# Theory of the Process $p+p \rightarrow d+W^{+}$†

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In this paper a simple theory for the process $p+p \rightarrow W+d$ is developed. Detailed numerical results are obtained by choosing particular wave functions. A discussion of the expected corrections to the calculation is also given.

THE discovery of the vector meson which mediates the weak interactions, $W$,[1] would be of extreme importance for weak interaction physics and for field theory in general. The $W$, if it exists, will be made in a variety of processes such as $\nu+n \rightarrow W^{+}+e^{-}+n$, or $\pi^{-}+p \rightarrow W^{-}+p$, or, as studied in this paper, $p+p \rightarrow d+W^{+}$. The $W$ couples to leptons with a dimensionless constant $g^2=GM_W{}^2/\sqrt{2}$, where $G$ is the Fermi constant defined so that $GM_n{}^2 \cong 10^{-5}$. Thus, for $M_W \simeq M_n$, $g^2 \simeq 10^{-5}$ and the smallness of this constant is, evidently, what makes any of the above processes difficult to detect. The $W$ may have a variety of decay modes,

$$\begin{aligned} W^{+} &\rightarrow e^{+}+\nu \\ &\rightarrow \mu^{+}+\nu \\ &\rightarrow \pi^{+}+\pi^{0} \\ &\rightarrow \pi^{+}+\pi^{+}+\pi^{-} \\ &\rightarrow K^{+}+\pi^{0}. \end{aligned}$$

As has been pointed out,[2] the dominance of a given mode is a sensitive function of the mass of the $W$, as the strongly interacting particles, into which the $W$ may decay, have resonant states in the energy region where the $W$ mass is likely to be; $M_W \geq 500$ MeV.

In this report we study the process $p+p \rightarrow W^{+}+d$. Despite the very strong background from processes such as $p+p \rightarrow n\pi$'s+nucleons, this reaction has the advantage that use may be made of the large proton fluxes in an accelerator like the Cosmotron.[3] Furthermore, if one attempts to detect the muonic decay mode of the $W$, it may be possible to shield out much of the direct pion contamination and to discriminate against unwanted $\mu$'s by energy resolution—the $W$ width is expected to be only a few kilovolts.

Encouraged by the experimental outlook, we have made a simple theory of $W$ production in association with deuterons. The theory can be improved in many places but we feel it is unlikely to be wrong by an order of magnitude.

The transition is described by

$$m=(\psi_f, T\psi_i), \tag{1}$$

where, introducing isotopic spin states $n_1$, $n_2$, $p_1$, $p_2$, explicitly,

$$\psi_f=\left[\frac{u(r)}{r}+S_{12}\frac{w(r)}{r}\right]\chi_m{}^1\frac{(n_1p_2-n_2p_1)}{\sqrt{2}} \tag{2}$$

is the deuteron $^3S_1$ and $^3D_1$ wave function normalized so that

$$\int_0^\infty [u(r)^2+w(r)^2]dr=1. \tag{3}$$

In the numerical work which we have done, so far the initial di-proton state, $\psi_i$, was taken to be a plane wave. The $T$ matrix will contain terms of the form

$$\begin{aligned} T=[g/(2E_W)^{1/2}][&J_\alpha(\mathbf{r}/2)_{(1)}{}^{(-)}\epsilon_\alpha \exp(i\mathbf{W}\cdot\mathbf{r}/2) \\ &+J_\alpha(-\mathbf{r}/2)_{(2)}{}^{(-)}\epsilon_\alpha \exp(-i\mathbf{W}\cdot\mathbf{r}/2)]. \end{aligned} \tag{4}$$

We have left out of Eq. (4) terms involving the direct coupling of $W$ mesons to pions and other particles. Thus, a calculation using Eq. (4) includes graphs of the type of Fig. 1(a) and leaves out graphs of the type of Fig. 1(b). The latter graphs correspond to "exchange current" corrections and are very difficult to evaluate reliably. In Eq. (4), $g$ is the dimensionless coupling constant, discussed above, $E_W$ is the $W$ energy, $J_\alpha{}^{(-)}$ is the isospin flipping weak current operator, evaluated at the position of a nucleon, either $\mathbf{r}/2$ or $-\mathbf{r}/2$ in center-of-mass coordinates, and $\epsilon_\alpha$ is the $W$ polarization

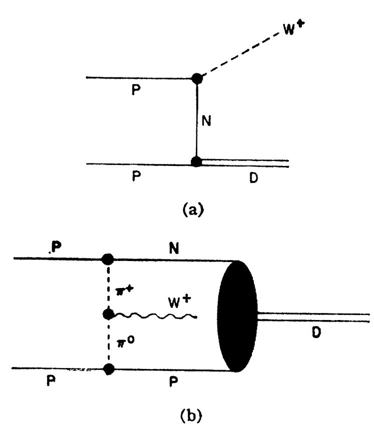


FIG. 1. (a) This diagram represents a contribution to the process $p+p \rightarrow d+W^{+}$ in which one of the incident protons is regarded as a spectator. (b) This diagram represents a meson exchange current contribution to the process $p+p \rightarrow d+W^{+}$.

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[1] See T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960) for a complete discussion of the expected properties of the $W$. See also, R. P. Feynman and N. Gell-Mann, *ibid.* **109**, 193 (1958).
[2] J. Bernstein and G. Feinberg, Phys. Rev. **125**, 1741 (1962).
[3] We have enjoyed numerous conversations with Dr. T. Ypsilantis on the experimental possibilities of finding $W$'s using $p$-$p$ reactions.

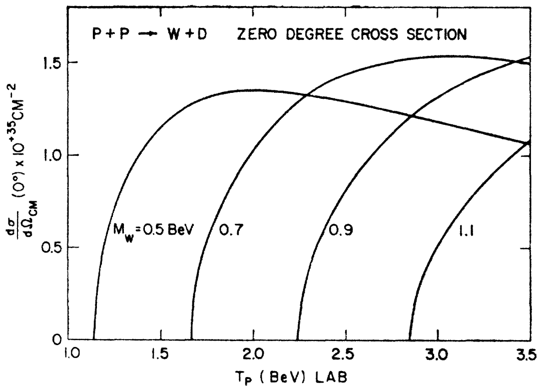


FIG. 2. A plot of the theoretical $W$ production cross section, Eq. (14) of the text, at 0° in the center-of-mass system as a function of the laboratory kinetic energy and the $W$ mass.

vector, which obeys the transversality condition $W_\alpha \epsilon_\alpha = 0$.

The current $J_{\alpha(1)}{}^{(-)}$ can be written, approximately, as

$$J_{\alpha(1)}{}^{(-)} = [F_V(-M_W{}^2)\gamma_\alpha + F_A(-M_W{}^2)\gamma_\alpha\gamma_5]_{(1)}\tau_{(1)}{}^{(-)}, \quad (5)$$

where $F_A$ and $F_V$ are the weak-interaction nucleon form factors. We have not included in Eq. (5) induced couplings, which might possibly be of some importance at large momentum transfers. Now, in the language of graph 1(a), the neutron in the intermediate state is off the mass shell. In the metric $(AB) = \mathbf{A}\cdot\mathbf{B} - A_0B_0$, taking $M_W \simeq 750$ MeV and an incident proton energy, in the center-of-mass system, of 1.5 BeV, the square of the effective intermediate neutron mass with a $W$ produced at 90° is about $N^2 = +\frac{1}{2}M^2$, and, hence, the neutron is rather far off the mass shell. Thus, the use of the form factors in Eq. (5), which are defined for the nucleons on the mass shell and the $W$ off the shell, must be regarded with some caution. If, on the other hand, one assumes that the nucleons can be treated approximately as if they were on the mass shell, then the form factors in Eq. (5) may give considerable *enhancement* to the $W$ production cross section. This could come about if the $W$ mass is close to that of the $\rho$. The form factor, $F_V$, on the conserved current theory, is proportional to the isovector electromagnetic form factor of the proton and which is greatly enhanced near the pole corresponding to the $\rho$ meson. It is quite possible that this enhancement will persist even when one of the nucleons is taken off of the shell, though, of course, one cannot prove this rigorously. However, if the $W$ mass is near the $\rho$ then the $W$ decay into $2\pi$'s is also enhanced by a corresponding factor[2] and, hence, if one detects the $W$ by its leptonic decay modes, the enhancement of the $W$ production cross section, if it exists, will not necessarily increase the observed counting rate of leptons coming from $W$ decay, i.e., the branching ratio into leptons is correspondingly decreased.

In the approximate calculation which we shall do with Eq. (5), we set $F_V(-M_W{}^2) = F_A(-M_W{}^2) = 1$ and take the nonrelativistic reduction of Eq. (5). There are relativistic corrections of order $p/(E_P+M) \simeq \frac{1}{2}$ and $W/E_W \simeq \frac{1}{2}$, assuming a $W$ mass of about 750 MeV and a total center-of-mass energy of 3 BeV. The terms of order $W/E_W$ come from the fourth component of the current $J_\alpha$ and the relation

$$\epsilon_4 = i\epsilon\cdot W/E_W, \quad (6)$$

which follows from the transversality condition, $W_\alpha\epsilon_\alpha = 0$. The leading term in Eq. (4) is given by[4]

$$T_{\text{approx}} = [g/(2E_W)^{1/2}][\boldsymbol{\sigma}_{(1)}\cdot\boldsymbol{\varepsilon}\exp(i\mathbf{W}\cdot\mathbf{r}/2)\tau_{(1)}{}^{(-)} + \boldsymbol{\sigma}_{(2)}\cdot\boldsymbol{\varepsilon}\exp(-i\mathbf{W}\cdot\mathbf{r}/2)\tau_{(2)}{}^{(-)}]. \quad (7)$$

Equation (7) is the nonrelativistic reduction of the axial vector current operator.

It would not be correct to attempt to expand the exponentials in Eq. (7) in a multipole expansion since $r \sim 1/\alpha \simeq 4.3\times10^{-13}$ cm, the deuteron radius, and $W \sim 1.6\times10^{13}$ cm$^{-1}$; that is, the $W$ produced in this reaction, at Cosmotron energies, acts like a "photon" with a wavelength much smaller than the deuteron size and, hence, all the multipoles must be retained. As a first orientation we may take the incident di-protons to be free and assume a very simple wave function for the deuteron. Let

$$\psi_d = \frac{e^{-\alpha r}}{r}\frac{(2\alpha)^{1/2}}{(4\pi)^{1/2}}\chi_m{}^1. \quad (8)$$

In this approximation[5] the matrix element for singlet-

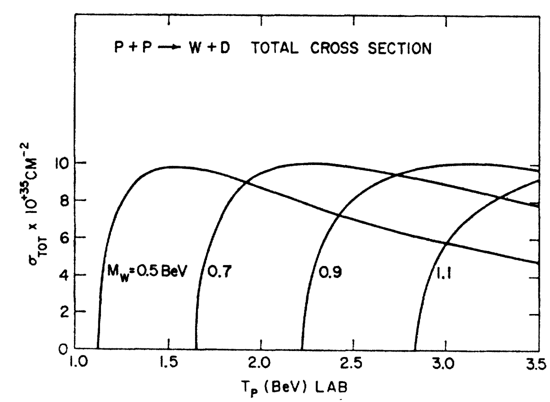


FIG. 3. A plot of the theoretical total $W$ production cross section, the integral of Eq. (14), as a function of the laboratory kinetic energy and the $W$ mass.

[4] We are indebted to Professor G. C. Wick for helpful discussions of the calculation of $m$.

[5] The choice of an exponential deuteron wave function corresponds to assuming a zero-range nuclear force. In the case of a zero-range force the nucleon-deuteron vertex in Fig. 1(a) is independent of whether or not the intermediate nucleon is on the mass shell. Thus, with the choice of an exponential wave function,

triplet transitions is given by

$$\frac{g(2\alpha)^{1/2}}{2(E_W)^{1/2}(4\pi)^{1/2}}\int d^3r\frac{e^{-\alpha r}}{r}\times\{\exp[i(\mathbf{p}+\mathbf{W}/2)\cdot\mathbf{r}]+\exp[i(\mathbf{p}-\mathbf{W}/2)\cdot\mathbf{r}]\}\times\chi_m{}^1\left(\frac{(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\boldsymbol{\varepsilon}}{2}\right)\chi_0=m_{\mathrm{I}},\quad(9)$$

and for triplet-triplet transitions by

$$\frac{g}{2}\frac{(2\alpha)^{1/2}}{(E_W)^{1/2}(4\pi)^{1/2}}\int d^3r\frac{e^{-\alpha r}}{r}\times\{\exp[i(\mathbf{p}+\mathbf{W}/2)\cdot\mathbf{r}]-\exp[i(\mathbf{p}-\mathbf{W}/2)\cdot\mathbf{r}]\}\times\chi_{m'}{}^1\left(\frac{(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{\varepsilon}}{2}\right)\chi_m{}^1=m_{\mathrm{II}}.\quad(10)$$

In obtaining Eqs. (9) and (10) we have taken the isotropic spin matrix elements of the operators $\boldsymbol{\tau}_{(i)}{}^{(-)}$. The differential cross sections which result from Eqs. (9) and (10) after summing over spins and polarizations are

$$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{I}}=\frac{3}{32\pi}g^2\frac{E_d\alpha W}{p}\times\left[\frac{1}{(\mathbf{p}+\mathbf{W}/2)^2+\alpha^2}+\frac{1}{(\mathbf{p}-\mathbf{W}/2)^2+\alpha^2}\right]^2,\quad(11)$$

and

$$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{II}}=\frac{3}{16\pi}g^2\frac{\alpha WE_d}{p}\times\left[\frac{1}{(\mathbf{p}+\mathbf{W}/2)^2+\alpha^2}-\frac{1}{(\mathbf{p}-\mathbf{W}/2)^2+\alpha^2}\right]^2,\quad(12)$$

with $p$ the incident proton momentum in the center-of-mass system and $E_d$ the final deuteron energy.

The angle-dependent terms in Eqs. (11) and (12) contain all powers of the production angle $\mathbf{p}\cdot\mathbf{W}$, as is to be expected, since we have included all multipoles in the $T$ matrix. A numerical calculation[6] shows that $d\sigma/d\Omega|_{\mathrm{II}}$, is smaller than $d\sigma/d\Omega|_{\mathrm{I}}$.

If we introduce the notation

$$A=p^2+\tfrac{1}{4}W^2+\alpha^2,\quad(13a)$$

and

$$B=pW,\quad(13b)$$

Eqs. (11) and (12) combine to give

$$\frac{d\sigma}{d\Omega}=\frac{3}{8\pi}g^2\frac{E_d\alpha W}{p}\frac{A^2+2B^2\cos^2\theta}{(A^2-B^2\cos^2\theta)^2}.\quad(14)$$

The presence of the terms in $B$, in Eq. (14), is due to the nondipole character of the $W$ emission and the denominator is a "retardation" correction of the usual form.

The total cross section, which follows from integration of Eq. (14) over angles, is

$$\sigma_{\mathrm{tot}}=\frac{3}{4}\left(\frac{g^2}{\alpha^2}\right)\frac{E_d\alpha^3}{p^3W}\left[\frac{3B^2}{A^2-B^2}-\frac{B}{2A}\ln\frac{A+B}{A-B}\right].\quad(15)$$

The rapid falloff of $\sigma_T$, at high energies, represents the use of the exponential wave function for the deuteron. This wave function has fewer high momentum components than, say, a hard-core wave function, and its use probably leads to an underestimate of the cross section. A program is under way, here, to use more realistic deuteron and di-proton wave functions and also to include effects of the $D$ state of the deuteron.

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it is consistent to regard Fig. 1(a) as a dispersion diagram with the nucleon on its mass shell. In this case the only real unknown in the problem is the value of $F_A(-M_W{}^2)$, assuming that the vector form factor is, by the conserved current theory, related to the isovector electromagnetic form factor of the nucleon. We have set both of these form factors equal to unity in the numerical work since we are interested in establishing an approximate lower bound to the production cross section.

[6] These calculations were done by Dr. H. Ruderman and Dr. T. Ypsilantis.