Helical Instabilities in Solid-State Plasmas

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(Received 29 October 1962)

The background of thermal carriers is included in a theory for helical instabilities in electron-hole plasmas. An important quantity in determining the stability criteria and frequency of the instability is the injection level. The theory applies to any injection level in n-type, p-type, and intrinsic semiconductors and to insulators. Comparison with Ancker-Johnson's experiment in p-InSb strongly supports the helical instability as the basic mechanism in the oscillistor. Experiments with different types of material should, according to the theory, give markedly different results. When the temperature dependence of the mobilities is known the plasma temperature can be determined from the electric field at the onset of instability.

INTRODUCTION

WHEN a magnetic field is applied parallel to a current in a semiconductor, oscillations develop when the magnetic field is increased above a certain critical value. Several authors[1–7] have observed this effect, called the oscillistor.[3] Recent experiments[5,7] confirm the suggestion made by Glicksman[8] that the oscillations are caused by helical instabilities. This type of instability was first proposed by Kadomtsev and Nedospasov[9] as an explanation of the anomalous diffusion in the positive column.[10] Recently, Johnson and Jerde[11] have given this theory a rigorous mathematical foundation.

In a semiconductor in thermal equilibrium there is a background of electrons (density n_0) and holes (density p_0). The composition of the thermal plasma depends on where the material is intrinsic ($n_0 \approx p_0$), p-type ($p_0 \gg n_0$), or n-type ($n_0 \gg p_0$). In insulators we have $n_0 = p_0 = 0$. By injection or ionization in the bulk, additional carriers can be introduced in the specimen. Electrons and holes are by these processes created in equal numbers, thus constituting an injected plasma.

In the paper by Glicksman[8] only the case of an injected plasma in the absence of a thermal background plasma was treated. However, the background plasma may be of great importance in a theory for the helical instability in a semiconductor plasma, as has been pointed out by several authors.[5,12,13] The purpose of this paper is to present a theory for helical instabilities in solid-state plasmas that covers all injection levels in intrinsic, n-type, and p-type semiconductors, as well as insulators.

BASIC EQUATIONS

We shall assume that the specimen is an infinitely long cylinder of radius R, and that the background carriers are uniformly distributed. The following condition must therefore be fulfilled:

$$\nabla n_0 = \nabla p_0 = 0. \tag{1}$$

The continuity equations for electrons and holes are

$$\partial n_i/\partial t + \nabla((n_0+n_i)\mathbf{v}_e) = \gamma n_i, \tag{2}$$

$$\partial p_i/\partial t + \nabla((p_0+p_i)\mathbf{v}_e) = \gamma p_i, \tag{3}$$

where n_i and p_i are the densities of injected electrons and holes, respectively. The velocity of the electrons is $\mathbf{v}_e$ and of holes $\mathbf{v}_h$. On the right-hand side we have included bulk generation and recombination in terms of the proportionality constant γ, which, then may be either positive or negative. In the case of injection without recombination γ is zero.

The equations of motion are (mks units)

$$(n_0+n_i)\mathbf{v}_e + D_e\nabla n_i + \mu_e(n_0+n_i)\mathbf{E} + \mu_e(n_0+n_i)\mathbf{v}_e \times \mathbf{B} = 0, \tag{4}$$

$$(p_0+p_i)\mathbf{v}_h + D_h\nabla p_i - \mu_h(p_0+p_i)\mathbf{E} - \mu_h(p_0+p_i)\mathbf{v}_h \times \mathbf{B} = 0, \tag{5}$$

where we have used (1). $\mathbf{E}$ is the electric field, $\mathbf{B}$ the magnetic field, μ_e and μ_h the electron and hole mobilities, and D_e and D_n their diffusion coefficients, respectively.

We will now make the following assumptions: The injected plasma is quasi-neutral $|n_i - p_i| \approx 0$ so that Poisson's equation is given as $\nabla^2 U = 0$, where U is the potential. The temperature of electrons and holes are equal. We then have

$$D_e = \mu_e V, \quad D_h = \mu_h V, \tag{6}$$

where V is the temperature in electron volts. The self-magnetic field caused by the current is negligible compared with the applied axial magnetic field $\mathbf{B}$.

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By putting $p_i = n_i$ and introducing the potential U, we derive from Eqs. (2) to (5) two equations which will form the basis of our investigation.

In cylindrical coordinates (r,θ,z) they are

$$(\kappa b-1)V\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n_i}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 n_i}{\partial\theta^2}\right]-\frac{1}{r}\frac{\partial}{\partial r}\left(r\epsilon_1\frac{\partial U}{\partial r}\right)-\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\epsilon_1\frac{\partial U}{\partial\theta}\right)$$
$$+(1-\kappa b^2)\mu_h B\left[\frac{1}{r}\frac{\partial n_i}{\partial\theta}\frac{\partial U}{\partial r}-\frac{1}{r}\frac{\partial n_i}{\partial r}\frac{\partial U}{\partial\theta}\right]+(b-1)V\frac{\mu_h}{\mu_h'}\frac{\partial^2 n_i}{\partial z^2}-\frac{\mu_h}{\mu_h'}\frac{\partial}{\partial t}\left(\epsilon_2\frac{\partial U}{\partial z}\right)=0, \quad (7)$$

$$\frac{\kappa b+1}{\mu_e}\frac{\partial n_i}{\partial t}-2V\frac{\mu_e'}{\mu_e}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n_i}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 n_i}{\partial\theta^2}\right]-\frac{\mu_e'}{\mu_e}(p_0-n_0)\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 U}{\partial\theta^2}\right]$$
$$+\frac{\mu_e'}{\mu_e}\mu_h B(b+1)\left[\frac{1}{r}\frac{\partial n_i}{\partial\theta}\frac{\partial U}{\partial r}-\frac{1}{r}\frac{\partial n_i}{\partial r}\frac{\partial U}{\partial\theta}\right]-(\kappa+1)V\frac{\partial^2 n_i}{\partial z^2}+\frac{\partial}{\partial z}\left(\epsilon_3\frac{\partial U}{\partial z}\right)=\frac{\kappa b+1}{\mu_e}\gamma n_i, \quad (8)$$

where $b=\mu_e/\mu_h$ is the mobility ratio, and the quantities μ_e' and μ_h' are defined by

$$\mu_e'=\frac{\mu_e}{1+\mu_e^2B^2},\quad \mu_h'=\frac{\mu_h}{1+\mu_h^2B^2}. \quad (9)$$

Further, we have defined

$$\kappa=\mu_e'\mu_h/\mu_h'\mu_e, \quad (10)$$

and

$$\epsilon_1=p_0+\kappa bn_0+(\kappa b+1)n_i,$$
$$\epsilon_2=p_0+bn_0+(b+1)n_i, \quad (11)$$
$$\epsilon_3=n_0-\kappa p_0+(1-\kappa)n_i.$$

THE STEADY STATE

In the steady state $\partial/\partial t=0$. We shall assume cylindrical symmetry (i.e., $\partial/\partial\theta=0$), and further that both the electric field E_{z0} and the steady-state current j_{0z} are constant in the z direction and zero in the r and θ directions.

The condition for constant current in the z direction gives when used in Eq. (7)

$$\frac{\partial U_0}{\partial r}=V\frac{\kappa b-1}{\epsilon_{1,0}}\frac{\partial n_{i,0}}{\partial r}, \quad (12)$$

where $n_{i,0}$ and U_0 are the steady-state density and potential, respectively, and $\epsilon_{1,0}$ the value of ϵ_1 with $n_i=n_{i,0}$.

Equation (12) is the condition for zero current in the r direction, and gives the radial variation of the steady-state potential in terms of $n_{i,0}$.

Inserting (12) into (8) yields

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\Lambda(r)\frac{\partial n_{i,0}}{\partial r}\right)+\frac{\gamma}{\mu_e'V}n_{i,0}+\frac{\mu_e}{\mu_e'}(\kappa b+1)^{-1}$$
$$\times\left[(\kappa+1)\frac{\partial^2 n_{i,0}}{\partial z^2}-(\kappa-1)E_{z0}\frac{\partial n_{i,0}}{\partial z}\right]=0, \quad (13)$$

where $\Lambda(r)$ is given as

$$\Lambda(r)=\frac{p_0+n_0+2n_{i,0}(r)}{p_0+\kappa bn_0+(\kappa b+1)n_{i,0}(r)}. \quad (14)$$

Since Λ is a function of $n_{i,0}$, Eq. (13) is nonlinear, and not separable into r and z dependencies. Since, in the two limiting cases $n_{i,0}\ll p_0+\kappa bn_0$ and $n_{i,0}\gg p_0+\kappa bn_0$, Λ will be a slowly varying function of $n_{i,0}$, we shall linearize Eq. (13) by taking Λ as a constant. We shall for the time being denote this value Λ by $\bar{\Lambda}$ without further specification as to how it should be determined.

Equation (13) can then be readily solved by separation, the solution being

$$n_{i,0}=N_{i,0}J_0(\beta_0 r)Z_0(z), \quad (15)$$

where $N_{i,0}$ is the value of $n_{i,0}$ at $r=0$ and $J_0(\beta_0 r)$ the zero-order Bessel function. $Z_0(z)$ is the z-dependent part of the solution which, however, need not be specified in this investigation. β_0 is given by

$$\beta_0^2=\frac{1}{\mu_e'V\bar{\Lambda}}\left(\gamma+\frac{\mu_e}{\kappa b+1}C\right), \quad (16)$$

where C is a separation constant. If the density is zero at the wall $\beta_0=2.4048/R$ where 2.4048 is the first zero of J_0.

It is apparent that taking Λ to be a constant is equivalent to assuming that the radial dependence of the steady-state density is a zero-order Bessel function.

In the next section, where quantities involving $\bar{\Lambda}$ will be transformed by finite Hankel transforms, the r-dependent value of Λ, Λ_0, given by (14) with $n_{i,0}=N_{i,0}J_0(\beta_0 r)$ will be used.

PERTURBATION THEORY

The steady-state solution may be perturbed by writing the density n_i and the potential U as

$$n_i=n_{i,0}+f(r,z)e^{i(kz+m\theta+\omega t)}, \quad (17)$$

$$U=U_0+g(r)e^{i(kz+m\theta+\omega t)}, \tag{18}$$

where f and g are small quantities compared to $n_{i,0}$ and U_0, k the wave number along the z axis, m the wave number in the azimuthal direction, and ω the frequency of the perturbation.

The z dependence for f is chosen to be the same as for $n_{i,0}$, i.e., $f(r,z)=f_1(r)Z_0(z)$, which gives the same loss rate in the z direction for f as for $n_{i,0}$.

By inserting (17) and (18) into (7) and (8) and by neglecting products of f and g, we obtain two first-order equations in f and g. Neglecting terms containing $\partial Z_0/\partial z$,[14] and elmininating g by introducing the function h defined by

$$g=(1/\epsilon_{1,0})[h+(\kappa b-1)Vf], \tag{19}$$

these equations take the form

$$\frac{\mu_h}{\mu_h'}\left[V\left(1-b+(\kappa b-1)\frac{\epsilon_{2,0}}{\epsilon_{1,0}}\right)k^2+i(1+b)kE_{z0}\right]f=-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right)-\frac{m^2}{r^2}h-\frac{u_h}{\mu_h'}\frac{\epsilon_{2,0}}{\epsilon_{1,0}}k^2h$$
$$-(\kappa b+1)\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{\epsilon_{1,0}}\frac{\partial n_{i,0}}{\partial r}h\right)-im(\kappa b^2-1)\mu_hB\frac{1}{\epsilon_{1,0}r}\frac{\partial n_{i,0}}{\partial r}h, \tag{20}$$

$$2V\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right)-\frac{m^2}{r^2}f\right]+(\kappa b-1)(p_0-n_0)\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(\frac{f}{\epsilon_{1,0}}\right)\right]-\frac{m^2}{r^2}\frac{f}{\epsilon_{1,0}}\right\}$$
$$-V\frac{\mu_e}{\mu_e'}\left[1+\kappa-(\kappa b-1)\frac{\epsilon_{3,0}}{\epsilon_{1,0}}\right]k^2f+\left[\frac{\kappa b+1}{\mu_e'}(\xi-i\omega)+(1-\kappa)\frac{\mu_e}{\mu_e'}kE_{z0}\right]f$$
$$=-(p_0-n_0)\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(\frac{h}{\epsilon_{1,0}}\right)\right]-\frac{m^2}{r^2}\frac{h}{\epsilon_{1,0}}\right\}-\frac{\mu_e}{\mu_e'}\frac{\epsilon_{3,0}}{\epsilon_{1,0}}k^2h-im(b+1)\mu_hB\frac{1}{r}\frac{\partial n_{i,0}}{\partial r}\frac{h}{\epsilon_{1,0}}, \tag{21}$$

where ξ is given by

$$\xi=\mu_e'V\bar{\Lambda}\beta_0^2. \tag{22}$$

Equations (20) and (21) are of the same type as those solved by Johnson and Jerde, and the same method of finite Hankel transforms can be used. Since a good approximation to the general solution is obtained by keeping only the first terms in the series for f and h, the solutions are carried out in that approximation.

A dispersion relation is derived from which the condition for stability is found by requiring $\mathrm{Im}(\omega)\geq 0$. At the onset of instability we also have the condition that the derivative of $\mathrm{Im}(\omega)$ with respect to the wave number k must equal zero. With these conditions satisfied, we are able to express the wavelength, the electric field, and the frequency of the oscillations at the onset of instability in terms of the magnetic field. In writing down these results, we introduce the following dimensionless quantities:

$$y=\mu_e\mu_hB^2,\quad Z=(1+y)\sigma^2(kR)^2, \mathcal{E}=E_{z0}R/V,$$
$$\Omega=\omega R^2/\mu_eV, \tag{23}$$

where σ is a numerical constant, determined by the first zero of $J_m(\beta_1 r)$.

[14] This is justified when the condition

$$\left|\frac{1}{Z_0}\frac{\partial Z_0}{\partial z}\right|\ll\left|\frac{b+1}{b(\kappa+1)}\mu_e'B\frac{m}{kr}\frac{E_{r0}}{V}\right|$$

is satisfied. This is a rather weak condition and is easily satisfied when density of the injected plasma decreases slowly in the axial direction, as is believed to be the case in oscillistor experiments (reference 8).

The equation determining Z is

$$3AGZ^4+(5A+BG)Z^3+(3B-CG)Z^2$$
$$-(3GD-C)Z-D=0. \tag{24}$$

The dimensionless electric field can be found from

$$AZ^3+BZ^2+CZ+D=mE(1+GZ)\sigma Z^{1/2}\mathcal{E}, \tag{25}$$

where the coefficients A to G are given by

$$\begin{aligned}
A&=(1+\nu^2)W_3W_5,\\
B&=(1+\nu)\{W_1+[(1+\nu y)W_1+(1+\nu)\\
&\qquad\times(1+(W_0-1)W_5)]W_5\}W_3,\\
C&=\{m^2\nu(1-\nu)W_2^2+[(1+\nu)(1-(1-2W_0)W_5)\\
&\qquad+(1+\nu y)W_1]W_1\}W_3,\\
D&=[W_1^2+m^2\nu W_2^2]W_0W_3,\\
E&=-\nu^{1/2}\left(\frac{1+\nu}{b-1}\right)(1+y)^{1/2}\\
&\qquad\times[(b+1)(1+y)W_1+(b-1)W_4]W_2,\\
G&=\frac{(b+1)(1+y)W_5+(b-1)W_4}{(b+1)(1+y)W_1+(b-1)W_4},
\end{aligned} \tag{26}$$

where

$$\nu=\frac{y}{b}\left(\frac{b-1}{1+y}\right)^2. \tag{27}$$

W_1 to W_5 are integrals defined as

$$W_1=1-(\kappa b+1)\frac{\alpha\eta}{\beta_1^2}\int_0^R \frac{r}{1+(\kappa b+1)\eta J_0}\frac{\partial J_0}{\partial r}J_m\frac{\partial J_m}{\partial r}dr,$$

$$W_2=(\kappa b+1)\frac{\alpha\eta}{\beta_1^2}\int_0^R \frac{J_m^2}{1+(\kappa b+1)\eta J_0}\frac{\partial J_0}{\partial r}dr,$$

$$W_3=(\kappa b+1)\alpha\int_0^R r\frac{\eta_1+2\eta J_0}{1+(\kappa b+1)\eta J_0}J_m^2dr, \tag{28}$$

$$W_4=(\kappa b+1)\alpha\eta_2\int_0^R \frac{r}{1+(\kappa b+1)\eta J_0}J_m^2dr,$$

$$W_5=(\kappa b+1)\alpha\int_0^R r\frac{\eta_4+(b+1)\eta J_0}{1+(\kappa b+1)\eta J_0}J_m^2dr,$$

where

$$\eta=\frac{N_{i,0}}{p_0+\kappa bn_0},\quad \eta_1=\frac{p_0+n_0}{p_0+\kappa bn_0},\quad \eta_2=\frac{p_0-n_0}{p_0+\kappa bn_0},$$

$$\eta_3=\frac{\kappa p_0-n_0}{p_0+\kappa bn_0},\quad \eta_4=\frac{p_0+bn_0}{p_0+\kappa bn_0}, \tag{29}$$

and finally

$$\alpha=2\{R[\partial J_m(\beta_1 r)/\partial r]_{r=R}\}^{-2}. \tag{30}$$

When the mobility ratio b and the values of $N_{i,0}$, n_0, and p_0 are known, the coefficients (26) can be calculated for a given value of y. A solution for the corresponding value of Z can then be found numerically from (24). Knowing Z we can then calculate $\mathcal{E}$, thus obtaining a stability curve relating the values of $\mathcal{E}$ and y at the onset of instability.

The frequencies of the oscillations are given by

$$\mathrm{Re}(\Omega)=\frac{\nu^{-1/2}W_2^{-1}}{m(b+1)(1+y)}\Big\{(1+\nu)W_3Z^2+[(1+\nu y)W_1+(1+\nu)(1+W_5)]W_3Z+W_0W_1W_3-m\frac{b+1}{b-1}\nu^{1/2}(1+y)^{3/2}W_2Z^{1/2}\sigma\mathcal{E}\Big\}. \tag{31}$$

Thus, the frequency can be calculated when Z and $\mathcal{E}$ are determined.

It can be shown that the $m=0$ mode is stable. The helical-type instability which corresponds to $m=1$ is considered in the remaining part of this paper.

ASYMPTOTIC SOLUTIONS

Equations (24), (25), and (31) are investigated in the limiting case of small and large magnetic fields, i.e., y. In these limits the quantities η to η_4 defined by (29) are constants. Thus, W_1 to W_5 defined by (28) are only functions of η.

1. $y\to 0$. In this limit we get $Z\approx$ const, and the wavelength $\lambda=1/k$ is, therefore, constant. The electric field is given as a function of y by

$$\mathcal{E}=K_0(\eta)y^{-1/2}, \tag{32}$$

where $K_0(\eta)$ is independent of y, but is a function of the injection level η.

The frequency Ω can be written in the form

$$\Omega=L_1(\eta)y^{-1/2}+L_2(\eta)y^{1/2}, \tag{33}$$

where $L_1=W_4L_0(\eta)$, and L_0, L_1, and L_2 are independent of y.

Two cases may be distinguished: For n- and p-type material $W_4\neq 0$, thus for sufficiently small values of y

$$\Omega=L_1(\eta)y^{-1/2}. \tag{34}$$

From (28) we see that W_4 has the opposite sign for n-type and p-type material. Thus, the frequencies have opposite signs, which means that the rotation of the helix is opposite in the two materials, as has been found experimentally by Okamoto *et al.*[5] Numerical calculations show that the frequency for p-type material is of the same sign as for intrinsic materials and insulators.

For intrinsic material and insulators $W_4=0$, and the first term in (33) is zero. Thus, the frequency for these materials varies with y as

$$\Omega=L_2(\eta)y^{1/2}, \tag{35}$$

where, for intrinsic materials $L_2(\eta)$ is a function of η, while for insulators L_2 is a constant.

We now have the interesting result that for n- and p-type material, the absolute value of the frequency will approach infinity as the magnetic field becomes small. This is in agreement with the results obtained by Misawa[13] for near intrinsic material. The frequency for intrinsic materials and insulators, however, in the same limit of magnetic field approaches zero.

2. $y\to\infty$. In this limit we also get $Z\approx$ const. From the definition (23) of Z, it then follows that the wavelength λ increases as $y^{1/2}$ for large y.

The electric field may be written

$$\mathcal{E}=K_\infty(\eta)y^{-1}, \tag{36}$$

and the frequency for all types of material is given by

$$\Omega=L_\infty(\eta)y^{-1/2}. \tag{37}$$

CALCULATION OF TEMPERATURE AND FREQUENCY

The passage of current through a semiconductor may cause heating of the plasma above the bath temperature. The temperature dependence of the mobilities may be expressed as[15]

$$\mu_e=\mu_{0e}(V_0/V)^{1/2},\quad \mu_h=\mu_{0h}(V_0/V)^{1/2}, \tag{38}$$

[15] R. A. Smith, *Semiconductors* (Cambridge University Press, New York, 1959), p. 160.

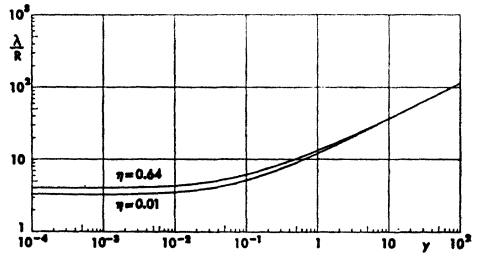

FIG. 1. Dimensions wavelength λ/R at onset of instability as a function of dimensionless magnetic field y for the injection levels $\eta=0.01$ and 0.64.

where V_0 is the bath temperature, μ_{0e} and μ_{0h} the mobilities at the temperature V_0, and $V\geq V_0$.

Then

$$y=y_0V_0/V, \tag{39}$$

where

$$y_0=\mu_{0e}\mu_{0h}B^2. \tag{40}$$

Further $\mathcal{E}_0$ is defined as

$$\mathcal{E}_0=E_{z0}R/V_0. \tag{41}$$

When the values of E_{z0} and B are determined experimentally, $\mathcal{E}_0$ and y_0 can be calculated.

To find the values of $\mathcal{E}$ and y corrections must be made for the difference between plasma temperature V and sample temperature V_0. From the third equation (23) and (39) V can be eliminated, giving

$$\mathcal{E}=\mathcal{E}_0\frac{y}{y_0}. \tag{42}$$

In the regions where the asymptotic results of the last section apply, the corrected values of $\mathcal{E}$ and y may easily be determined, and hence the temperature and frequency calculated.

We do this for the case $y\rightarrow 0$, which is the one of greatest experimental interest.

From Eqs. (23), (32), and (42) we calculate the temperature of the plasma

$$V=[y_0V_0(E_{z0}R/K_0)^2]^{1/3}. \tag{43}$$

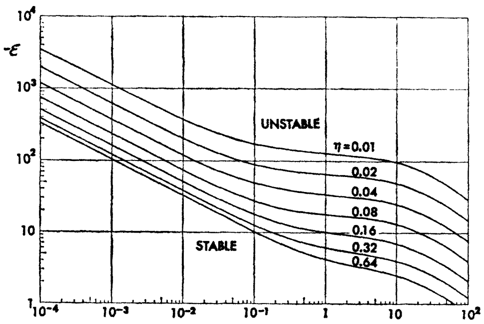

FIG. 2. Dimensionless electric field $\mathcal{E}$ at onset of instability as a function of dimensionless magnetic field y for different injection levels η.

This expression for the plasma temperature as a function of the electric field, is based upon the assumption that the helical instability is the origin of the oscillations in the oscillistor. The validity of the expressions is limited to situations where the temperature-dependent mobilities are given by (38).

When the injection ratio has been experimentally determined together with the electric and magnetic fields at the onset of oscillations, the temperature can be calculated from (43). It should be noted that the temperature determined this way is independent of the specific properties of the material.

For p- and n-type material we obtain the frequency corrected for plasma temperature

$$\omega=\mu_{0e}L_1[V_0y_0^{-1/2}(E_{z0}/K_0R^2)^2]^{1/3}. \tag{44}$$

For intrinsic material and insulators we get

$$\omega=\mu_{0e}L_2V_0y_0^{1/2}R^{-2}. \tag{45}$$

K_0, L_1, and L_2 can be calculated numerically by taking a value $y=y_1\ll 1$ in the asymptotic region, and for this

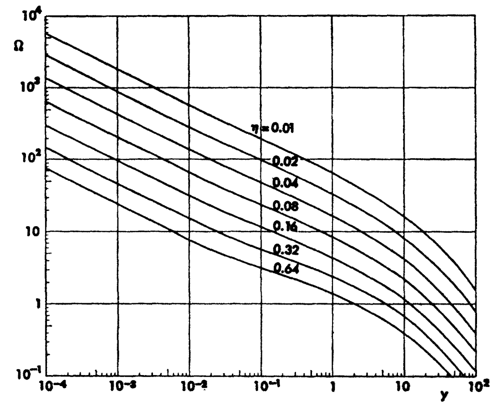

FIG. 3. Dimensionless frequency Ω at the onset of instability as a function of dimensionless magnetic field y for different injection levels.

value y, calculate the corresponding values $\mathcal{E}=\mathcal{E}_1$ and $\Omega=\Omega_1$ as functions of injection level η. Then

$$K_0=\mathcal{E}_1(\eta)y_1^{1/2},\quad L_1=\Omega_1(\eta)y_1^{1/2},\quad L_2=\Omega_1(\eta)y_1^{-1/2}. \tag{46}$$

COMPARISON WITH EXPERIMENT

To the author's knowledge only one experiment has been published containing sufficient data to check this theory. In the experiments reported by Ancker-Johnson,[6,7] the electric and magnetic fields and the oscillation frequency at the onset of instability were measured in p-InSb.

The sample had a cross section 0.78×0.71 mm. As the value for R we have taken $R=3.7\times10^{-2}$ cm. The sample temperature was 77°K ($V_0=6.6\times20^{-3}$ eV). The electron mobility and the mobility ratio were $\mu_{0e}\approx 2\times10^5$ cm²/V-sec and $b\approx 30$, respectively. The injection level has been estimated by using the current-voltage characteristic in the absence of applied magnetic field. When I_T is the total current and I_Ω the Ohmic current,

$$\eta \approx (I_T - I_\Omega)/(b+1)I_\Omega.$$

Equations (24), (25), and (31) have been solved for a number of different injection levels in p-type material with mobility ratio $b=30$.[16]

There is very little variation in the wavelength for different injection levels, and in Fig. 1 only the curves corresponding to $\eta=0.01$ and 0.64 are drawn. As expected the lowest injection levels give the most stable situation (Fig. 2). As the injection level increases to high values, the separation between the curves approaches zero. The dimensionless frequency in Fig. 3 shows a very strong dependence on injection level. For small values of y the decrease in frequency is of the order $\sim 10^2$ when η varies between 0.01 and 0.64. In Fig. 4 we have plotted $\mathcal{E}=\mathcal{E}_1$ and $\Omega=\Omega_1$ corresponding to $y=y_1=10^{-3}$ as functions of injection level η.

The experiment[6,7] was done using values $y_0 \lesssim 5\times10^{-2}$.

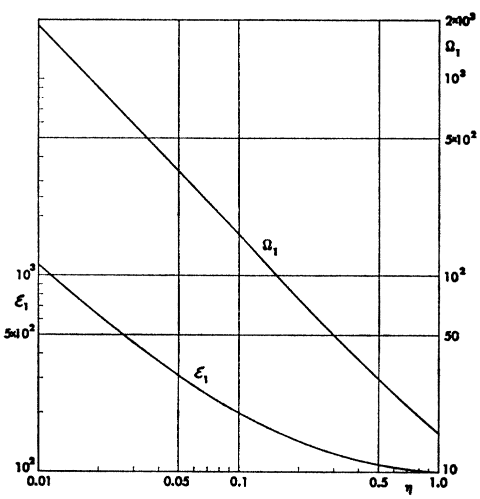

FIG. 4. Dimensionless electric field $\mathcal{E}_1$ and frequency Ω_1 corresponding to $y_1=10^{-3}$ as function of injection level η.

[16] Numerical calculations for a number of different materials are given by Ø. Holter, Årbok Univ. Bergen. Mat.-Naturv. Serie 1963 (in press).

Judging from the curves for $\mathcal{E}$ and Ω (Fig. 2 and 3) the asymptotic formulas corresponding to $y\rightarrow 0$ are valid in this region.

In the first three columns of Table I are listed the experimentally measured values of the magnetic field B, the electric field E_{z0} and the frequency f. The fourth column gives the estimated values of the injection level η. The two last columns give the values of the temperature V and the frequency f calculated by using Eqs. (43) and (44), respectively.

TABLE I. Experimental and theoretical quantities related to onset of instability.

Experimental[a]				Theoretical	
B (G)	E_{z0} (V/cm)	f (Mc/sec)	η	V (eV)	f (Mc/sec)
620	40	27.5	0.03	1.5×10^{-2}	30.1
490	65	27.0	0.08	2.8×10^{-2}	24.6
435	82	25.0	0.14	3.8×10^{-2}	20.1
285	103	25.0	0.24	3.9×10^{-2}	17.0
170	152	22.5	0.56	4.1×10^{-2}	13.1

[a] See references 6 and 7.

The temperature rises to a value corresponding to the optical phonon energy ($\sim 2.5\times10^{-2}$ eV) at an electric field ~ 55 V/cm. Higher electric fields produce saturation at a value $\sim 4\times10^{-2}$ eV.

The agreement between measured and calculated frequencies is good at low electric fields. The calculated frequencies, however, decrease more rapidly than the measured values. It has been observed that the self-magnetic field, which has been neglected in the theory, is sufficient to cause pinching of the plasma for $E \gtrsim 10^2$ V/cm.

Although the only experimental results available currently for check on the theory relate to p-InSb, the agreement for this case strongly indicates the helical instability as the basic mechanism in the oscillistor. However, the theory predicts markedly different results in other types of material, hence, further experimental checks on the theory are of interest.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge many stimulating and helpful discussions with B. Ancker-Johnson and R. Johnson.