Methods for Testing the CPT Theorem*

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The decay and interactions of the neutral K mesons are shown to provide some direct tests of the validity of the CPT theorem for weak interactions of strange particles. The connection with related tests of CP and T invariance is also discussed. It is found that the absence of a 2π decay mode of the K_2 is an indication of CP invariance of the nonleptonic interactions of the K meson but not a proof thereof. Even if CPT invariance is assumed, the 2π mode does not provide an unambiguous test of CP invariance. It is shown that the absence of a detectable charge asymmetry in the leptonic decay of the K_2 indicates either CPT or CP invariance of the leptonic interactions if the nonleptonic interactions are assumed to be CP invariant. A direct test of CP invariance would be provided by a measurement of the interference term in the leptonic decay curve of the K^0, as noted by Sachs and Treiman. A direct test of CPT invariance would be provided by an accurate measurement of the interference term for each of the charge states in the leptonic decay of the K^0.

It is shown that a specific test of combined CPT and CP invariance of the weak interactions would be provided by a measurement of the elastic and regenerative scattering of K_1 and K_2 mesons by nuclei.

The value of the K_1-K_2 mass difference determined by the interference method is found to be independent of the question of CPT or CP invariance. On the other hand the evidence for $\Delta S=-\Delta Q$ transitions obtained from the ratio of the leptonic decay rates of K_1 and K_2 mesons does depend on the assumption of either CPT or CP invariance of the nonleptonic interactions. This ambiguity can be removed by an accurate determination of the charge asymmetry in the leptonic decays.

I. INTRODUCTION

EVER since a clear statement and proof[1] of the CPT theorem has been given, it has assumed a central role in physics. The postulates underlying the theorem are of such a fundamental nature that the theorem itself is treated as a fundamental truth in almost all physical arguments. For his original proof, Pauli[1] made explicit use of three postulates: invariance under the proper Lorentz group, the connection between spin and statistics, and a postulate concerning the local nature of the field theory. Subsequently, Jost[2] gave a proof of the theorem which circumvented the postulated locality of the theory but made use of a very weak condition he calls "weak locality" and a postulate concerning the analytic properties of vacuum expectation values, which is evidently a less restrictive assumption than that of Pauli.

Because of the basic nature of these underlying postulates, it would be most interesting to have some direct experimental tests of the theorem. In fact, a failure of the postulated analyticity or locality of the theory seems possible, or perhaps even likely, in the case of the weak interactions since the weak interaction theories, as they are presently constituted, are not renormalizable. Therefore, the influence of weak interactions at high momentum transfers on the structure of field theories is certainly not well understood. It will be assumed here that the *only* place in which a failure of CPT invariance is likely to occur is for the weak interactions, and our attention will be directed to the problem of detecting such a failure more or less directly, especially in the case of the strange particles.

Lee and Yang have remarked[3] that CPT invariance guarantees the equality of lifetimes for weakly decaying particles and antiparticles. It is important to recognize that the converse is not true, the fact that particle and antiparticle are found to have the same lifetime does not constitute a proof of CPT invariance, it implies only that *either CP or CPT* invariance holds. On the other hand, if the lifetimes are found to be different, it follows that *both* CP and CPT must fail. This is an illustration of a relationship between tests of CPT and CP which will appear frequently. A direct test of CPT is made difficult because CP invariance alone will usually mask a failure of the CPT theorem. However, there do exist tests of CP which are independent of the question of CPT invariance. If such a test demonstrates that a given interaction is CP invariant, then a standard test of T invariance will serve as a test of the CPT theorem. On the other hand, if it is established that CP fails, then a combined test of CPT and CP, such as a comparison of particle and antiparticle properties, will serve to resolve the question of CPT invariance.

The tool best adapted for a variety of tests of this kind appears to be the neutral K meson. It has already been shown[4] that the K^0, $\bar{K}^0$ system provides a unique means for testing CP invariance as a consequence of the failure of the $\Delta S=\Delta Q$ rule. Here, we shall look into the decay modes under the assumption that CPT does not necessarily apply and show that a combination of

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[1] G. Luders, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 28, No. 5 (1954); W. Pauli, in *Neils Bohr and the Development of Physics* (Pergamon Press, New York, 1955), p. 30.

[2] R. Jost, Helv. Phys. Acta 30, 409 (1957). *Theoretical Physics in the Twentieth Century,* edited by M. Fierz and V. Weisskopf (Interscience Publishers, Inc., New York, 1960), p. 107.

[3] T. D. Lee and C. N. Yang, Phys. Rev. 104, 254 (1956).

[4] R. G. Sachs and S. B. Treiman, Phys. Rev. Letters 8, 137 (1962).

tests may be used to separate the question of CPT invariance from that of CP invariance. We shall also examine the K-mesonic nuclear scattering phenomena and demonstrate that they provide a direct test of combined CPT and CP invariance of the weak interactions.

2. DEFINITION OF K_1 AND K_2

The invariance principles under consideration manifest themselves in two rather different ways, one is through the complex mass matrix of the K^0, $\bar{K}^0$ system and the other is through the decay amplitudes themselves. The condition imposed on the mass matrix, M, by CPT invariance is that the diagonal elements (K^0 to K^0 and $\bar{K}^0$ to $\bar{K}^0$) of $W=M^2$ should be equal, while that imposed by CP invariance is that the diagonal elements be equal and that the off-diagonal elements be equal. CPT invariance was assumed by Lee, Oehme, and Yang[5] in their analysis of the problem so that they took the diagonal elements to be equal. This assumption simplifies the resulting expressions for the K_1 and K_2 states which are defined as those states that diagonalize the matrix W. In the most general case the states may be written in the form

$$\begin{aligned} |K_1\rangle &= [s^2|K^0\rangle + rs|\bar{K}^0\rangle](1+s^2)^{-1/2}, \\ |K_2\rangle &= [|K^0\rangle - rs|\bar{K}^0\rangle](1+s^2)^{-1/2}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} s &= (1+\eta^2)^{1/2}-\eta, \\ \eta &= (\langle\bar{K}|W|\bar{K}\rangle - \langle K|W|K\rangle) \\ &\quad \div 2(\langle K|W|\bar{K}\rangle\langle\bar{K}|W|K\rangle)^{1/2}, \\ r &= (\langle\bar{K}|W|K\rangle / \langle K|W|\bar{K}\rangle)^{1/2}, \end{aligned} \tag{2}$$

and W is the square of the complex mass matrix.

When CPT invariance holds we have $\eta=0$ or

$$s=1,$$

and then Eq. (1) is in agreement with the result of Lee, Oehme and Yang.[5] When CP holds, $r=1$ and $s=1$. Therefore, a determination of s would provide a combined test of CPT and CP invariance: If $s\neq 1$, both CPT and CP must fail while, if $s=1$, it can only be concluded that one of the two invariance conditions is valid. On the other hand, a measurement of r would provide an independent test of CP invariance without reference to the CPT theorem.

The values of r and s are governed by the weak-coupling contributions to the mass matrix, that is, to the self-energy matrix of the K^0, $\bar{K}^0$ system. Self-energy contributions arise from virtual states of two pions, three pions, four pions, etc. and from virtual states of the leptonic modes. In general, the self-energy is of second order in the corresponding weak coupling constant; hence, the ratio of nonleptonic to leptonic contributions may be of the same order of magnitude as the branching ratio for decay.[6] Therefore, the contribution of the leptonic modes to r and s may be quite small, and it will be assumed in the following that a large deviation from $r=1$ and $s=1$ is to be expected only if the *nonleptonic* interactions violate the corresponding invariances. However, it should be kept in mind that this is an assumption which might go awry if there is a significant difference in the energy dependence of the leptonic and nonleptonic interactions.

3. NONLEPTONIC MODES

Information concerning the coefficients appearing in Eq. (1) may be obtained directly from the rate of the decay $K_2\to 2\pi$, which is known to be small. To obtain the amplitude for this decay process, let us denote the 2π decay amplitude of the K^0 by d and that of the $\bar{K}^0$ by ζd. Since our notation implies that

$$|\bar{K}^0\rangle = CP|K^0\rangle,$$

we have

$$\zeta=1,$$

when the nonleptonic weak interaction is CP invariant.

From Eq. (1), we find the 2π decay amplitude, $D^{(1)}(2\pi)$ of the K_1:

$$D^{(1)}(2\pi)=d(s^2+rs\zeta)(1+s^2)^{-1/2}, \tag{3}$$

and that of the K_2:

$$D^{(2)}(2\pi)=d(1-rs\zeta)(1+s^2)^{-1/2}. \tag{4}$$

Thus, we may write

$$D^{(2)}(2\pi)=\xi D^{(1)}(2\pi) \tag{5}$$

with

$$\xi=s^{-1}(1-rs\zeta)(s+r\zeta)^{-1}. \tag{6}$$

The experimental results[7] on the 2π decay mode of the K_2 indicate that the branching ratio of the 2π mode of the K_2 is less than one percent. Thence, it follows that

$$|\xi|^2<10^{-5}, \tag{7}$$

since $|\xi|^2$ is the ratio of the rates $K_2\to 2\pi$ and $K_1\to 2\pi$. Hence we find that, in order of magnitude,

$$|rs\zeta-1|<3\times 10^{-3}. \tag{8}$$

Since only the product $rs\zeta$ appears in Eq. (8), we cannot draw the positive conclusion that any one of the factors (much less, each of them) is close to unity. In this connection, it is important to recognize the mutual independence of the factors. The quantity ζ

[5] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).

[6] However, the ratio of 3π to 2π self-energy contributions is not simply related to the branching ratio for decay since the decay rate is governed largely by the statistical weight in this case.

[7] D. Neagu, E. O. Okonov, N. I. Petrov, A. M. Rosanova, and V. A. Rusakov, Phys. Rev. Letters **6**, 552 (1961). A slightly higher estimate than that given by these authors is used here because this is of the order of the relative number of $K^0{}_{\mu 3}$ events with slow neutrinos that are expected, and it is not clear how these could have been distinguished from $K^0{}_{\pi 2}$ events in the experiment cited above. Evidently no such events were observed. This comment is due to W. F. Fry (private communication).

depends only on the coupling to the real 2π mode. It is related to the mass matrix through this coupling. But the mass matrix, which governs the values of r and s, depends both on the virtual 2π states and on all other virtual states which are coupled to the K^0 and $\bar{K}^0$. Hence, the three parameters are essentially independent. Nevertheless, the most natural conclusion to draw from Eq. (8) is that

$$|r-1|<3\times10^{-3},\quad |s-1|<3\times10^{-3},\quad |\zeta-1|<3\times10^{-3}, \tag{9}$$

namely, that CP is valid to the indicated approximation. A method for determining directly the value of s is discussed in Sec. 5.

With respect to the tenuous character of the solution Eq. (9), we note that *even if CPT is valid* the $K_2\to 2\pi$ rate is *not* a direct test of CP invariance since it gives only the product of the independent quantities r and ζ.

Our conclusion then is that the $K_2\to 2\pi$ rate provides a strong indication but not a clearcut proof that CP is valid for the *nonleptonic* weak interactions. However, the limit 3×10^{-3} is of the same order as the branching ratio for the leptonic mode of the K_1, hence of the same order as leptonic effects on the mass matrix. Thus, there remains the possibility that a maximal failure of either CP or CPT, or both, may occur in the leptonic interactions of strange particles.

4. LEPTONIC MODES

The decay amplitudes for the leptonic modes[8] will be denoted by f, f', etc., as follows:

$$\begin{array}{ll} f: & K^0\to\pi^-e^+\nu \\ f': & \bar{K}^0\to\pi^+e^-\bar{\nu} \end{array}\Big\}\ \Delta S=\Delta Q,\qquad \begin{array}{ll} g': & K^0\to\pi^+e^-\bar{\nu} \\ g: & \bar{K}^0\to\pi^-e^+\nu \end{array}\Big\}\ \Delta S=-\Delta Q. \tag{10}$$

The consequences of each of the various invariance conditions for the amplitudes are as follows:

$$CPT:\quad f'=f^*,\quad g'=g^*, \tag{11a}$$

$$CP:\quad f'=f,\quad g'=g, \tag{11b}$$

$$T:\quad f^*=f,\quad g^*=g,\quad f'^*=f',\quad g'^*=g'. \tag{11c}$$

The leptonic decay amplitudes for the K_1 and K_2 are

$$D^{(1)}(\pi^-e^+)=(s^2f+rsg)(1+s^2)^{-1/2}, \tag{12a}$$

$$D^{(1)}(\pi^+e^-)=(s^2g'+rsf')(1+s^2)^{-1/2}, \tag{12b}$$

$$D^{(2)}(\pi^-e^+)=(f-rsg)(1+s^2)^{-1/2}, \tag{12c}$$

$$D^{(2)}(\pi^+e^-)=(g'-rsf')(1+s^2)^{-1/2}. \tag{12d}$$

Consider first the charge asymmetry in K_2 decay,

$$|D^{(2)}(\pi^-e^+)|^2-|D^{(2)}(\pi^+e^-)|^2 =(|f-rsg|^2-|g'-rsf'|^2)/|1+s^2|. \tag{13}$$

From the conditions for CP invariance, $r=s=1$ and Eq. (11b), it would follow that the charge asymmetry, Eq. (13), vanishes, but the conditions for CPT invariance alone are not sufficient to guarantee that it vanishes. On the other hand, if use is made of the tentative result Eq. (9) that $rs\approx1$, either CPT or CP invariance of the leptonic interactions alone would lead to a very small asymmetry, of the order of $rs-1$. T invariance, Eq. (11c), does not by itself, place a limitation on the asymmetry.

Attempts to observe an asymmetry in the K_2 decay indicate that it is quite small,[7,9] which could be a consequence of either over-all CP invariance or of CPT invariance of the leptonic interactions coupled with CP invariance of the nonleptonic interactions ($rs\approx1$).

A separate test of CP invariance is provided by the interference effects discussed in reference 4. It is pointed out there that a failure of CP invariance will lead to an interference term in the curve giving the rate of leptonic decay of the K^0 as a function of time if $\Delta S=-\Delta Q$ transitions occur in the leptonic decay, as indeed seems to be the case.[10,11] In the more general case under consideration here, the interference term is governed by the amplitude

$$D^{(1)*}(\pi^-e^+)D^{(2)}(\pi^-e^+)+D^{(1)*}(\pi^+e^-)D^{(2)}(\pi^+e^-) =[(s^2f+rsg)^*(f-rsg)+(s^2g'+rsf')^*(g'-rsf')]/|1+s^2|. \tag{14}$$

If the conditions for CP invariance hold, the amplitude of the interference term vanishes. However, neither the conditions for CPT nor the condition Eq. (11c) for T implies that the amplitude be small even when use is made of the observed $K_2\to 2\pi$ rate expressed in the form Eq. (9). Hence, a measurement of the interference term is only a test of CP invariance even when CPT invariance fails.

The more detailed experiment on the rate of leptonic decay into *each* charge state as a function of time may be used to separate the question of CPT invariance from CP invariance. The interference term in the rate of decay into the $\pi^-e^+\nu$ mode is proportional to

$$\mathrm{Re}[D^{(1)}(\pi^-e^+)D^{(2)*}(\pi^-e^+)e^{-i\Delta t}] =|1+s^2|^{-1}\,\mathrm{Re}[(s^2f+rsg)(f-rsg)^*e^{-i\Delta t}], \tag{15}$$

[8] The electron mode is considered here since it is better known and it involves only one amplitude in the V, A theory.

[9] D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. Letters 7, 255 (1961).

[10] R. P. Ely, W. M. Powell, H. White, M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, O. Fabbri, F. Farini, C. Filippi, H. Huzita, G. Miari, U. Camerini, W. F. Fry, and S. Natali, Phys. Rev. Letters 8, 132 (1962).

[11] A. Barbaro-Galtieri, W. H. Barkas, H. H. Heckman, J. W. Patrick, and F. M. Smith, Phys. Rev. Letters 9, 26 (1962); G. Alexander, S. P. Almeida, and F. S. Crawford, Jr., *ibid.* 9, 69 (1962).

where Δ is the K_1-K_2 mass difference and t is the time after production of a K^0. Similarly, the interference term for the $\pi^+e^-\bar{\nu}$ mode is proportional to

$$\mathrm{Re}[D^{(1)}(\pi^+e^-)D^{(2)*}(\pi^+e^-)e^{-i\Delta t}] = |1+s^2|^{-1}\,\mathrm{Re}[(s^2g'+rsf')(g'-rsf')^*e^{-i\Delta t}]. \quad (16)$$

If the mass difference is known with sufficient accuracy, a measurement of each of the interference terms Eq. (15) and Eq. (16) may be used to determine

$$(s^2f+rsg)(f-rsg)^*+(s^2g'+rsf')^*(g'-rsf') \approx (f+g)(f-g)^*-(f'+g')^*(f'-g'), \quad (17)$$

in the approximation of Eq. (9). According to Eq. (11), this difference vanishes if, and only if, the leptonic interactions are *CPT* invariant. Therefore, this measurement evidently offers a unique opportunity for testing *CPT* invariance of the leptonic interactions without reference to *CP* invariance.

5. NUCLEAR SCATTERING EFFECTS

Pais and Piccioni[12] have shown that the amplitudes for the scattering of K_1 and K_2 mesons by nuclei depend on the way in which the states of well-defined strangeness are mixed to form the K_1 and K_2 states. Since the amount of mixing, which is given by Eq. (1), is sensitive to the question of *CPT* and *CP* invariance of the weak interactions, the scattering may be expected to provide a test of the invariance principles.

If T is the scattering matrix, then in the K^0, $\bar{K}^0$ representation its matrix elements will be denoted as follows:

$$\langle K^0|T|K^0\rangle=\tau,\qquad \langle\bar{K}^0|T|\bar{K}^0\rangle=\tau'. \quad (18)$$

Furthermore, from conservation of strangeness in scattering,

$$\langle K^0|T|\bar{K}^0\rangle=\langle\bar{K}^0|T|K^0\rangle=0. \quad (19)$$

To calculate the matrix elements in the K_1, K_2 representation, it is necessary to construct states $\langle K_1|$ and $\langle K_2|$ which are adjoint to those given by Eq. (1). Since the transformation, Eq. (1), is not, in general, unitary, the required transformation is given by the inverse of Eq. (1), namely,

$$\langle K_1|=[\langle K^0|+(sr)^{-1}\langle\bar{K}^0|](1+s^2)^{-1/2},\qquad \langle K_2|=[\langle K^0|-sr^{-1}\langle\bar{K}^0|](1+s^2)^{-1/2}. \quad (20)$$

Then the desired matrix elements are given by

$$\begin{aligned}\langle K_1|T|K_1\rangle&=(1+s^2)^{-1}(s^2\tau+\tau'),\\ \langle K_2|T|K_2\rangle&=(1+s^2)^{-1}(\tau+s^2\tau'),\\ \langle K_1|T|K_2\rangle&=(1+s^2)^{-1}(\tau-\tau'),\\ \langle K_2|T|K_1\rangle&=s^2(1+s^2)^{-1}(\tau-\tau').\end{aligned} \quad (21)$$

[12] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).

We note first that Eqs. (21) are independent of r, hence these amplitudes depend only on the question of combined *CPT* and *CP* invariance.[13] A value of $s\neq 1$, which would indicate a violation of *CPT* and *CP* invariance, leads to a difference between the $K_1\to K_1$ and $K_2\to K_2$ elastic cross sections as well as a difference between the $K_1\to K_2$ and $K_2\to K_1$ regenerative cross sections. In fact the ratio of the two regenerative cross sections,

$$\sigma(K_1\to K_2)/\sigma(K_2\to K_1)=|s^2|^2, \quad (22)$$

provides a direct measure of $|s^2|$. Thus, a measurement and comparison of these cross sections would provide a test of combined *CPT* and *CP* invariance and it would help to unravel the ambiguities involved in the interpretation of Eq. (8), thereby serving as a means for verifying in part the conclusion Eq. (9).

6. REMARKS ON THE MASS DIFFERENCE AND $\Delta S=\pm\Delta Q$ EXPERIMENTS

The experimental methods for determining the K_1-K_2 mass difference, Δ, and for testing the relationship between ΔS and ΔQ in K^0 decays depend on interference phenomena of the type that have been under discussion here. Therefore, it is relevant to question the influence of a failure of *CPT* and *CP* invariance on conclusions drawn from these experiments.

First, it may be noted that the interference method[14] for measuring Δ does not depend on the question of either *CPT* or *CP* invariance. This method is based on the determination of the intensity of $\bar{K}^0$ mesons as a function of the time after production of a K^0 beam. An expression for the intensity may be obtained from the state vector as a function of time which, according to Eq. (1), takes the form

$$\psi(t)=(1+s^2)^{-1}[(s^2|K^0\rangle+rs|\bar{K}^0\rangle)e^{-i\omega_1 t} +(|K^0\rangle-rs|\bar{K}^0\rangle)e^{-i\omega_2 t}]. \quad (23)$$

Here $\omega_1=\frac{1}{2}(\Delta-i\Gamma_1)$ and $\omega_2=-\frac{1}{2}(\Delta+i\Gamma_2)$, where Γ_1 and Γ_2 are the total decay rates of the K_1 and K_2 mesons. The time dependence of the probability for observing a $\bar{K}^0$ interaction is therefore proportional to

$$|e^{-i\omega_1 t}-e^{-i\omega_2 t}|^2, \quad (24)$$

which involves only the parameters Γ_1, Γ_2, and Δ. Therefore, the measurement of Δ by Birge *et al.*[15] based on this method is independent of the invariance properties considered here.

In contrast, it has been noted by Whatley[16] that the forward regenerative scattering method for measuring

[13] M. L. Good, Phys. Rev. **106**, 591 (1957) has noted that these amplitudes are independent of the question of *CP* invariance in the case that the *CPT* theorem is valid.

[14] W. F. Fry and R. G. Sachs, Phys. Rev. **109**, 2212 (1958).

[15] U. Camerini, W. F. Fry, J. A. Gaidos, H. Huzita, S. V. Natali, R. B. Willmann, R. W. Birge, R. P. Ely, W. M. Powell, and H. S. White, Phys. Rev. **128**, 362 (1962).

[16] M. Whatley, Phys. Rev. Letters **9**, 317 (1962).

Δ suggested by M. L. Good[17] and applied by R. H. Good *et al.*[18] would be modified by a failure of CP. In fact, there is, in general, an interference term proportional to ξ, given by Eq. (6), in the apparent intensity of the forward regenerative scattering when the 2π decay mode is used to detect the regenerated K_1 mesons. As we have seen in Sec. 3, $\xi \neq 0$ if CP fails so that the value of Δ obtained from this experiment depends on the assumption of CP invariance.

The relationship between ΔS and ΔQ in K^0 decays has been determined by Ely *et al.*[10] and Alexander *et al.*[11] by measuring the ratio of the partial leptonic decay rates of the K_1 and K_2 mesons. This ratio is given by

$$R=\frac{|D^{(1)}(\pi^- e^+)|^2+|D^{(1)}(\pi^+ e^-)|^2}{|D^{(2)}(\pi^- e^+)|^2+|D^{(2)}(\pi^+ e^-)|^2}=\frac{|s^2 f+rsg|^2+|s^2 g'+rsf'|^2}{|f-rsg|^2+|g'-rsf'|^2}. \quad (25)$$

If CPT is valid for the nonleptonic interactions, $s\approx 1$ and

$$R\approx\frac{|f+rg|^2+|g'+rf'|^2}{|f-rg|^2+|g'-rf'|^2}, \quad (26)$$

while CP invariance of the nonleptonic interactions ($r\approx s\approx 1$) leads to the result

$$R\approx\frac{|f+g|^2+|f'+g'|^2}{|f-g|^2+|f'-g'|^2}. \quad (27)$$

Thus, if *either* CPT *or* CP is valid for the nonleptonic interactions, the $\Delta S=\Delta Q$ rule ($g=g'=0$) would imply $R=1$. Hence, the fact that $R\neq 1$ has been interpreted as strong evidence for the existence of $\Delta S=-\Delta Q$ transitions. It is interesting to note that there remains an ambiguity here. If neither CPT nor CP holds, then the $\Delta S=\Delta Q$ rule gives

$$R=|s|^2\frac{|s|^2|f|^2+|r|^2|f'|^2}{|f|^2+|r|^2|s|^2|f'|^2}, \quad (28)$$

which may be quite different from 1 if $|s|\neq 1$. This possibility can be tested directly by the scattering experiment suggested in Sec. 5.

A more direct measure of the relationship between ΔS and ΔQ is given by the charge asymmetry experiment.[19] The asymmetry as a function of time is given by the ratio

$$A(t)=\frac{|D^{(1)}(\pi^+ e^-)e^{-i\omega_1 t}+D^{(2)}(\pi^+ e^-)e^{-i\omega_2 t}|^2}{|D^{(1)}(\pi^- e^+)e^{-i\omega_1 t}+D^{(2)}(\pi^- e^+)e^{-i\omega_2 t}|^2}=\frac{|(s^2 g'+rsf')e^{-i\omega_1 t}+(g'-rsf')e^{-i\omega_2 t}|^2}{|(s^2 f+rsg)e^{-i\omega_1 t}+(f-rsg)e^{-i\omega_2 t}|^2}. \quad (29)$$

At $t=0$, this is simply the branching ratio of the K^0 meson into the two charge states,

$$A(0)=|g'|^2/|f|^2, \quad (30)$$

which vanishes if $\Delta S=-\Delta Q$ transitions are forbidden. Since $A(0)$ is independent of r and s, the test of the $\Delta S=\Delta Q$ rule obtained by extrapolating the charge asymmetry to $t=0$ is largely independent of the question of CPT or CP invariance. Note, however, that the use of an extrapolation in time means that this result, too, could be misleading if

$$s^2\approx -1. \quad (31)$$

Then the ratio $A(t)$ would be quite different from 0 for values of t satisfying $t\gtrsim (s^2+1)/\Delta\approx 0$ even if $\Delta S=\Delta Q$. However, the determination of R already establishes that $|s|\neq 1$ if $\Delta S=\Delta Q$, hence the measurement of the charge asymmetry, when combined with a measurement of R, would remove any ambiguity in the test of the $\Delta S=\Delta Q$ rule. The experimental results[10] on the charge asymmetry lack the required statistical significance but they do indicate that $A(0)\neq 0$.

7. CONCLUSIONS

It has been shown that the small branching ratio for the decay mode $K_2\to 2\pi$ suggests, rather than proves, that the nonleptonic interactions are CP invariant since it indicates that the product $rs\zeta\approx 1$, while CP invariance requires that $r=s=\zeta=1$. Independent information concerning s, which is different from one only if both CPT and CP invariances fail, may be obtained directly from K^0 scattering experiments, as shown in Sec. 5. However, even if $s=1$ the interpretation of the above experiment is ambiguous because the parameters r and ζ are essentially independent. More detailed information on the 3π mode would probably serve to resolve this ambiguity.

The small charge asymmetry in the leptonic decay of the K_2^0 is an indication of either CPT or CP invariance of the leptonic interactions if the CP-invariance of the nonleptonic interactions is accepted. A very direct test of over-all CP invariance is offered by the method of reference 4, a determination of the rate of leptonic decay of the K^0 meson as a function of time. By determining separately the partial rates of decay into the leptonic modes of different charge, it is possible to obtain a unique test of CPT invariance of the leptonic interactions. Detailed experiments of this kind

[17] M. L. Good, Phys. Rev. **110**, 550 (1958).
[18] R. H. Good, R. P. Matsen, F. Muller, O. Piccioni, W. M. Powell, H. S. White, W. B. Fowler, and R. W. Birge, Phys. Rev. **124**, 1223 (1961).
[19] S. B. Treiman and R. G. Sachs, Phys. Rev. **103**, 1545 (1958).

should eventually make it possible to unravel all the parameters involved in the leptonic decay.

Fortunately, it turns out that the most direct measurement of the K_1-K_2 mass difference is independent of the question of either *CPT* or *CP* invariance. On the other hand, the test of the $\Delta S=\Delta Q$ rule based on leptonic decay rates of K^0 mesons does depend on these questions so that there exists an ambiguity in the interpretation of the presently available experimental data. However, a more accurate determination of the charge asymmetry in these decay processes will serve to resolve the ambiguity.

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K^+-p and K^--p Total Cross Sections in the Momentum Range 3–19 BeV/c*

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The total cross sections of K^+ and K^- mesons on protons have been measured between 3 and 19 BeV/c at the Brookhaven alternating gradient synchrotron. The data were obtained from measurements of attenuation in a liquid hydrogen target. The K mesons were defined in momentum by magnetic deflection and a scintillation counter telescope. They were identified by measuring the velocity with a high-pressure gas-filled Čerenkov counter. Values of the cross sections were obtained at intervals of $\sim$1 BeV/c with an error for each value of approximately 5%. $\sigma_t(K^+p)$ is, within errors, constant over the entire momentum range with a value of 18.4 mb. $\sigma_t(K^-p)$ decreases gradually from a value of 28 mb at 4 BeV/c to 21.6 mb at 19 BeV/c. The cross sections are significantly different up to the highest momenta reached in this experiment.

I. INTRODUCTION

THE successful operation of proton accelerators in the 30-BeV range at CERN and at the Brookhaven National Laboratory has made it possible to increase substantially the energy range of total cross-section measurements. We report here measurements of total cross sections of K^+ and K^- mesons on protons between 3 and 19 BeV/c at the Brookhaven alternating gradient synchrotron (AGS). At momenta below 5 BeV/c, data have been published by a number of authors.[1–10] More recently some data above 5 BeV/c have been reported from the CERN laboratory.[9,10]

Advances in the theory of strong interactions have given added impetus to attempts to extend cross-section measurements to the highest available energies. On the basis of general field-theoretical arguments, Pomeranchuk[11] has shown that, in the high-energy limit, particle and antiparticle total cross sections should become equal. More recently Chew and Frautschi[12] have proposed that the elastic scattering amplitude for any process is dominated by the Regge pole[13] in the crossed channel. Within this framework, Udgaonkar[14] has shown that it may be possible to understand how the different cross sections approach the Pomeranchuk limit.

In Secs. II and III we describe the apparatus and

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