculated by the methods of Talmi⁴ and Edmonds and Flowers.³

According to Talmi's method the states are described by *ij*-coupling wave functions and it is assumed that the only effective interaction (assumed to be the same within a subshell) is a two-body force. The radial wave functions are not specified, but it is assumed that they are not changed by addition of nucleons within the subshell. Using this model, it is, for example, possible to

²⁶ B. G. Harvey (private communication).

²⁷ J. Jänecke, Nucl. Phys. 30, 328 (1962).

²⁸ See compilation in Landolt-Börnstein, Numerical Data and Functional Relationships in Science and Technology, New Series, Group I: Nuclear Physics and Technology, Vol. 1, Energy Levels of Nuclei: A = 5 to A = 257, edited by A. M. Hellwege and K. N. Hellwege (Springer-Verlag, Berlin, 1961). ²⁹ L. T. Dillman, J. J. Kraushaar, and J. D. McCullen (to be

published)

³⁰ R. R. Wilson, A. A. Bartlett, J. J. Kraushaar, J. D. McCullen, and R. A. Ristinen, Phys. Rev. 125, 1655 (1962).

predict the positions of the $f_{7/2}^n$ levels in various $f_{7/2}$ s.c.s. nuclides from the positions of the $f_{7/2}^2$ levels. Using the prescriptions given by de-Shalit,³¹ we have calculated the predicted positions of $f_{7/2}^n$ levels in nuclides of this type based on the levels of $f_{7/2}^2$ in Ca⁴². (The same type of calculation with essentially the same results has been reported in reference 32.) The results for even nuclides are shown in Fig. 5. According to the model, the 2+, 4+, and 6+ seniority 2(v=2) levels should occur at the same heights above the 0+ (v=0)ground states in all of the even nuclides. Also, the 2+, 4+, 5+, and 8+ (v=4) levels in Ca⁴⁴ and Cr⁵² would be expected to lie at the same energies.

It is seen that, where one can clearly identify the levels, there is rather good agreement in the positions of the v=2 levels from one nuclide to another, with the exception of those of Ca44. Note, however, that with the new spin assignments²⁹ among the Ca⁴⁴ levels, agreement with the other $f_{7/2}$ nuclides has been considerably improved, although the first excited state still falls at a surprisingly low energy. The v=4 levels of Cr⁵² having an $f_{7/2}^4$ proton configuration, appear to agree reasonably well with the predictions based on the Ca42 levels. It has been noted³² that the lower 4+ level in Cr⁵² probably has seniority 4. In Ca^{44} , the 4+ level at 2.28 MeV may also have v=4, but there is not enough information to determine the seniority with certainty. We have also calculated the predicted positions for levels of $f_{7/2}^3$. There is little experimental information for comparison; however, the level ordering in V⁵¹ appears to be in good agreement with predictions, but the level spacings are generally smaller in the experimental level structure than in that predicted from the Ca⁴² levels.

Edmonds and Flowers³ have calculated the relative positions of levels in $f_{7/2}$ nuclides using the *jj*-coupling model, oscillator wave functions of the form $\phi_i \exp \phi_i$ $\left[-\frac{1}{2}(r/a_0)^2\right]$, and a residual two-body interaction of Gaussian shape described by $V(r) = D \exp[-(r/a)^2]$. In Fig. 5, we have shown their calculated level positions for a range parameter (a/a_0) of 1.4, normalized to the Ca⁴² first excited state. Their predictions are in rather good agreement with those based on Talmi's model and with the experimental data, although the positions of the two 4+ levels are inverted in the Edmonds-Flowers predictions. There is an inversion of the 4+ levels upon going to the Edmonds-Flowers range parameter of 2.0; however, the other levels are not well fitted using this value of the parameter. If the level in Cr⁵² at 2,965 MeV is the 2+ (v=4) level, its position is reproduced better by the Edmonds-Flowers predictions than by those obtained using Talmi's method and the Ca42 levels.

The *jj*-coupling model accounts rather well for the general features of levels in $f_{7/2}$ nuclides. However, in the forms presented by Talmi and Edmonds and

Flowers, it does not seem capable of explaining the rather different position of the first excited state in Ca44. An additional feature that has not been explained is the origin of the 0+ and 2+(1+) levels in Ca⁴² at 1.84 and 2.42 MeV, respectively. One might suspect that they are the $(p_{3/2})_{0,2+}$ levels; however, on the assumption that the 3/2- level at 1.95 MeV in Ca⁴¹ is the $p_{3/2}$ level, one would expect the $(p_{3/2}^2)_0 - (f_{7/2}^2)_0$ energy difference to be of the order of 3.9 MeV. If the levels at 1.84 and 2.42 MeV in Ca⁴² are mainly $p_{3/2}^2$, considerable mixing of other configurations would be needed to bring the energy levels down to such low values. It is not clear whether or not corresponding levels are present in the other even $f_{7/2}$ nuclides. There are poorly established levels in Fe⁵⁴ at 1.95 and 2.17 MeV, and a wellestablished, but unclassified, level at 2.55 MeV. The (0+) level in Ca⁴⁴ at 1.88 MeV is possibly an analog of the 0+ level at 1.84 MeV in Ca⁴². Of the two 2+ levels at 2.66 and 3.35 MeV in Ca⁴⁴, one is probably the $f_{7/2}^4$, v=4 level and the other may be analogous to the 2.42-MeV level of Ca⁴². In Ti⁵⁰ and Cr⁵², there are no unexplained levels below 2.5 MeV, although in Cr⁵² there are unexplained levels at 2.65 and 3.16 MeV. (Zaika and Nemets³³ note a possible unresolved level in Ti⁵⁰ "at low energy" from (d,p)-reaction data.) Thus, if the jj-coupling model is to be able to give a more complete description of the $f_{7/2}$ nuclides, the origin of the Ca⁴² levels at 1.84 and 2.42 MeV must be considered and the absence or shift of the corresponding levels to higher energies in the other even $f_{7/2}$ nuclides explained.

The possible influence of collective effects in this region is not clear. In Ca⁴² one might be tempted to describe the levels as being mainly vibrational, with the 1.84-, 2.42-, and 2.75-MeV levels representing the 2phonon state, and the 3.19-MeV 6+ level being one member of the 3-phonon state. However, the mean position of the assumed 2-phonon levels, 2.5 MeV, is considerably less than twice the energy of the first excited (1.52-MeV) level. Raz⁵ has calculated the positions of levels obtained in $f_{7/2}$ nuclides when a weak or intermediate surface interaction is added to the twobody interaction. The ratios of the energies of upper levels to that of the 2+ first excited state are given as a function of the strength of the two-body force, D, and a surface-interaction parameter, x. Using the corrected curves³⁴ for D=0.2, excellent fits to the ratios E_{4+}/E_{2+} and E_{6+}/E_{2+} in Ca⁴², Ca⁴⁴, and Cr⁵² are obtained using x values of 0.02, 0.32, and 0.08, respectively, assuming for Ca⁴⁴ and Cr⁵² that the upper 4+ levels have v=2. Note that the value of x is determined by the ratio E_{4+}/E_{2+} , and the ratio E_{6+}/E_{2+} provides an independent check. Although, in their present formulation, Raz' calculations do not explain the anomolous levels (e.g.,

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³⁴ We are indebted to Professor Raz for pointing out the error

³⁴ We are indebted to Professor Raz for pointing out the error in the reported calculations. Corrections to the calculations have been made and will be published.

the 1.84- and 2.42-MeV levels in Ca42), the good fits obtained suggest that the model deserves further attention. Kisslinger and Sorensen⁶ have made an extensive study of levels using a model based on strong shortrange (pairing) forces and longer range (P_2) forces; however, they have not extended their model to include nuclides below the N = 28 shell.

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Kinetic Energy Release in 23-MeV Deuteron Fission of U²³⁸⁺

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Average recoil ranges have been measured for six fission products formed by reactions of 23-MeV H² with U²³⁸. The ranges of products from near-symmetric fission (Cd¹¹⁵ and Ag¹¹¹) are longer than for neutron fission of U²³⁵, while the ranges of asymmetric products (I¹³¹, Ba¹⁴⁰, Mo⁹⁹, and Sr⁸⁹) are shorter. The kineticenergy deficit for near-symmetry fission is 15 ± 6 MeV smaller for this system than for thermal-neutron fission of U²³⁶. The magnitude of the kinetic-energy deficit for U²³⁶ fission is re-examined by comparing range data with recent time-of-flight measurements and neutron emission probabilities. This comparison leads to a kinetic-energy deficit of approximately 23 MeV for U²³⁶ fission.

I. INTRODUCTION

ANY different measurements have been made of velocities, energies and ranges of fission products from various kinds of fission.^{1,2} One interesting feature of many of these measurements is that the kinetic energies of near-symmetric fission products seem to be significantly smaller than the asymmetric products. We define the term "kinetic-energy deficit" as the difference between maximum kinetic-energy release and that for symmetric fission. This kinetic-energy deficit has been reported for several fissile nuclei at excitation energies near the threshold.¹⁻³ A few experiments have been reported at very large excitation energies,^{4,5} but there is very little information about this effect at excitation energies a few tens of a MeV greater than threshold.^{3,4} The quantitative evaluation of this kinetic energy

deficit has not, as yet, been established for low-energy fission. Coincidence-counting techniques have been very successful for measurements of the energies of asymmetric products.^{1,2} But various difficulties have prevented these techniques from obtaining unambiguous results for the symmetric fission products of much lower yield. Radiochemical recoil range measurements have perfect resolution, but conversion from range to energy requires some assumptions.

Several workers have reported range measurements for thermal-neutron induced fission of Pu²³⁹ and U²³⁵ and spontaneous fission of $\mathrm{Cf}^{252.\,6-10}$ The analysis of these data is based on a comparison with velocity measurements of the fragments of high yield.8-10 From the velocities and an assumption of the number of neutrons emitted per fragment, one can obtain final kinetic energies after neutron emission. Then one obtains range-energy relationships for products of asymmetric fission. These range-energy relationships are extrapo-

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