Experimental Limits for the Electron-Proton Charge Difference and for the Charge of the Neutron*†

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Upper limits for the electric charges on the atoms of cesium and potassium and the molecules of hydrogen and deuterium have been measured by a molecular beam deflection method. The results are: $q(\mathrm{Cs}) = (1.3\pm5.6)\times10^{-17}\,q_e$, $q(\mathrm{K}) = (-3.8\pm11.8)\times10^{-17}\,q_e$, $|q(\mathrm{H_2})| < 2\times10^{-15}\,q_e$, and $|q(\mathrm{D_2})| < 2.8\times10^{-15}\,q_e$ (q_e is the absolute value of the electron charge). They are consistent with the usual view that atoms and molecules are electrically neutral. The results on H_2 and D_2 provide very direct upper limits of $1\times10^{-15}\,q_e$ for the electron-proton charge difference and $2.4\times10^{-15}\,q_e$ for the neutron charge. The cesium result can be interpreted as showing that the magnitude of the electron charge differs from that of the proton by less than 5 parts in 10^{19} and that the neutron charge is less than $5\times10^{-19}\,q_e$. The relevance of charge equality to cosmological theory, and to the theory of elementary particles is discussed.

1. INTRODUCTION

AN experiment to test on a microscopic level for the presence of a very minute electric charge on atoms and molecules is reported in this paper. The method of the experiment is to study the deflection of a molecular beam in a homogeneous electric field. The results are consistent with the principle that atoms and molecules have no net electric charge, and they can be interpreted to provide an upper limit to the difference between the magnitudes of the electron and proton charges and also to the charge of the neutron.

The present view that any electric charge is an integral multiple of a basic unit is a conclusion which gradually evolved from the experimental findings of the late 19th and early 20th centuries and which has been supported by subsequent observations. However, there have been occasional speculations about the consequences to large scale matter of a small difference, δq, between the electron and proton charge magnitudes. In 1924, Einstein remarked[1] that if δq were only 3 parts in 10^{19} of the electron charge, the magnetic fields of the earth and the sun might be understood as the fields associated with rotating charged bodies. In 1959, Lyttleton and Bondi made the interesting suggestion[2] that if δq were 2 parts in 10^{18} of the electron charge, then the observed rate of expansion of the universe could be explained as an electric repulsion. Indeed, their suggestion provided the specific stimulus for our present experiment.

The equality of electron and proton charges also has importance for the theory of elementary particles,[3,4] as will be discussed later. Considerations here are of a qualitative nature and do not suggest any specific value for the charge difference.

Experimental comparisons of the magnitudes of the electron and proton charges to a greater precision than is possible from their individually measured values[5] have been done by methods which measure the total charge Q of a number M of molecules. Such measurements[6–8] have been consistent with the condition $Q=0$ within the experimental errors, and they have been interpreted to establish an upper limit to the total charge of a single molecule and then also an upper limit to the electron-proton charge difference and to the neutron charge.

A microscopic experiment on individual molecules, in contrast to these macroscopic bulk matter experiments, can be done by observing the trajectory of a beam of molecules passing through a homogeneous electric field (Fig. 1). This method was first used by Hughes with the CsI molecule[9] and then by Shapiro and Estulin with thermal neutrons.[10] Since the trajectory of an individual molecule is essentially independent of the other beam molecules, the microscopic molecular beam experiment provides a very simple situation which is quite free of ambiguities of interpre-

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[1] Remark reported by A. Piccard and E. Kessler, Arch. Sci. Phys. et Nat. (Geneva) **7**, 340 (1925). Some other speculations related to the effects an electron-proton charge difference are given in the following references: W. F. G. Swann, Phil. Mag. **3**, 1088 (1927); Astrophys. J. **133**, 733 (1961). P. M. S. Blackett, Nature **159**, 658 (1947). V. A. Bailey, J. and Proc. Roy. Soc. New South Wales **94**, 77 (1960).

[2] R. A. Lyttleton and H. Bondi, Proc. Roy. Soc. (London) **A252**, 313 (1959); F. Hoyle, *ibid.* **A257**, 431 (1960); L. G. Chambers, Nature **191**, 1082 (1961); W. F. G. Swann, Astrophys. J. **133**, 738 (1961).

[3] G. Feinberg and M. Goldhaber, Proc. Natl. Acad. Sci. U. S. **45**, 1301 (1959).

[4] M. Gell-Mann, in *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 792.

[5] R. A. Millikan, *The Electron* (University of Chicago Press, Chicago, Illinois, 1917), 1st ed., pp. 80–83.

[6] A. Piccard and E. Kessler, reference 1.

[7] A. M. Hillas and T. E. Cranshaw, Nature **184**, 892 (1959); **186**, 459 (1960); H. Bondi and R. A. Lyttleton, *ibid.* **184**, 974 (1959).

[8] J. G. King, Phys. Rev. Letters **5**, 562 (1960).

[9] V. W. Hughes, Phys. Rev. **76**, 474 (1949); **105**, 170 (1957).

[10] I. S. Shapiro and I. V. Estulin, Zh. Eksperim. i Teor. Fiz. **30**, 579 (1956) [translation: Soviet Phys.—JETP **3**, 626 (1957)].

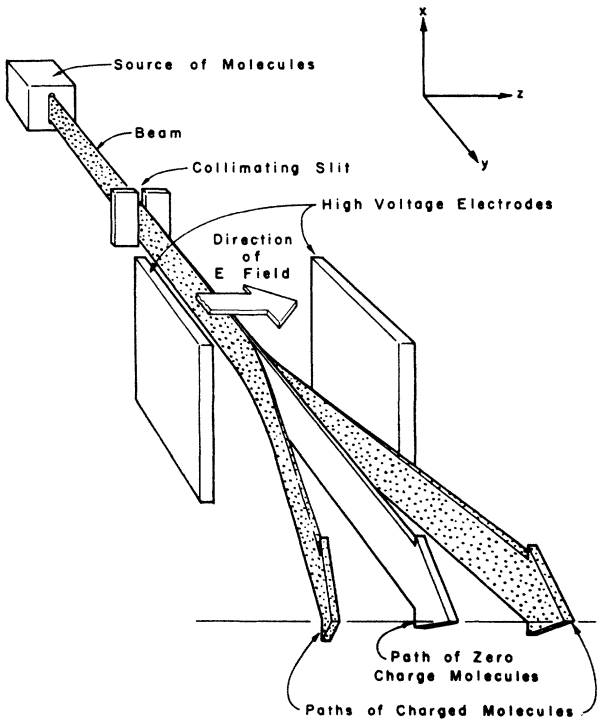

FIG. 1. Molecular beam measurement of atomic or molecular charge.

tation. The measurements reported in this paper have been done on beams of Cs and K atoms and of H_2 and D_2 molecules, and they represent an improvement in sensitivity by a factor of 10^4 over the original experiment of Hughes. Brief accounts of the present work have already appeared.[11,12]

The remainder of this paper is divided into four parts. In Sec. 2 we discuss the principles of the experiment; in Secs. 3 and 4, the measurements on alkali atoms and on the hydrogen molecules, respectively; in Sec. 5, the results and their interpretation. Two Appendixes treat quadrupole effects and deflection analysis.

2. EXPERIMENT: PRINCIPLES

2.1 Deflection of Molecular Beam by an Electric Field

The general principle of the experiment can be discussed with reference to Fig. 2. A well-collimated beam of molecules passes through a region in which a strong electric field may be applied transverse to the direction of beam propagation and the influence of the field on the beam trajectory is studied. Any deflection of the beam by the electric field is then ascribed to forces of the field on a net charge q and of the field gradient on the induced electric dipole moment. In this subsection it is shown that these two effects can be distinguished by comparing deflection measurements with the field in one direction to deflections with the field in the opposite direction; the neglect of the effect of an induced electric quadrupole moment is justified. The method by which extremely small deflections of the beam can be measured is described in subsection 2.2.

Molecules effuse from the oven with a modified Maxwellian velocity distribution characteristic of the oven temperature T. If I designates the total number of molecules incident on the detector per second, and if $I_v dv$ is the number of these molecules which have velocities between v and $v+dv$, then

$$I_v dv = (2I/\alpha^4)v^3 e^{-v^2/\alpha^2} dv, \tag{1}$$

where $\alpha = (2kT/m)^{1/2}$ is the most probable velocity for molecules of mass m inside an enclosure at absolute temperature T.

Molecules pass through the collimator and the field region, and then strike the detector. Suppose the source and collimator slits are of infinitesimal width and lie parallel to one another in the vertical direction. Choose a coordinate system with the x axis in the vertical direction, and with the y axis along a line joining the two slits. We are interested in deflections, s, in the z direction given by

$$s = \int_0^{t_0} \int_0^{t} a_z(x,y,z)dt'dt, \tag{2}$$

where a molecule leaves the source at time $t=0$ and arrives at the detector plane at $t=t_0$, and experiences an acceleration

$$\mathbf{a} = \frac{1}{m}[q\mathbf{E} + \nabla(\boldsymbol{\mu}\cdot\mathbf{E}) + \nabla(\tfrac{1}{6}\mathbf{Q}:\nabla\mathbf{E}) + \cdots] \tag{3}$$

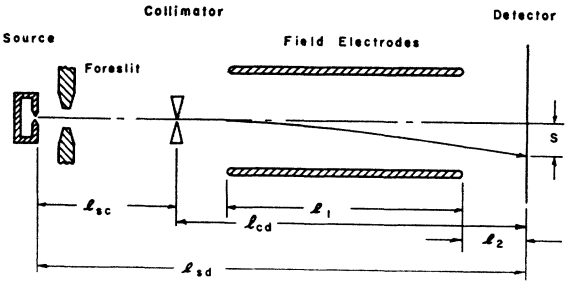

FIG. 2. Geometry of apparatus, showing trajectory of an atom which has been deflected by the electric field. Dimensions (in centimeters) are:

	Alkali experiment	Hydrogen experiment
l_1	200	94
l_2	30	14.6
l_{sc}	193	41
l_{sd}	455	163

[11] J. C. Zorn, G. E. Chamberlain, and V. W. Hughes, Bull. Am. Phys. Soc. 6, 63 (1961); *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 790.

[12] J. C. Zorn, G. E. Chamberlain, and V. W. Hughes, Bull. Am. Phys. Soc. 5, 36 (1960).

in which $\boldsymbol{\mu}$ and $\mathbf{Q}$ are the electric dipole and quadrupole moments[13] of the molecule in the field $\mathbf{E}$. Although the trajectory of an individual molecule has quantum mechanical uncertainties, it follows from the correspondence principle that the maximum of the spatial distribution of these molecules will coincide with the trajectory calculated classically as in Eq. (2).

It is useful to consider the deflections s for the cases in which the electric field has opposite directions $-\mathbf{E}$ (designated case A) and $+\mathbf{E}$ (case B). Since the direction of the induced dipole moment changes sign upon reversal of the field direction, we have

$$s(B)-s(A)=\frac{2}{m}\int_0^{t_0}\int_0^{t}[q\mathbf{E}+\nabla(\tfrac{1}{6}\mathbf{Q}:\nabla\mathbf{E})]_z dt'dt. \quad (4)$$

The quadrupole term is negligible in our work (Appendix I); hence, we retain only the $q\mathbf{E}$ term of the integrand.

The characteristic lengths of our apparatus are much larger than the dimensions of the source and collimator slits, so the molecular trajectories are all very nearly parallel to the y axis. Moreover, all components of acceleration are small, so the x and y components of velocity are virtually constant and we can write $dt=dy/v$. If we assume that the electric field is in the z direction, is uniform within the electrode gap, and is zero elsewhere, then Eq. (4) may be written for molecules of velocity α as

$$s_\alpha(B)-s_\alpha(A)=(qE/2kT)l_1(l_1+2l_2). \quad (5)$$

A measurement of $s_\alpha(B)-s_\alpha(A)$, together with the known values of E, T, l_1, l_2, and the Boltzmann constant k, determines the net molecular charge q.

The complete expression for the force on a beam molecule must include gravitational and magnetic terms. However, the gravitational force is not important since the deflections of interest are in the horizontal plane, and all magnetic forces are negligibly small.

2.2 Measurement of Small Deflections of the Beam

The oven and collimator slits and the detector are of finite width, of course, and the detector signal vs detector position is typically as shown in Fig. 3. The dependence of signal on detector position is particularly strong at z_1 and z_2, and we exploit this characteristic to detect and measure very small shifts in the position of the beam. We now derive the relationship between the deflections of the molecules and the change in observed signal.

The signal obtained from a detector of finite width is designated $I(z,E)$, where z is the coordinate indicating the position of the detector and E is the constant value

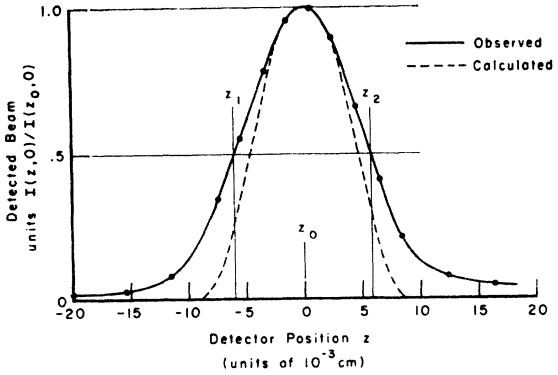

FIG. 3. Theoretical beam shape (calculated on the basis of classical trajectories and ideal geometry) compared to an observed $I(z,0)$ for K atoms.

of the field in the electrode gap. The contribution of molecules with velocities between v and $v+dv$ is designated $I_v(z,E)dv$, so

$$I(z,E)=\int_{v=0}^{\infty} I_v(z,E)dv. \quad (6)$$

Application of the electric field E will deflect every molecule of velocity v by the amount s_v, and hence

$$I_v(z,E)=I_v(z-s_v,0). \quad (7)$$

Note that

$$s_v=(\alpha/v)^2 s_\alpha. \quad (8)$$

The change in intensity at the detector position z due to the deflection of the beam molecules when the field E is applied is designated as $\delta(z,E)$:

$$\delta(z,E)\equiv I(z,E)-I(z,0). \quad (9)$$

Use of Eqs. (6) and (7) and a Taylor's series expansion gives

$$\delta(z,E)=-\int_{v=0}^{\infty}[I_v(z,0)-I_v(z-s_v,0)]dv$$

$$=-\int_{v=0}^{\infty}\left[\frac{\partial I_v(z,0)}{\partial z}s_v-\frac{1}{2}\frac{\partial^2 I_v(z,0)}{\partial z^2}s_v^2+\cdots\right]dv. \quad (10)$$

It is shown in Appendix II that only the first term in the series need be retained when s_α is small and when the detector position z is near the half-maximum intensity coordinates z_1 or z_2; hence, use of Eqs. (1) and (8) gives

$$\delta(z,E)=-[\partial I(z,0)/\partial z]s_\alpha. \quad (11)$$

This intuitively reasonable result permits us to write

$$s_\alpha(B)-s_\alpha(A)=\frac{\delta(z,A)-\delta(z,B)}{\partial I(z,0)/\partial z}. \quad (12)$$

Measurements are performed on both sides of the beam at z_1 and z_2. Because $(\partial I/\partial z)_{z_1}\simeq-(\partial I/\partial z)_{z_2}$, it is

[13] J. H. Van Vleck, *Electric and Magnetic Susceptibilities* (Oxford University Press, New York, 1932). J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, *Molecular Theory of Gases and Liquids* (John Wiley & Sons, New York, 1954), p. 839.

possible to use the mean value of the slope of the beam shape as an input value for the calculation of results:

$$\langle dI/dz\rangle \equiv \frac{1}{2}\left[\left(\frac{\partial I(z,0)}{\partial z}\right)_{z_1}-\left(\frac{\partial I(z,0)}{\partial z}\right)_{z_2}\right]. \quad (13)$$

The average value of $s_\alpha(B)-s_\alpha(A)$, as determined at both z_1 and z_2 will then be, to a good approximation,

$$s_\alpha(B)-s_\alpha(A) = \frac{1}{2}\frac{[\delta(z_1,A)-\delta(z_1,B)]-[\delta(z_2,A)-\delta(z_2,B)]}{\langle dI/dz\rangle}. \quad (14)$$

An advantage of this deflection analysis as compared to the conventional one[9,14] is that only directly observed quantities appear, and that slit dimensions are not parameters in the final result. Furthermore, the analysis is valid even in the presence of slit misalignments and of molecular beam scattering by the background gas in the apparatus. If slit dimensions are known and if slit alignments are perfect, the two methods give identical results. The present deflection analysis has also been used in a measurement of the polarizability of alkali atoms, and the agreement of the results[15] with those of Salop *et al.*, whose atomic beam experiment[16] did not require measurement of beam deflections, gives confidence in the analysis.

When high potentials are applied, a current flows across the electrode gap. This current, which can be as large as 150 μA for fields of 100 kV/cm, depends on the applied voltage in a nonlinear manner, and also on the immediate past history of the electrodes. The current causes an over-all attenuation of the molecular beam, as is discussed in Sec. 3.2 below.

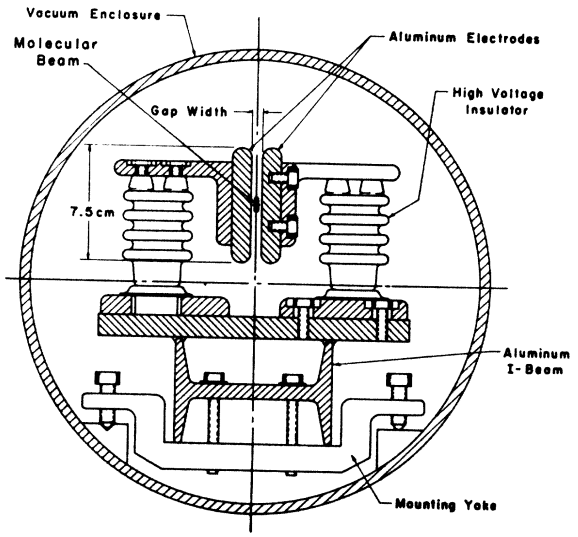

FIG. 4. Cross section of electrode assembly.

We write the observed change of intensity $\Delta(z,E)$ at detector position z as a result of applying field E as the sum of the change due to deflection $\delta(z,E)$ and that due to attenuation $D(z,E)$:

$$\Delta(z,E)=\delta(z,E)+D(z,E). \quad (15)$$

In general, $D(z,A)\neq D(z,B)$, but it seems reasonable to suppose that for a given field the attenuation will be relatively constant over the width of the beam, so $D(z_1,A)=D(z_2,A)$ and $D(z_1,B)=D(z_2,B)$. From Eq. (15) we then have

$$[\Delta(z_1,A)-\Delta(z_1,B)]-[\Delta(z_2,A)-\Delta(z_2,B)] = [\delta(z_1,A)-\delta(z_1,B)]-[\delta(z_2,A)-\delta(z_2,B)] \quad (16)$$

and the contributions of $D(z,E)$ can be avoided. Use of Eqs. (5), (14), and (16) gives the result:

$$q=\frac{kT}{l_1(l_1+2l_2)E} \times\left(\frac{[\Delta(z_1,A)-\Delta(z_1,B)]-[\Delta(z_2,A)-\Delta(z_2,B)]}{\langle dI/dz\rangle}\right). \quad (17)$$

3. EXPERIMENT: ALKALI ATOMS

3.1 Apparatus

The vacuum envelope is a 490-cm-long brass tube of 30-cm diam divided by a single bulkhead into a 28-cm source chamber and a 462-cm main chamber. The dividing bulkhead has a foreslit aperture of 0.05 cm^2 through which the beam passes. The main chamber is evacuated to a pressure of about 3×10^{-7} mm of Hg by two oil diffusion pumps, having a combined rated speed of 1000 liter/sec, used together with water-cooled baffles and liquid-nitrogen traps. The source chamber is maintained at a pressure of about 10^{-5} mm of Hg by a 300 liter/sec oil pump with a water-cooled baffle. All demountable vacuum joints are sealed with neoprene O-ring gaskets.

The beam of atoms is generated in an electrically heated nickel oven of conventional design.[17] The oven slit is 0.004 cm wide, 1.5 cm high, and 0.01 cm thick. The oven temperature is measured with a chromel-alumel thermocouple.

The collimator is a pair of ground steel edges (thin razor blades) mounted 0.004 cm apart on an all-metal support. The collimator slits are 2 cm high and do not limit the beam height of 0.5 cm which is determined by the vertical dimensions of the foreslit and detector openings.

The field electrodes are mounted with adjustable insulating supports on an aluminum I beam as shown in Fig. 4. This method of construction allows assembly

[14] O. Stern, Z. Physik **41**, 563 (1927). R. G. J. Fraser, *Molecular Rays* (Cambridge University Press, New York, 1931), pp. 127 ff.
[15] G. E. Chamberlain and J. C. Zorn, Bull. Am. Phys. Soc. **7**, 70 (1962); Phys. Rev. **129**, 677 (1963).
[16] A. Salop, E. Pollack, and B. Bederson, Phys. Rev. **124**, 1431 (1961).
[17] P. Kusch and V. W. Hughes, in *Handbuch der Physik*, edited by S. Flügge (Springer Verlag, Berlin, 1959), Vol. 37, Part 1, p. 6.

and adjustment before the field is placed as a unit on a three-point suspension inside the vacuum envelope. Because this I-beam structure is very rigid, it is not difficult to maintain the electrode spacing uniform to within ± 0.003 cm and their straightness to within ± 0.006 cm over the 200-cm length of the electrodes.

The electrodes themselves are made from 1.25-cm-thick aluminum tool and jig plate which is very flat as delivered from the mill and which will retain its flatness after machining, because it is free of strains. However, jig plate is manufactured by a casting process that leaves minute holes throughout the metal which prevent one from polishing to a really smooth surface; these surface irregularities may be the points at which high voltage discharges originate, and the castings may outgas. The insulating support for the electrodes is made by affixing a suitable baseplate and top bracket to an alumina high voltage terminal; six such insulators hold each electrode firmly in place.

The electrode voltages are derived from two regulated high voltage power supplies; the voltages are measured with the meters on these supplies which are in turn checked frequently with a calibrated vacuum tube voltmeter.

The surface ionization beam detector uses a 92% platinum, 8% tungsten alloy[18] hot wire of 0.0025-cm diam; the wire is heated to about 1300°K with a dc current of 28 mA. The ionized beam atoms are drawn from the detector wire by a 12-V potential to a collector surrounding the wire; the ion current is measured with a modified DuBridge-Brown[19] electrometer having a 10^{11}-Ω grid input resistor. A galvanometer gives the readout of the beam detection system. The maximum sensitivity of the beam detector is 6×10^{-16} A of ion current for 1 cm of galvanometer scale deflection; the characteristic time constant is 5 sec. The usual beam is 1.2×10^{-12} A [corresponding to $I(z_0,0)=2000$ cm of galvanometer deflection]. Aside from the discharge effects mentioned below, there is no observable coupling between the electrode voltages and the electrometer circuitry; isolation is achieved with an electrostatic shield of 2-mm mesh wire screen between the detector and the electrodes.

3.2 Procedure

Because the apparatus is so sensitive to changes in geometry that a change in the relative positions of beam and detector of as small as 10^{-5} cm is noticeable, effort is made to keep the laboratory at a reasonably constant temperature. The apparatus is optically aligned so that the beam runs about midway between the electrodes. Only minor difficulty was experienced in the optical alignment of the slits looking through the 200-cm-long, 1-mm-wide electrode gap. A Keuffel and Esser 9092-1A jig transit is used for this work.

The magnitude and direction of the dipole polarizability deflection depend critically on the position of the collimator, and indeed by suitable choice of position this deflection can be made to vanish. It is convenient to adjust the collimator position until a small deflection occurs, because this allows one to observe a definite deflection signal each time the field is turned on or off. In preliminary experiments a collimator of quartz edges mounted on a Vycor rod was placed in the electrode gap (a 1-cm gap was used) and the anomalously large dipole polarizability deflection observed was ascribed to accumulation of charge on the glass edges. Only a small amount of polarizability deflection is observed in the final version of the experiment with the all-metal collimator mounted in a region essentially free of electric fields. We observe no increase in this polarizability deflection when the electrodes are adjusted to dimensional tolerances about twice those mentioned in Sec. 3.1. From our estimates it seems reasonable that the dipole polarizability deflections arise from the field inhomogeneities at the ends of the electrodes.

Initially a careful measurement of the beam shape, $I(z,0)$ vs z, is made (see Fig. 3). The detector is then set at z_1 or z_2 and many deflection readings $\Delta(z_i,E)$ are taken with both polarities of applied voltage. About a dozen readings of Δ are made for each z_i and each polarity at several different voltages. Finally, another measurement of $I(z,0)$ vs z is made to assure that no excessive changes in geometry have occurred over the hour or so needed for the deflection measurements.

The agreement between the calculated and observed curves of $I(z,0)$ is good; the small discrepancy is of the usual size and is attributed to atomic beam scattering, slit misalignment, and imperfect knowledge of slit dimensions. For a collimator-detector distance of l_{cd} and a collimator width w_c, diffraction of the beam is appreciable only when w_c^2/l_{cd} is of the same order as the De Broglie wavelength of the beam molecules.[20] In this experiment $(w_c^2/l_{cd})\gg\lambda$ (molecule), so diffraction does not contribute significantly to the observed width of $I(z,0)$.

The deflection of the beam due to a net atomic charge would be directly proportional to E and, at the field strengths used in this experiment, the deflection from the induced dipole moment is proportional to E^2. The observed dependence of $\Delta(z_i,E)$ is shown in Fig. 5. It is seen that $\Delta(z_i,E)$ is linearly proportional to E^2 up to a field E of about 10^5 V/cm, as expected for deflections due to dipole polarizability alone. At still higher fields Δ is no longer proportional to E^2; indeed both

[18] Alloy No. 479, Sigmund Cohn, Inc., Mt. Vernon, New York. Use of this material as a detector of alkali atom beams is discussed by S. Datz and E. Taylor, J. Chem. Phys. **25**, 389 (1956).

[19] L. DuBridge and H. Brown, Rev. Sci. Instr. **4**, 532 (1933); F. O'Meara, *ibid.* **22**, 106 (1951).

[20] In an unpublished experiment done by P. R. Fontana and J. C. Zorn with the present apparatus ($l_{cd}=262$ cm) the width [(z_1-z_2) of Fig. 3] of a beam of potassium atoms was measured as a function of collimator width. The results showed a broadening of the beam for collimator widths less than 8×10^{-4} cm in agreement with the predictions of single slit diffraction theory.

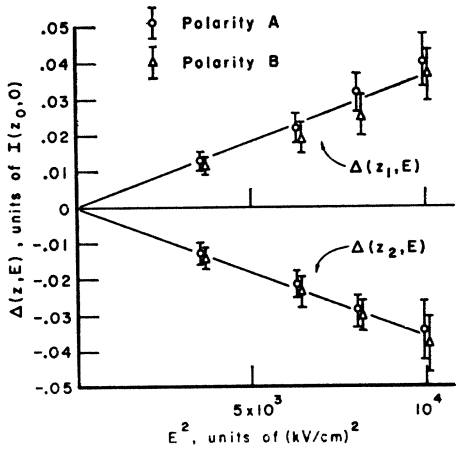

FIG. 5. $\Delta(z,E)$ vs E^2 for potassium.

$\Delta(z_1,E)$ and $\Delta(z_2,E)$ decrease with increase in E at sufficiently high values of E. This behavior is not consistent with deflection due to a net atomic charge and a dipole polarizability but rather is explained by an attenuation of the atomic beam at the higher fields. The beam appears to be attenuated in proportion to the gap current, and this gives rise to a field dependent signal change $D(z_i,E)$ not associated with an electric deflection of the beam atoms.

On the basis of our estimates, which are limited by a lack of detailed knowledge of conditions in the electrode gap, it is unlikely that direct collisions of beam atoms with electrons of the gap current are the cause of $D(z_i,E)$. It seems more probable that the impacts of the energetic electrons drive impurities out of the aluminum surfaces which increases the pressure in the electrode gap and attenuates the beam. Confirmatory evidence for this view is provided by the observation that a change in pressure of 1×10^{-8} mm of Hg (as by warming a liquid-nitrogen trap), causes more than a 1% change in $I(z_0,0)$.

Electric fields higher than 100 kV/cm can be obtained with the electrodes described set at spacings of 1 or 2 mm. For a given electric field value, the current which flows in the gap is considerably less when a 1-mm gap and a lower voltage are used. Discharges across the electrode gap become more frequent as the applied voltage is increased; a discharge will usually drive the galvanometer off scale and so disable the detector for about 30 sec. In the run illustrated in Fig. 5, 60 kV/cm was a useful compromise between high field strength and the absence of discharges. All data used for the computation of atomic charge were taken at values of E below where nonlinearity of the $\Delta(z_i,E)$ vs E^2 plot is noticeable.

3.3 Data, Analysis, and Results

The experimental data needed for calculation of the charge of the atom with Eq. (17) and for estimation of the statistical error, as described below, are listed in Table I.

$\bar{\Delta}(z_i,E)$ is the average galvanometer scale deflection

$$\bar{\Delta}(z_i,E)\equiv\frac{1}{n_{iE}}\sum_{r=1}^{n_{iE}}\Delta_r(z_i,E),$$

where $\Delta_r(z_i,E)$ is the rth member of the sample of n_{iE} deflection readings. The sample variance, s_{iE}^2, is

$$s_{iE}^2\equiv\frac{1}{n_{iE}}\sum_{r=1}^{n_{iE}}[\Delta_r(z_i,E)-\bar{\Delta}(z_i,E)]^2.$$

TABLE I. Data for alkali atoms.

Run	Cesium measurements 1[a]	2[a]	3	4	5	Potassium measurements 1	2	3
$\Delta(z_1,A)$[b]	−26.4	6.0	−15.0	−10.7	−11.2	−9.9	32.2	14.7
n_{1A}	8	5	10	9	20	19	6	22
s_{1A}^2	21.0	1.2	6.0	1.3	11.2	4.6	8.5	8.6
$\Delta(z_1,B)$[b]	−35.2	0	−15.4	−11.3	−14.0	−10.6	37	13.8
n_{1B}	8	6	8	8	18	17	6	22
s_{1B}^2	13.0	1.2	6.25	1.2	9.2	5.5	33	10.9
$\Delta(z_2,A)$[b]	17.3	−9.25	13.5	11.8	35.4	9.5	−38	−8.3
n_{2A}	7	4	8	6	14	8	5	12
s_{2A}^2	6.6	5.0	5.75	1.8	14.1	1.5	6.0	6.6
$\Delta(z_2,B)$[b]	13.2	−15.75	9.6	11.0	35	10.4	−38.3	−7.5
n_{2B}	8	4	7	10	13	8	6	16
s_{2B}^2	13.8	2.25	6.6	2.1	12.4	2.5	3.3	6.8
$\langle dI/dz\rangle$	2.85×10^5	2.34×10^5	2.48×10^5	1.84×10^5	4.5×10^5	3.05×10^5	3.55×10^5	6.5×10^5
E (kV/cm)	80	70	90	90	70	70	60	80
T (°K)	477	465	476	478	486	506	515	515
$q_j(10^{-17}q_e)$	+16.3±10.7	−2.4±7.8	−12.4±6.9	−0.9±4.8	+6.1±4.7	+6.3±5.2	−20.5±28	+2.8±2.5

[a] Runs 1 and 2 for cesium were done with field gap set at 2 mm; for all other runs a 1-mm gap was used.
[b] $\Delta(z_i,E)$ is given in units of centimeters of galvanometer scale deflection.

In our work, the output signals are given in centimeters of deflection on a galvanometer scale. With the beam off and the detector wire unheated, less than 0.1 cm of noise is seen. Under the best operating conditions the deflections $\Delta_r(z_i,E)$ are reproducible to about 1 cm, but often the scatter of the readings is larger than this, as indicated by the sample variances in Table I. We believe that the main causes of the scatter are ion bursts from the detector wire and fluctuations in the vacuum. The uncertainty to be quoted in our final result for the net charge of cesium corresponds to 0.5 cm of deflection.

With the mean observed deflections $\bar{\Delta}(z_i,E)$ and other measured quantities, the charge q_j for the jth run is computed from Eq. (17). The average value of charge observed from n runs is

$$\bar{q}\equiv\frac{1}{n}\sum_{j=1}^{n}q_j, \tag{18}$$

and the sample variance is

$$s^2\equiv\frac{1}{n}\sum_{j=1}^{n}(q_j-\bar{q})^2. \tag{19}$$

The true charge of the atom is designated $q(\text{atom})$; from the theory of small samples[21] we expect the variable

$$t\equiv\frac{\bar{q}-q(\text{atom})}{s/(n-1)^{1/2}}$$

to have Student's distribution with $n-1$ degrees of freedom. The confidence interval

$$\left(\bar{q}-t_1\frac{s}{(n-1)^{1/2}}\right)<q(\text{atom})<\left(\bar{q}+t_1\frac{s}{(n-1)^{1/2}}\right)$$

is associated with the confidence coefficient

$$P(t_1)=\int_{-t_1}^{t_1}f(t,n-1)dt,$$

where $f(t,n-1)$ is the frequency function for Student's t with $n-1$ degrees of freedom. The limit t_1 of the integral is tabulated[22] as a function of $(n-1)$ and $P(t_1)$. We choose $P(t_1)=0.7$ since this corresponds closely in the limit of large n to the confidence coefficient for the standard error of the mean. The expression used to calculate the net charge of Cs and K atoms is

$$q(\text{atom})=\bar{q}\pm t_1\frac{s}{(n-1)^{1/2}}. \tag{20}$$

The uncertainty in the result q_j of a single run will now be discussed. We assume that the individual deflections $\Delta_r(z_i,E)$ are normally distributed about their mean $\bar{\Delta}(z_i,E)$. Because of apparatus instabilities it was not possible to get enough data for a sensitive chi-square test of normality, but no obvious contradiction to the normal hypothesis was observed. Small sample theory is used to analyze the data from a single run. The random error for an individual q_j comes mainly from uncertainty in the correct value for $[\Delta(z_1,A)-\Delta(z_1,B)]-[\Delta(z_2,A)-\Delta(z_2,B)]$ in Eq. (17). The theory for estimation of the sum (or difference) of two mean values[23] has been extended to estimate the sum of any number of means. The result, applied to the data from a single run, gives

$$q_j=\frac{kT\langle\Delta\rangle}{l_1(l_1+2l_2)E\langle dI/dz\rangle}, \tag{21}$$

where

$$\langle\Delta\rangle\equiv[\bar{\Delta}(z_1,A)-\bar{\Delta}(z_1,B)]-[\bar{\Delta}(z_2,A)-\bar{\Delta}(z_2,B)]$$
$$\pm t_1\left[\left(\frac{1}{n_{1A}}+\frac{1}{n_{1B}}+\frac{1}{n_{2A}}+\frac{1}{n_{2B}}\right)\times\left(\frac{n_{1A}s_{1A}^2+n_{1B}s_{1B}^2+n_{2A}s_{2A}^2+n_{2B}s_{2B}^2}{n_{1A}+n_{1B}+n_{2A}+n_{2B}-4}\right)\right]. \tag{22}$$

Here, t_1 is Student's t for $(n_{1A}+n_{1B}+n_{2A}+n_{2B}-4)$ degrees of freedom. For presentation of results in Table I we again choose $P(t_1)=0.7$.

Important sources of systematic errors can be discussed with reference to Eq. (21). The source temperature T is subject to about 4% error, primarily because of thermal gradients in the oven. A 6% error in the electric field E comes from uncertainties in the field gap dimension and in the value of the applied voltage. Lengths l_1 and l_2 are measured to better than 1%. The uncertainty in $\langle dI/dz\rangle$ is estimated at 5%. All of these effects contribute to an estimate of 10% for the systematic error.

The derivation of Eq. (21) assumes a Maxwellian velocity distribution in the beam [Eq. (1)]. Deviations from this distribution can arise from lack of ideal effusion from the source, polymer content of the beam, and scattering. From the experiments of Miller and Kusch[24] the finite thickness of our source slit is expected to affect the velocity distribution by an amount corresponding to less than a 3% increase in source temperature. Velocity analyses of molecular beams[24,25] and experiments[26] on Cs_2 and K_2 indicate that the polymer content of beams of Cs and K is about 1%, and this is small enough to be negligible in the present experiment. Slow atoms will be preferentially scattered

[21] H. Cramér, *Mathematical Methods of Statistics* (Princeton University Press, Princeton, New Jersey, 1946), pp. 237 ff and pp. 512–18.

[22] R. A. Fisher, *Statistical Methods for Research Workers* (Oliver and Boyd, Edinburgh, 1950), 11th ed.

[23] H. Cramér, reference 21, pp. 520–23.

[24] R. C. Miller and P. Kusch, Phys. Rev. **99**, 1314 (1955).

[25] G. M. Rothberg, M. Eisenstadt, and P. Kusch, J. Chem. Phys. **30**, 517 (1959).

[26] P. Kusch, S. Millman, and I. I. Rabi, Phys. Rev. **55**, 1176 (1939).

from the beam,[24,27] but this effect appears to be too small to be important here. Since these systematic effects on the velocity distribution are small, no corrections have been applied for them.

For the apparatus used in the alkali experiment, $l_1=200$ cm and $l_2=30$ cm; from Eq. (17) we have

$$q=1.66\times10^{-9}\frac{T\langle\Delta\rangle}{E\langle dI/dz\rangle}q_e, \tag{23}$$

where $\langle\Delta\rangle$ is defined by Eq. (22), E is in V/cm, and q_e is the absolute value of the electron charge. Input values for Eq. (23) and results for q_j(atom) are given in Table I. From Eq. (20) we compute q(atom):

$$q(\mathrm{Cs})=(13\pm56)\times10^{-18}q_e,$$
$$q(\mathrm{K})=(-38\pm118)\times10^{-18}q_e.$$

As discussed above, the quoted uncertainty corresponds to the standard error of the mean. Use of the conventional deflection analysis[9,14] gives values for q(atom) smaller than the above values by about a factor of 2.

4. EXPERIMENT: HYDROGEN MOLECULES

A measurement of the net charge of H_2 provides a direct test for an electron-proton charge difference; a measurement on D_2, in addition, will then give an upper limit for the electric charge of the neutron. Hence, a measurement of the net electric charge of H_2 and D_2 by the beam method is of fundamental importance.

The gas molecular beam apparatus is a modification of the system described in Sec. 3.1. Since the generation of an atomic hydrogen beam requires the use of a dissociator in which stability problems may arise, a molecular beam is used for the charge measurements. Because the Pirani gauge used for the detection of beams of H_2 and D_2 is far less efficient than the surface ionization detector used for the alkalis, the gas apparatus is shorter and less sensitive to small deflections than the alkali apparatus. As a result, our limits for the charge of the gas molecules are appreciably larger than those obtained for cesium and potassium.

The vacuum envelope is divided into a source chamber, an interchamber and a main chamber. Between the source chamber and interchamber is a 0.025-cm×0.625-cm aperture, and between the interchamber and main chamber is a 0.050-cm×0.4-cm opening for passage of the beam. The source chamber is evaluated by a 700 liter/sec oil diffusion pump; the typical operating pressure in this chamber with gas beam flowing is 2×10^{-4} mm of Hg as read on an ionization gauge. The interchamber and main chamber are each pumped by 300 liter/sec oil pumps. All three diffusion pumps have water-cooled baffles and liquid-nitrogen traps. The main chamber operating vacuum of 2×10^{-7} mm of Hg is virtually unaffected by the presence of the beam.

The gas beam issues from a 0.005-cm×1.1-cm slit of 0.01-cm thickness in a thermally conducting cavity which can be cooled to liquid-nitrogen temperature. Gas is fed to the cavity through a capillary tube, and a flow of about 12 cc/min STP of H_2 gives a satisfactory beam.

The collimator is as described in Sec. 3.1 except that the slit is 0.005 cm wide. The beam height is determined by the vertical dimensions of the foreslit (0.4 cm) and detector (0.33 cm). The electrode assembly is as used in the alkali experiment except that the electrodes are 94 cm long. All H_2 and D_2 work is done with a 1-mm electrode gap.

The gas beam detector is a Pirani gauge built after the pattern of Prodell and Kusch[28] by Drake[29] of this laboratory. The slit opening of the gauge is 0.0025 cm wide and 0.33 cm high. With the Pirani detector at 300°K, the sensitivity is such that 3.6×10^9 H_2 molecules/sec into the gauge gives 1 cm of deflection on the galvanometer scale used for readout. Cooling the entire Pirani gauge to dry ice–acetone temperature (195°K) improves the sensitivity by a factor of 2. A very simple electrostatic shield between the Pirani gauge and the electrodes prevents observable coupling between the high-voltage fields and the detector.

Initial alignment is made so the molecular beam runs in the center of the electrode gap. Because the dipole polarizability of H_2 is small[30] there is no observable polarizability deflection.[31] The minimum detectable s_α is about 0.8×10^{-4} cm which corresponds to a change in signal of about 1/200th of the total beam intensity. The gap currents observed in the gas experiment were very much lower than those seen under similar field conditions in the alkali work. We believe this is because the electrodes were repolished when being shortened for the gas experiments, and were not subsequently exposed to a beam of condensable atoms.

Several runs were taken with the source cooled by liquid nitrogen, but the advantage associated with lower temperature is offset by the lower beam intensity. Because drifts in the Pirani detector are less troublesome at higher beam intensities, the data given here are all taken with the source at 300°K.

The actual charge measurement proceeds in about the same way as is outlined in Sec. 3.2 for the alkalis.

[27] I. Estermann, O. C. Simpson, and O. Stern, Phys. Rev. **71**, 238 (1947); P. M. Marcus and J. H. McFee, in *Recent Research in Molecular Beams*, edited by I. Estermann (Academic Press Inc., New York, 1959), pp. 43 ff.

[28] A. G. Prodell and P. Kusch, Phys. Rev. **88**, 184 (1952).

[29] The details of the Pirani gauge used in this experiment are given by C. W. Drake, Yale Ph.D. dissertation, 1958 (unpublished).

[30] L. Essen, Proc. Phys. Soc. (London) **B66**, 189 (1953).

[31] We have used a modification of the present gas apparatus to observe the deflection of an H_2 beam under the influence of an inhomogeneous electric field of known properties. The results are in agreement with what one expects from the known polarizability of H_2 and assure that the system, as used for the charge measurements, is adequately sensitive to small deflections.

In all cases the observed signal $\Delta(z_i,E)$ is less than the 1-mm noise level of the Pirani gauge, so a statistical analysis of deflections is not applicable.

For the gas apparatus, the lengths are $l_1=94$ cm and $l_2=14.6$ cm, and Eq. (17) may be written

$$q=7.44\times10^{-9}\frac{T\langle\Delta\rangle}{E\langle dI/dz\rangle}q_e, \tag{24}$$

where E is in V/cm and q_e is the absolute value of the electron charge. The data and derived results are given in Table II and from these we conclude that

TABLE II. Data for hydrogen molecules.[a]

	Hydrogen		Deuterium	
Run	1	2	1	2
$I(z_0,0)$ (cm)	20	19	14	13
$\langle dI/dz\rangle$	1180	1100	867	750
$q_j(10^{-15}q_e)$	<1.9	<2.0	<2.6	<3.0

[a] For all runs: $T=300°$K, $E=100$ kV/cm, $\langle\Delta\rangle<1$ mm.

$$|q(H_2)|<2\times10^{-15}q_e, \quad |q(D_2)|<2.8\times10^{-15}q_e.$$

5. RESULTS AND INTERPRETATION

5.1 Limits for the Electron-Proton Charge Difference and the Electric Charge of the Neutron

The charge of an atom or molecule is assumed to be completely given by the scalar sum $q=Z\delta q+Nq_n$, where Z is the number of electron-proton pairs, $\delta q\equiv q_p-q_e$ is the electron-proton charge difference, N is the number of neutrons, and q_n is the neutron charge. The most direct determination of a limit for δq is obtained from the measurement of the net charge of the hydrogen molecule:

$$|\delta q|=|q(H_2)|/2<1\times10^{-15}q_e. \tag{25}$$

In addition, the result from deuterium gives a limit for q_n:

$$|q_n|<2.4\times10^{-15}q_e. \tag{26}$$

Smaller limits than the above can be obtained from the experimental values for the charges of cesium and potassium given in Sec. 3:

$$q(\mathrm{Cs})=55\delta q+78q_n=(13\pm56)\times10^{-18}q_e. \tag{27}$$

$$q(\mathrm{K})=19\delta q+20q_n=(-38\pm118)\times10^{-18}q_e. \tag{28}$$

As simultaneous equations in δq and q_n, the solution gives

$$\delta q=(-8.5\pm27)\times10^{-18}q_e \tag{29}$$

independently of the value of q_n, and

$$q_n=(6.1\pm20)\times10^{-18}q_e \tag{30}$$

independently of the value for δq.

A still smaller limit for the electron-proton charge difference can be given if one assumes that $\delta q=q_n$. This relation follows from the usual assumption that charge is conserved in beta decay of the neutron ($n\rightarrow p+e^-+\bar{\nu}$) and that the charge of the antineutrino is zero.[32] Then $\delta q=q(\mathrm{atom})/(Z+N)$ and we obtain from $q(\mathrm{Cs})$:

$$\delta q=(1.0\pm4.2)\times10^{-19}q_e. \tag{31}$$

On the other hand, a limit to the antineutrino charge, $q_{\bar{\nu}}$, can be obtained from the results of Eqs. (29) and (30) and from the assumption of charge conservation in the reaction $n\rightarrow p+e^-+\bar{\nu}$:

$$|q_{\bar{\nu}}|<4\times10^{-17}q_e. \tag{32}$$

As interpreted thus far, our molecular beam deflection experiment measures the molecular charge q expressed as the constant of proportionality between the force $\mathbf{F}$ and the applied electric field $\mathbf{E}$ in the relation $\mathbf{F}=q\mathbf{E}$. We may also point out that the molecular charges quoted are averages over a trajectory of several meters and a time of a few milliseconds.

If the different molecules in the beam had different charges due, for example, to different electrons having different charges, then the molecular beam would spread in a transverse electric field. The absence of observed spreading sets a limit to the molecular charge distribution. Specific analysis of the H_2 data under the assumption of similar normal distributions of the charges of electrons and protons yields an upper limit of about $8\times10^{-15}q_e$ to the widths of these distributions. This experimental result appears to be the strongest available evidence against any hypothesis of a distribution in charge for the electron and the proton.

5.2 Comparison with Other Experiments

Table III lists experimental data on the electron-proton charge difference obtained by the gas efflux and beam deflection methods. Note that all the results give $\delta q=0$ with a conservative interpretation of the limits of experimental errors. In the macroscopic gas efflux method the total charge Q of M gas molecules is measured by observing the change in potential of a metal container relative to its surroundings when gas effuses from the container. Mechanical distortion effects and difficulties in preventing efflux of ions and electrons limit the sensitivity of this experiment. As for any other macroscopic experiment, the interpretation must consider possible charge compensation effects associated with the net charge of a large number of molecules. The limit on δq from the gas efflux experiments is between 10^{-21} and $10^{-20}q_e$, whereas the limit from the

[32] An upper limit to the neutrino charge can be obtained by considering that the neutrino is a Dirac particle with a mass of 500 eV (upper limit to the allowed neutrino mass) and computing the upper limit to the charge that is consistent with neutrino cross-section data [J. S. Allen, *The Neutrino* (Princeton University Press, Princeton, New Jersey, 1958)]. The limit found for the neutrino charge in this way is about $10^{-9}q_e$.

TABLE III. Measured limits for the charge of molecules.

Method	Molecule	q(molecule) in units of q_e	$\delta q = q$(molecule)$/(Z+N)$	Investigators
By gas efflux	CO_2	$<2.2\times10^{-19}$	$<5\times10^{-21}$	Piccard and Kessler[a]
	A	$(4\pm4)\times10^{-20}$	$(1\pm1)\times10^{-21}$	Hillas and Cranshaw[b]
	N_2	$(6\pm6)\times10^{-20}$	$(2.1\pm2.1)\times10^{-21}$	Hillas and Cranshaw
	H_2	$(-2.5\pm1.5)\times10^{-20}$	$(-1.3\pm0.8)\times10^{-20}$	King[c]
	He	$(4\pm2)\times10^{-20}$	$(1\pm0.5)\times10^{-20}$	King
By beam deflection	CsI	$<4\times10^{-13}$	$<1.5\times10^{-15}$	Hughes[d]
	Free neutron	$<6\times10^{-12}$		Shapiro and Estulin[e]
	CsF	$<2\times10^{-14}$	$<1.3\times10^{-16}$	Zorn, Chamberlain and Hughes[f]
	KF	$<1\times10^{-13}$	$<1.7\times10^{-15}$	Zorn, Chamberlain and Hughes
	H_2	$<2\times10^{-15}$	$<1\times10^{-15}$	Present work
	D_2	$<2.8\times10^{-15}$	$<0.7\times10^{-15}$	Present work
	K	$(3.8\pm11.8)\times10^{-17}$	$(1\pm3)\times10^{-18}$	Present work
	Cs	$(1.3\pm5.6)\times10^{-17}$	$(1\pm4.2)\times10^{-19}$	Present work

By Mössbauer effect, a limit of $1\times10^{-15}q_e$ for the charge of the photon has been established.[g]

[a] See reference 6. [c] See reference 8. [e] See reference 10.
[b] See reference 7. [d] See reference 9. [f] See reference 12.
[g] L. Grodzins, D. Engelberg, and W. Bertozzi, Bull. Am. Phys. Soc. 6, 63 (1961).

beam deflection experiment is $5\times10^{-19}q_e$. It appears that the gas efflux and the molecular beam experiments are complementary in the sense that the gas experiment measures the change in potential produced by the charge, whereas the molecular beam experiment measures the charge itself through the force exerted on it by an externally applied electric field.

Millikan's oil drop experiment can be interpreted to yield[2,9] a limit on δq of about $10^{-16}q_e$. An extension of this type of experiment using a small magnetically suspended metal sphere has been proposed[33] to achieve a higher sensitivity in the determination of δq.

A substantial increase in sensitivity with the beam deflection method also appears possible. Use of an ultra-high vacuum system should improve beam stability and may allow for the application of higher electric fields. Optimum choice of electrode material together with adequate care about surface condition[34]—perhaps even the use of heated glass as a cathode material[35]—should allow an increase in the applied electric field. Use of a mass spectrometer together with the surface ionization detector and collection of adequate data so that the statistical error is small should improve the detection sensitivity. A dual detector to read $\Delta(z_1,E)$ and $\Delta(z_2,E)$ simultaneously would help to discriminate against noise, because meaningful signals satisfy $\Delta(z_1,E)=-\Delta(z_2,E)$. Altogether, improvement of the sensitivity of the beam deflection method by a factor of 100 over the present result seems possible.

5.3 Interpretation of Results

The atomic beam deflection experiment on the alkali atoms reported in this paper provides a limit for δq of $5\times10^{-19}q_e$. This limit is about $\frac{1}{4}$ the value of δq required by the theory of the expanding universe proposed by Lyttleton and Bondi.[2] Furthermore, the macroscopic experiments by the gas efflux method provide the even smaller limit of $10^{-21}q_e$ to $10^{-20}q_e$. All of these results provide strong evidence against the form of the Lyttleton-Bondi proposal which requires $\delta q\simeq2\times10^{-18}q_e$; they do not test the alternative, though less attractive, form of the Lyttleton-Bondi proposal which requires a greater number of protons than electrons in the universe.

Ideally, elementary particle theory should predict the observed spectrum of the elementary particles including their charge and mass ratios. Modern quantized field theory can describe discrete particles, but it requires that values of a particle's mass and charge be obtained from experiment. The invariance of the theory under charge conjugation or the interchange of particle and antiparticle does provide the theoretical prediction that a particle and its antiparticle should have charges which are equal in magnitude but opposite in sign, for example, that the electron and positron charges should have the same magnitude and also the proton and antiproton charges should have the same magnitude. However, theory does not predict the ratio of the magnitudes of the charges on two different particles, for example, that of the electron and the proton.

Indeed, in view of modern charge renormalization theory the question of the electron-proton charge ratio becomes rather deep and somewhat ambiguous. If the bare charges of the electron and proton were equal, then conventional renormalization theory with gauge invariance would require that the renormalized electron and proton charges should also be equal. However, Gell-Mann[4] and Nambu have remarked that if in addition to the photon there is another neutral vector particle which is coupled to the proton but not to the electron, then even though the bare charges of the electron and the proton were equal, the renormalized charges are expressed in terms of ambiguous, quadratically divergent integrals and might not be equal.

[33] J. W. Trischka and T. I. Moran, Bull. Am. Phys. Soc. 5, 298 (1960); P. Franken (private communication).
[34] J. G. Trump and R. J. Van de Graaff, J. Appl. Phys. 18, 327 (1947).
[35] J. J. Murray, University of California Radiation Laboratory Report UCRL-9506, 1960 (unpublished).

Feinberg and Goldhaber have discussed[3] the connection between the conservation laws and charge equalities of particles. At present, the absolute conservation laws of charge, baryon number, and lepton number are all independent and are believed valid for any particle reaction. Because of the independent conservation laws for baryons and leptons, use of charge conservation in the known reactions involving elementary particles does not, of itself, determine the ratios of the charges of all the elementary particles. For example, then the apparent absence of the reaction $p \rightarrow e^+ + \pi^0$ leaves the ratio of the electron to proton charges undetermined. Conversely, if the electron (lepton) and proton (baryon) charge magnitudes were different, then the absence of such a reaction, or, more generally, the conservation of baryons would follow from the conservation of charge instead of being an independent principle.

The equality of the electron and proton charge magnitudes has been established with unusually high precision in this and other recent experiments, hence they offer no support for the suggestion that baryon conservation might be simply a consequence of charge conservation. Furthermore, it would seem that any theory of elementary particles should require that the renormalized electron and proton charge magnitudes be equal.

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APPENDIX I: QUADRUPOLE EFFECTS

If a charge distribution of density ρ having a quadrupole dyadic $\mathbf{Q}$ with components

$$Q_{ij}=\int (x_i x_j-\delta_{ij}r^2)\rho d\tau$$

is in an electric field with gradient $\nabla\mathbf{E}$, then the quadrupole contribution to the total energy is

$$W_Q=-\tfrac{1}{6}\mathbf{Q}:\nabla\mathbf{E}\equiv-\tfrac{1}{6}\textstyle\sum_{ij} Q_{ij}(\nabla\mathbf{E})_{ji}.$$

For our case (see below), $\mathbf{Q}$ is diagonal if the z axis is chosen to be in the direction of the electric field. The z component of force arising from the quadrupole moment is then

$$(\mathbf{F}_Q)_z=(-\nabla W_Q)_z=-\frac{1}{6}\frac{\partial}{\partial z}\left(Q_{zz}\frac{\partial E}{\partial z}\right).$$

Alkali atoms in their ground $^2S_{1/2}$ states are used in our experiment. They have no quadrupole moment in zero field, and in the fields used, the induced quadrupole moments will be proportional to E^2 to a good approximation: $Q_{zz}=\alpha_Q E^2$. The quadrupole polarizability α_Q has been calculated to be $\alpha_Q=4.88\times10^{-39}$ esu for the hydrogen atom,[36] and it is estimated to be of the order 10^{-36} esu for K and Cs. Estimates of our field inhomogeneities, together with the approximate values of α_Q, show that the quadrupole forces can be neglected in comparison to the forces on atomic charges of the size measured in our experiment. The deflection associated with an atomic charge could also be distinguished from that associated with an induced quadrupole moment by their different dependences on E.

APPENDIX II: APPROXIMATION IN DEFLECTION ANALYSIS

We now show that in the Taylor series expansion of Eq. (10)

$$\delta(z,E)=-\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n!}\frac{\partial^n I(z,0)}{\partial z^n}s_\alpha{}^n\int_{v=0}^{\infty}\frac{2e^{-v^2/\alpha^2}d(v/\alpha)}{(v/\alpha)^{2n-3}}$$

only the $n=1$ term need be retained. Because the individual integrals diverge for $n\geq2$, it is convenient to divide the range of integration into two parts: the first from $v=0$ to $v=v_1$, where v_1/α is a small number ($\sim1/10$), and the second from $v=v_1$ to $v=\infty$. We show (1) that the contribution to δ of the entire series for the lower range of integration is negligible and (2) that the contribution to δ of each individual term with $n\geq2$ for the upper range of integration is negligible.

The change in the number of slow ($v\leq v_1$) atoms striking the detector can be no larger than the maximum detectable number of slow atoms in the beam. Hence,

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n!}\frac{\partial^n I(z,0)}{\partial z^n}s_\alpha{}^n\int_{v=0}^{v_1}\frac{2e^{-v^2/\alpha^2}d(v/\alpha)}{(v/\alpha)^{2n-3}}$$
$$<I(z_0,0)\int_{v=0}^{v_2}\frac{v^3}{\alpha^4}2e^{-v^2/\alpha^2}dv\simeq\tfrac{1}{2}I(z_0,0)\left(\frac{v_1}{\alpha}\right)^4,$$

which equals $5\times10^{-5}I(z_0,0)$ for $v=\alpha/10$. The maximum contribution of slow atoms is, therefore, at most of the same order as the minimum detectable signal.

The contribution of the $n=2$ term for the upper range of integration

$$\frac{+1}{2}\frac{\partial^2 I(z,0)}{\partial z^2}s_\alpha{}^2\int_{v=\alpha/10}^{\infty}\frac{2e^{-v^2/\alpha^2}d(v/\alpha)}{(v/\alpha)}$$

can be estimated. From an examination of plots of $I(z,0)$ vs z we estimate that $|(\partial^2 I/\partial z^2)|<10^7$ near z_1 and z_2. The integral has the value 4. For the dipole polarizability deflections obtained, $s_\alpha=10^{-4}$ cm typically, so the galvanometer signal associated with the $n=2$ term is 0.2 cm or less compared to the $n=1$ contribution of about 30 cm. The smallness of s_α assures that terms for $n\geq3$ will also be negligible.

[36] A. D. Buckingham, C. A. Coulson, and J. T. Lewis, Proc. Phys. Soc. (London) **A69**, 639 (1956).