## IV. CONCLUSIONS

One aim of this study was to match the spectra of ${}_{20}Ca^{42}$ and ${}_{22}Ti^{50}$; to some extent this has been successful. The known levels in these nuclei and the corresponding predictions from this work are listed in Table IV.

*Note added in proof.* The low excited states of $Ti^{48}$ have been experimentally observed by R. A. Ristinen, A. A. Bartlett, and J. J. Kraushaar (to be published). They find the levels with the following energy, spin, and energy ratio ($E_I/E_2$): (a) 0 keV, $0^+$, 0; (b) 983.3 keV, $2^+$, 1; (c) 2295.0 keV, $4^+$, 2.33; (d) 2430 keV, $(1,2^+)$, 2.48; (e) 3223.9 keV, $4^+$, 3.28; (f) 3239.9 keV, $(5^+)$, 3.30; (g) 3340 keV, $6^+$, 3.40; (h) 3620, ?, 3.78. This can be compared with the theoretical predictions for $D=1.0$, $X=0.4$ where the following spins and energy ratios ($E_I/E_2$) are computed: (a) $0^+$, 0; (b) $2^+$, 1; (c) $4^+$, 2.30; (d) $2^+$, 2.48; (e) $0^+$, 3.00; (f) $2^+$, 3.05; (g) $6^+$, 3.40; (h) $4^+$, 3.58; (i) $1^+$, 3.76. I wish to thank the authors for permission to use their results before publication.

(It is also of interest that these results fit many of the levels in $Ca^{44}$, $Cr^{52}$, and $Fe^{54}$. These are also listed in Table IV.)

As can be seen, the second excited state of spin $0^+$ in $Ca^{42}$ cannot be matched with any choice of the parameters. An obvious solution to this problem is suggested by the work of Thankappan and Pandya[6] who have used this same model with $1d_{3/2}$ and $2s_{1/2}$ orbitals to fit the levels in $Si^{30}$. Their results have two shell-model $0^+$ states $(d_{3/2}{}^2)_0$ and $(s_{1/2}{}^2)_0$. This gives a low-lying $0^+$ state for a variety of parameters. Similar calculations are now underway adding $2p_{3/2}$ orbitals to the present calculation. Hopefully these results will give a much better fit to all the levels and perhaps also account for the very large $E0$ transition in[7] $Ca^{42}$ from the 1.84-MeV $0^+$ level to the ground state.

## V. ACKNOWLEDGMENTS

The author wishes to thank N. MacDonald for his great assistance in many phases of this work and J. Weneser for many stimulating suggestions. He also wishes to thank J. W. Calkin, K. Fuchel, and N. Reiss of the Applied Mathematics Division, Brookhaven National Laboratory for their kind aid in diagonalizing the matrices involved in this work. The aid of P. Zimmer and the members of the Computational Center of the State University of New York at Stony Brook is greatly appreciated.

[6] V. K. Thankappan and S. P. Pandya (to be published). I am indebted to S. P. Pandya for making this work available to me before publication.

[7] N. Benczer-Koller, M. Nessin, and T. H. Kruse, Phys. Rev. **123**, 262 (1961).

PHYSICAL REVIEW VOLUME 129, NUMBER 6 15 MARCH 1963

# Beta-Decay Matrix Elements in $Sb^{122}$†*‡

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(Received 6 November 1962)

An electronic computer has been used to investigate the six nuclear matrix elements which enter into the $2^-$ to $2^+$ 1.40-MeV beta transition in the decay of $Sb^{122}$. Data from beta-gamma angular correlation, beta-circularly polarized gamma angular correlation, nuclear orientation, and nuclear resonance experiments were used in this analysis. As a further aid, the Feenberg-Ahrens relations between certain of the nuclear matrix elements were employed to catalog the solutions and to simplify the search problem. In order to discover how the remaining ambiguity of these solutions could most easily be reduced, for each of the solutions calculations were made of the predicted results of all possible experiments on this beta transition. These calculations show how sufficient experimental data can be obtained to determine unambiguously all six nuclear matrix elements. In an appendix all the theoretical formulas which give the experimental observables for a first forbidden $2^-$ to $2^+$ beta transition in terms of the nuclear matrix elements are summarized.

## INTRODUCTION

THERE are, in general, six nuclear matrix elements which can contribute to a $2^-$ to $2^+$ first forbidden beta transition.[1] The possibility of experimentally determining so many overlap integrals for the same two nuclear configurations makes these transitions particularly attractive for studying nuclear structure. Although a large amount of both experimental and theoretical work has been reported on transitions of this type, no one has succeeded in finding a unique solution for the six nuclear matrix elements.[2] One of the principal reasons

† This research was supported by a grant from the National Science Foundation.

* This work was done in part at the Computation Center at the Massachusetts Institute of Technology, Cambridge, Massachusetts.

‡ A preliminary account of part of this work has been given: Bull. Am. Phys. Soc. **7**, 34 (1962).

§ Alfred P. Sloan Research Fellow, 1961–63.

[1] For a recent comprehensive review of first forbidden beta decay see H. A. Weidenmüller, Rev. Mod. Phys. **33**, 574 (1961).

[2] Some recent papers not in reference 1 in which attempts have been made to find the nuclear matrix elements for first forbidden transitions are: $Sb^{124}$—P. Alexander and R. M. Steffen, Phys. Rev. **124**, 150 (1961); R. M. Steffen, *ibid.* **124**, 145 (1961). $Eu^{152}$—H. Dulaney, C. H. Braden, and L. D. Wyly, *ibid.* **125**, 1620 (1961). $Eu^{154}$—S. K. Bhattacherjee and S. K. Mitra, *ibid.* **126**, 1154 (1962). $I^{126}$, $Rb^{84}$, $As^{74}$—D. S. Harmer and M. L. Perlman, *ibid.* **122**, 218 (1961).

that the nuclear matrix elements have not been determined is the manner in which the observables such as the shape of the beta spectrum, the beta-gamma angular correlation, and the beta-circularly polarized gamma angular correlation depend upon the nuclear matrix elements. If the energy of the electron at the end point of the spectrum ($W_0$) is much less than one-half the Coulomb energy of the electron at the nuclear radius ($\alpha Z/2R$), then, unless there is some special cancellation between the nuclear matrix elements, the observables depend mainly on two linear combinations of the nuclear matrix elements, and it appears very difficult to determine all six of the matrix elements. The approximation ($\alpha Z/2R \gg W_0$) for the observables in which only the first term in an expansion in powers of the nuclear radius is retained is called the $\xi$ approximation.[3,4] A considerable amount of the literature has been devoted to investigations as to whether or not particular $2^-$ to $2^+$ first forbidden transitions show deviations from the $\xi$ approximation.[5] The view has been expressed that if the beta transition can be adequately described in the $\xi$ approximation, then it is almost impossible to determine all six of the nuclear matrix elements without making very precise measurements.[1] In view of this general difficulty, serious attempts to find various of the nuclear matrix elements have resorted to crutches from the theories of nuclear structure.[6] This is unfortunate as one would rather use the beta-decay interaction, which is now quite well understood, to unambiguously investigate nuclear structure.

This paper reports an analysis of the first forbidden $2^- \rightarrow 2^+$, 1.40 MeV beta transition in the decay of $Sb^{122}$ which shows that a unique solution for all six nuclear matrices can be found even in the case of a beta transition where the spectrum shape and the beta-gamma angular correlation show no deviation from the $\xi$ approximation. This analysis differs from most previous analyses of similar transitions in two ways. In the first place it employs data from nuclear orientation experiments. The power of the nuclear orientation experiments for reducing the ambiguities involved in the analysis has not been sufficiently appreciated in the published literature. In the second place this analysis utilizes the Ahrens-Feenberg relations[7] between certain of the nuclear matrix elements to catalog the solutions and to simplify the search problem. Not only is the beta-decay interaction now known, but there is also good evidence that the conserved current hypothesis for the vector part of the beta-decay interaction is valid.[8–10] It has been pointed out by Fujita[11] that if the conserved current hypothesis is true, then a more precise expression, which does not depend upon the details of the nuclear force, can be derived for certain of the Ahrens-Feenberg ratios.

In conjunction with the analysis reported in this paper, we have also calculated for the various sets of nuclear matrix elements the predicted results for all the presently known experiments that can be performed on this $Sb^{122}$ beta transition. Results are reported for the beta-gamma angular correlation, the beta-circularly polarized gamma correlation, the anisotropy of beta emission from polarized nuclei, and the longitudinal polarization of the beta rays. It is pointed out that either a measurement of the beta-circularly polarized gamma correlation coefficient as a function of energy or a measurement of the anisotropy of beta emission from polarized nuclei as a function of energy would make it possible to determine more precisely the nuclear matrix elements which transform under spatial rotations like a vector and to see if the conserved current hypothesis for the vector portion of the beta decay interaction gives the correct ratio for the nuclear matrix elements. Precise measurements of the longitudinal polarization of the beta ray would make it possible to determine more precisely the matrix elements which transform under spatial rotations like a pseudoscalar and to investigate the conserved current hypothesis for the axial vector interaction.

## SUMMARY OF RELEVANT DATA

The decay scheme of 65-h $Sb^{122}$ which is given by the most recent set of Nuclear Data Cards is shown in Fig. 1.[12] For the analysis reported in this paper we are interested in the 1.40-MeV beta transition, and we shall summarize here only the data relevant to this analysis. The shape factor for the spectrum has been measured and found to be the same as that for an allowed transition.[13,14] An early measurement by Shaknov[15] showed that the beta-gamma angular correlation coefficient was not zero. Recently, Steffen[5] has remeasured the beta-gamma angular correlation as a function of the energy of the beta ray. It is convenient to write the beta-gamma angular correlation in the form

$$N(W,\theta)=1+\epsilon P_2(\cos\theta). \qquad (1)$$

The portion of Steffen's results used in this analysis are summarized in Table I.

Deutsch and Lipnik reported a measurement of the beta-circularly polarized gamma angular correlation for this beta transition.[16] When the beta-circularly polarized

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[12] *Nuclear Data Cards* (National Research Council, Washington, D. C.), NRC 60-4-85 through 60-4-94.
[13] M. J. Glaubman, Phys. Rev. **98**, 645 (1955).
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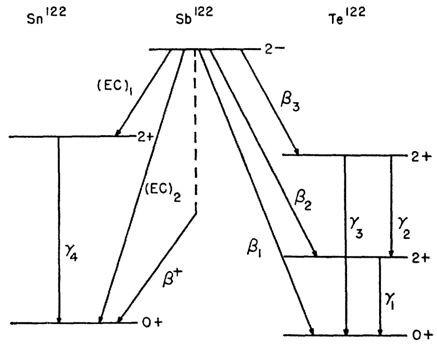
[15] I. Shaknov, Phys. Rev. **82**, 333(A) (1951).
[16] J. P. Deutsch and R. Lipnik, J. Phys. Radium **21**, 806 (1960).




| RADIATION | ENERGY | INTENSITY | LOG ft |
|---|---|---|---|
| $\beta_1$ | 1.97 | 30.0 % | 8.4 |
| $\beta_2$ | 1.40 | 62.9 % | 7.6 |
| $\beta_3$ | 0.74 | 4.0 % | 7.7 |
| $\beta^+$ | 0.57 | 0.0063 % | — |
| $(EC)_1$ | 0.45 | 0.7 % | — |
| $(EC)_2$ | 1.45 | 2.3 % | — |
| $\gamma_1$ | 0.564 | 66.4 % | EQ |
| $\gamma_2$ | 0.686 | 3.4 % | EQ+MD |
| $\gamma_3$ | 1.26 | 0.7 % | EQ |
| $\gamma_4$ | 1.14 | 0.73 % | EQ |

FIG. 1. The nuclear decay scheme for $Sb^{122}$.

gamma angular correlation is expressed in the form

$$N(W,\theta)=1+\omega(W,\theta)(p/W)\cos\theta, \tag{2}$$

then their result is

$$\omega=-0.034\pm0.034. \tag{3}$$

For this measurement the average $(v/c)$ for the electrons was $0.890\pm0.025$ and the angle $\theta$ was such that $\cos\theta=0.960\pm0.005$.

Bradley, Pipkin, and Simpson[17] reported a dynamic nuclear orientation experiment for $Sb^{122}$ in a doped silicon crystal. If the angular distribution of the gamma ray ($\gamma_1$) following $\beta_2$ is written in the form

$$N(\theta)=1-(10/7)B_2f_2P_2(\cos\theta)-(40/3)B_4f_4P_4(\cos\theta), \tag{4}$$

where $f_2$ and $f_4$ are the orientation parameters and $B_2$ and $B_4$ are the attenuation factors which depend upon the angular momentum carried off by the electron-neutrino system, then the results of their orientation experiment can be summarized by the equation

$$B_2/B_4=1.2\pm0.2. \tag{5}$$

They were unable to obtain independent values for $B_2$ and $B_4$ because they did not know the fraction of the radioactive nuclei in the sample which was being dynamically oriented.

Somoilov, Sklyarevskii, and Stepanov[18] oriented $Sb^{122}$ when it was present as an impurity atom in an iron foil by cooling the iron foil to 0.02°K. They found the angular distribution of the gamma rays (principally $\gamma_1$) was given by the expression

$$N(\theta)=1-\frac{(2.2\pm0.2)\times10^{-5}}{T^2}P_2(\cos\theta), \tag{6}$$

where $T$ is the absolute temperature. They were unable to derive a value for $B_2$ because they did not know the magnetic field at the antimony nucleus when the antimony atom was present in the iron lattice. They could, however, derive a relationship between the magnetic field, $H$, at the antimony nucleus and the $B_2$ attenuation coefficient. This relationship is

$$B_2=[(1.9\pm0.2)\times10^5/H]^2. \tag{7}$$

Bradley *et al.*[17] showed from an analysis of the two different orientation experiments together with the beta gamma-angular correlation measurement of Shaknov that the magnetic field at the antimony nucleus in the iron sample was either 190 or 340 kG. Acting upon this information, Sloan,[19] working in this laboratory, used a super-regenerative oscillator to search for the nuclear resonances of $Sb^{121}$ and $Sb^{123}$ when the antimony was present as an impurity atom in an iron lattice. He found the resonances for both isotopes and showed that the field at the nucleus was

$$H=193\pm3\text{ kG}. \tag{8}$$

When this result is inserted into Eq. (7), one obtains for $B_2$ the expression

$$B_2=0.9\pm0.1. \tag{9}$$

For our analysis we have taken the end point of the beta spectrum to be $W_0=3.74mc^2$ and the partial half-life for the 1.40-MeV beta transition to be $3.72\times10^5$ sec. It has been assumed that the nuclear radius is given by the expression $R=1.20A^{1/3}(mc/\hbar)\times10^{-13}$ electron Compton wavelengths.

TABLE I. A summary of part of the results from the beta-gamma angular correlation measurements of Steffen. This portion of his results was used in the analysis reported in this paper.

| Electron energy in $mc^2$ units | Anisotropy $\epsilon$ |
|---|---|
| 1.96 | $0.035\pm0.003$ |
| 2.50 | $0.052\pm0.005$ |
| 3.00 | $0.066\pm0.007$ |
| 3.50 | $0.081\pm0.004$ |

## SUMMARY OF THE THEORY

In a notation similar to that employed by Kotani[3,4,20] the six nuclear matrix elements which can contribute to

[17] G. E. Bradley, F. M. Pipkin, and R. E. Simpson, Phys. Rev. **123**, 1824 (1961).

[18] B. N. Somoilov, V. V. Sklyarevskii, and E. P. Stepanov, Soviet Phys.—JETP **11**, 261 (1960).

[19] E. Sloan, Harvard University thesis, 1962 (unpublished). Some uncertainty in interpretation would arise if the field at the antimony nuclei in the domain walls (nuclei supposedly seen by nuclear resonance) is not the same as that at the antimony nuclei in the main part of the sample.

[20] It should be noted here that Ross and Kotani, whose notation we are following, used electron wave functions such that $\int i\alpha$ has the opposite sign from that of Ahrens and Feenberg.

this beta transition are

$$\left.\begin{aligned} zw&=C_A\int \boldsymbol{\sigma}\cdot\mathbf{r} \quad (10)\\ zv&=C_A\int i\gamma_5 \quad (11)\end{aligned}\right\},\quad \lambda=0$$

$$\left.\begin{aligned} zu&=C_A\int i(\boldsymbol{\sigma}\times\mathbf{r}) \quad (12)\\ zy&=-C_V\int i\boldsymbol{\alpha} \quad (13)\\ zx&=-C_V\int \mathbf{r} \quad (14)\end{aligned}\right\},\quad \lambda=1$$

$$z=C_A\int B_{ij},\quad \lambda=2. \tag{15}$$

Here we have introduced the parameters $w$, $v$, $u$, $y$, and $x$ in such a fashion as to express five of the nuclear matrix elements in terms of the unique one, $z$. Two combinations of the nuclear matrix elements which occur frequently and which dominate the expressions for the various observables are

$$V=v+\xi w,\qquad \lambda=0 \tag{16}$$

$$Y=y-\xi(u+x),\quad \lambda=1. \tag{17}$$

Here $\xi=\alpha Z/2R$, $R$ is the nuclear radius in units of the electron compton wavelength, $\alpha$ is the fine structure constant, and $Z$ is the charge on the daughter nucleus.

For our analysis we need expressions for the various observables such as the angular correlation in terms of these nuclear matrix elements. In the Appendix to his paper Kotani[4] summarizes for first forbidden transitions the general formulas for the angular correlation, the shape factor of the spectrum, the longitudinal polarization of the beta rays, the beta-circularly polarized gamma angular correlation, the longitudinally polarized beta-gamma correlation, and the transversely polarized beta-gamma correlation. Kotani's formulas are advantageous because they are conveniently written for writing a computer program. In the Appendix to this paper we have summarized the explicit formulas for a $2^-$ to $2^+$ first forbidden transition and have also given the expressions for the $B_2$ and $B_4$ coefficients and for the angular distribution of the electrons from polarized nuclei. Where possible all of the formulas have been checked against the corresponding ones of Morita and Morita.[21] An error in Eq. (A5) of Kotani's paper has been corrected.

From general considerations concerning commutation relations, Ahrens and Feenberg[7] derived theoretical expressions for the ratios of certain nuclear matrix elements. Two of these relations are[20]

$$\Lambda_V\xi\left(\int \mathbf{r}\right)=\int i\boldsymbol{\alpha}, \tag{18}$$

and

$$\Lambda_A\xi\int \boldsymbol{\sigma}\cdot\mathbf{r}=-\int i\gamma_5. \tag{19}$$

According to Ahrens and Feenberg, for electron decay,

$$\Lambda_A=\Lambda_V=\left(2.4+\frac{W_i-W_f-2.5mc^2}{mc^2}\frac{A^{1/3}}{Z}\right)$$
$$+\text{nuclear potential term.} \tag{20}$$

Here $W_i$ is the total energy of the parent nucleus, $W_f$ the total energy of the daughter nucleus, $A$ the atomic number of the daughter nucleus, and $Z$ the charge on the daughter nucleus. Feenberg and Ahrens estimated the value of the nuclear potential term to be $-1.4$.

Recently, Fujita[11] pointed out that if the conserved current hypothesis is valid for the vector part of the beta-decay interaction, then the Siegert theorem for radiative transitions of nuclei can be generalized to beta decay and an expression for $\Lambda_V$ which does not depend upon the details of the nuclear force can be derived. Fujita obtained the expression

$$\Lambda_V=\left[2.4+\left(\frac{W_i-W_f-2.5mc^2}{mc^2}\right)(2R/\alpha Z)\right]. \tag{21}$$

Fujita pointed out that this expression is the same as that of Ahrens and Feenberg when one sets the nuclear potential term equal to zero and uses the old value for the nuclear radius $[R=(\alpha/2)A^{1/3}]$. If the conserved current theory and Fujita's conjecture are correct, then one of the nuclear matrix elements can be expressed in terms of another one and the problem is reduced from six to five unknown matrix elements.

Fujita showed that this relationship was consistent with the first forbidden transition in RaE ($Bi^{210}$).[11] Spector and Blin-Stoyle[22] have reported calculations which indicate that for RaE

$$2.0\leq\Lambda_V\leq5.3, \tag{22}$$

and they assert that RaE confirms the conserved current theory. Ullman[23] reinvestigated the RaE problem using newer data for the electron longitudinal polarization, and he found that a larger value of $\Lambda_V$ was required to fit all the data. On this basis he concluded that it was optimistic to say that the data for RaE confirm the conserved current theory. This only points out the desirability of more unambiguous determinations of the nuclear matrix elements. In our analysis we have used the Ahrens-Feenberg relations as a guide in determining what is the interesting range of variation of the nuclear matrix elements.

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[22] R. M. Spector and R. J. Blin-Stoyle, Phys. Letters **1**, 118 (1962).

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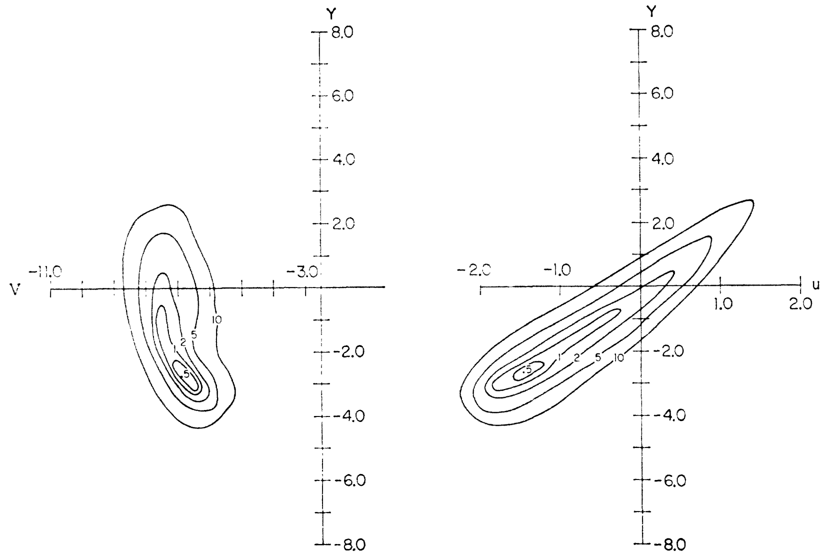


FIG. 2. A contour map of the $VY$ and $uY$ planes showing the region of good solutions. The various contours are the lines of constant $\chi^2$ for the best solution at each point. The labels on the contour map give the value of the $\chi^2$. For the calculations summarized in these figures, the search increment on $V$ and $Y$ was 0.5 and that on $u$, 0.1.

## MODE OF ANALYSIS AND RESULTS

An IBM 7090 computer was employed to find the set of nuclear matrix elements which best fit the data. For fitting the beta-gamma angular correlation, the beta-circularly polarized gamma angular correlation, the $B_2/B_4$ ratio, and the $B_2$ value, it is only necessary to deal with the parameters $w$, $u$, $x$, $V$, and $Y$. Once a solution for these parameters has been found the value of $z$ and hence the value of each of the nuclear matrix elements can be calculated from the expression for the half-life of the beta transition. Values for the parameters $\Lambda_V$ and $\Lambda_A$ in the range $1.5 \leq \Lambda_V, \Lambda_A \leq 5$ were selected and the parameters $V$, $Y$, and $u$ were taken as independent variables. The computer was instructed to compute the predicted values of the experimentally measured quantities for values of $V$, $Y$, and $u$ in the region

$$\begin{aligned} -20 \leq V \leq 0, \\ -10 \leq Y \leq 10, \\ -5 \leq u \leq 5. \end{aligned} \tag{23}$$

An interval of 0.5 was used for each of the three parameters. Calculations were also made with a $u$ interval of 0.1 to make sure that no solutions were being missed because the search mesh was too coarse. Those $V$, $Y$, $u$ points which gave $B_2 \leq 0.8$ were immediately rejected. For the other six measurements (4 beta-gamma angular correlation measurements, 1 beta-circularly polarized gamma correlation measurement, and 1 value for $B_2/B_4$) the quantity

$$(\chi_i)^2 = \left(\frac{\text{predicted value} - \text{measured value}}{\text{experimental error}}\right)^2 \tag{24}$$

was calculated. Those points with $\chi_i^2 \geq 10$ for any one of the six measured quantities were rejected. All of the other solutions were recorded together with the values for the total $\chi^2$

$$\chi^2 = \sum_{i=1}^{6} (\chi_i)^2. \tag{25}$$

In order to see the structure of the regions of good solutions, a plot on the $VY$ plane of the minimum $\chi^2$ for each $VY$ point was made. A similar plot was made on the $uY$ plane. Figure 2 shows a $VY$ plane and a $uY$-plane plot for the case $\Lambda_A = \Lambda_V = 2.5$. These plots are typical in that the $VY$ plot shows one region of good solutions and the $uY$ plot shows a region of correlated

TABLE II. A summary of the solutions found for the various $\Lambda_A$, $\Lambda_V$ pairs. The $\chi^2$ listed is the total $\chi^2$ for that particular solution. If the increment in any one of the independent search parameters, $V$, $Y$, or $u$, required to increase the $\chi^2$ by a factor of 4 from the minimum value is taken as a measure of the error, then for all solutions the errors are less than $\pm 1$, $\pm 1$, and $\pm 0.5$ on $V$, $Y$, and $u$, respectively. In each case only one minimum for $\chi^2$ was found.

| $\Lambda_A$ | $\Lambda_V$ | $V$ | $Y$ | $w$ | $u$ | $x$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|
| 1.5 | 1.5 | −8.0 | −1.00 | 1.270 | 0.000 | −0.159 | 1.287 |
| 1.5 | 2.5 | −8.0 | −2.00 | 1.270 | −1.000 | −0.772 | 0.768 |
| 1.5 | 3.0 | −8.5 | −1.00 | 1.349 | −0.500 | −0.290 | 1.227 |
| 1.5 | 5.0 | −8.5 | 0.00 | 1.349 | 3.50 | 0.875 | 0.961 |
| 2.5 | 2.5 | −7.0 | −2.50 | 0.370 | −1.50 | −1.130 | 0.762 |
| 3.0 | 3.0 | −7.5 | −1.00 | 0.298 | −0.50 | −0.290 | 1.086 |
| 5.0 | 1.5 | −7.0 | −0.50 | 0.139 | 0.00 | −0.079 | 1.455 |
| 5.0 | 5.0 | −8.0 | 0.00 | 0.159 | 4.00 | 1.00 | 1.016 |
| Modified $B_{ij}$ | | −7.0 | −0.60 | 0.000 | 0.00 | 0.00 | 2.17 |

TABLE III. A summary of the nuclear matrix elements found for the various solutions. The errors in the matrix elements can be estimated from the information in Table II.

| $\Lambda_A$ | $\Lambda_V$ | $\frac{1}{R}\int B_{ij}$ | $\frac{1}{R}\int \mathbf{r}$ | $\int i\boldsymbol{\alpha}$ | $\frac{1}{R}\int i\boldsymbol{\sigma}\times\mathbf{r}$ | $\int i\gamma_5$ | $\frac{1}{R}\int \boldsymbol{\sigma}\cdot\mathbf{r}$ |
|---|---|---|---|---|---|---|---|
| 1.5 | 1.5 | $\pm 9.90\times10^{-2}$ | $\mp 1.87\times10^{-2}$ | $\mp 5.37\times10^{-3}$ | 0.0 | $\mp 3.61\times10^{-2}$ | $\pm 1.25\times10^{-1}$ |
| 1.5 | 2.5 | $\pm 9.63\times10^{-2}$ | $\mp 8.84\times10^{-2}$ | $\mp 4.21\times10^{-2}$ | $\mp 9.63\times10^{-2}$ | $\mp 3.51\times10^{-2}$ | $\pm 1.22\times10^{-1}$ |
| 1.5 | 3.0 | $\pm 9.32\times10^{-2}$ | $\mp 3.22\times10^{-2}$ | $\mp 1.83\times10^{-2}$ | $\mp 4.66\times10^{-2}$ | $\mp 3.61\times10^{-2}$ | $\pm 1.26\times10^{-1}$ |
| 1.5 | 5.0 | $\pm 8.58\times10^{-2}$ | $\pm 8.93\times10^{-2}$ | $\pm 8.47\times10^{-2}$ | $\pm 3.00\times10^{-1}$ | $\mp 3.32\times10^{-2}$ | $\pm 1.16\times10^{-1}$ |
| 2.5 | 2.5 | $\pm 9.45\times10^{-2}$ | $\mp 1.27\times10^{-1}$ | $\mp 6.04\times10^{-2}$ | $\mp 1.42\times10^{-1}$ | $\mp 1.68\times10^{-2}$ | $\pm 3.50\times10^{-2}$ |
| 3.0 | 3.0 | $\pm 9.18\times10^{-2}$ | $\mp 3.17\times10^{-2}$ | $\mp 1.81\times10^{-2}$ | $\mp 4.59\times10^{-2}$ | $\mp 1.58\times10^{-2}$ | $\pm 2.74\times10^{-2}$ |
| 5.0 | 1.5 | $\pm 9.68\times10^{-2}$ | $\mp 9.10\times10^{-3}$ | $\mp 2.62\times10^{-3}$ | 0.0 | $\mp 1.30\times10^{-2}$ | $\pm 1.35\times10^{-2}$ |
| 5.0 | 5.0 | $\pm 7.71\times10^{-2}$ | $\pm 9.18\times10^{-2}$ | $\pm 8.70\times10^{-2}$ | $\pm 3.08\times10^{-1}$ | $\mp 1.18\times10^{-2}$ | $\pm 1.23\times10^{-2}$ |
| Modified $B_{ij}$ | | $\pm 9.46\times10^{-2}$ | 0.00 | $\mp 1.04\times10^{-3}$ | 0.0 | $\mp 1.02\times10^{-2}$ | 0.0 |

solutions. Table II summarizes the best sets of parameters found for the various $\Lambda_A$, $\Lambda_V$ pairs. Also listed in Table II is the solution obtained in the modified $B_{ij}$ approximation. For the modified $B_{ij}$ approximation it is assumed that

$$Y\neq 0,\quad V\neq 0,\quad z\neq 0,$$
$$u=w=x=0. \tag{26}$$

This is one of the approximations commonly used in the literature.[1,6] In order to estimate the error in the solutions, the increment in each of the $V$, $Y$ and $u$ parameters required to increase the $\chi^2$ by a factor of 4 from the minimum value was found for each set of solutions. It was found that in all cases the required value lay within the range

$$\bar{V}\pm 1,\quad \bar{Y}\pm 1,\quad \text{and}\quad \bar{u}\pm 0.5,$$

where $\bar{V}$, $\bar{Y}$, $\bar{u}$ is the solution with the minimum $\chi^2$. Table II also shows that for all the $\Lambda_A$, $\Lambda_V$ pairs there is only one general region of good solutions.

For each of the solutions in Table II, calculations were made of the values of the six nuclear matrix elements. These results are summarized in Table III. A calculation was also made of the $B_{ij}$ matrix element of the unique first forbidden, 1.97-MeV beta transition to the ground state of the $Te^{122}$ daughter. This matrix element is

$$\left(\frac{1}{R}\int B_{ij}\right)_{\beta_1}=(2.45\pm 0.03)\times 10^{-1}. \tag{27}$$

Table III indicates that the value for the $B_{ij}$ matrix element does not depend very sensitively upon the values of $\Lambda_A$ and $\Lambda_V$ used. This is quite interesting as the ratio

$$\left(\frac{1}{R}\int B_{ij}\right)_{\beta_2}\Big/\left(\frac{1}{R}\int B_{ij}\right)_{\beta_1} \tag{28}$$

is quite useful in determining which nuclear model best describes these beta transitions.[6]

In order to discover which experiments could most easily further reduce the ambiguity of the solutions, the predicted results for all presently possible measurements were calculated for each of the solutions in Table II. Calculations were made for the shape factor of the beta spectrum, the beta-gamma angular correlation, the beta-circularly polarized gamma correlation, the longitudinal polarization of the beta rays, the longitudinally polarized beta-gamma angular correlation, the transversely polarized beta-gamma angular correlation, the angular distributions of the electrons from polarized nuclei, and the $B_2$ and $B_4$ parameters. Table IV summarizes the $B_2$ and $B_4$ parameters and the longitudinal polarization of the beta rays at the end point of the spectrum for various of the solutions listed in Table II. Figure 3 shows the beta gamma angular correlation predicted for these same solutions. The numbers beside the various curves are explained in Table IV. Figure 4 shows the predicted longitudinal polarization of the beta rays for this same set of solutions. Figure 5 shows the beta-circularly polarized

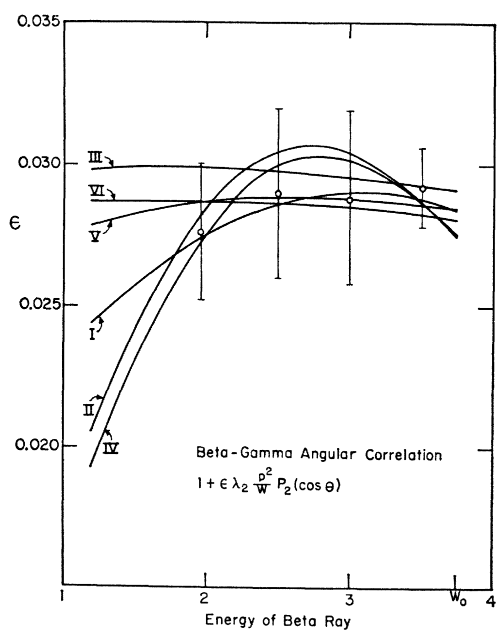


FIG. 3. A summary of the beta-gamma angular correlation for the various solutions listed in Table IV. The Roman numerals serve to identify the various solutions. Also shown on this graph are the results from the beta-gamma angular correlation experiments of Steffen which were used in determining the solutions. The $\epsilon$ used in this figure is not the same as the $\epsilon$ employed in the text.

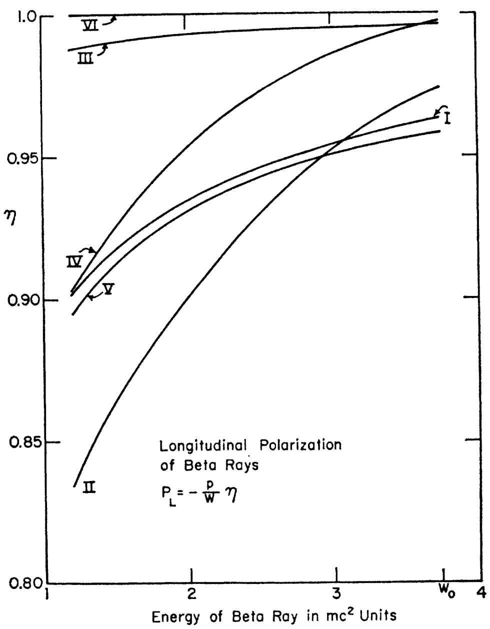


FIG. 4. The values of the longitudinal polarization of the beta rays as a function of energy for each of the solutions listed in Table IV.

TABLE IV. A summary of the $B_2$ and $B_4$ parameters and the longitudinal polarization of the $\beta$ rays at the end points of the beta spectrum for some of the solutions listed in Table II. The solution number serves to identify the curves in Figs. 3 through 7.

| $\Lambda_A$ | $\Lambda_V$ | Solution number | $B_2$ | $B_4$ | Longitudinal polarization for $W=W_0$ |
|---|---|---|---|---|---|
| 2.5 | 2.5 | I | 0.929 | 0.800 | −0.929 |
| 1.5 | 5.0 | II | 0.906 | 0.715 | −0.939 |
| 5.0 | 1.5 | III | 0.985 | 0.986 | −0.960 |
| 5.0 | 5.0 | IV | 0.903 | 0.700 | −0.961 |
| 1.5 | 1.5 | V | 0.978 | 0.965 | −0.924 |
| Modified $B_{ij}$ | | VI | 0.984 | 0.980 | −0.964 |

gamma angular correlation as a function of energy for $\theta=180°$; Fig. 6 shows the beta-circularly polarized gamma angular correlation as a function of angle for a fixed electron energy; Fig. 7 shows the anisotropy of the beta rays from polarized nuclei as a function of energy. A study of Table IV and Figs. 4 through 7 shows that one set of experiments would increase the precision with which $\Lambda_V$ and the $\lambda=1$ matrix elements are known and another set would determine the $\lambda=0$ matrix elements and $\Lambda_A$. A measurement of the beta-circularly polarized gamma correlation coefficient as a function of the energy of the beta ray or a measurement of the anisotropy of beta emission from polarized nuclei as a function of energy would determine the $\lambda=1$ matrix elements and $\Lambda_V$. The simplest experiment which gives a measure of the $\lambda=0$ matrix elements is a precise (1 to 2%) measurement of the longitudinal polarization of the beta rays.

The computer program was written in Fortran and the entire computation including debugging took approximately 4 h on a 7090. We will furnish copies of our computer program to any interested parties.

## CONCLUSIONS

This analysis indicates that even for a beta transition such as $Sb^{122}$ where the spectrum has an allowed shape and the angular correlation shows no deviation from the $\xi$ approximation, one can still determine all six of the nuclear matrix elements. The most useful experiments for this purpose are the nuclear orientation experiments, the beta-circularly polarized gamma correlation experi-

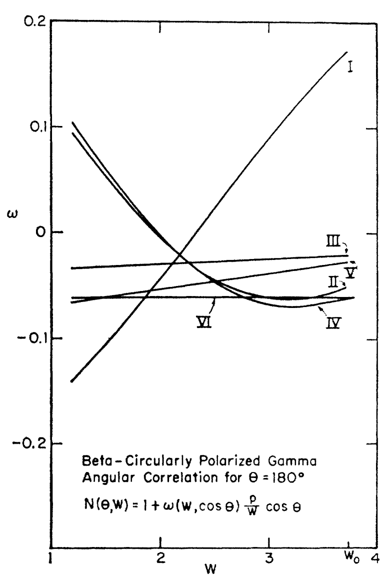


FIG. 5. The beta-circularly polarized gamma angular correlation as a function of electron energy for a fixed angle of 180° for each of the solutions listed in Table IV.

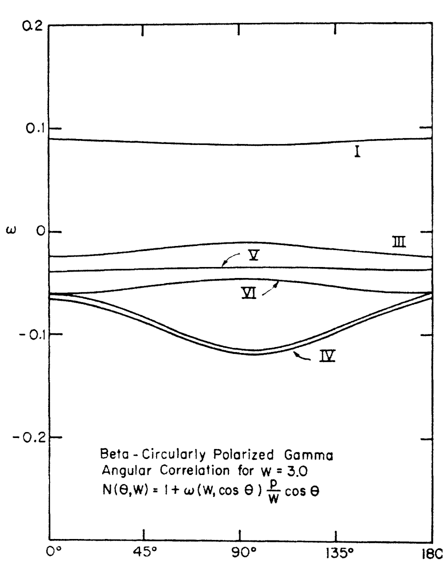


FIG. 6. The beta-circularly polarized gamma angular correlation as a function of angle for a fixed electron energy of $W=3.0mc^2$ for each of the solutions listed in Table IV.

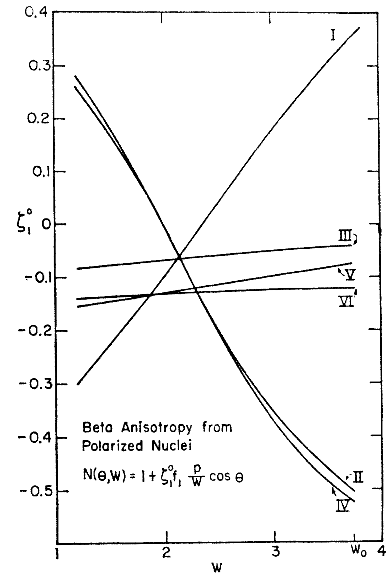


FIG. 7. The anisotropy of the beta rays from polarized nuclei as a function of the electron energy for each of the solutions listed in Table IV.

ments, and the measurements of the longitudinal polarization of the beta rays. This analysis indicates that the use of the Ahrens-Feenberg relations can simplify the analysis. It is important to investigate the validity of these relationships and their connection with the conserved current theories of the beta interaction. This analysis also indicates that the $B_{ij}$ approximation gives a good value for the $B_{ij}$ matrix element and that the character of the $Sb^{122}$ decay is most probably due to a cancellation effect rather than a selection rule. It is instructive to note that for a decay such as $Sb^{122}$ where the nuclear orientation experiments show that $V\gg Y$, one can obtain a good idea of the behavior as a function of energy of the circular polarization correlation and the anisotropy from oriented nuclei by retaining those terms which depend on $V^2$, $VY$, $xV$, and $uV$.

## ACKNOWLEDGMENTS

We are particularly indebted to the MIT Computation Center for providing us with the 7090 time and facilities with which to perform this calculation.

## APPENDIX

In this Appendix we summarize the explicit formulas for a $2^-\to 2^+$ first forbidden beta transition which is followed by a $2^+\to 0^+$ electric quadrupole gamma ray. In addition to those formulas which appear in the paper of Kotani,[4] we give here the expressions for the anisotropy of electrons and the anisotropy of the gamma rays when the nuclei are oriented.[24] Whenever possible we have checked these formulas against those of Morita and Morita.[21] The Morita and Morita formulas can be obtained from those of Kotani by setting $\lambda_1$ through $\lambda_8$ equal to one. The primary reason for listing the formulas here is to provide in one place a complete list of the explicit expressions for this particular decay scheme. Since the advent of the IBM 7090 computer it is simpler to compute all the various functions such as the Fermi function rather than to use the published tables. The formulas listed here are in such a form as to make programming straightforward. Only one error was found in Kotani's formulas. His Eq. (A5) should be replaced by our Eq. (A16).

### *A. Summary of the notation:*

| | |
|---|---|
| $p$ | momentum of the electron in $mc^2$ units; |
| $W$ | total energy of the electron in $mc^2$ units $[W=(p^2+1)^{1/2}]$; |
| $W_0$ | total energy of the electron at the end point of the beta spectrum; |
| $R$ | radius of the daughter nucleus in electron compton wavelengths; |
| $Z$ | charge on the daughter nucleus; |
| $\alpha$ | fine structure constant; |
| $F(Z,W)$ | Fermi function; |
| $\Gamma(x+iy)$ | gamma function of a complex argument; |
| $m$ | mass of the electron; |
| $\tau$ | circular polarization of gamma ray (+1 for right, −1 for left) |

### *B. List of special functions:*

$$y=\alpha ZW/p, \tag{A1}$$

$$F(Z,W)=2(1+\gamma_1)(2pR)^{2(\gamma_1-1)}\times(\exp\pi y)\frac{|\Gamma(\gamma_1+iy)|^2}{|\Gamma(2\gamma_1+1)|^2}, \tag{A2}$$

$$\theta_k=\arg(\Gamma(\gamma_k+iy))+\tfrac{1}{2}\pi(\gamma_k-k), \tag{A3}$$

$$A=\tfrac{1}{4}(3+\gamma_2-\gamma_1)[(2+2\gamma_1)/(2+\gamma_2)]^{1/2}, \tag{A4}$$

$$\gamma_k=[k^2-(\alpha Z)^2]^{1/2}, \tag{A5}$$

$$\lambda_1=\frac{2+\gamma_2}{2(1+\gamma_1)}(2pR)^{2(\gamma_2-\gamma_1-1)}\times(12\Gamma(1+2\gamma_1)/\Gamma(1+2\gamma_2))^2\times\left|\frac{\Gamma(\gamma_2+iy)}{\Gamma(\gamma_1+iy)}\right|^2, \tag{A6}$$

$$\lambda_2=A(\lambda_1)^{1/2}[\cos(\theta_2-\theta_1)+y\sin(\theta_2-\theta_1)/(\gamma_2+2\gamma_1)], \tag{A7}$$

$$\lambda_4=A(\lambda_1)^{1/2}[\cos(\theta_2-\theta_1)+[(\alpha Z)^2/y(\gamma_2+2\gamma_1)]\sin(\theta_2-\theta_1)], \tag{A8}$$

$$\lambda_6=A(\lambda_1)^{1/2}[(1+\gamma_2+\gamma_1)/(3+\gamma_2-\gamma_1)]\times\cos(\theta_2-\theta_1), \tag{A9}$$

$$\lambda_8=\tfrac{4}{3}[(\gamma_2+\gamma_1+3)/(1+\gamma_1)(1+\gamma_2+\gamma_1)]\lambda_6. \tag{A10}$$

[24] General expressions for these quantities are given in references 1, 17, and 21.

*C. The beta-decay interaction Hamiltonian density,* $\mathcal{H}_\beta$:

$$\mathcal{H}_\beta=\left(\frac{G}{\sqrt{2}}\right)G(\bar{\psi}_p\gamma_u(C_V-C_A\gamma_5)\psi_N)\times(\bar{\psi}_e\gamma_u(1+\gamma_5)\psi_V)+\text{H.c.}, \quad (A11)$$

where

$$C_V=1,$$
$$C_A=-1.19\pm0.04,$$
$$G=2.97\times10^{-12} \text{ (atomic units)}.$$

*D. The number of electrons emitted per second as a function of energy,* $N(W)$:

$$N(W)dW=\left(\frac{mc^2}{\hbar}\right)\left(\frac{G^2}{2\pi^3}\right)\times z^2F(Z,W)C(W)pW(W_0-W)^2dW, \quad (A12)$$

where $C(W)$ is the shape factor for the beta spectrum (see $F$ below).

*E. The half-life of the beta transition,* $\tau_{1/2}$:

$$\frac{1}{\tau_{1/2}}=\left(\frac{1}{\ln 2}\right)\left(\frac{mc^2}{\hbar}\right)\left(\frac{G^2}{2\pi^3}\right)z^2\times\int_1^{W_0}F(Z,W)C(W)pW(W_0-W)^2dW. \quad (A13)$$

*F. The shape factor for the beta spectrum,* $C(W)$:

$$C(W)=k+kaW+(kb/W)+kcW^2, \quad (A14)$$

where

$$k=\left[\zeta_0^2+\left(\frac{w}{3}\right)^2\right]+\left[\zeta_1^2+\frac{W_0^2}{18}(2x+u)^2-\frac{1}{18}(2x^2+7u^2)\right]+\left(\frac{W_0^2-\lambda_1}{12}\right), \quad (A15)$$

$$\zeta_1=Y+(u-x)W_0/3, \quad (A16)$$

$$\zeta_0=V+w(W_0/3), \quad (A17)$$

$$ak=-\tfrac{4}{3}uY-(W_0/9)[(4x^2+5u^2)+\tfrac{3}{2}], \quad (A18)$$

$$bk=\tfrac{2}{3}[-w\zeta_0+(u+x)\zeta_1], \quad (A19)$$

$$ck=\tfrac{1}{9}(4x^2+5u^2)+(1+\lambda_1)/12. \quad (A20)$$

*G. The longitudinal polarization of the electrons,* $P_L$:

$$P_L=-\frac{p}{W}\left[1+\left(\frac{-(bk/W)+dk}{C(W)}\right)\right], \quad (A21)$$

where $b$ is given by (A19) and

$$dk=(2/9)[-w^2+(u^2-x^2)]. \quad (A22)$$

*H. The β-γ angular correlation:* The angular correlation between the beta ray and a gamma ray whose circular polarization is $\tau$ can be written in the form

$$N(\theta)=1+\tau A_1\frac{p}{W}P_1(\cos\theta)+A_2\frac{p^2}{W}P_2(\cos\theta)+\tau A_3\frac{p^3}{W}P_3(\cos\theta), \quad (A23)$$

where

$$A_1=(R_4k+gkW+hkW^2)/C(W), \quad (A24)$$

$$A_2=(R_3k+ekW)/C(W), \quad (A25)$$

$$A_3=lk/C(W), \quad (A26)$$

and the various auxiliary functions are

$$R_4k=-(\tfrac{1}{6})^{1/2}[2\zeta_0\zeta_1+\tfrac{2}{3}xw]+(\tfrac{1}{6})[\zeta_1^2-(W_0/6)^2(2x+u)^2-(u/2)^2]+(1/36)(\tfrac{7}{2})^{1/2}[(2x+u)W_0^2-\tfrac{3}{5}(4x+3u)]+(1/240)(5W_0^2-3\lambda_1), \quad (A27)$$

$$gk=\tfrac{2}{3}(\tfrac{1}{6})^{1/2}(2x+u)\zeta_0-(1/18)\times[(5u-2x)Y-3W_0u(x-\tfrac{1}{2}u)]-\tfrac{1}{6}(\tfrac{7}{2})^{1/2}[\lambda_4Y+\tfrac{1}{3}W_0(x+2u)]-W_0/24, \quad (A28)$$

$$hk=-\tfrac{1}{9}u(x-u)+(\sqrt{14}/30)(x+2u)+(1/48)(1+\tfrac{3}{5}\lambda_1), \quad (A29)$$

$$R_3k=\lambda_2[-(1/21)^{1/2}\zeta_0+\tfrac{1}{6}(2x-u)\zeta_1+\tfrac{1}{2}(1/14)^{1/2}\zeta_1], \quad (A30)$$

$$ek=-(1/72)(2x+7u)(2x-u)-(1/12)(1/14)^{1/2}(5u-2x)-(1/112)\lambda_1, \quad (A31)$$

$$lk=-\tfrac{1}{5}(2/7)^{1/2}(2x-u)-(2/35)\lambda_1. \quad (A32)$$

It is convenient to speak of the angular correlation coefficient, $\epsilon$, where

$$\epsilon=(p^2/W)A_2, \quad (A33)$$

and the $\beta^-$ circularly polarized gamma coefficient, $\omega$, where

$$\omega=\left(\frac{1}{\cos\theta}\right)\frac{A_1P_1(\cos\theta)+p^2A_3P_3(\cos\theta)}{1+\epsilon P_2(\cos\theta)}. \quad (A34)$$

*I. The longitudinally polarized β-γ correlation,* $P_L^\gamma$:

$$P_L^\gamma=-\frac{p}{W}\times\left\{\frac{k+[-(bk/W)+dk]+[(R_5k/W)+nk]P_2(\cos\theta)}{C(W)[1+\epsilon P_2(\cos\theta)]}\right\}, \quad (A35)$$

where

$$R_5k=(\lambda_4W^2-\lambda_2p^2)R_3k, \quad (A36)$$

$$nk=\tfrac{1}{3}(1/21)^{1/2}w+(1/18)(2x-u)(x-u)+\tfrac{1}{6}(1/14)^{1/2}(x-u). \quad (A37)$$

*J. The transversely polarized β-γ correlation, $P_T$:* The transverse β polarization in or perpendicular to the plane of the β and γ rays is expressed as follows:

$$P_{T\parallel}=-\frac{3}{2}\frac{p}{W}\sin\theta\cos\theta\left[\frac{R_6k+nkW}{C(W)[1+\epsilon P_2(\cos\theta)]}\right], \quad (A38)$$

$$P_{T\perp}=(\alpha Z)\left(\frac{9}{8}\right)\left(\frac{p}{W}\right)\sin\theta\cos\theta \times\left[\frac{R_8k+nkW}{C(W)[1+\epsilon P_2(\cos\theta)]}\right], \quad (A39)$$

where

$$R_6=(\lambda_6/\lambda_2)R_3, \quad (A40)$$

$$R_8=(\lambda_8/\lambda_2)R_3. \quad (A41)$$

*K. The angular distribution of the electrons from oriented nuclei:*

$$N(\theta,W)=1+\frac{p}{W}\zeta_1{}^0f_1P_1(\cos\theta)+\frac{p^2}{W}\zeta_2{}^0f_2P_2(\cos\theta) +\frac{p^3}{W}\zeta_3{}^0f_3P_3(\cos\theta), \quad (A42)$$

where the orientation parameters are

$$f_1=\tfrac{1}{2}\sum_{m_i} m_ia_{m_i}, \quad (A43)$$

$$f_2=\tfrac{1}{4}[\sum_{m_i}(m_i)^2a_{m_i}-2], \quad (A44)$$

$$f_3=\tfrac{1}{8}[\sum_{m_i}(m_i)^3a_{m_i}-(17/5)\sum_{m_i} m_ia_{m_i}]. \quad (A45)$$

The $a_{m_i}$ are the relative populations of the nuclear sublevels normalized so that

$$\sum_{m_i} a_{m_i}=1 \quad (A46)$$

The other parameters are

$$\zeta_1{}^0=2(\zeta_{11}+\zeta_{12}W+\zeta_{13}W^2)/C(W), \quad (A47)$$

$$\zeta_{11}=-(\tfrac{1}{6})^{1/2}[2\zeta_0\zeta_1+\tfrac{2}{3}xw] -\tfrac{1}{6}[\zeta_1{}^2-(W_0/6)^2(2x+u)^2-(u/2)^2]+(1/36) \times(\tfrac{7}{2})^{1/2}[(2x+u)W_0{}^2-\tfrac{3}{5}(4x+3u)] -(1/240)(5W_0{}^2-3\lambda_1), \quad (A48)$$

$$\zeta_{12}=(\tfrac{2}{3})(\tfrac{1}{6})^{1/2}(2x+u)\zeta_0 +(1/18)[(5u-2x)Y-3W_0u(x-\tfrac{1}{2}u)] -(\tfrac{1}{6})(\tfrac{7}{2})^{1/2}[\lambda_4Y+\tfrac{1}{3}W_0(x+2u)]+W_0/24, \quad (A49)$$

$$\zeta_{13}=\tfrac{1}{9}u(x-u)+(\sqrt{14}/30)(x+2u) -(1/48)(1+\tfrac{3}{5}\lambda_1), \quad (A50)$$

$$\zeta_2{}^0=4(\zeta_{21}+W\zeta_{22})/C(W), \quad (A51)$$

$$\zeta_{21}=\lambda_2[(1/21)^{1/2}\zeta_0-\tfrac{1}{6}(2x-u)\zeta_1+\tfrac{1}{2}(1/14)^{1/2}\zeta_1], \quad (A52)$$

$$\zeta_{22}=(1/72)(2x+7u)(2x-u) -\tfrac{1}{12}(1/14)^{1/2}(5u-2x)+(\lambda_1/112), \quad (A53)$$

$$\zeta_3{}^0=\tfrac{1}{3}(2/7)^{1/2}(2x-u)-2\lambda_1/21. \quad (A54)$$

*L. The angular distribution of the electric quadrupole gamma rays following the beta ray when the nuclei are oriented:*

$$N_\gamma(\theta)=1-(10/7)B_2f_2P_2(\cos\theta) -(40/3)B_4f_4P_4(\cos\theta), \quad (A55)$$

where

$$f_4=\frac{1}{16}\left[\sum_{m_i}(m_i)^4a_{m_i}-\frac{31}{7}\sum_{m_i}(m_i)^2a_{m_i}+\frac{72}{35}\right], \quad (A56)$$

$$B_2=(\beta_0+\beta_1/2-3\beta_2/14)/(\beta_0+\beta_1+\beta_2), \quad (A57)$$

$$B_4=(\beta_0-2\beta_1/3+2\beta_2/7)/(\beta_0+\beta_1+\beta_2), \quad (A58)$$

$$\beta_0=\int_1^{W_0}\left(\alpha_{00}+\frac{\alpha_{02}}{W}\right)F(Z,W)pW(W_0-W)^2dW, \quad (A59)$$

$$\alpha_{00}=[\zeta_0{}^2+(w/3)^2], \quad (A60)$$

$$\alpha_{02}=-\tfrac{2}{3}w\zeta_0, \quad (A61)$$

$$\beta_1=\int_1^{W_0}\left(\alpha_{10}+\alpha_{11}W+\frac{\alpha_{12}}{W}+\alpha_{13}W^2\right) \times F(Z,W)pW(W_0-W)^2dW, \quad (A62)$$

$$\alpha_{10}=\zeta_1{}^2+\frac{(W_0)^2}{18}(2x+u)^2-1/18(2x^2+7u^2), \quad (A63)$$

$$\alpha_{11}=-\tfrac{4}{3}uY-\tfrac{1}{9}W_0(4x^2+5u^2), \quad (A64)$$

$$\alpha_{12}=\tfrac{2}{3}(u+x)\zeta_1, \quad (A65)$$

$$\alpha_{13}=\tfrac{1}{9}(4x^2+5u^2), \quad (A66)$$

$$\beta_2=\int_1^{W_0}(\alpha_{20}+\alpha_{21}W+\alpha_{23}W^2) \times F(Z,W)pW(W_0-W)^2dW, \quad (A67)$$

$$\alpha_{20}=(W_0{}^2-\lambda_1)/12, \quad (A68)$$

$$\alpha_{21}=-\tfrac{1}{6}W_0, \quad (A69)$$

$$\alpha_{23}=(1+\lambda_1)/12. \quad (A70)$$