# Measurement of the Decay Parameters of the $\Lambda^0$ Particle\*

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The decay parameters of  $\Lambda^0 \to \pi^- + p$  have been measured by observing the polarization of the decay protons by scattering in a carbon-plate spark chamber. The experimental procedure is discussed in some detail. A total of 1156 decays with useful proton scatters was obtained. The results are expressed in terms of polarization parameters,  $\alpha$ ,  $\beta$ , and  $\gamma$  given below:

$$\alpha = 2 \operatorname{Res} p^*/(|s|^2 + |p|^2) = +0.62 \pm 0.07,$$
  
 $\beta = 2 \operatorname{Ims} p^*/(|s|^2 + |p|^2) = +0.18 \pm 0.24,$   
 $\gamma = |s|^2 - |p|^2/(|s|^2 + |p|^2) = +0.78 \pm 0.06,$ 

where s and p are the s- and p-wave decay amplitudes in an effective Hamiltonian  $s + p \boldsymbol{\sigma} \cdot \boldsymbol{p} / |\boldsymbol{p}|$ , where  $\boldsymbol{p}$  is the momentum of the decay proton in the center-of-mass system of the  $\Lambda^0$ , and  $\sigma$  is the Pauli spin operator. The helicity of the decay proton is positive. The ratio |p|/|s| is  $0.36_{-0.06}^{+0.05}$  which supports the conclusion that the KAN parity is odd. The result  $\beta = 0.18 \pm 0.24$  is consistent with the value  $\beta = 0.08$  expected on the basis of time-reversal invariance.

### I. INTRODUCTION

SOON after the discovery of an up-down asymmetry in the decay  $\Lambda^0 \to \pi^- + p$ , Lee and Yang<sup>2</sup> discussed in the most general form the phenomenological description of the decay. They pointed out that the decay of each nonleptonic channel of the  $\Lambda^0$  (either  $\pi^- + p$  or  $\pi^0 + n$  final state) is specified by three real parameters. These are the magnitudes of the s- and p-wave amplitudes and their relative phase. They also pointed out that a different set of parameters  $\alpha$ ,  $\beta$ , and  $\gamma$ , along with the decay rate, also specify each  $\Lambda^0$  decay mode completely. The parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  are related to the s- and p-wave amplitudes, and are accessible to experimental determination by measurement of the various components of the polarization of the decay nucleon. We describe here an experiment which has determined these coefficients. The determination of the  $\Lambda^0$  decay amplitudes bears on the question of the form of the weak interaction for strangeness-changing decays. The determination of the decay amplitudes also has a direct bearing on the parity of the  $K\Lambda N$  system.

#### II. PHENOMENOLOGICAL DESCRIPTION OF A0 DECAY

We shall discuss the decay of the charged mode of the  $\Lambda^0$ ,  $\Lambda^0 \to \pi^- + p$ . The discussion for the neutral decay mode has the same form. The decay can be specified by the complex amplitudes s and p for the s- and p-wave

same problem was discussed independently by J. Leitner, Nuovo Cimento 8, 68 (1958).

decay channels. If the  $\Lambda^0$  sample has an initial polarization  $P_{\Lambda}$ , the decay protons will have an angular distribution in the rest frame of the  $\Lambda^0$  given by

$$N(\theta)d\Omega = (1/4\pi)[1 + \alpha P_{\Lambda} \cos\theta]d\Omega, \tag{1}$$

where  $\cos\theta = \mathbf{n} \cdot \mathbf{p}$ , where **n** is a unit vector in the direction of the  $\Lambda^0$  polarization and  $\mathbf{p}$  is a unit vector in the direction of emission of the proton.  $\alpha$  is the decay asymmetry parameter. All the above quantities refer to the center-of-mass system of the  $\Lambda^0$ . Note that our definition of  $\alpha$  is opposite in sign to that used in previous literature.

Note added in proof. The convention used for  $\alpha$  has the merit that now  $\alpha$  is directly the nucleon helicity, a parameter of more direct physical meaning. A similar convention has been adopted by R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi, Phys. Rev. Letters 9, 66 (1962).

An unpolarized sample of  $\Lambda^0$  hyperons will decay with a longitudinal polarization of the proton equal to  $\alpha$ . The sign of  $\alpha$  specifies the helicity of the proton; protons of positive helicity  $(\boldsymbol{\sigma} \cdot \mathbf{p} > 0)$  have a positive value for  $\alpha$ .  $\alpha$  is related to the decay amplitudes s and p by the equation

$$\alpha = 2 \operatorname{Res} p^* / (|s|^2 + |p|^2).$$
 (2)

Examination of the polarization of the decay proton for a polarized sample gives a more complicated dependence. We shall resolve the proton polarization in the center-of-mass system of the  $\Lambda^0$  into components along orthogonal axes defined in the following way: The z axis will be taken along the  $\Lambda^0$  polarization direction, the x axis will be chosen so that in the center-ofmass system the x-z plane contains the decay proton, and the y axis will be chosen so that x, y, z forms a right-handed orthogonal coordinate system. If the initial  $\Lambda^0$  has a polarization  $P_{\Lambda}$  directed along the positive z axis, then we have the following expressions for the

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¹ F. Eisler, R. Plano, A. Prodell, N. Samios, M. Schwartz, J. Steinberger, P. Bassi, V. Borelli, G. Puppi, H. Tanaka, P. Waloschek, V. Zaboli, M. Conversi, P. Franzini, I. Mannelli, R. Santangelo, V. Silvestrini, D. A. Glaser, C. Graves, and M. L. Perl, Phys. Rev. 108, 1353 (1957); F. S. Crawford, M. Cresti, M. L. Good, K. Gottstein, E. M. Layman, F. T. Solmitz, M. L. Stevenson, and H. K. Ticho, ibid. 108, 1120 (1957).

² T. D. Lee and C. N. Yang, Phys. Rev. 108, 1645 (1957). The same problem was discussed independently by J. Leitner, Nuovo

proton polarizations along the three axes:

$$P_{z} = \left[ \gamma P_{\Lambda} + \alpha \cos \theta + P_{\Lambda} (1 - \gamma) \cos^{2} \theta \right] / \left[ 1 + \alpha P_{\Lambda} \cos \theta \right], \quad (3)$$

$$P_x = \left[\alpha \sin\theta + P_{\Lambda}(1 - \gamma) \sin\theta \cos\theta\right] / \left[1 + \alpha P_{\Lambda} \cos\theta\right], \quad (4)$$

$$P_{y} = [\beta P_{\Lambda} \sin \theta] / [1 + \alpha P_{\Lambda} \cos \theta], \tag{5}$$

where

$$\gamma = (|s|^2 - |p|^2)/(|s|^2 + |p|^2) \tag{6}$$

and

$$\beta = 2 \text{ Im} s p^* / (|s|^2 + |p|^2). \tag{7}$$

The parameter  $\gamma$  is positive or negative depending on whether the s- or p-wave decay amplitude is larger. If  $\gamma$  is negative, indicating a predominance of p wave, then the direction of  $P_z$  at  $\theta=90^\circ$  will be opposite to the original direction of the  $\Lambda^0$  polarization. This is a consequence of the fact that the p-wave amplitude contains a spin-flip term.

The parameter  $\beta$  is closely related to time-reversal invariance. Time-reversal invariance implies that the amplitudes s and p are real, relative to each other, in which case  $\beta$  vanishes. The strong final-state interaction between the pion and nucleon will alter this condition and  $\beta$  will be expected to have a small but nonzero value. If time reversal is valid, s and p may be rewritten

$$s = s' \exp i\delta_s,$$

$$p = p' \exp i\delta_p,$$
(8)

where s' and p' are now real amplitudes, and  $\delta_s$  and  $\delta_p$  are, respectively, the s- and p-wave pion-nucleon scattering phase shifts at the appropriate center-of-mass energy and isotopic spin state. With these definitions  $\alpha$  and  $\beta$  become

$$\alpha = \lceil 2s' p' / (s'^2 + p'^2) \rceil \cos(\delta_s - \delta_p), \tag{9}$$

$$\beta = [2s'p'/(s'^2 + p'^2)] \sin(\delta_s - \delta_p). \tag{10}$$

The parameters are related by the equation

$$\alpha^2 + \beta^2 + \gamma^2 = 1.$$

The analysis above also applies to the neutral decay mode  $\Lambda^0 \to \pi^0 + n$ . There is fairly strong evidence that the nonleptonic decay of the  $\Lambda^0$  hyperon is governed by the  $|\Delta T| = \frac{1}{2}$  rule.<sup>3</sup> For this circumstance the neutral decay mode amplitudes are related to the charged mode amplitudes by

$$s^{0} = (1/\sqrt{2})s^{-},$$
  

$$p^{0} = (1/\sqrt{2})p^{-},$$
(11)

where the superscripts "0" and "—" refer, respectively, to the neutral and charged modes. To the extent that the  $|\Delta T|=\frac{1}{2}$  rule is valid for the  $\Lambda^0$  decay, the specification of the charged mode amplitudes specifies the neutral decay mode amplitudes.

### III. EXPERIMENTAL APPARATUS

The observation of an up-down asymmetry of the  $\Lambda^0$  decay pions in the reaction  $\pi^- + p \to \Lambda^0 + K^0$  demonstrates that the reaction produces polarized  $\Lambda^0$  hyperons.1 The topology of the reaction allows it to be singled out from many kinds of possible  $\pi^- + p$  interactions. Ionization, initially due to the incoming pion, is interrupted at the interaction point and only resumes when the  $\Lambda^0$  and/or the  $K^0$  decay. Since the  $\Lambda^0$  and  $K^0$ always go in the forward direction near threshold, there is a downstream gap in the ionization. Figure 1 shows the schematic arrangement of the experiment. A count in the counters  $C_1C_2C_4$  with an anticoincidence in  $C_3$ (indicated by  $C_1C_2\bar{C}_3C_4$ ) is the signature of a gap in ionization and a possible  $\Lambda^0$ -producing interaction. This signature triggers a spark chamber array which displays the event. The decay protons pass into an array of carbon spark chamber plates where some will scatter allowing polarization analysis.

The apparatus for visual observation of the  $\Lambda^0$  event consisted of three distinct spark chambers all contained in one large gas-tight box. The first chamber consisted of eleven 6-in.×4-in.× $\frac{1}{4}$ -in. plates separated by  $\frac{1}{4}$ -in. gaps. These plates were made of hollow aluminum frames with surfaces of 0.003-in. aluminum foil secured on the frames with Mylar tape. In the hollow centers of the last four plates a  $\frac{1}{4}$ -in,×3-in.×3-in. piece of polyethylene was placed. These polyethylene inserts served as a target for the production of the  $\Lambda^0$  hyperons. Each target being only  $\frac{1}{4}$  in. thick enabled the stopping point of the pion to be determined to an accuracy of  $\pm \frac{1}{8}$  in. The effective thickness of the target was 2 in. which is about 1.5  $\Lambda^0$  mean decay lengths.

The production chamber was followed by a 12-in.  $\times 12$ -in.  $\times \frac{1}{4}$ -in. anticoincidence counter, which was then followed by a decay chamber whose plates were 0.001-in. aluminum foil. This chamber was constructed from  $\frac{3}{8}$ -in.-thick Lucite frames. The frames were 10 in.  $\times$  10 in. on the inside, and 12 in.  $\times$  12 in. on the outside. The foils were attached to the frames with double-sided tape and then the frames were glued together. The gas

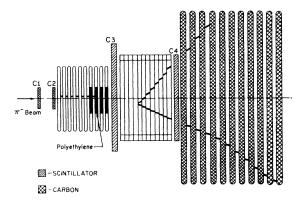


Fig. 1. Schematic diagram showing arrangement of apparatus.

An example of an event has been sketched in

<sup>&</sup>lt;sup>3</sup> F. S. Crawford, M. Cresti, R. L. Douglass, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters 2, 266 (1959).

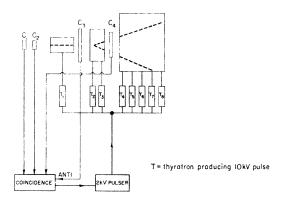


Fig. 2. Schematic view of spark chamber triggering system.

volume of the small chamber communicated with the larger box by small holes drilled in the sides.

The decay chamber was designed to contain as little mass as possible. The competing background events in the experiment were: (1)  $\pi^- + p \rightarrow \pi^0 + n$  with a subsequent conversion of one of the  $\pi^0 \gamma$  rays in the space between  $C_3$  and  $C_4$ , and (2) the production of neutrons which interact in the space between  $C_3$  and  $C_4$  to produce a count in  $C_4$ . Both backgrounds go to zero in the limit of no material between  $C_3$  and  $C_4$  and in the limit of infinitesimally thin counters. Aluminum foils 0.001 in. thick were as thin as we felt could operate satisfactorily as spark chamber plates. The limitation on the thinness of the counters was the requirement of good efficiency; in particular, as is discussed below, the anticounter must be very efficient. The counter  $C_4$ , referred to as the transmission counter, was  $\frac{1}{8}$  in. thick and  $9\frac{1}{2}$  in.  $\times 9\frac{1}{2}$  in. in area. The aluminum foil decay chamber was 4.5 in. thick which was approximately 2.5 mean decay lengths for  $\Lambda^0$  decay. Additional thickness of this chamber would have made the ratio of  $\Lambda^0$  particles to background decrease since background is proportional to the interaction material present whereas the additional number of  $\Lambda^0$  particles to be gained after 2.5 mean decay lengths is negligible.

The spark chamber which followed the transmission counter was made of  $2\text{-ft}\times2\text{-ft}\times\frac{1}{2}\text{-in}$ . carbon plates separated by  $\frac{3}{8}$ -in. gaps. This spark chamber consisted of 10 gaps. The density of the carbon was  $1.785 \text{ g/cm}^3$  which meant that a 600-MeV/c proton stopped in the last plate after passing normally through the 10 gaps. The number of carbon plates was not sufficient to stop all of the  $\Lambda^0$  decay protons, and, as will be discussed later, the momentum of the protons was determined from the decay kinematics and not the range. The scattering of the  $\Lambda^0$  decay protons was observed in the carbon plates. The area of the plates was made sufficiently large to contain all possible useful proton scatters with no requirement of any geometrical correction.

All three spark chambers were contained in a 30-in.  $\times$  30-in.  $\times$  30-in. aluminum box with 2-ft  $\times$  2-ft viewing windows for 90° stereoscopic observation. The chambers

were photographed through 30-in.-diam Lucite lenses of 10-ft focal length. The cameras were placed 10 ft from the principal plane of the Lucite lens. The image of the spark chamber was a true orthogonal projection on the plane perpendicular to the lens axis. The entire box was filled with 1 atm of neon with the addition of 1% argon. The entire spark chamber array was pulsed by 8 separate 5C22 thyratrons which were driven by a common 6130 thyratron. The first production chamber was pulsed by a single 5C22 thyratron. The 6 high-voltage plates of the decay chamber were driven by two 5C22 thyratrons, each thyratron pulsing 3 plates. Each of the 5 high-voltage plates of the carbon chamber were driven by an individual thyratron. Figure 2 shows a schematic view of the operation of the spark chambers.

### IV. EXPERIMENTAL PROCEDURE

The pion beam was produced by protons from the external beam of the Brookhaven Cosmotron striking an 8-in. $\times 1$ -in. $\times \frac{1}{2}$ -in. carbon target. The negative pion beam was selected in the forward direction by a system of momentum analyzing and focusing magnets. A beam of  $1.07\pm0.03$  BeV/c mesons at an intensity of  $10^4$  pions per pulse was obtained for an internal circulating beam of  $10^{11}$  protons per pulse. The primary proton beam energy was 2 BeV. This lower Cosmotron energy allows the repetition rate of the machine to increase to one pulse every 3 sec.

The pion beam was selected by two  $2\frac{3}{4}$ -in.-diam  $\times \frac{1}{8}$ -in.-thick scintillators. The size of the focused beam was 2 in. in the horizontal direction and 1 in. in the vertical direction. The counting rate for  $C_1C_2\bar{C}_3C_4$  was 1.6 counts per  $10^4 \, \pi^-$  mesons. Suitable dead time circuits allowed only one spark chamber pulse per Cosmotron pulse. Because of the Poisson distribution of the counting rate only about 80% of the Cosmotron pulses yielded a spark chamber pulse when 104 pions per pulse passed through the chamber. The sensitive time of the spark chamber was  $\approx 0.7 \,\mu \text{sec}$  so that an instantaneous rate greater than 106 pions per second gave an excessive number of double tracks in the chamber. The duty cycle of the Cosmotron was such that 104 pions per pulse corresponded to an instantaneous beam rate of 106 pions per second. The distribution of kinds of events which triggered the spark chambers is given in Table I.

The  $\Lambda^0$ ,  $K^0$ , and  $\Lambda^0 K^0$  (events which show both a  $\Lambda^0$  and  $K^0$  decay) decays were separated by the kine-

Table I. Distribution of events photographed in experiment.

Event	Percent of pictures
$\Lambda^0$	30
$K^0$	5
$\Lambda^0 K^0$	4
Blank	27
Electron-positron pair	31
Electron-positron pair Straight-through	3



Fig. 3. Photograph of  $\Lambda^0$  decay with scattered proton.

matic analysis described in Sec. V, and together made up 39% of the total pictures taken. The number of  $\Lambda^0$  events which had a decay proton scatter in a useful angular and momentum range was about 2% of all  $\Lambda^0$  decays. Figure 3 shows a  $\Lambda^0$  event with a proton scatter in the carbon. Figure 4 shows a double event with both a  $\Lambda^0$  and a  $K^0$  decay.

The remaining 61% of the pictures were due to backgrounds of various kinds. The category designated as blanks were a combination of neutron-induced events and accidental triggers. The accidental triggers occurred when a beam pion ( $C_1C_2$  coincidence) interacted to form a zero prong star, combined with a chance count in  $C_4$ . The chance counts which contribute to this rate could not be beam counts since the beam counts in  $C_4$  must pass through the anticoincidence counter  $C_3$ . Thus, only neutron background from the accelerator and photomultiplier noise contributed to the blank rate. Most of the blank pictures showed a pion stopping in the first chamber with practically nothing in the remaining two chambers. The blank rate was also checked electronically by delaying  $C_3$  and  $C_4$  with respect to  $C_1$  and  $C_{2*}$ .

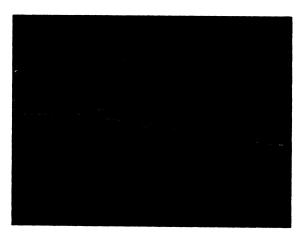


Fig. 4. Photograph of  $\Lambda^0$  and  $K^0$  decay. The lower decay is the  $\Lambda^0$ .

The rate for these conditions gave the accidental counts between  $C_4$  and  $C_1C_2$  of nonbeam particles since the rate in  $C_4$  from the beam was still anticoincided. This measured rate was  $\approx 0.5 \times 10^{-4}$  counts per incident pion. The  $C_1C_2(\bar{C}_3C_4)_{\text{\tiny delayed}}$  rate was nearly independent of the pion beam intensity which indicated that this rate was due in part to photomultiplier tube noise. The other principal source of background was electronpositron pairs due to  $\gamma$  rays from  $\pi^0$  decays interacting in the aluminum foils and counters. Most of these events were conversions either in the last part of  $C_3$  or in counter  $C_4$ . These electron pairs have characteristic small opening angles and presented no difficulty in identification. Figure 5 shows a typical  $e^+e^-$  pair. The inefficiency of the anticoincidence counter  $C_3$  accounted for the 3\% of the pictures which just showed a straightthrough track. This rate amounted to an inefficiency of one lost count per  $2 \times 10^5$  incident pions.

In addition to the particular event registered, 24% of all pictures showed an additional straight-through track. This rate is compatible with the chance that two pions arrived within the clearing time of the chamber. In most cases the additional straight-through track did not affect the ability to analyze the event.

The measured rate of  $\Lambda^0$  or  $K^0$  production was 0.6×10<sup>-4</sup> per incident pion. A calculation was made of the expected rate using the known cross sections for production on hydrogen,4 the known lifetimes of the  $\Lambda^0$  and  $K^0$ , and the fact that in propane the ratio of hydrogen events to total events is 0.40.5 This calculation gave a predicted rate of  $1.1 \times 10^{-4} \Lambda^0$  or  $K^0$  per pion. The fact that the observed rate was lower than the calculated rate was due in part to anticoincidence accidentals, where a chance count in  $C_3$  anticoincides a good event. During the run this accidental rate varied from 20 to 40%, so that the expected rate was of the order of  $0.8 \times 10^{-4}$ , which is consistent with the observed rate. The calculated fractions of single  $\Lambda^0$  events,  $\Lambda^0 K^0$ events, and single  $K^0$  events were 70, 15, and 15%, respectively. The experimental observations are in approximate agreement.

A number of the  $\Lambda^0$  hyperons will come from the reaction

$$\pi^- + p \rightarrow \Sigma^0 + K^0$$
,  
 $\Sigma^0 \rightarrow \Lambda^0 + \gamma$ ,

which has a threshold at 1030 MeV/c incident pion momentum. These  $\Lambda^0$  particles presumably have only a small polarization since they carry only  $\frac{1}{3}$  of the  $\Sigma^0$  polarization, which is unknown at the present time. These  $\Lambda^0$  particles dilute the polarization of the  $\Lambda^0$ 

<sup>&</sup>lt;sup>4</sup>L. Bertanza, P. L. Connolly, B. B. Culwick, F. R. Eisler, T. Morris, R. Palmer, A. Prodell, and N. P. Samios, Phys. Rev. Letters 8, 332 (1962).

<sup>&</sup>lt;sup>5</sup> F. Eisler, R. Plano, A. Prodell, N. Samios, M. Schwartz, J. Steinberger, P. Bassi, V. Borelli, G. Puppi, H. Tanaka, P. Waloschek, V. Zaboli, M. Conversi, P. Franzini, I. Manelli, R. Santangelo, and V. Silvestrini, Nuovo Cimento 10, 468 (1958).



Fig. 5. Photograph of an  $e^+e^-$  pair.

sample, but otherwise are quite acceptable since the kinematics of the  $\Lambda^0$  decay are completely determined by the angle the two decay prongs make with the  $\Lambda^0$  line of flight.

In the total experiment about 250 000 photographs were taken of which more than 60 000 contain a  $\Lambda^0$  decay. The experiment was carried out in a run of 2-weeks duration during the summer of 1961.

### V. KINEMATIC ANALYSIS OF EVENTS

The photographs of the spark chamber were taken in two 90° stereoscopic views. The use of spherical Lucite field lenses gave orthogonal projections of the events which made the analysis rather simple. In addition to the event, each picture contained fiducial lines from an illuminated Lucite reticule as well as a register number.

The two films were projected to 70% of actual size in a film viewer. All measurements on the events were made in terms of angles of the tracks with respect to the fiducial lines. The angle measurements were made with a digitized protractor. The protractor consisted of two Lucite arms, one coupled to the shaft of a Datex 1000 count decimal encoder, the other arm to the body of the encoder. The latter arm was attached to a drafting machine so that it remained fixed in orientation when the protractor was moved over the screen. Five angles were encoded for each view: (1) incident  $\pi^-$  direction, (2) line of flight of  $\Lambda^0$ , (3) pion decay direction, (4) proton decay direction, and (5) scattered proton direction. All directions could be recorded to an accuracy of  $\pm 0.3^{\circ}$  except for the  $\Lambda^{0}$  line of flight where the accuracy was  $\pm 1.0^{\circ}$ . The poorer accuracy for the  $\Lambda^0$  flight path was due to uncertainty in the locations of the stopping point of the incident pion and of the decay vertex, as well as the generally short length of the  $\Lambda^0$  flight path. Additional information concerning the event, such as the plate in which the proton scattered, the proton range, the pion range, event number, etc., was also recorded. The angular data were recorded on IBM cards directly from the digitizer, and the additional data from a set of dials on a parameter board.

All events which appeared to be a  $\Lambda^0$  decay with the decay proton scattering in the carbon were recorded. The visual criteria used were:

- (1) The assumed proton track which showed a scatter had to make a smaller angle with respect to the  $\Lambda^0$  flight path than the pion. In the range of  $\Lambda^0$  momenta considered (400–1000 MeV/c), it is impossible for the proton to be emitted at the larger angle to the  $\Lambda^0$  line of flight.
- (2) The proton had to have a residual range after scattering of at least three carbon plates. This criterion was to ensure that the scattering occurred at a momentum of 400 MeV/c or greater so that the proton scatter had a reasonable sensitivity to the polarization of the proton.
- (3) The scatter point had to be well defined, and the angle of scatter had to be at least 4° in one view.
- (4) At the point of scatter there had to be no evidence of extra prongs. This condition was imposed to reduce the number of inelastic scatters.

It should also be noted that due to the greater proton ionization, these tracks were readily distinguishable because of their brightness and uniform high efficiency. The pion tracks were by contrast weak and showed inefficiencies. The event in Fig. 3 demonstrates this ionization effect.<sup>6</sup>

The IBM cards for the measured events were processed by an IBM 650 computer programmed to spatially reconstruct each event and to calculate the momenta of the  $\Lambda^0$  and its decay products from the decay angles of the pion and the proton. For each event the following quantities were computed:

 $\Phi=$  decay angle of the proton with respect to the  $\Lambda^0$  line of flight in the center-of-mass system of the  $\Lambda^0$ .  $\mathbf{p}_{\Lambda}$ ,  $\mathbf{p}_{\pi}{}^l$ ,  $\mathbf{p}_{\pi}{}^l=$  momenta of the  $\Lambda^0$ , decay proton, and decay pion, respectively, in the laboratory system. The superscript l denotes laboratory.

 $\Theta$ = polar scattering angle of the decay proton in the carbon.

 $\cos\theta = \mathbf{n} \cdot \hat{p}$ , where  $\mathbf{n}$  is a unit vector in the direction  $-(\mathbf{p}_{\pi_{\text{inc}}} \times \mathbf{p}_{\Lambda})$ .  $\mathbf{p}_{\pi_{\text{inc}}}$  is the momentum of the incoming pion, and  $\hat{p}$  is a unit vector in the direction of the momentum of the decay proton in the center-of-mass system.

Also the coplanarity of the  $\Lambda^0$  decay was computed.

The results of these calculations were then examined to select the acceptable events from which the values of the decay parameters were determined. Any event which was ambiguous or had some uncertainty was removed from the sample. The acceptable events were selected before any angles which were relevant to the

<sup>&</sup>lt;sup>6</sup> E. Engels, D. Roth, J. W. Cronin, and M. Pyka, I.R.E. Trans. Nucl. Sci. **NS-9**, 256 (1962).

various expected asymmetries were computed. This step eliminated any subjective bias which might be present in the selection. The following selection criteria were applied:

- (1) The trajectory of the  $\Lambda^0$  and its decay products had to be coplanar to within 2°.
- (2) The polarization analyzing power S for the event had to be greater than 0.2. This requirement imposed restrictions on the values of  $\Theta$  and on the momentum of the proton at the point of scatter. A detailed discussion of the assignment of analyzing power is given in Sec. VI A.
- (3) The momentum of the decay products as calculated from the opening angles had to be consistent with the range observed for these particles in the carbon chamber.
- (4) All selected events were required to have an angle between the  $\Lambda^0$  line of flight and the decay proton direction of less than 16°.

The momentum distribution of the decay protons was such that approximately 50% of them stopped in the carbon plates of the scattering spark chamber. For these events a fairly rigorous check on the momentum calculated from the decay angles could be made. For the other half of the data the calculated proton momentum had to be sufficient to pass through all of the carbon plates.

In order to check the accuracy of the determination of the proton momentum from kinematics, a sample of events was selected for which the proton stopped in the carbon chamber. The momentum of the protons for this sample was calculated from both decay kinematics and the range measurement. A comparison was then made of the momentum of the  $\Lambda^0$  for each event calculated using the proton momentum determined by each of these two methods. Figure 6 shows a plot of the deviation of the  $\Lambda^0$  momentum determined purely by kinematics with that determined using the proton momentum determined by range measurement. The determination of the  $\Lambda^0$  momentum from proton range measurements has an uncertainty of only  $\pm 30 \text{ MeV}/c$ , so that the spread in the curve of  $\pm 80 \text{ MeV/}c$  is due to the errors inherent in determination of  $\Lambda^0$  momentum from kinematics. The curve is centered about zero which indicates there is no systematic error in the determination of the momentum.

Events selected with the above criteria can be demonstrated to be free of  $K_1^0$  contamination. The requirement that the angle between the  $\Lambda^0$  line of flight and the proton be less than 16° restricts the number of  $K_1^0$  charged decays to about 15% of the  $\Lambda^0$  decays, allowing for a half of the  $K^0$  mesons to escape as  $K_2^0$  particles. Of these 15%, only those  $K_1^0$  mesons with large momentum will be confused with  $\Lambda^0$  particles. This is because, if a  $K_1^0$  decay had been mistaken for a  $\Lambda^0$  decay, in most cases the particle identified as the proton

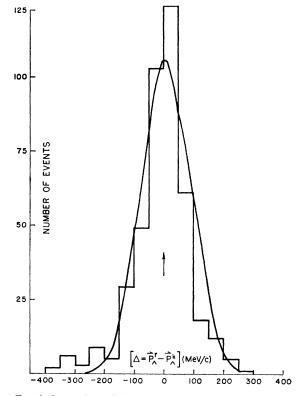


Fig. 6. Comparison of  $\Lambda^0$  momentum calculated from the decay angles,  $P_{\Lambda}{}^k$ , with that computed from proton range and decay opening angle,  $P_{\Lambda}{}^r$ . The abscissa is  $\Delta = P_{\Lambda}{}^r - P_{\Lambda}{}^k$  and the ordinate is the number of events in a given interval of  $\Delta$ .

(one with smallest opening angle) would, as a result of the kinematic calculation, be expected to come to rest in the carbon chamber. However, the pion with smallest opening angle from  $K_1^0$  decay has sufficient range to penetrate all the carbon plates. From this range inconsistency most of the  $K_1^0$  decays are eliminated. A detailed calculation based on the momentum spectrum of the  $K_1^0$  mesons indicates that the  $K_1^0$  contamination in the final sample is less than 1%, which contributes a negligible error to the results.

Events which satisfied the selection criteria were returned to the computer for computation of the relevant azimuthal scattering angles. The details of these computations are discussed in the next section.

## VI. DETERMINATION OF DECAY PARAMETERS

# A. Measurement of $\alpha$ from Unpolarized $\Lambda^0$ Particles

If no reference is made in the analysis to the production plane of the  $\Lambda^0$  particles, the sample is unpolarized since there is no preferred orientation of the production plane. Consider the decay of a  $\Lambda^0$  as shown in Fig. 7. The proton has a longitudinal polarization  $\alpha$ , and the figure is drawn for the case of positive helicity. In the transformation from the center-of-mass system to the

laboratory system the proton momentum is swept forward due to the motion of the  $\Lambda^0$ . For our sample the average angle that the proton makes in the laboratory with the line of flight of the  $\Lambda^0$  was  $10^\circ$ . In the nonrelativistic limit the spin component remains fixed in space during the transformation to the laboratory system. The relativistic transformation of the spin vector has been discussed by Wigner.<sup>7</sup> The angle  $\epsilon$  between the spin component and the proton momentum after the transformation to the laboratory system is given by

$$\epsilon = \tan^{-1} \left[ \frac{(1 - \beta_0^2)^{1/2} \sin \Phi}{\cos \Phi + \beta_0 / \beta} \right], \tag{12}$$

where  $\beta_0 = v_0/c$ ,  $v_0$  being the velocity of the decay proton in the rest system of the  $\Lambda^0$ , and  $\beta = v/c$ , v being the velocity of the  $\Lambda^0$  in the laboratory. Thus, in the laboratory system a transverse component of spin, given by  $\alpha \sin \epsilon$ , is produced. For our sample the difference between the exact transformation and the nonrelativistic transformation of the spin direction averaged 1.2°, with the exact spin direction making the smaller angle with the proton momentum. In all our calculations the exact transformation was used except in the determination of  $\beta$  as discussed in Sec. VI D. The average value of the transverse polarization for the sample was  $0.84\alpha$ . This value is essentally unchanged if one uses the approximation that the spin remains fixed in the center-of-mass to laboratory transformation.

The polarization of the  $\Lambda^0$  decay protons was detected by their scattering in the carbon plates. The analyzing power  $S(\Theta, \mathbf{p}_s)$  was obtained from a compilation of polarization data given by Birge and Fowler. Here  $\Theta$ is the polar angle of scattering of the proton and  $\mathbf{p}_s$  is the momentum of the proton at the point of scatter. Figure 8 shows the angles involved in the analysis of polarization. The z-x plane is the decay plane of the  $\Lambda^0$ . The expectation value of the spin along the z axis is  $\alpha \sin \epsilon$ . This polarization results in an asymmetry about

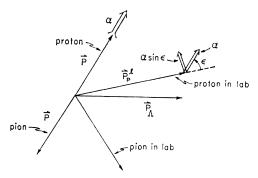


Fig. 7. Schematic view of the decay of an unpolarized A<sup>o</sup>.

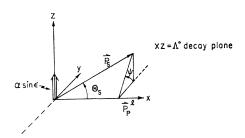


Fig. 8. Angles involved in analysis of polarization of decay protons from unpolarized  $\Lambda^0$  particles.

the azimuthal angle of scattering  $\psi$  given by

$$1 + \alpha S \sin \epsilon \cos \psi$$
. (13)

Experimentally it has been found that the analyzing power S is positive, i.e., in Fig. 8 a beam of protons polarized in the positive z direction will scatter preferentially in the positive y direction. Thus, if a predominance of events is found with  $\psi$  having a positive cosine over the number of events with  $\psi$  having a negative cosine, the helicity of the decay proton is positive.

In order to determine the analyzing power S for an event it is necessary to know the proton momentum at the point of scattering as well as the polar angle of scatter. The momentum at point of scatter was determined in two ways. For those events in which the proton came to rest in the carbon chamber, the momentum was determined from the residual range of the proton after scattering. The uncertainty in this momentum determination was  $\pm 50 \text{ MeV}/c$ . For those events for which the proton passed out of the carbon chamber the momentum was determined by subtracting the momentum lost in traversing the plates to the scatter point from the original momentum determined for the decay proton from kinematics. The uncertainty in this determination was  $\pm 80 \text{ MeV}/c$ .

A total of 1156 events satisfied all the criteria and were used to determine  $\alpha$ . The average analyzing power for these events was 0.565. Of these events, 686 had a value of  $\psi$  with a positive cosine, and 470 had a value of  $\psi$  with a negative cosine. Thus, it is clear that the  $\Lambda^0$  decay proton has positive helicity and the value of  $\alpha$  is positive as we now define it. The value of  $\alpha$  may be determined from the relation

$$\alpha = \frac{2}{\pi} \frac{1}{\langle S \rangle \langle \sin \epsilon \rangle} \frac{N_{+} - N_{-}}{N_{+} + N_{-}},\tag{14}$$

where  $N_{+}$  is the number of events with positive cosine, and  $N_{-}$  is the number of events with negative cosine. Using the average values  $\langle S \rangle = 0.565$  and  $\langle \sin \epsilon \rangle = 0.84$ , this formula gives  $\alpha = 0.62$ .

In order to obtain the best estimate for the magnitude of  $\alpha$  we use the maximum likelihood method and maximize the function

$$\mathcal{L}(\alpha) = \prod_{i} (1 + \alpha S_{i} \sin \epsilon_{i} \cos \psi_{i}) \tag{15}$$

E. P. Wigner, Rev. Mod. Phys. 29, 255 (1957).
 R. W. Birge and W. B. Fowler, Phys. Rev. Letters 5, 254 (1960).

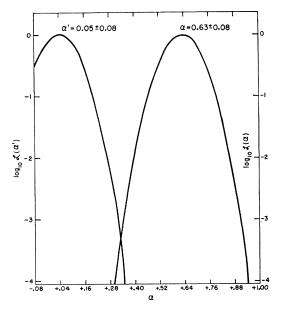


Fig. 9. Plot of likelihood function for the determination of  $\alpha$  from the unpolarized  $\Lambda^0$  sample. Also shown is the likelihood function for  $\alpha'$ , the equivalent parameter characterizing a polarization transverse to the decay plane.  $\alpha'$  is expected to be zero.

for a variation of  $\alpha$ . The result of this computation is given in Fig. 9. The value of  $\alpha$  for the best fit is  $+0.63\pm0.08$ . The errors are taken to be the  $e^{-1/2}$  points on the curve, since the curve has a Gaussian shape.

In order to check that the polarization does indeed lie in the plane of decay of the  $\Lambda^0$  we have made a similar analysis for the component of polarization transverse to the  $\Lambda^0$  decay plane. This is done by forming the likelihood function

$$\mathcal{L}(\alpha') = \prod_{i} (1 + \alpha' S_i \sin \epsilon_i \sin \psi_i), \tag{16}$$

where  $\alpha'$  is expected to be zero. The result found is  $\alpha' = 0.05 \pm 0.08$ , which gives confidence that there are no geometrical biases which would influence the value of  $\alpha$  determined by this method. The likelihood curve for  $\alpha'$  is also plotted in Fig. 9.

We will defer discussion of the uncertainties in the analyzing power until we have completed the analysis of the polarized  $\Lambda^0$  particles in Sec. VI D.

## B. Determination of $\alpha P_{\Lambda}$

From Sec. II, the angular distribution of decay protons in the center-of-mass system with respect to the initial polarization direction is

$$N(\theta)d\Omega = (1/4\pi)[1 + \alpha P_{\Lambda} \cos\theta]d\Omega, \tag{1}$$

where we have taken  $-(\mathbf{p}_{r_{\text{ine}}} \times \mathbf{p}_{\Lambda})$  as the direction of expected polarization, i.e., we define  $P_{\Lambda}$  as positive if it lies along this direction.

The value of the up-down asymmetry was determined from the data used to measure the decay proton polarization plus an additional sample of data for which all the features were identical except that the polar scattering angles of the protons were too small to give a useful analyzing power. The additional data consisted of 738 events resulting in a total of 1894 events used for the determination of  $\alpha P_{\Lambda}$ .

A likelihood function of the form

$$\mathcal{L}(\alpha P_{\Lambda}) = \prod_{i} \lceil 1 + \alpha P_{\Lambda} \cos \theta_{i} \rceil \tag{17}$$

was constructed. The likelihood curve is plotted in Fig. 10 with the result  $\alpha P_{\Lambda} = +0.355 \pm 0.037$ . The positive value for  $\alpha P_{\Lambda}$  means that  $P_{\Lambda}$  is positive since  $\alpha$  has already been determined to be positive. Thus, in the production reaction, the polarization is along the direction  $-(\mathbf{p}_{\tau_{\rm ino}} \times \mathbf{p}_{\Lambda})$ . The positive value for  $\alpha$  means the protons decay preferentially in this direction.

### C. The Determination of $\beta$

The polarization  $P_{\nu}$ , which is perpendicular to the decay plane, is sensitive to the time-reversal parameter  $\beta$ .  $P_{\nu}$  contributes a transverse polarization to the decay proton in the direction normal to the plane containing  $\mathbf{n}$ , the normal to the production plane, and  $\mathbf{p}_{\nu}^{l}$ , the direction of the laboratory momentum of the proton.

In the analysis to follow, we make two approximations which simplify the computations. First, we assume

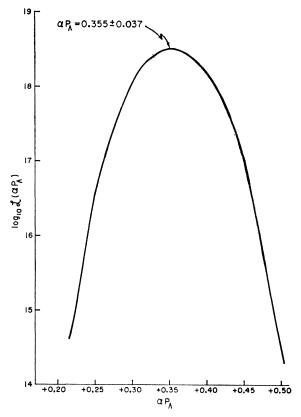


Fig. 10. Plot of likelihood function for  $\alpha P_{\Lambda}$ .

that the spin direction is unchanged when referred either to the coordinates of the center-of-mass system or the laboratory system. Second, we assume that the component of spin projected on the actual laboratory direction of the decay proton is the same as the projection onto the line of flight of the  $\Lambda^0$ . The net effect of these assumptions is negligible since when averaged over all the events the value of the projection is unchanged. The approximations have the effect of weighting some events a bit too heavily in the likelihood analysis while others are not weighted sufficiently. The result of the likelihood calculations is not influenced by slight errors in weights if the average weight for the entire sample is correct.

Inspection of Fig. 11(a) shows that the projection of  $P_{\nu}$  transverse to the proton line of flight is given by  $P_{\nu} \cos \chi$ . Thus, the contribution of  $\beta$  to the transverse polarization of the proton is

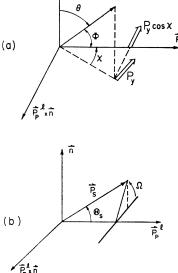
$$P_{\nu} \cos \chi = \frac{\beta P_{\Lambda} \sin \theta \cos \chi}{1 + \alpha P_{\Lambda} \cos \theta} = \frac{\beta P_{\Lambda} \cos \Phi}{1 + \alpha P_{\Lambda} \cos \theta}$$
(18)

in the approximation stated above.

The azimuthal scattering angle of the proton is now defined with respect to  $\mathbf{n}$  and  $\mathbf{p}_p{}^l \times \mathbf{n}$ . Figure 11(b) shows how this angle  $\Omega$  is defined. The definition is such that for a positive value of  $\beta$  there will be a predominance of events with a negative value of  $\sin\Omega$  for forward decaying protons, and a positive value of  $\sin\Omega$  for backward decaying protons.

There is also a transverse component of polarization in the same plane due to the component  $P_x$ . In this analysis no explicit reference is made to the sign of the angle  $\chi$ , so that this component of polarization averages

Fig. 11. (a) Angles involved in the analysis (a) of the time-reversal term  $\beta$ . The angle  $\chi$  is actually the angle between the projection of  $\beta$  on the  $\mathbf{p}_{\Lambda}$  and  $\mathbf{p}_{\Lambda} \times \mathbf{n}$  plane, and the direction of  $\mathbf{p}_{\Lambda}$ . As explained in the text we assume  $\chi$  is measured with respect to  $\mathbf{p}_{p}^{l}$ . (b) Diagram showing the azimuthal angle  $\Omega$  used in the polarization analysis to determine  $\beta$  and  $P_{z}$ .



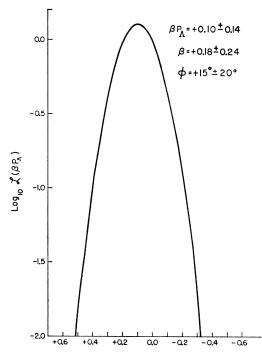


Fig. 12. Likelihood function for  $\beta P_{\Lambda}$ .

out since the number of events on each side of the  $(\mathbf{p}_{\Lambda},\mathbf{n})$  plane is equal.

The following likelihood function was formed for  $\beta P_{\Lambda}$ :

$$\mathcal{L}(\beta P_{\Lambda}) = \prod_{i} \left[ \frac{1 + \beta P_{\Lambda} \cos \Phi_{i} \sin \Omega_{i}}{\left[ 1 + \alpha P_{\Lambda} \cos \theta_{i} \right]} \right]. \tag{19}$$

The likelihood curve is shown in Fig. 12. The result is  $\beta P_{\Lambda} = +0.10\pm0.14$ . The poor statistical accuracy is due to the fact that events for which the proton decays either forward or backward in the center-of-mass system are somewhat suppressed because of the way the sample was chosen to yield proton scattering events. The backward protons did not generally give useful scattering events because they had too little momentum in the laboratory. The forward protons were accompanied by backward pions which triggered the anticoincidence counter.

The ratio  $\beta P_{\Lambda}/\alpha P_{\Lambda} = \tan\phi$ , where  $\phi = (\delta_s - \delta_p)$ , is the difference in phase between the s-wave and p-wave decay amplitudes. Using the above results we find  $\phi = +15^{\circ} \pm 20^{\circ}$ . If the  $\Lambda^0$  decays by a  $|\Delta T| = \frac{1}{2}$  transition, the final-state interaction of the pion-nucleon scattering will produce a phase shift of  $\phi = +7^{\circ}$ . A violation of time-reversal invariance would be detected by a deviation from this phase. There is no evidence in this experiment for a strong violation of time-reversal invariance.

Using the value +0.63 for  $\alpha$ , we find  $P_{\Lambda}=0.56$  from  $\alpha P_{\Lambda}=0.355$ .  $\beta$  is then  $+0.18\pm0.24$ .

<sup>&</sup>lt;sup>9</sup> S. W. Barnes, B. Rose, G. Giacomelli, J. Ring, K. Miyake, and K. Kinsey, Phys. Rev. 117, 226 (1960).

### D. Determination of $\alpha$ and $\gamma$

The result of the previous section indicates that there is no large violation of time-reversal invariance. The analysis that follows is based on the assumption that time-reversal invariance is valid, and that the final-state pion-nucleon phase shift gives the most accurate value for  $\beta$ , i.e.,  $\beta = \alpha \tan^{\circ} = 0.077$ . Then  $\beta^2 = 0.006$  which is a negligible contribution to the sum  $\alpha^2 + \beta^2 + \gamma^2 = 1$ , and hence we will assume that  $\alpha^2 + \gamma^2 = 1$ .

The polarization  $P_z$  of the proton along the initial direction of the  $\Lambda^0$  polarization is a function of both  $\alpha$  and  $\gamma$ .  $P_z$  contributes a transverse polarization of the decay proton equal to  $P_z \cos \sigma$ , where  $\sigma$  is the angle between the direction  $\mathbf{n}$  and the direction  $(\mathbf{p}_p{}^l \times \mathbf{n}) \times \mathbf{p}_p{}^l$ . This is the angle between the normal to the production plane and the normal to the laboratory direction of the decay proton in the  $(\mathbf{n}, \mathbf{p}_p{}^l)$  plane. For our sample the average value of  $\sigma$  was  $7^\circ$ .

Taking  $\alpha^2 + \gamma^2 = 1$ ,  $P_z$  is completely determined by the values of  $\alpha$ ,  $\alpha P_A$ , and the sign of  $\gamma$ . The polarization  $P_z \cos \sigma$  is detected by an asymmetry in the azimuthal scattering angle  $\Omega$  defined in Fig. 11(b). A predominance of events with positive  $\cos \Omega$  indicates a polarization  $P_z$  which is positive, i.e., along  $\mathbf{n}$ . The value of  $P_z$  is a strong function of the angle  $\theta$ . We have divided the data into four equal intervals of  $\cos \theta$  and have determined the value of  $P_z$  for each interval by maximizing the likelihood function

$$\mathcal{L}(P_z) = \prod_i \left[ 1 + P_z(\cos\sigma_i) S_i \cos\Omega_i \right] \tag{20}$$

in each case. In Fig. 13 we compare these polarizations with the computed values of  $P_z(\alpha,\gamma,\theta)$  for several values of  $\alpha$ . It is evident that this analysis is very sensitive to the value of  $\alpha$  where the sensitivity comes from the wide separation in expected polarization in the region  $-0.5 > \cos\theta > -1.0$ . It is clear that a value of  $\alpha \approx +0.6$  gives a good fit to the data. It is also evident that a negative value for  $\gamma$  cannot possibly fit the polarizations.

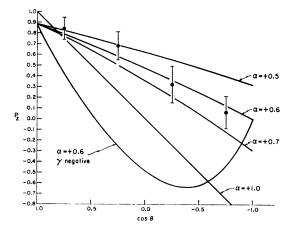


Fig. 13. Comparison of experimental polarizations  $P_x$  with those calculated for various values of  $\alpha$ . All curves except the one indicated are for  $\gamma$  positive.

There are unique features of this analysis which make the result relatively independent of the analyzing power used to evaluate the polarization. If the analyzing powers used were too high by 20% (a situation which could exist if there were a large number of grossly inelastic scatters which have no analyzing power), then all the points would have to be raised by 20%. The polarization in the interval  $-0.5 > \cos\theta > -1.0$  would then be increased from 0.10 to 0.12, which is a very slight change compared to the statistical error. One can see that since the value of  $\alpha$  is most sensitive to this point, the value of  $\alpha$  thus determined is quite insensitive to the analyzing power. On the other hand, for the interval  $1.0 > \cos\theta > 0.5$  the polarization is rather insensitive to the value of  $\alpha$ . The fact that the measured polarization in that interval agrees with the predicted value is further confirmation that our choice of analyzing power assignments is correct.

The features which are clearly displayed in the graph can be put in the form of a likelihood analysis. We form the function

$$\mathfrak{L}(\alpha, \operatorname{sign}\gamma) = \prod_{i} \left[ 1 + P_{z}(\alpha, \operatorname{sign}\gamma, \theta_{i}) S_{i} \cos \sigma_{i} \cos \Omega_{i} \right], \quad (21)$$
where

nere

$$\begin{split} P_z &= \frac{\gamma(\alpha P_{\rm A}) + \alpha^2 \, {\rm cos}\theta + (\alpha P_{\rm A}) \, (1 - \gamma) \, {\rm cos}^2\theta}{\alpha \big[ 1 + \alpha P_{\rm A} \, {\rm cos}\theta \big]}, \\ \gamma &= \pm \, (1 - \alpha^2)^{1/2}, \end{split}$$

and  $\alpha P_{\Lambda} = 0.355$ .

In Fig. 14 this function is plotted in the neighborhood of the best fit. This solution gives  $\alpha = +0.61 \pm 0.07$  with

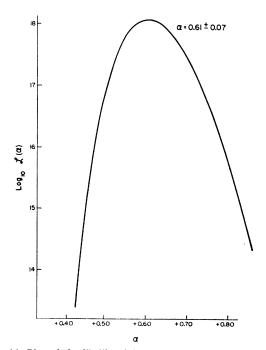


Fig. 14. Plot of the likelihood function for  $\alpha$  based on analysis of polarized  $\Lambda^0$  particles in the neighborhood of the best fit.

 $\gamma$  positive. The quoted error is the 1/e point on the likelihood curve. This is chosen as an error since the likelihood curve is not quite symmetrical about the peak.

In Fig. 15 we plot the likelihood curve as a function of  $\gamma$  for the assumed signs of  $\alpha$  both positive and negative. The relative probability that  $\alpha = +1$  compared to the best solution +0.61 is  $10^{-11}$ . The relative probability that  $\alpha$  is negative is less than  $10^{-31}$ . The relative probability that  $\gamma$  is negative is less than  $10^{-11}$ . Thus, the experiment is quite conclusive in establishing the signs of  $\alpha$  and  $\gamma$ .

The likelihood curves were computed using the value of  $\alpha P_{\Lambda}$  as a fixed constant. By repeating the computation with varying  $(\alpha P_{\Lambda})$ , we find  $d\alpha/d(\alpha P_{\Lambda}) = +0.80$ . This contributes an error to  $\alpha$  of  $\pm 0.03$ . When this error is folded into the error  $\pm 0.07$ , its contribution is negligible.

The sensitivity of the result to analyzing power has also been investigated. This test was done by first replacing the analyzing power assigned to each event with the average analyzing power of the entire sample,  $\langle S \rangle = 0.565$ , and repeating the likelihood analysis. This gave the same result as the assignment of individual analyzing powers. Then the analyzing power was varied and the results gave a value for  $d\alpha/dS = +0.25$ . Thus, if the analyzing power were changed by  $\pm 0.10$ ,  $\alpha$  would change by 0.025, so that we conclude from this measurement that  $\alpha$  is unaffected by any reasonable error in the analyzing power. The consistency between the value of  $\alpha$  determined from the unpolarized sample (Sec. VI A), and that determined in the present analysis, is strong support that the analyzing power has been correctly chosen. Any a priori uncertainty in the analyzing power would be in the direction of assigning too large values, since it is unlikely that an inelastic contamination can result in anything but a lowered analyzing power. A lowered analyzing power would reduce  $\alpha$  slightly in the present analysis and increase  $\alpha$  in the analysis from the unpolarized  $\Lambda^0$  particles, making the consistency poorer.

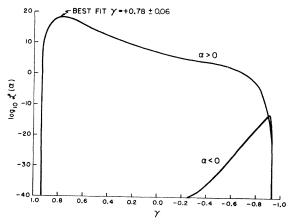


Fig. 15. Plot of the likelihood function for  $\gamma$  for both positive and negative values of  $\alpha$  and  $\gamma$ .  $|\alpha|$  must be greater than 0.355 since  $\alpha P_{\Lambda} = 0.355$ .

TABLE II. Results of likelihood calculation removing constraint  $\alpha^2 + \gamma^2 = 1$ .

$\alpha$	γ	$\alpha^2 + \gamma^2$	$[\log_{10} \mathcal{C}(\alpha)]_{\max}$
+0.54	+0.56	0,60	17.2
+0.58	+0.68	0.80	17.8
+0.61	+0.78	1.00	18.1
+0.63	+0.87	1.16	18.2

Our concern over the analyzing power has been stimulated by the fact that in the experiment of Engels et al. 10 a spark chamber was calibrated in a polarized proton beam. In that case the observed analyzing power was 25% lower than that predicted from the elastic curves. In that calibration there was absolutely no range requirement so that inelastic scatters with large energy losses which appeared as kinked tracks could not be separated from elastic events. In this present experiment we did have some check on the elasticity and all events with a range inconsistency were rejected. The consistency of our results for the assignment of elastic analyzing powers leads us to the conclusion that the inelastic contamination in this experiment was small. The insensitivity of the results to the analyzing power gives us confidence that our result is free from any systematic error that might depend on the amount of inelastic scattering.

Finally, we have removed the constraint that  $\alpha^2+\gamma^2=1$ , and have computed the likelihood for  $\alpha^2+\gamma^2=0.6$ , 0.8, and 1.16, the latter sum being physically impossible. The results of these calculations are given in Table II. One can see there is very little sensitivity to  $\beta$  in this analysis. The solution for  $\alpha^2+\gamma^2=0.8$  is certainly possible. This would yield a value of  $\beta^2=0.2$ , and hence  $\beta=0.45$ . This value of  $\beta$ , though large, is little more than one standard deviation away from our result from the direct determination of  $\beta$ . Our results for  $\alpha$  and  $\gamma$  are not changed significantly even if such a large time reversal violation would occur. In particular, the conclusions we will draw from our value for  $\gamma$  are not altered even if  $\beta=0.45$ .

### VII. CONCLUSIONS AND DISCUSSION

The values of the decay parameters measured in this experiment are given in Table III.

TABLE III. Results of measurements.

 $\alpha = +0.62 \pm 0.07$   $\beta = +0.18 \pm 0.24$   $\gamma = +0.78 \pm 0.06^{a}$   $|p/s| = 0.36_{-0.06}^{+0.05}$   $p^{2}/(s^{2} + p^{2}) = 0.11_{-0.03}^{+0.04}$ 

<sup>\*</sup> Result based on assumption that  $\beta = +0.08$ , the value expected for time-reversal invariance.

<sup>&</sup>lt;sup>10</sup> E. Engels, T. Bowen, J. W. Cronin, R. L. McIlwain, and L. G. Pondrom, Phys. Rev. (to be published).

TABLE IV. Summary of determinations of  $\alpha$ .

α	Reference
$-0.85 \pm 0.2$	Boldt et al.a
$+0.45 \pm 0.5$	Birge et at.b
$+0.75_{-0.50}^{+0.15}$	Leitner et al.c
$+0.67_{-0.18}^{+0.24}$	Beall et al.d
$+0.62 \pm 0.07$	present experiment

E. Boldt, H. S. Bridge, D. O. Caldwell, and Y. Pal, Phys. Rev. Letters

B. Boldt, H. S. Bridge, D. O. Caldwell, and Y. Pal, Phys. Rev. Letters 1, 256 (1958).
R. W. Birge and W. B. Fowler, Phys. Rev. Letters 5, 254 (1960).
J. Leitner, L. Gray, E. Harth, S. Lichtman, J. Westgard, M. Block, R. Brucker, A. Engler, R. Gessaroli, A. Kovacs, T. Kikuchi, C. Meltzer, H. O. Cohn, W. Bugg, A. Pevsner, P. Schlein, M. Meer, N. T. Grinellini, L. Lendinara, L. Monari, and G. Puppi, Phys. Rev. Letters 7, 264 (1961).
E. F. Beall, Bruce Cork, D. Keefe, P. G. Murphy, and W. A. Wenzel, Phys. Rev. Letters 8, 75 (1962).

The result  $\alpha = +0.62 \pm 0.07$  is an average of the two methods of determination, a direct method using  $\Lambda^0$ particles as an unpolarized source, and a more sensitive method using the knowledge of the  $\Lambda^0$  polarization. The error was not decreased since the two methods of determination are not completely independent. The positive sign for  $\alpha$  means that the protons in  $\Lambda^0$  decay have their spin preferentially oriented in the direction of their motion. This result agrees in sign with all but one of the previous measurements, which are listed in Table IV.

In addition to the determination of the sign of  $\alpha$  we have made a determination of its magnitude. We point out again that the result is quite insensitive to the polarization analyzing power. It is highly improbable that  $\alpha$  can be exactly +1.0, so that any simple ideas concerning a maximum parity violation in which the s- and p-wave amplitudes are equal cannot be correct. The value  $\alpha = +0.62 \pm 0.07$  is not in disagreement with the lower limit for  $\alpha$  obtained in bubble chamber measurements.11

The result  $\beta = +0.18 \pm 0.24$  is in agreement with the value  $\beta = +0.08$  expected on the basis of time reversal invariance. This result is the first experimental check of time reversal for the nonleptonic hyperon decays. The relative phase between the s- and p-wave amplitude was found to be +15°±20°, compared to an expected  $+7^{\circ}$ . The statistical error is such that a large time reversal violation is unlikely.

The value  $\gamma = +0.78 \pm 0.06$  yields a ratio of p- to s-wave amplitude

$$|p|/|s| = 0.36_{-0.06}^{+0.05}$$
.

A similar result with less statistical accuracy has been obtained by Beall et al.12 The predominance of s-wave decay over p-wave decay has long been suspected on the basis of hyperfragment evidence.<sup>13</sup> With the present

<sup>13</sup> See, for example, R. H. Dalitz, Rev. Mod. Phys. 31, 823 (1959).

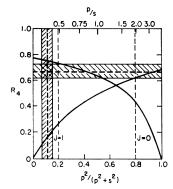


Fig. 16. Figure taken from Dalitz and Liu (reference 15). The graph shows relation |p|/|s| and  $R_4$ , the twobody decay mode ratio of ΛH4, for assumed values of J=0 and J=1 for the spin of  $_{\Lambda}H^4$ . The horizontal shaded strip gives the experimental limits of  $R_4$ . The vertical strip gives the experimental limits for the /|s| ratio determined in this experiment.

result for |p|/|s| the arguments can be turned about and provide information concerning hyperfragments and  $\Lambda^0$ -nucleon forces. The small value for |p|/|s| makes it quite certain that the  $\Lambda^0$ -nucleon force is stronger in the singlet than in the triplet state.

The preponderance of s wave over p wave has a direct bearing on the relative  $K\Lambda N$  parity. The observation by Block et al.14 of the reaction

$$K^- + \text{He}^4 \rightarrow {}_{\Lambda}\text{He}^4 + \pi^-$$
 (22)

proves that the  $K\Lambda N$  parity is odd providing that the spin of  $_{\Lambda}$ He<sup>4</sup> is J=0. Dalitz and Liu<sup>15</sup> have computed the relation between the ratio

$$R_4 = (_{\Lambda}H^4 \rightarrow \pi^- + He^4)/\text{all decay modes}$$
 (23)

and the |p|/|s| ratio for  $\Lambda^0$  decay for J=0 or J=1 for the ground state of AH4 or AHe4. The curve of Dalitz and Liu is plotted in Fig. 16 along with the experimental value  $R_4 = 0.67_{-0.05}^{+0.06}$  of Ammar et al., 16 and our result  $|p|/|s| = 0.36_{-0.06}^{+0.05}$ . It is clear that J = 0 is the spin of the  ${}_{\Lambda}\mathrm{He^4}$ , and that the relative  $K\Lambda N$  parity is odd. 17 The only objection to the conclusion is the possibility of a bound excited state of  $_{\Lambda}$ He<sup>4</sup> with J=1 through which the reaction observed by Block et al. proceeds.

Since there now exists a large amount of experimental data on both  $\Lambda^0$  and  $\Sigma^{\pm}$  decay, it is of interest to consider the relations between  $\Lambda^0$  and  $\Sigma^{\pm}$  decays predicted by various theories. The most successful theories are based either on global symmetry or the doublet approximation and the  $|\Delta T| = \frac{1}{2}$  rule.<sup>18</sup>

no statistical uncertainty that the spin of AHe<sup>4</sup> is J=0.

18 See, for example, S. B. Treiman, Nuovo Cimento 15, 916 (1960); A Pais, Phys. Rev. 122, 317 (1961). Reference to many other authors who have developed similar theories are given in this paper. Also theories have been developed on the basis of an

<sup>&</sup>lt;sup>11</sup> F. S. Crawford, Lawrence Radiation Laboratory, University of California (private communication). The most recent published results give  $|\alpha_A| > 0.66 \pm 0.13$ . F. S. Crawford, M. Cresti, M. L. Good, F. T. Solmitz, and M. L. Stevenson, Phys. Rev. Letters 2,

<sup>11 (1959).

12</sup> E. F. Beall, Bruce Cork, D. Keefe, P. G. Murphy, and W. A. Wenzel, Phys. Rev. Letters 8, 75 (1962).

<sup>&</sup>lt;sup>14</sup> M. M. Block, E. B. Brucker, C. C. Chang, R. Gessaroli, T. Kikuchi, A. Kovacs, C. M. Meltzer, A. Pevsner, P. Schlein, R. Strand, H. O. Cohn, E. M. Harth, J. Leitner, L. Monari, L. Lendinara, and G. Puppi, Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 419.
<sup>15</sup> R. H. Dalitz and L. Liu, Phys. Rev. 116, 1312 (1959).
<sup>16</sup> R. G. Ammar, R. Levi Setti, W. E. Slater, S. Limentani, P. E. Schlein, and P. H. Steinberg, Nuovo Cimento 19, 20 (1961).
<sup>17</sup> If we form a likelihood analysis in a manner similar to Beall et al.. (reference 12) we find the relative probability that the

Beall et al., (reference 12) we find the relative probability that the spin of  $_{\Lambda}$ He<sup>4</sup> is J=1 rather than J=0 is less than  $10^{-10}$ . There is

These theories predict that  $\alpha_{\Lambda} \approx -\alpha_{\Sigma}^{0}$ , where  $\alpha_{\Sigma}^{0}$  is the asymmetry parameter for  $\Sigma^+ \to \pi^0 + p$ . With the result  $\alpha_{\Sigma}^0 = -0.73_{+0.11}^{-0.16}$  of Beall *et al.*<sup>12</sup> the prediction of the theory is fulfilled. (Note that our sign convention is used here. A negative sign means negative proton helicity.) Precise numerical agreement cannot be expected since the theories assume no mass difference between the  $\Lambda^0$  and  $\Sigma^{\pm}$  hyperons. Further, the theories assume the  $|\Delta T| = \frac{1}{2}$  rule for nonleptonic decays which appears to be satisfied experimentally, although there appears to be one discrepancy which should be explored. That is the prediction of the  $|\Delta T| = \frac{1}{2}$  rule that  $|\alpha_{\Sigma}^{0}|$ =  $0.99_{-0.05}^{+0.01}$ , <sup>19</sup> compared with the experimental result of  $-0.73_{+0.11}^{-0.16}$ . Corrections to the theory because of

odd  $\Sigma$ - $\Lambda$  relative parity which predict the same result, i.e.,  $\alpha \Sigma^0 \approx -\alpha_{\Lambda}$ . See Jugoro Iizuka and Reinhard Oehme, Phys. Rev. 126, 787 (1962).

<sup>19</sup> J. W. Cronin, Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 590.

mass differences will tend to reduce the p-wave amplitude relative to the s-wave amplitude in the  $\Lambda^0$  decay because of angular momentum barriers. This effect may produce the low |p|/|s| rato found for the  $\Lambda^0$  decay in this experiment.

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## K\* Spin and the Isovector Kaon Charge\*

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The isovector kaon charge is calculated using an unsubtracted dispersion relation for the kaon form factor. The  $\pi\pi$ - $K\bar{K}$  amplitude appearing in the form factor discontinuity is evaluated by using unitarity and crossing symmetry, and thus is related to the experimental values of the  $\rho$  and  $K^*$  energies and widths. It is shown that the correct order of magnitude for the kaon charge is obtained if the K\* spin is one and not if it is zero.

HERE has been considerable discussion recently on the spin assignment to the  $K^*$  resonance based on the analysis of various experiments. 1-8 The purpose

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Argentina.

<sup>1</sup> M. Alston, G. Kalbsleisch, H. Ticho, and S. Wojcicki have reported on a study of some angular distributions which are conconference on High-Energy Physics at CERN (CERN Scientific Information Service, Geneva, 1962); also Lawrence Radiation Laboratory Report UCRL-10232, 1962 (unpublished)].

 $^2$  R. Armenteros et al. have ruled out spin zero in studying the  $K^*$  production in (pp) annihilation at rest. See R. Armenteros, L. Montanet, D. R. O. Morrison, A. Shapira, S. Nilsson, J. Vandermeulen, Ch. D'Andlau, A. Astier, C. Ghesquiere, B. P. Gregory, D. Rahm, P. Rivet, and F. Solmitz [Proceedings of the 1962 International Conference on High Energy Physics at CERN (CERN Scientific Information Service, Geneva, 1962); also CERN/TC/PHYSICS 62-9 (unpublished)].  $^3$  W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee and T. O'Halloran also ruled out spin zero by studying the process  $K^++p\to K^*+N_{22}^*$  [Phys. Rev. Letters 9, 330 (1962)]. <sup>2</sup> R. Armenteros et al. have ruled out spin zero in studying the

of this note is to show that if the spin of the  $K^*$  is assumed to be one, good agreement is obtained for the isovector charge of the kaon,4 while no such agreement can be obtained if the  $K^*$  spin is zero. Throughout this work the approximation of retaining only the  $\rho$ -meson contribution in the I=1, J=1 channel is performed, and the  $K^*$  is assumed to be the only effective  $\pi K$ resonance.

The isovector kaon form factor satisfies the dispersion relation<sup>5</sup>

$$F_K(t) = \frac{1}{\pi} \int_4^\infty \frac{2q'^3 F_{\pi}^*(t') B_1^{(-)}(t')}{(t')^{1/2} (t'-t)} dt', \tag{1}$$

where 
$$q' = [(t'/4) - 1]^{1/2}$$
,  $B_1^{(-)}(t)$  is the  $I = 1$ ,  $J = 1$ 

<sup>&</sup>lt;sup>4</sup> A qualitative argument in this direction was given by G. Frye [Nuovo Cimento 18, 282 (1960)]. S. K. Bose recently studied the isovector kaon form factor, using a subtracted dispersion relation [Nuovo Cimento 24, 970 (1962) and errata (to be published)]. The method used involves a divergency in the case of a spin-one  $K^*$ , so that a cutoff is needed. No conclusion is drawn on the spin of the  $K^*$ .

F. Ferrari, G. Frye, and M. Pusterla, Phys. Rev. 123, 308

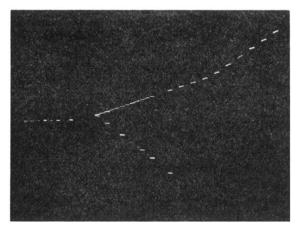


Fig. 3. Photograph of  $\Lambda^0$  decay with scattered proton.

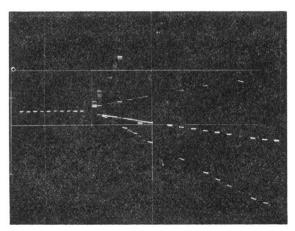


Fig. 4. Photograph of  $\Lambda^0$  and  $K^0$  decay. The lower decay is the  $\Lambda^0$ .

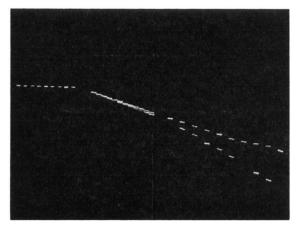


Fig. 5. Photograph of an  $e^+e^-$  pair.