PHYSICAL REVIEW VOLUME 129, NUMBER 4 15 FEBRUARY 1963

Gamma-Ray Spectrum from Thermal-Neutron Capture in Hf^{177} and Associated Energy Levels in Hf^{178}†

R. K. SMITHER
Argonne National Laboratory, Argonne, Illinois

(Received 10 May 1962; revised manuscript received 26 October 1962)

The gamma-ray spectrum resulting from thermal-neutron capture in Hf^{177} was investigated with the Argonne 7.7-m bent-crystal spectrometer. The observed spectrum consisted of 217 gamma rays with energies between 50 keV and 1.6 MeV. These precision measurements of gamma-ray energies and intensities were used to modify and extend the level scheme of Hf^{178}. The proposed level scheme of Hf^{178} contains the following 20 levels (the energy of the level being given in keV followed by the K value, spin, and parity of the level): ground state $[0,0^+]$; 93.17 $[0,2^+]$; 306.59 $[0,4^+]$; 632.10 $[0,6^+]$; 1058.3$[[0,8^+]]$; 1196.27 $[0,0^+]$; 1237.26 $[[0,0^+]]$; 1269.34 $[0,2^+]$; 1322.85 $[0,2^+]$; 1384.48 $[[\ ,4]]$; 1402.25 $[2,2^+]$; 1420.64 $[2,2^+]$; 1431.25 $[0,0^+]$; 1473.13 $[(1,1^+)$ or $(2,2^+)]$; 1473.32 $[0,4^+]$; 1512.92 $[0,4^+]$; 1514.56 $[[\ ,2^+]]$; 1550.02 $[[2,3^+]]$; 1569.78 $[[2,3^+]]$; and 1575.73 $[[0,2^+]]$. The double brackets indicate an uncertainty in the assignments.

I. INTRODUCTION

THIS investigation of the level scheme of Hf^{178} is the first of a series of similar investigations of deformed even-Z even-N nuclei being performed with the Argonne 7.7-m bent-crystal spectrometer.[1] The level schemes of these nuclei exhibit rotational bands built on low-lying intrinsic states as would be expected from the collective model.[2–4]

The ground-state rotational band of a deformed even-Z even-N nucleus is always a $K=0$ positive-parity band. The spin sequence is 0^+, 2^+, 4^+, 6^+, etc., while the energies of levels are given to the first approximation by

$$E=(\hbar^2/2\mathcal{J})I(I+1),$$

where I is the spin of the level and $\mathcal{J}$ is a constant with the mathematical character of a moment of inertia.

Additional rotational bands are present in the level schemes of these deformed even-even nuclei.[5–7] Some of these excited-state bands are believed to be associated with the ground-state band and are referred to as β and γ vibrational bands. In addition, it is possible to have excited-state bands based on configurations other than that of the ground state. The lowest energy β vibrational band is a $K=0$ positive-parity band with a spin sequence 0^+, 2^+, 4^+, 6^+, $\cdots$. The lowest energy γ vibrational band is a $K=2$ positive-parity band with the spin sequence 2^+, 3^+, 4^+, 5^+, $\cdots$. The energies of the individual members of an excited-state band are given to the first approximation by

$$E=E_0+(\hbar^2/2\mathcal{J})[I(I+1)-(I_0+1)],$$

where E_0 us the energy and I_0 is the spin of the lowest level in the band.

If the nuclear deformation associated with the excited-state band is similar to that associated with the ground-state rotational band, then the level spacings of the excited-state band and the ground-state band will be similar as well. In general, therefore, it may be difficult to tell the difference between a β or γ vibrational band built on the ground-state configuration and a rotational band based on a configuration which is different from that of the ground state. An alternative interpretation of some of the excited-state bands may be found in the axially-asymmetric-rotor model.[8,9]

The object of the work described in this paper is to find and study excited-state rotational bands in the Hf^{178} nucleus.

The isotope Hf^{178} was selected for investigation because the first five members of the ground-state rotational band (0^+, 2^+, 4^+, 6^+, 8^+) are believed to have been identified,[10,11] through the study of the β decay of Ta^{178}.

In the work of Felber, Stephens, and Asaro,[10] a precision β-ray spectrometer was used to measure the energies of the internal-conversion lines associated with the gamma transitions between the members of the ground-state rotational band in Hf^{178}. The precision of these energy measurements appeared to be sufficient to allow one to identify the corresponding gamma rays in the spectrum obtained with the bent-crystal spectrometer.

---

† Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] B. Hamermesh, D. Rose, and H. Ostrander, Rev. Sci. Instr. **28**, 233 (1957).

[2] For a compilation of some of the experimental data available, see G. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 (1956); A. Bohr and B. Mottelson, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), Chap. 17; A. K. Kerman, in *Nuclear Reactions*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Company, Amsterdam 1959), Vol. I, Chap. 10.

[3] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **26**, No. 14 (1952).

[4] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **27**, No. 16 (1953).

[5] B. Harmatz, T. H. Handley, and J. W. Mihelich, Phys. Rev. **123**, 1758 (1961).

[6] C. J. Gallagher, H. L. Nielsen, and O. B. Nielsen, Phys. Rev. **122**, 1590 (1961).

[7] For a compilation of some of the excited-state data, see C. A. Mallman, Nucl. Phys. **25**, 266 (1961).

[8] A. S. Davydov and G. F. Filippov, Nucl. Phys. **8**, 237 (1958); A. S. Davydov and V. S. Rostrosky, *ibid.* **12**, 58 (1959).

[9] C. A. Mallman, Phys. Rev. Letters **2**, 507 (1959); C. A. Mallman and A. K. Kerman, Nucl. Phys. **16**, 105 (1960); C. A. Mallman, *ibid.* **25**, 266 (1961).

[10] F. F. Felber, Jr., Master's thesis, University of California Radiation Laboratory Report UCRL-3618, January, 1956 (unpublished); F. F. Felber, Jr., F. S. Stephens, Jr., and F. Asaro, J. Inorg. Nucl. Chem. **7**, 153 (1958).

[11] M. Deutsch and R. W. Bauer, Nucl. Phys. **21**, 128 (1960).

This identification of gamma rays associated with the ground-state rotational band would allow one to define the energies of the levels in the ground-state band with increased precision. In addition, the intensities of these gammas could be used to predict to what extent the multiple-gamma cascades proceed through each of the levels in the ground-state band. With this information on energies and intensities, the ground-state band could then be used as a basis for the development of the rest of the level scheme.

The production of excited Hf^{178} through thermal-neutron capture in Hf^{177} leads to an excited state with a spin of 3^- or 4^-. The three- and four-step gamma cascades that originate with these capture states can populate levels with a wide range of spins (0 to 8). Without this effect, the high-spin members of a rotational band would be populated very weakly and most probably would not be observable.

The previous neutron-capture gamma-ray work[12–14] indicated that an appreciable fraction of the multiple-gamma cascades passed through the 1–2-MeV region of the level scheme of Hf^{178}. This suggested that the low-energy gamma rays associated with this region of the level scheme might have appreciable intensities and therefore might be detectable with the bent-crystal spectrometer. The recent β-decay experiments of Gallagher, Nielsen, and Nielsen[6] indicate that there are at least five excited states in Hf^{178} between 1.1 and 1.5 MeV. Although more levels are found in this region with the bent-crystal spectrometer, good agreement exists between the above-mentioned β-decay work and the capture γ-ray work reported in this paper. This agreement between the two experiments serves as an important external check on the method used to develop decay schemes and gives more confidence in the proposed level scheme.

II. EXPERIMENTAL METHOD

The gamma-ray spectrum resulting from thermal-neutron capture in a sample of natural Hf and a sample enriched to 94% Hf^{178} were investigated with the Argonne 7.7-m bent-crystal spectrometer.[1] The γ rays associated with neutron capture in Hf^{177} (depopulating excited levels in the Hf^{178}) were identified from the relative intensities of the gamma rays as observed with the natural and enriched samples. The observed spectrum associated with $Hf^{177}(n,\gamma)Hf^{178}$ consisted of 217 gamma rays between the energies of 40 keV and 1.6 MeV. A list of the energies and intensities of these gamma rays appears in Table I. The quoted errors are a combination of the statistical error derived from the rms deviation of the data (8–17 runs) and an estimated systematic error. The relative values of the energies should be somewhat better than indicated by the quoted errors because of the cancellation of systematic errors. Many of the large errors assigned to the gamma rays above 1 MeV are due to the complicated nature of the spectrum rather than to the limitations in the determination of the Bragg angle. The doublet and triplet structures indicated in the "remarks" column should be interpreted to mean that a change in the energy and/or intensity of one member of the group will change the energies and intensities of the others. Although the stated energy values are believed to correspond to the best fit to the data, the statistical uncertainty in the experiments allows some variation in the interpretation of these doublet and triplet structures. This is reflected in the larger errors assigned to their energies.

There is some evidence of a continuum of weak (<0.5%) gamma rays between 1.5 and 2.0 MeV. Some unresolved structure occurs at 1665 keV. The limited resolution and poorer statistics of the 1600-keV region make this region diffcult to interpret.

The average precision of the measurements of the gamma-ray energies was one part in 2500 at 1 MeV and one part in 10 000 at 100 keV. The corresponding errors in the gamma-ray energies are 0.4 keV and 0.01 keV, respectively. The fractional line width $\Delta E/E$ (where ΔE = full width at half-maximum) of the spectrometer, also varies with the energy and is given by the relation $\Delta E/E = 0.01\ E$, where E is the gamma-ray energy in MeV. The line width during this experiment was, therefore, 10 keV at 1 MeV and 0.1 keV at 100 keV.

The formula used to convert the wavelength of the gamma rays into energy is

$$\text{Energy} = 12\,372.4\ \text{keV-xu/wavelength}.$$

The wavelength of the gamma ray was obtained from

$$\text{wavelength} = 2d \sin\theta,$$

where $\sin\theta$ (the measured quantity in this experiment) is the sine of the Bragg angle for first-order diffraction and d is the spacing between the (310) planes of quartz. The value used for d in this experiment was 1177.70 xu (Siegbahn scale, 22°C).

The intensities of the gamma rays (Table I) are given in photons per 100 neutron captures in Hf^{177}. The average absolute error in these intensities is believed to be between 10 and 20%, with the larger errors at the extremes of the spectrum. The relative intensities are believed to be good to 5% for the strong lines and to 10–20% for the weak ones. Some additional uncertainty arises in the resolution of a close doublet or triplet.

In order to obtain an absolute value (photons per 100 neutron captures) for the intensities of the observed gamma rays, it is necessary to know the number of neutrons captured per second by the Hf^{177} in the sample. The number of neutron captures per second in the sample is often difficult to obtain in experiments of this

---

[12] L. V. Groshev, A. M. Demidov, V. N. Lutsenko, and V. I. Pelekhov, Atomnaya Energiya (USSR) 4, 5 (1958); J. Nucl. Energy 9, 50 (1959).

[13] P. J. Campion and G. A. Bartholomew, Can. J. Phys. 35, 1361 (1957).

[14] C. A. Fenstermacher, J. E. Draper, and C. K. Bockelman, Nucl. Phys. 14, 693 (1960).

TABLE I. A list of the energies, errors, and intensities of the thermal-neutron capture-gamma-ray spectrum resulting from $Hf^{177}(n,\gamma)Hf^{178}$ as measured with the Argonne 7.7-m bent-crystal spectrometer. The energies and errors (keV) are given in columns 1 and 2 while the intensities (in photons per 100 neutron captures in Hf^{177}) appear in column 3. The errors are meant to be probable errors and to reflect both the statistical and systematic errors of the measurement. The probable error in the intensities is 10–20%, with the latter prevailing at the extremes of the spectrum. The gamma-ray intensities have not been corrected for internal conversion. Column 4 indicates if the gamma ray appears in the final scheme of Hf^{178} (Fig. 5).

(1) Energy (keV)	(2) Error (keV)	(3) γ's/100 neutron captures in Hf^{177}	(4) In scheme (Fig. 5)	(5) Remarks
1552.1	±2.8	0.40		
1500.4	±2.7	0.76		Doublet structure
1473.9	±2.7	0.32	Yes	
1420.9	±2.0	2.14	Yes	Doublet structure with possible line at ≈1419 keV (Intensity ≈0.8)
1402.9	±2.6	0.81	Yes	
1381.8	±3.0	0.27	Yes	
1350.5	±3.0	0.44		
1338.2	±2.0	4.65	Yes	Doublet structure with possible line at ≈1322 keV (Intensity ≈0.8)
1327.5	±2.2	2.05	Yes	
1310.3	±2.4	1.24	Yes	
1291.9	±2.3	0.57	Yes	
1269.5	±1.6	1.80	Yes	
1228.8	±1.4	7.30	Yes	Possible symmetric doublet (separation ≈1−2 keV)
1206.4	±0.6	5.98	Yes	
1176.3	±1.2	9.08	Yes	Doublet structure
1167.4	±1.2	4.71	Yes	
1143.7	±0.9	1.82	Yes	
1125.4	±1.2	0.19	?	
1103.1	±0.5	3.56	Yes	
1081.8	±1.0	2.80		Doublet structure
1077.8	±1.0	2.90	Yes	
1057.7	±1.2	0.24		
1032.3	±2.0	0.27		
1016.5	±0.7	0.56	Yes	
1006.9	±1.0	0.38		← Troublesome impurity line at 1003.5 keV
983.5	±1.0	0.27		
969.5	±0.8	0.40		Doublet structure
962.1	±0.7	1.09	Yes	
949.09[a]	±1.6	0.24		
937.97[a]	±1.6	0.21		
921.26	±0.4	1.09		
901.30	±0.4	0.36		
881.27	±1.2	0.15	Yes	
870.11[b]	±1.4	0.11		Overlapping structure
861.84	±1.2	0.28		
855.87[b]	±1.5	0.15		
847.71	±1.5	0.12		
841.11	±0.9	0.17	Yes	
831.65[a]	±1.2	0.08		
822.05	±1.2	0.09		Doublet structure
817.50	±1.2	0.11		
792.54	±1.0	0.10		
780.58	±1.0	0.10		Doublet structure
775.68	±1.0	0.22		
763.11	±0.9	0.13		Triplet structure
758.01	±0.9	0.31		
752.35	±0.4	0.36	Yes	
744.15	±0.6	0.19		
736.90	±0.4	0.19		
716.33	±0.4	0.21		
704.24	±0.4	0.14		
698.17	±0.6	0.11		Doublet structure
694.82	±1.2	0.07		
687.44	±0.6	0.10		
682.37	±0.6	0.16		
672.35	±0.8	0.08		Doublet structure
668.69	±0.5	0.10		
662.40	±0.5	0.10		Doublet structure with troublesome impurity at 654.8 keV
658.29	±0.6	0.14		
649.43	±0.4	0.13		Doublet structure
645.37	±0.2	0.33		
632.93[b]	±0.3	0.13		
628.32	±0.3	0.23		
617.81	±0.3	0.23		
611.10	±0.4	0.11		
606.84	±0.3	0.39		
597.64	±0.3	0.41		Doublet structure
596.33	±0.3	0.31		
590.33	±0.4	0.09		Doublet structure
587.20	±0.4	0.14		
583.07	±0.4	0.12		Doublet structure
580.45	±0.4	0.08		
572.23	±0.3	0.11		
567.78	±0.5	0.03		
563.08[a]	±0.5	0.017		
559.38	±0.5	0.035		
550.10[a]	±0.5	0.016		
547.36	±0.2	0.51		
542.99	±0.3	0.12		
534.66	±0.2	0.21		
522.08	±0.1	0.36		← Annihilation radiation at 510.94 keV; troublesome impurity at 507.87 keV
503.90	±0.2	0.10		
497.62	±0.1	1.46		
494.22	±0.1	0.17		
488.70	±0.3	0.09		← Troublesome impurity at 486.2 keV
483.92	±0.3	0.046		
481.40	±0.2	0.09		
478.31	±0.3	0.047		Doublet structure
477.31	±0.3	0.039		
472.91	±0.1	0.23		
466.83	±0.4	0.28		
459.91	±0.1	0.12		
453.99	±0.1	0.16		
449.79	±0.2	0.042		
438.84	±0.2	0.07		
435.57	±0.14	0.21		
430.81	±0.30	0.19		
427.71	±0.20	0.07		Triplet structure ground-state band ($8^+ \rightarrow 6^+$)
426.21	±0.14	0.58	Yes	
424.39	±0.17	0.23		
420.87	±0.20	0.08		← Possible doublet
417.00	±0.30	0.021		
406.70	±0.20	0.024		← Troublesome impurity at 408.07 keV
403.64	±0.09	0.16		
399.84	±0.09	0.17		
397.48	±0.20	0.025		
395.65[b]	±0.14	0.040		← Possible doublet
390.90	±0.20	0.015		
388.04	±0.20	0.015		
383.26	±0.07	0.26		
379.44	±0.07	0.11	Yes	
376.45	±0.10	0.016		
374.62	±0.10	0.027		
371.74	±0.10	0.012		
364.33	±0.14	0.011		Doublet structure
363.24	±0.15	0.06		

---

[a] Questionable line (poor statistics or complicated structure).

[b] Questionable isotopic assignment (usually due to the low intensity of the gamma when observed with the natural abundance sample.

TABLE I (*continued*).

(1) Energy (keV)	(2) Error (keV)	(3) γ's/100 neutron captures in Hf^{177}	(4) In scheme (Fig. 5)	(5) Remarks
356.83	±0.05	0.38		
354.59	±0.10	0.025		
350.98	±0.10	0.046		
348.26	±0.05	0.50		
345.43	±0.15	0.040		
341.98[b]	±0.05	0.020		
339.08	±0.04	1.14		
337.51	±0.05	0.05		
331.60	±0.06	0.049		
328.59	±0.08	0.042		
325.51	±0.04	6.82	Yes	Ground-state band ($6^+ \rightarrow 4^+$)
321.46	±0.08	0.015		
318.89	±0.08	0.017	Yes	Triplet structure
318.29	±0.08	0.017		
317.28	±0.03	0.12		
313.38	±0.05	0.15		Doublet structure with possible impurity at 312.5 keV
312.42	±0.05	0.026		
311.16	±0.07	0.042		
308.38	±0.10	0.009		
306.37	±0.04	0.10	Yes	
296.72	±0.03	0.06		
289.47	±0.03	0.60		
280.63	±0.03	0.10	Yes	
279.31	±0.06	0.036		Doublet structure
278.58	±0.06	0.021		
277.28	±0.06	0.22	Yes	Doublet structure
276.88	±0.06	0.22	Yes	
274.69	±0.04	0.047		
273.09	±0.03	0.25		
270.52	±0.08	0.06		Doublet structure with troublesome impurity at 269.85 keV
270.06	±0.08	0.027		
268.69	±0.04	0.012		
264.59	±0.02	0.025		
259.99	±0.02	0.05		
256.56	±0.05	0.34		Doublet structure
256.01	±0.05	0.28		
249.37	±0.03	0.07		
247.54	±0.04	0.015		
246.88	±0.04	0.024	Yes	
245.23	±0.02	0.51	Yes	
244.23	±0.02	0.52		
242.55	±0.03	0.029		
241.95	±0.04	0.08		
234.49	±0.03	0.10		
230.74[a]	±0.07	0.010		
230.22	±0.04	0.045		
229.31	±0.02	0.15		
227.20	±0.02	0.19	Yes	
224.37[b]	±0.06	0.006	Yes	
223.96	±0.06	0.027		
223.32	±0.14	0.006		
218.93	±0.02	0.09		
217.76	±0.03	0.044		
216.63	±0.09	0.014		Doublet structure
216.29	±0.02	0.15		
213.42	±0.02	41.30	Yes	Ground-state band ($4^+ \rightarrow 2^+$)
208.26	±0.03	0.012		
204.05	±0.04	0.13	Yes	Doublet structure
203.75	±0.04	0.20	Yes	
201.04[b]	±0.03	0.009		
190.07	±0.01	0.083	Yes	
187.59	±0.01	0.071		
185.31[b]	±0.06	0.007	Yes	
180.39	±0.02	0.021		
176.19	±0.02	0.051		
173.63	±0.03	0.014		Doublet structure
173.31	±0.03	0.038		
169.47	±0.02	0.031		
166.63	±0.01	0.014		
161.92	±0.02	0.037	Yes	
157.86	±0.02	0.023		
151.29	±0.01	0.063	Yes	
149.14	±0.01	0.10	Yes	
148.07	±0.03	0.042		
147.78	±0.01	0.16	Yes	
144.49	±0.01	0.27	Yes	
140.86	±0.01	0.19		
137.67	±0.01	0.05		
129.33	±0.01	0.04	?	
128.09	±0.03	0.018		
126.58	±0.01	0.25	Yes	
125.84	±0.04	0.01		← Troublesome impurity at 125.94 keV
124.09	±0.01	0.022		
122.85	±0.01	0.39		
116.58	±0.01	0.082		
112.63	±0.01	0.043		
111.66	±0.01	0.092		
107.31	±0.01	0.14		
105.14	±0.01	0.12		
102.58	±0.01	0.017	Yes	
100.94[b]	±0.02	0.023		
97.91	±0.01	0.066		
93.17	±0.005	18.09	Yes	Ground-state band ($2^+ \rightarrow 0^+$)
88.84	±0.01	0.35	Yes	
86.97	±0.02	0.023		
85.59	±0.01	0.43	Yes	
82.70	±0.01	0.40		
73.07	±0.01	0.077	Yes	
71.05	±0.01	0.030	Yes	
53.90[b]	±0.01	0.012		
52.74[b]	±0.01	0.018		

type. The calculation of this number requires knowledge of both the absolute value of the neutron flux at the sample position and the energy spectrum of the neutron flux. One is also required to know the absolute value and energy dependence of the neutron-capture cross section. Both the cross section and flux measurements usually have errors of 10% or more and this sets a lower limit of 15 to 20% on the accuracy of the intensity measurements. In this experiment, the number of neutron captures per second in the sample was obtained in a manner that did not need the absolute value of the neutron-capture cross section in Hf^{177} or the absolute value of the neutron flux. The number of neutron captures per second in Hf^{177} was obtained instead from the relative increase in the intensities of the gamma rays associated with $\mathrm{Hf}^{178}(n,\gamma)\mathrm{Hf}^{179}$ as a function of integrated neutron flux. The change in the intensities of these gamma rays reflects the change in the number of Hf^{178} atoms present in the sample and thus the number of neutrons captured in the Hf^{177} isotope which resulted in the formation of Hf^{178} nuclei. It was possible to obtain a large percentage change (250%) in the number of Hf^{178} atoms for a rela-

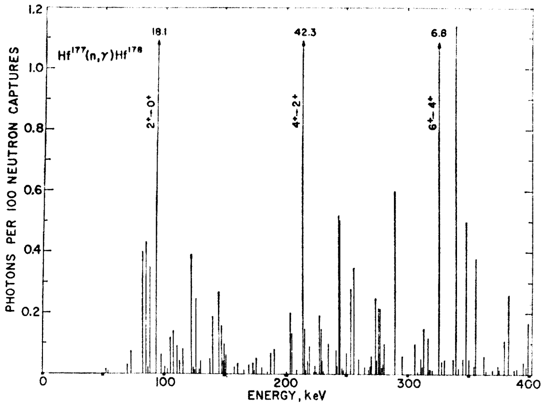

FIG. 1. Line graph of the $\mathrm{Hf}^{177}(n,\gamma)\mathrm{Hf}^{178}$ gamma-ray spectrum below 400 keV, observed with the Argonne 7.7-m bent-crystal spectrometer. The intensities of the gamma rays are given in photons per 100 neutron captures in Hf^{177}.

tively small change (≈8%) in the number of Hf^{177} atoms by using a sample which was highly enriched in Hf^{177}. The percentage change in the number of Hf^{178} atoms multiplied by the initial number of Hf^{178} atoms is equal to the number of neutron captures in Hf^{177} minus a small correction (≈3%) for the loss of Hf^{178} through $\mathrm{Hf}^{178}(n,\gamma)\mathrm{Hf}^{179}$. The change in the number of Hf^{178} atoms is then used to obtain the number of neutron captures per second in Hf^{177}. This number is, of course, a function of the operating power level of the reactor. A check on this method can be made by dividing the capture rate by the product of the estimated neutron flux at the sample position[15] times the number of Hf^{177} atoms to obtain an effective thermal-neutron cross section. The estimate of the thermal neutron flux comes from the experiments[16,17] with Cd and Sm in which absolute values can be obtained for both the effective cross section of the samples and the neutron flux. The value of (350±20) b obtained by this method for the effective thermal-neutron-capture cross section of Hf^{177} agrees quite well with the published value[18] of (380±30) b.

The observed gamma-ray spectrum is shown in the form of a line graph in Figs. 1, 2, and 3. The intensities of the γ rays shown in these figures are those listed in Table I. They have been corrected for the self-absorption in the sample, the absorption of the material (He, air, Al, Li, paraffin, quartz, ⋯) between the sample and detector, the reflectivity of the quartz crystal, the transmission of the collimator, and the efficiency of the NaI detector. All of these quantities except the self-absorption correction were determined by experiment. No correction was made for the internal conversion of the gamma rays. The observed relative intensities of the gamma rays originating from the natural-abundance sample (Hf metal) and of the gamma rays originating from the enriched sample ($\mathrm{HfO_2}$) form a good check on the self-absorption calculations. The predicted intensity ratios (enriched/unenriched) agreed with the observed ratios to within 10% throughout the energy range.

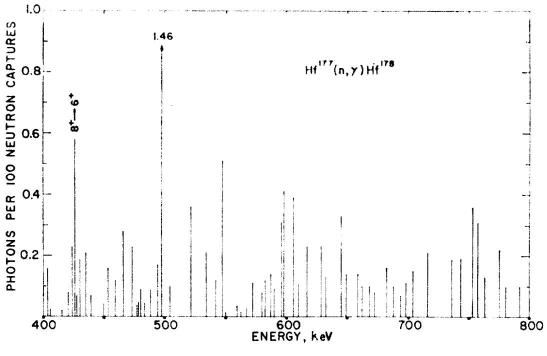

FIG. 2. Line graph of the $\mathrm{Hf}^{177}(n,\gamma)\mathrm{Hf}^{178}$ gamma-ray spectrum between 400 and 800 keV. The intensities of the gamma rays are given in photons per 100 neutron captures in Hf^{177}.

A number of interesting features of the spectrum are visible in the line graphs. The three gamma-ray transitions between the lowest four members of the ground-state rotational band (0+, 2+, 4+, 6+) are evident as strong lines at 93, 213, and 325 keV in Fig. 1. The strength[18] of the 213-keV γ-ray transition (41%)[19] from the 4+ to the 2+ member of the ground-state band reflects the previously mentioned expectation that high-spin states are well populated in the decay scheme. A second prominent group of strong transitions occurs between 1.0 and 1.5 MeV in Fig. 3. In the proposed level scheme, the strong gamma rays forming this group appear as transitions from levels of excited-state rotational bands to the levels of the ground-state band. The relative intensities of these 1–2-MeV gamma transitions are of considerable interest in that they are used to determine the *K* values of many of the excited states.

A third group of gamma rays stands out in the line

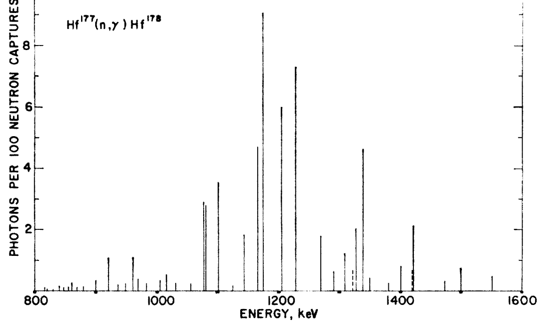

FIG. 3. Line graph of the $\mathrm{Hf}^{177}(n,\gamma)\mathrm{Hf}^{178}$ gamma-ray spectrum between 800 and 1600 keV. The intensities of the gamma rays are given in photons per 100 neutron captures in Hf^{177}.

---

[15] The sample used to create a gamma-ray source for the spectrometer was located at the center of a "through" beam hole tangent to the core of the Argonne research reactor, CP-5. The thermal-neutron flux at this position is approximately 3.7×10^{13} neutrons/cm^2/sec.

[16] R. K. Smither, Phys. Rev. **124**, 183 (1961).

[17] R. K. Smither, Bull. Am. Phys. Soc. **7**, 316 (1962).

[18] H. Pomerance, Phys. Rev. **88**, 412 (1952).

[19] The gamma intensities given in percent are equivalent to intensities given in photons per 100 neutron captures in Hf^{177}.

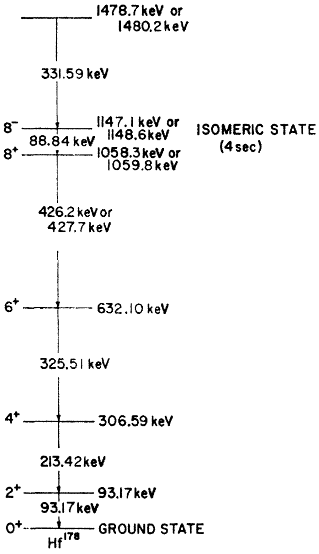

FIG. 4. Level scheme of He^{178} proposed by Felber (reference 10), modified to agree with the energy values of the cascade gammas as measured with the Argonne 7.7-m bent-crystal spectrometer.

graph (Fig. 1) between 70 and 140 keV. Their proximity to the 93.17-keV gamma ray suggests that they may be the transitions between the lower members of the excited-state rotational bands. Many of these low-energy gammas are used in the proposed level scheme as transitions between the upper levels (1–2 MeV). The relative intensities or branching ratios of these low-energy gammas furnish the clues that allow one to sort out the individual levels into separate rotational bands despite the fact that many of the levels have similar spins, parities, and K values. If these low-energy gamma rays are $M1$ or $E2$ transitions, then their total intensities (γ+conversion electron) are four or five times the γ intensity shown. They then become 1–2% transitions, which are comparable to the intensities observed for the γ transitions between 1.0 and 1.5 MeV (Fig. 3).

III. COMPARISON WITH β-DECAY EXPERIMENTS

Two isomeric states in Ta^{178} have been observed to decay to excited states in Hf^{178}. The decay of the 9.3-min isomer[6] appears to populate only low-spin states (0, 1, and 2) in Hf^{178}. The 2.1-h isomer, however, appears to populate a high-spin negative-parity state (8^-)[10,11] in Hf^{178} at 1147 keV. This negative-parity state is believed to decay to the 8^+ member of the ground-state rotational band at 1058 keV.[10,11] In the proposed scheme of Felber *et al.*,[10] this 8^+ state decays to a 6^+ level at 632 keV which in turn decays to a 4^+ level at 306 keV, etc. In this manner, the levels of the ground-state rotational band are identified up to the 8^+ level. If this set of levels could be observed as part of the gamma cascade following neutron capture in Hf^{177}, it would be a considerable help in identifying other states in the scheme.

In an attempt to do this, the conversion-electron measurements were compared with the capture-gamma-ray spectrum observed with the Argonne bent-crystal spectrometer. A comparative list of the energies of the gamma transitions appears in Table II. The energies of the gamma transitions as obtained from the conversion-electron work of Felber, Stephens, and Asaro[10] are found to agree quite well with the capture-gamma-ray energies obtained from the bent-crystal data. In all but one case it is possible to make a relatively unambiguous association between the two experiments. In the case of the proposed $8^+ \to 6^+$ transition of ≈ 427 keV (gamma energy), two choices are possible in the capture-gamma-ray data.

The level scheme proposed by Felber[10] is a single-chain cascade in which all of the decay passes through a 4-sec isomeric state in Hf^{178}. This scheme appears in Fig. 4. The energies in Fig. 4 are those obtained with the Argonne bent-crystal spectrometer. The assumption that the cascade is a single chain with no major branches puts a restriction on the intensities of gamma-ray transitions, namely, that the intensity of each succeeding member of the chain must be equal to or greater than that of the member just above it. The "greater than" is possible in the capture-gamma-ray work since the individual levels can be fed from levels other than those shown in Fig. 4. Care must be taken to include both the gamma-ray intensity and the conversion-electron intensity in these intensity comparisons. The only place in Fig. 4 where this intensity rule is in conflict with the intensities of the assigned gamma rays is at the 1058-keV level. Here the total intensity (gamma and conversion electron) of the 88.84-keV transition is 10 times as large as that of the 427.7-keV transition. The previous statement assumes that the 88.84-keV gamma is $E1$ and the 427.7-keV gamma is $E2$.[10,11] If the 88.84-keV gamma is assumed to be any other multipole, the ratio is much larger and thus an even larger discrepancy occurs. If one uses the alternative choice, the 426.2-keV gamma, for the $8^+ \to 6^+$ transition, then

TABLE II. A comparative list of the transition energies obtained by Felber[a] and the pertinent capture-gamma-ray energies and intensities as measured with the Argonne 7.7-m bent-crystal spectrometer. The intensities quoted in column 3 are in photons per 100 neutron captures in Hf^{177} and are the intensities observed with the bent-crystal spectrometer when the reactor is operating. The total intensities quoted in column 4 are equal to the gamma intensity from column 3 plus the estimated conversion-electron intensity (reference 22) for the K and L shells. Column 5 contains the assumed multipolarity of the radiation. Column 8 is the ratio of the transition energy obtained from the conversion electron work (column 6) to the γ energy (column 1). This is a check for systematic differences.

Argonne bent-crystal data					Felber *et al.*[a]		
Energy (keV)	Error (keV)	Intensity (%) γ	total	Assumed multi-polarity	Energy (keV)	Error (keV)	Ratio of energies (Felber/Smither)
331.60	±0.06	0.049	0.051	$E2$	331.9	±0.33	1.0009
88.84	±0.01	0.35	0.51	$E1$	88.81	±0.08	0.9997
427.71	±0.20	0.07	0.07	$E2$	427.0	±0.43	0.9983
426.21	±0.14	0.58	0.59	$E2$			1.0019
325.51	±0.04	6.82	7.12	$E2$	325.8	±0.33	1.0009
213.42	±0.02	41.3	50.2	$E2$	213.70	±0.21	1.0013
93.17	±0.01	17.9	89.5	$E2$	93.17	±0.09	1.0000

[a] Reference 10.

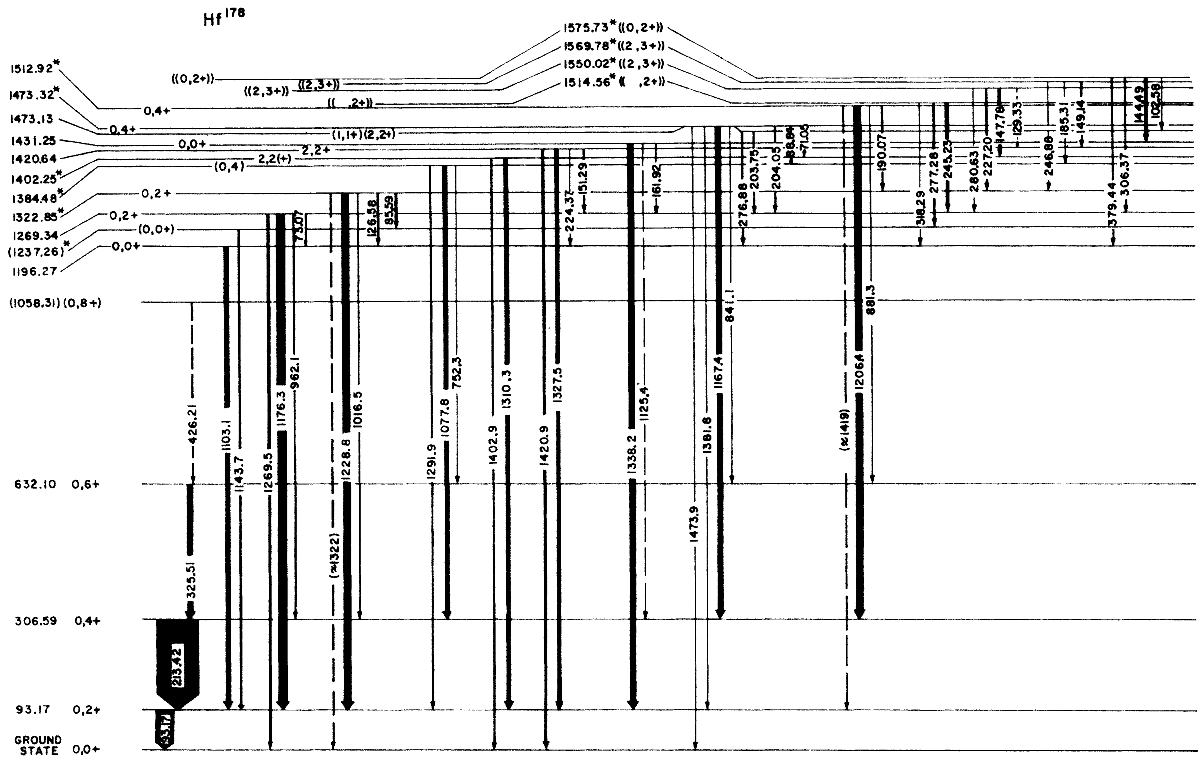

FIG. 5. Level scheme of Hf^{178} as deduced from the gamma-ray measurements made with the Argonne 7.7-m bent-crystal spectrometer. The energies of the levels and gamma rays are given in keV. The K-value, spin, and parity of the level is given on the left in that order. The width of a line in the level scheme is meant to reflect the relative intensity of the gamma ray. The width scale used for the low-energy gammas between the upper states (energies between 1195 and 1575 keV) is four times that used for the gamma rays with energies between 1 and 2 MeV and for the ground-state band. No corrections have been made for internal conversion. An asterisk following the energy of a level indicates that this level has not been observed previously. The parentheses and double parentheses indicate uncertainty in the assignments.

the intensity balance is proper but the energies measured in the present experiment are in poorer agreement with those of Felber *et al.* The energy difference between the (426.21±0.14)-keV gamma ray and the (427.0 ±0.4)-keV transition energy as obtained from the conversion-electron work[10] is (0.8±0.4) keV. This energy difference is twice as large as the probable error and, therefore, less satisfactory than the energy difference of (0.7±0.5) keV obtained with the (427.71±0.20)-keV gamma.

A possible solution to the intensity mismatch of the 88.84- and the 427.7-keV gamma transitions would be to assume that two unresolved transitions are present in the gamma-ray spectrum of Hf^{178} at 88.84-keV. The line shape of the 88.84-keV as observed with the Argonne bent-crystal spectrometer shows no sign of a doublet structure; but if the second gamma were only a tenth as intense as the stronger, the two gamma-ray energies would have to be separated by 0.06 keV for the weaker line to be observable.

If some fraction of the 88.84-keV gamma is a transition between two levels other than the 8^- and 8^+ assumed in Fig. 4, then it is possible that it will be observed somewhere else in the decay scheme. A level spacing of 88.84 keV appears in the final decay scheme between two levels (1384.48 and 1473.32 keV) with the same spin (4). The 88.84-keV gamma or at least part of its intensity is tentatively placed in this position in the level scheme.

Because of the conflict in the gamma-ray assignment for the $8^+ \rightarrow 6^+$ transition, only the first four members of the ground-state rotational band (0^+, 2^+, 4^+, and 6^+) were used in the construction of the level scheme from the gamma-ray measurements. The 426.2-keV gamma does appear in the final level scheme (Fig. 5), however, as a possible candidate for the $8^+ \rightarrow 6^+$ transition. This assignment, which should be considered as only tentative, would at least allow the full intensity of the 88.84-keV gamma to be placed above the 8^+ level. If the full intensity of the 88.84-keV gamma is placed above the 426.2-keV gamma as a transition from an 8^- level to an 8^+ level (see Fig. 4), then additional problems arise. One must now explain why the sum of the intensities of the transitions which end at the 8^- level is ten times as large as the sum of the intensities of the transitions which proceed to the nearby 8^+ level. Because of this unusual situation and the uncertainty in the $8^+ \rightarrow 6^+$ assignment, the author hesitates to add the 88.84-keV gamma to the final level scheme as a transition to the 8^+ level at 1058 keV.

In the recent conversion-electron studies of Gallagher, Nielsen, and Nielsen,[6] five levels were found in the Hf^{178} level scheme with excitation energies (in keV) of 1197±12 (0^+), 1277±10 ((2^+)), 1430±14 ((1^+)), 1440±14 (0^+), and 1483±12 (2^+). (The double parentheses indicate an uncertainty in the assignment.) These levels were excited through the electron-capture decay of the 9.3-min, spin $=1^+$ isomeric state of Ta^{178}. These authors obtained much information about the level scheme of Hf^{178} through a series of coincidence experiments (γ-conversion electron) with a high-transmission magnetic spectrometer with good resolution. This conversion-electron work has proved to be a considerable help in unraveling the neutron-capture gamma-ray spectrum observed with the bent-crystal spectrometer and in the subsequent construction of an extended level scheme of Hf^{178}. In particular, the spin assignments of two 0^+ levels come directly from this conversion-electron work. Further, the fact that the rest of the level scheme agrees very well with the conversion-electron work gives one added confidence in the method used in this paper to generate the level scheme of Hf^{178}. In Table III the gamma rays observed with the Argonne bent-crystal spectrometer are compared with the transitions observed following the decay[6] of the 1^+ isomer of Ta^{178}. It is important to note the systematic energy difference (an average difference of about 8 keV) between the two sets of data if confusion is not to result in the later discussions.

TABLE III. A comparative list of the transition energies observed following the decay of the 1^+ isomeric state[a] of Ta^{178} and the gamma-ray energies from the bent-crystal data which appear to be associated with the same transitions. The last column is the difference between the energies obtained by Gallagher, Nielsen, and Nielsen[6] and those obtained with the bent-crystal spectrometer. Above 900 keV, the average energy difference is 7 keV.

Argonne bent-crystal data			Gallagher, Nielsen, and Nielsen[a]			Comparison
Energy (keV)	Error (keV)	Intensity (γ in %)	Energy (keV)	Error (keV)	Intensity (K conversion)	$E_{GNN}-E_S$ (keV)
93.17	±0.01	17.9	93	±2	11	0
203.75	±0.04	0.20	203	±4	0.022	−1
213.42	±0.02	41.3	214	±4	0.089	1
962.1	±0.7	1.09	970	±10	0.0012	8
1103.1	±0.5	3.56	1105	±11	0.0085	2
(1167.4)[b]	±1.2	(4.71)	1176	±12	0.0087	9
1176.3	±1.2	9.08	1180	±12		4
1269.5	±1.6	1.80	(1260)[c]	±25	⩽0.0015	(−9)
1327.5	±2.2	2.1	1335	±13	0.012	8
1338.2	±2.0	4.6	1345	±13	0.011	7
1381.8	±3.0	0.3	1390	±14	⩽0.018	8
1420.9	±2.0	2.1	1430	±14	0.013	9
1473.9	±2.7	0.3	1483	±15	⩽0.004	9

[a] Reference 6.
[b] May not be the same transition.
[c] Energy obtained from γ ray rather than conversion electron.

IV. NEW LEVEL SCHEME

A. Ground-State Band

The precision measurements of the energies and intensities of the capture gamma rays, performed with bent-crystal spectrometer, were used to modify and extend the level scheme of Hf^{178}. This new level scheme appears in Fig. 5. In developing a decay scheme of this type, it is very convenient to start with a set of established levels. In this case, the first four levels of the ground-state rotational band (0^+, 2^+, 4^+, 6^+) were used. The positions of these levels and a check on their spin assignments were obtained in the following way.

A comparison of the capture-gamma data (Argonne bent-crystal spectrometer) with the Coulomb-excitation work of Chupp, DuMond, Gordon, Jopson, and Mark[20] leaves no doubt that the strong gamma observed[21] at 93.17±0.005 keV is indeed the transition from the first 2^+ state to the ground state. If one uses the theoretical $(K+L+M)$ conversion coefficient[22] for this $E2$ transition, then its total intensity (γ+conversion electron) is 90% of the neutron-capture rate in Hf^{177}. Four other gamma rays proceed directly to the ground state in the final decay scheme. The sum of their intensities is about 5%.[13] This suggests that relatively few additional strong gamma rays proceed directly to the ground state.

The energy of the (213.42±0.02)-keV gamma ray identifies it as the same transition that is observed[10] following the electron-capture decay of Ta^{178}. An $E2$ assignment for this transition is strongly supported by the conversion-electron data.[10] The large intensity (41%) of the 213-keV gamma ray, as observed with the crystal spectrometer, coupled with the coincidence experiments performed elsewhere[6,10] as well as those performed in this work, lead to the conclusion that the 213-keV gamma ray proceeds directly to the 2^+ state at 93 keV. The 4^+ spin assignment for the 306-keV level is very attractive from the point of view of the collective model in that the energy of the 306-keV level is very close to the theoretically predicted value. This spin assignment is substantiated by the lack of a transition from this level to the 0^+ ground state. The high sensitivity of the crystal spectrometer allows one to set an upper limit on the gamma intensity of this possible crossover transition. The calculated energy of the level, 306.59±0.03 keV, comes directly from the sum of the two cascade gammas (93+213 keV). An upper limit of 0.08 photons per 100 neutron captures is set for the possible crossover gamma from the 306.59-keV level to the ground state. This upper limit would be much lower were it not for a gamma ray in the spectrum very close to this energy. The gamma ray that comes closest to matching this energy is the one of 306.37±0.04 keV.

---

[20] E. Chupp, J. Dumond, F. Gordon, R. Jopson, and H. Mark, Phys. Rev. **112**, 518 (1958).

[21] The energy or energies quoted here are the values obtained from the bent-crystal work at Argonne (this paper). For the corresponding conversion-electron transitions, see Tables II and III.

[22] L. A. Sliv and I. M. Band, Leningrad Physic-Technical Institute Reports, 1956 and 1958 [translation: Reports 57 ICC K1 and 58 ICC $L1$ issued by the Physics Department of the University of Illinois, Urbana, Illinois (unpublished)].

The intensity of this gamma ray is 0.16 photons per 100 neutron captures. The energy difference (level spacing minus gamma-ray energy) of (0.22±0.05) keV is small but is still four times as large as the probable error in the measurement. As it is, the 306.37-keV gamma ray fits very nicely higher up in the level scheme.

The energy of the (325.51±0.04)-keV gamma agrees very well with the value found for this transition in the electron-conversion studies[10] (Table II). Its intensity of 6.3% strongly suggests that it is in the lower part of the level scheme. The γ-γ coincidence studies involving the 325-keV transition support the assumption that this transition decays directly to the 4^+ level at 306.59 keV. The level thereby suggested at 632.10±0.05 keV is assumed to have a spin of 6^+. This would be consistent with the interpretation that it is the fourth member of the ground-state rotational band. The experimentally measured electron-conversion coefficient[10] for this transition ($\alpha_k=0.06$) is consistent with the theoretical value[22] of $\alpha_k=0.045$ for an $E2$ multipole and thus supports the 6^+ assignment for the 632.10-keV level. Again, the bent-crystal spectrometer can be used to set an upper limit on the intensity of the possible crossover transitions to the lower levels. The intensity limits obtained are $\leqslant 0.1\%$ for the $6^+ \rightarrow 0^+$ transition and $\leqslant 0.02\%$ for the $6^+ \rightarrow 2^+$ transition. Because of these low limits on the intensities of the crossover transitions and the above-mentioned agreement with previous work, the 632.10-keV level is treated as a 6^+ level in the development of the decay scheme.

The usefulness of the proposed 8^+ level at 1058.3 keV is limited by the previously mentioned uncertainty (Sec. III) as to which gamma ray to use as the $8^+ \rightarrow 6^+$ transition. Placing the 8^+ level at 1058.3 keV corresponds to assigning the 426.2-keV gamma to the $8^+ \rightarrow 6^+$ transition. If the alternative 427.6-keV gamma is used, then the energy of the 8^+ level becomes 1059.8 keV.

The energy precision of the bent-crystal work allows one to check the observed level spacings in the ground-state rotational band to a rather high degree of accuracy. Of particular interest here is the determination of how well the usual correction term[2,4] $BI^2(I+1)^2$ predicts the position of the upper levels of the ground-state band. If the energy of a level in the ground-state band is assumed to be given by

$$E=AI(I+1)+BI^2(I+1)^2,$$

where I is the spin of the level and A and B are constants with the units of energy, it is possible to solve for A and B by using only the first two level spacings ($0^+ \rightarrow 2^+$ and $2^+ \rightarrow 4^+$). The energies of the 6^+ and 8^+ levels can then be predicted and compared with the observed values. This is done in Table IV. From the energies of the 2^+ and 4^+ excited states one obtains 15.614 keV for A and -0.01420 keV for B. On the basis of these constants, the predicted energy of the 6^+ level is 632.12 keV, in excellent agreement with the measured value of 632.10 keV.[21] The predicted value for the 8^+ level, based on the same values of A and B, is 1050.55 keV. This does not agree very well with either the 1058.3 or the 1059.8 keV levels suggested in Sec. III. The error in the predicted value is believed to be between 1 and 2 keV, while the errors in the suggested experimental values are between 0.1 and 0.2 keV. The difference between the predicted value and the two experimental values appears to be outside the estimated errors. This could result from the distortion of the position of the 8^+ level by other nearby levels or from a wrong identification of the 8^+ member of the ground-state band. A better candidate (energy-wise) for the $8^+ \rightarrow 6^+$ transition would be either the 420.87- or the 417.00-keV gamma. This would place the 8^+ level at 1052.97 or 1049.10 keV, respectively.

TABLE IV. Comparison of the observed and predicted energies of the 6^+ and 8^+ members of the ground-state rotational band of Hf^{178}. The theoretical predictions assume that the excitation energy of each state is given by the relation $E(I)=A(I)(I+1)+B(I)^2(I+1)^2$, where I is the spin of the state and A and B are constants with the units of energy. The theoretical prediction for the 6^+ and 8^+ levels use 15.614 keV for the value of A and $-0.014.20$ keV for the value of B. These numbers are obtained from the first two level spacings in the band ($4^+ \rightarrow 2^+$ and $2^+ \rightarrow 0^+$).

	Excitation energy $E(I)$ of the level (in keV)				
	$I=0^+$	$I=2^+$	$I=4^+$	$I=6^+$	$I=8^+$
Experimental	0	93.17	306.59	632.10	1058.31
Theoretical				632.12	1050.55

B. Development of Upper Levels

An attempt to develop the level scheme of Hf^{178} with a relatively limited use of the conversion-electron data was felt to be worth while. If this approach reproduced most of the levels found in the conversion-electron work,[6] then some confidence could be put in the new levels (Fig. 5) found by this method.

The basic assumption in the method used to develop this level scheme of Hf^{178} was that the energies and spin assignments of the ground-state rotational band were those discussed in Sec. IV A, namely, a 0^+ ground state, a 2^+ level at 93.17 keV, a 4^+ level at 306.59 keV, and a 6^+ level at 632.10 keV. The upper levels (which appear in Fig. 5) were then built on the ground-state band in a straightforward manner. The energy differences between all strong gamma-ray transitions above 700 keV were calculated and compared with the energies of the three cascade gammas (93, 213, and 325 keV) in the ground-state band. About 16 such energy differences matched the energies of the three gamma rays. Combinations of these differences were compared until a consistent level scheme was obtained. In this manner 8 levels were suggested at energies of approximately 1269, 1323, 1384, 1402, 1421, 1431, 1473, and 1513 keV. The low-energy part of the gamma-ray spectrum was then investigated for possible interconnecting transitions. Ten such interconnections were found.

TABLE V. Theoretical ratios of reduced[a] gamma transition probabilities from states with $K=0$, 1, and 2 to the levels of the $K=0$ ground-state band of even-even nuclei. The K value and spin of the initial state is given at the left under K_i and I_i, respectively. The columns are labeled by I_f/I_f', where I_f and I_f' are the two spins of the final states under consideration. The transition probability of the transition to the final state of higher spin is always in the numerator of the ratio. The left side of the table contains all those ratios that can be unambiguously predicted from the theory. The right side of the table contains those ratios that may contain more than one type of multipole radiation. Each column in this part of the table contains two ratios. The first ratio corresponds to quadrupole radiation ($E2$ or $M2$) while the second ratio, in parentheses, corresponds to dipole radiation ($M1$ or $E1$). When the dipole radiation is forbidden by the collective model, an arrow is placed in parentheses to indicate the direction in which the ratio will be changed if the forbidden radiation is present. The question marks indicate that dipole radiation is forbidden in both transitions and, if band mixing is considered, both transitions could contain dipole radiation.

		$E2$ radiation only			$E2+M1$ admixtures possible			
K_i	$I_i \diagdown I_f/I_f'$	4/0 $E2$	6/2 $E2$	8/4 $E2$	2/0 $E2(M1)$	4/2 $E2(M1)$	6/4 $E2(M1)$	8/6 $E2(M1)$
0	0				∞			
1	1				∞ (0.5)			
0	2	2.6			1.4 (↑)	1.8 (↓)		
1	2	1.1			0.28(∞)	3.5 (0)		
2	2	0.07			1.4 (↑)	0.05(↓)		
1	3					2.6 (0.2)		
2	3					0.4 (?)		
0	4		1.6			0.9 (↑)	1.8 (↓)	
1	4		1.0			0.8 (∞)	12.0 (0)	
2	4		0.25			2.9 (↑)	0.08(↓)	
1	5						1.8 (0.85)	
2	5						0.56(?)	
0	6			1.4			0.8 (↑)	1.7(↓)
1	6			1.0			0.04(∞)	25.0(0)
2	6			0.4			3.6 (↑)	0.1(↓)

[a] The "reduced ratio" is the ratio of the gamma transition probabilities with the energy factor removed. To obtain the observed intensity ratio from the "reduced ratio" in Table V, it is necessary to multiply the ratio given in the table by the fifth power of the ratio of the gamma-ray energies if $E2$ radiation is considered and by the third power of the ratio of the gamma-ray energies for $M1$ radiation.

Although the above method appears to be effective for states with spins of 1, 2, 3, and 4, it will not detect 0^+ states. A spin-0 state will have only one gamma transition to the ground-state band ($0^+ \rightarrow 2^+$). All other transitions are highly forbidden. In some cases the 0^+ levels will be suggested by gamma transitions from the upper levels (see Sec. IV D), but positive identification of a 0^+ spin assignment must come from other experimental work.

Four additional levels were established above 1.5 MeV through a detailed analysis of the low-energy gamma rays.[23] If this process is repeated, more levels are suggested at higher energies and the level spacings become smaller. Unfortunately, the likelihood of making an error (placing a gamma ray in the wrong position) in the decay scheme increases with the square of the number of levels in the scheme. For this reason it was felt advisable to limit the level scheme to levels below 1.6 MeV. This limited extension of the level scheme is very interesting, however, in that the added levels can be interpreted as members of the rotational bands which were suggested by the better established levels.

C. Character of Individual Levels

(Spins, Parities, and K Values)

Much information about the spins and parities of the levels in Fig. 5 can be obtained from the detailed consideration of the gamma-ray transitions that connect them to the rest of the scheme. For transitions between levels of the same parity, only $M1$ and $E2$ transitions are expected to have observable intensities. The competing $M3$ and $E4$ transitions are expected to be slower by at least a factor of 10^5. The assignment of positive parity to most of the levels in Fig. 5 depends strongly on the previously mentioned conversion-electron work.[6] Negative-parity assignments cannot be ruled out for the levels not seen in the decay[6,10] of Ta^{178} because no conversion coefficients are available for the gamma-ray transitions that connect them to the ground-state band. Tentative parity assignments were made for some of these levels by considering the relative intensities of the gamma rays that connect the levels to the rest of the scheme.

In the development of the spin assignments of the levels in Fig. 5, it has been assumed that observed gamma rays correspond to a spin change $\Delta S \leqslant 2$. The spin assignments for positive-parity levels were made in the following manner. A 1^+ state will have two transitions to the ground-state band, $1^+ \rightarrow 2^+$ and $1^+ \rightarrow 0^+$; a 2^+ state will have three transitions to the ground-state band, $2^+ \rightarrow 4^+$, $2^+ \rightarrow 2^+$, and $2^+ \rightarrow 0^+$; a 3^+ state will have two transitions, $3^+ \rightarrow 4^+$ and $3^+ \rightarrow 2^+$; a 4^+ state will have three transitions, $4^+ \rightarrow 6^+$, $4^+ \rightarrow 4^+$, and $4^+ \rightarrow 2^+$; etc. A similar set of rules can be made for negative-parity states.

The collective model predicts that relative intensities of gamma-ray transitions (of the same multipolarity) originating at one level and proceeding to different levels of another band is a simple function of their

---

[23] This analysis made use of the IBM-704 computer located at the Argonne National Laboratory.

energies and Clebsch-Gordan coefficients. These Clebsch-Gordan coefficients are functions of the initial and final K values as well as the spins of the levels and the multipolarities of the transitions. If the spins of the levels, multipolarities of the transitions, and the K value of the final states are known, then it is possible to determine K_i, the K of the initial state, from the relative gamma-ray intensities. This is the principal method used to establish the K values that appear in Fig. 5.

Before considering the individual levels in Fig. 5, it is instructive to consider in detail what the collective model predicts for the relative intensities of gamma transitions leading to the ground-state rotational band. The ratio of the gamma transition probabilities for two transitions originating at the same level and proceeding to two lower levels of another rotational band is given by

$$\frac{T}{T'}=\frac{(I_i\lambda I_fK_f|I_iK_i\lambda K_f-K_i)^2E^{2\lambda+1}}{(I_i\lambda I_f'K_f|I_iK_i\lambda K_f-K_i)^2(E')^{2\lambda+1}},$$

where the primed and unprimed quantities correspond to the two different final states. The theoretical ratios of the reduced transition probabilities for gamma-ray transitions that can proceed to the ground-state rotational band ($K_f=0$) are compiled in Table V. The word "reduced" in "reduced transition probabilities" means that the energy dependence has been removed from the quoted ratios. The observed gamma-ray intensity ratios must, therefore, be multiplied by the inverse ratio of the gamma-ray energies raised to the $(2\lambda+1)$ power, where λ is the multipolarity of the radiation before they can be compared with the table. The labels in Table V indicate only $M1$ and $E2$ transitions, but the ratios are equally valid for transitions with a change in parity ($E1$ and $M2$). Replacing the labels $M1$ and $E2$ by $E1$ and $M2$, respectively, would yield the table for transitions that require a change in parity. This insensitivity of the intensity ratio to the type of radiation (electric or magnetic) means that it is possible to determine the K value for a given state without knowing the parity of the state.

The theoretical ratios in Table V are divided into two groups. The ratios on the left-hand side involve only $E2$ radiation and can, therefore, be predicted uniquely. The ratios on the right-hand side contain transitions which can be mixtures of two multipoles ($E2$ and $M1$). When both multipolarities are possible for both transitions considered in the ratio, two ratios are listed. The $E2$ ratio comes first, followed by the $M1$ ratio in parentheses. In many cases $M1$ radiation will be possible in only one of the transitions considered in the ratio. In these cases the ratio is, therefore, either ∞ or 0, depending on whether the possible $M1$ radiation is in the numerator or the denominator. The transition to the higher spin level of the ground-state band is always in the numerator in Table V. In many cases the $M1$ radiation is forbidden by the collective model. The selection rule $\Delta K\leqslant\lambda$, where λ is the multipolarity of the radiation and $\Delta K=|K_i-K_f|$, forbids $M1$ radiation between $K=2$ and $K=0$ bands. The symmetry of certain transitions also forbids $M1$ radiation. Of particular interest is the case in which $K_i=K_f=0$ and $I_i=I_f$. In this case dipole radiation is forbidden. These selection rules can, however, be broken down by a small amount of mixing of one band with the other, either directly or through the levels of a third band. The presence of forbidden dipole radiation will tend to raise or lower the observed intensity ratio. In those cases in which $M1$ radiation is forbidden unless some mixing of the bands occurs, an arrow is placed in the parentheses following the $E2$ ratio to indicate the effect (increase ↑, decrease ↓) of the $M1$ radiation on the ratio. The degree to which the forbidden $M1$ radiation is present will reflect the mixing of the two bands, or in some cases the mixing with a third band.

TABLE VI. A list of the energies, K values, spins, and parities of the levels in Hf^{178} (Fig. 5) as deduced from the capture-gamma-ray measurements and their comparison with previous data. The parentheses indicate uncertainties in the number or assignment as discussed in the text. The errors in the spacings between the upper levels is 0.01–0.04 keV, while the error in the spacings between the upper levels and the ground-state band is ≈0.5 keV. The following abbreviations are used in the remarks column: GSRB—ground-state rotational band; A—also observed following Coulomb excitation (reference 15); B—also observed following the 2.2-h decay of Ta^{178} (reference 10); and C—also observed following the 9.3-min decay of Ta^{178} (reference 6).

Energy (keV)	Assignment[a] (K, spin, π)	Remarks	
0.0	$0, 0^+$	Ground state	
93.17	$0, 2^+$	GSRB,	A, B, C
306.59	$0, 4^+$	GSRB,	B, C
632.10	$0, 6^+$	GSRB,	B
1058.31	$(0, 8^+)$	(GSRB),	B
1196.27	$0, 0^+$		C
1237.26	$(0, 0^+)$	Tentative	
1269.34	$0, 2^+$		C
1322.85	$0, 2^+$		
1384.48	$(0, 4^+)$		
1402.25	$2, 2^+$		
1420.64	$2, 2^{(+)}$		C
1431.25	$0, 0^+$		C
1473.13	$(1, 1^+); (2, 2^+)$		(C)
1473.32	$0, 4^+$		
1512.92	$0, 4^+$		
1514.56	$((\ , 2^+))$	Tentative	
1550.02	$((2, 3^+))$	Tentative	
1569.78	$((2, 3^+))$	Tentative	
1575,73	$((0, 2^+))$	Tentative	

[a] The parentheses and double parentheses indicate an uncertainty in the assignment.

Comparison of the observed gamma intensity ratios with the ratios given in Table V has led to the assignment of the K and I values which appear at the left of each level in Fig. 5.

A list of the energies, K values, spins, and parities of the levels in Fig. 5 appears in Table VI. All energies are given in keV. The errors in the level spacings are of the order of 0.5 keV when the members of the ground-state band are compared with the upper levels (above 1100 keV), while the errors in the spacings between these upper levels are about 0.03 keV. The parentheses around an assignment indicate uncertainty in the assignment. The double parentheses indicate assignments based solely on the relative intensities of low energy gamma rays.

D. Individual Levels

1. The 1196.27-keV Level

Gallagher, Nielsen, and Nielsen[6] propose a 0^+ level at an excitation energy of 1197 ± 10 keV. The only gamma decay of this level observed by the above authors was an (1105 ± 11)-keV transition to the 2^+ first excited state. A strong gamma transition was observed at 1103.1 ± 0.5 keV in the gamma-ray spectrum obtained with the Argonne bent-crystal spectrometer (see Table I). This gamma ray was well resolved from its nearest neighbors (1125 and 1079 keV) and could therefore be uniquely associated with the 1105-keV transition mentioned above. The 1103.1-keV gamma-ray energy was, therefore, added to the energy of the 2^+ first excited state (93.17 keV) to obtain the energy (1196.27 keV) of the 0^+ level. Four observed low-energy gamma transitions connect this level to the rest of the scheme.

The experiments of Gallagher, Nielsen, and Nielsen[6] set a lower limit on the conversion coefficient of the transition from the 1196-keV level to the 0^+ ground state, namely $\alpha_K \geqslant 0.017$. This limits the multipolarity of the 1196-keV transition to $E0$, $M2$, or some higher multipole. By comparing the data from the bent-crystal spectrometer with the conversion-electron data,[6] a new lower limit of 0.035 is obtained for the conversion coefficient of the 1196-keV transition. With the exception of $E0$, this eliminates all multipoles below $M4$ or $E6$ and confirms the 0^+ assignment. This argument assumes that the 1103 keV transition is an $E2$ transition. The positive parity of the 1196-keV level is based on the conversion coefficient of the 1103-keV transition, for which α_K (experimental)[6]$=0.0026$ and α_K (theoretical $E2$)$=0.0027$.[22] The error in the energy of the 1196.27-keV level in Fig. 5 is the error associated with the energy of the 1103.1-keV gamma, namely, ± 0.5 keV. The failure to find any trace of a gamma transition to the 4^+ level at 306.59 keV is further support for the 0^+ assignment

2. The 1237.26-keV Level

The possible existence of another 0^+ level in this region of the level scheme was suggested by the spin and K-value assignments and the energy spacing of the two levels at 1323 and 1513 keV (see parts 4 and 10 of this section). The K-value and spin assignments for 1323 and the 1513 keV levels were [0,2] and [0,4], respectively. The $K=0$ assignment for both of these levels strongly suggested that one should find a $K=0$, spin$=0$ level somewhat below these states. The energy spacing between the 1323- and 1513-keV levels was used to calculate the approximate position of the 0^+ level. This calculation placed the 0^+ level at 1241 keV. If the calculation were to include a small correction term proportional to $I^2(I+1)^2$, where I is the spin of the state, then the predicted value could be lower. If the correction term B obtained from the levels in the ground-state band is used (see Sec. IV A) then the predicted energy of the 0^+ level is 1239 keV. Although the correction for the upper band may not be the same as that needed for the ground-state band, it is not likely to be more than four times as large. The level might therefore have an energy as low as 1230 keV. The data were searched for all possible two-step cascades that would start with the level at 1323 keV, pass through a level between 1200 and 1260 keV, and end at the 2^+ level at 93 keV. Only one such cascade was found. The 85.59-keV gamma formed the transition from the spin$=2$ level at 1322.85 keV to a proposed 0^+ level at 1237.26 keV. The 1143.7-keV gamma formed the link from this level to the 2^+ member of the ground-state band at 93 keV. The energy difference between the predicted position of the 0^+ level and the position obtained above from the two-step cascade is less than 2 keV. This means that the correction term for this rotational band is quite similar to the value obtained for the ground-state band. The only other two-step cascade that might conceivably be used (105 keV followed by the 1125-keV gamma) was ruled out on the basis of the relative intensities of the gamma-ray transitions. The total intensity (gamma+conversion electron) of the upper (105 keV) transition was three times as large as the lower (1125 keV) transition. Since no levels other than the 2^+ level at 93 keV are available for the gamma decay of a 0^+ level at 1218 keV, it was rejected as a cadidate. The next closest possible candidate for a 0^+ level came at 1175 keV. This level missed the predicted position by 65 keV. This energy discrepancy was felt to be too large, so the 1175-keV level was rejected as well.

3. The 1269.34-keV Level

The spin$=2$ assignment of the 1269.34-keV level is derived from the existence of gamma transitions to the first three levels ($0^+,2^+,4^+$) of the ground-state band. The positive-parity assignment for this level is based on the conversion-electron coefficient of the 962.1-keV transition, as measured by Gallagher, Nielsen, and Nielsen.[6] This argument assumes that the (970 ± 10)-keV transition observed in the conversion-electron work[6] is the same transition as the 962.1-keV transition observed with the bent-crystal spectrometer. A check on this assumption is dicussed later.

The assignment of $K=0$ for this level is derived from the relative intensities of the three gammas that proceed to the ground-state rotational band. The ratio of the intensity of the $2^+ \rightarrow 4^+$ (962.1 keV) transition to that of the $2^+ \rightarrow 0^+$ (1269.5 keV) transition, after the removal of the E^5 energy factor,[24] is 2.4 ± 0.3. Comparison with

---

[24] All intensity ratios in this paper have had an energy factor

Table V (column $I_f/I_f'=4/0$) shows that this ratio agrees quite well with the predicted ratio 2.6 for $K=0$. The theoretical ratios for $K=1$ ($R=1.1$) and $K=2$ ($R=0.07$) are considerably smaller than the measured ratio. The ratio of the intensity of the $2^+ \rightarrow 2^+$ (1176 keV) transition to the $2^+ \rightarrow 0^+$ (1261 keV) transition is found to be 7.4±0.6. This is compatible with an assignment of $K=0$ or 2 if one assumes that some $M1$ radiation is present in the $2^+ \rightarrow 2^+$ transition. A relatively small amount of band mixing is needed to account for this forbidden $M1$ radiation. The ratio is not compatible with a $K=1$ assignment, however, unless a large hindrance factor is assumed for the allowed $M1$ radiation. Only a $K=0$ assignment is, therefore, consistent with all of the above information. The best check on the K value of this level, as discussed previously, is the ratio of the intensities of the two $E2$ transitions (962.1- and 1269.5-keV) to the 4^+ and 0^+ members of the ground-state band. No confusion about multipoles can arise here because both transitions are expected to consist entirely of $E2$ radiation. The 73.07-keV transition is then considered to be the $E2$ transition from the 2^+ state to the 0^+ base state of a $K=0$ band. This 2^+ state (1269 keV) is believed[6] to be strongly populated by the decay of the 1^+ isomer of Ta^{178}. In this respect it is similar to the 0^+ state at 1196 keV, but dissimilar to the 0^+ state at 1237 keV. The bent-crystal data can be used to predict relative conversion-electron intensities provided that the multipolarities and admixtures of the transitions are known. If the level scheme in Fig. 5 and the theoretical $E2$ inetnsity ratios in Table V are used to obtain these quantities (1269 keV, 100% $E2$; 1176 keV, 20% $E2$+80% $M1$; 962 keV, 100% $E2$) the predicted ratios of the conversion electron intensities agree with the experimentally measured values[6] to within 20%. This agreement between the two experiments constitutes a necessary check on the assumption that the 1269-keV level seen with the crystal spectrometer is the 1277-keV level observed in the conversion-electron work. This association is important because the conversion coefficient ($E2$) for the 962-keV transition measured by Gallagher, Nielsen, and Nielsen[6] is used to obtain the positive-parity assignment for this level. If this association is assumed to be correct, the full assignment for this level is $K=0$, spin=2, and positive parity.

4 *The 1322 85-keV Level*

The gamma decay of this level proceeds to 0^+, 2^+, and 4^+ states. This leads to a spin assignment of 2^+ for the 1322.85-keV level. The positive parity assignment is a direct result of the relatively large intensity (0.25%) of the 126.58-keV gamma ray which proceeds to the 0^+ level at 1196.27 keV. If the parity of the 1322.85-keV level were negative, then this transition would be $M2$ in character and would therefore be expected to be very weak. The ratio of the intensity of the $2^+ \rightarrow 4^+$ (1016 keV) transition to that of the $2^+ \rightarrow 2^+$ (1228.8 keV) transition is 0.20±0.03.[19] This ratio is compatible only with a $K=0$ assignment (see Table V). The missing $E2$ transition to the 0^+ ground state (see dashed transition in Fig. 5) is believed to be hidden under the wing of the 1327.5-keV gamma ray. A reexamination of the bent-crystal data indicated that the intensity of the missing $2^+ \rightarrow 0^+$ transition (1322 keV) was ≈0.7%.[19] This gives a value of ≈2.6 for the ratio of the intensity of the $2^+ \rightarrow 4^+$ (1016 keV) transition to that of the $2^+ \rightarrow 0^+$ (≈1322 keV) transition. This is consistent only with a $K=0$ assignment.

The 85.59-keV gamma ray is inferred to be the $E2$ transition between the 2^+ and 0^+ members of a $K=0$ band with an intrinsic or base state at 1237.3 keV. Since a low-energy gamma ray (126 keV) also decays to a 0^+ state at 1196 keV, additional arguments are needed to ensure that the right 2^+ level is associated with each 0^+ level. Associating the 0^+ state at 1237.26 keV with the 2^+ state at 1269.34 keV, and the 0^+ state at 1196.27 keV with the 2^+ state at 132.285 keV, would result in widely different energy spacings (32 and 126 keV) between the 2^+ and 0^+ states of each band. The relative enhancement of the 85.59-keV gamma is 10 times that of the 126.58-keV gamma if one assumes that both transitions are $E2$. The much stronger enhancement of the 85.59-keV gamma would be expected only if the two associated levels (1322.85 and 1237.26 keV) were members of the same band. The presence of the 126.58-keV ($E2$) transition between the levels of two different $K=0$ bands suggests that some mixing of the two bands does occur.

The 1228.8-keV transition was not observed in the conversion-electron data.[6] The 1143-keV ($E2$) and the 1236-keV ($E0$) transitions were not observed as well. However, the K-conversion line for the 1228-keV transition falls very close to the energy of the L-conversion line of the strong doublet at 1176 keV (gamma transition energy). Thus, a weak trace of the 1228-keV transition in the conversion electron work cannot be ruled out. The K-conversion line of the 1143-keV transition would likewise be hidden under the L-conversion line of the 1103.1-keV transition; and a small trace of the K-conversion lines of the 1237.26- and 1322.85-keV transitions may be hidden under the L-conversion line of the 1176.3-keV transition and the K conversion line of the 1327.5-keV transition, respectively.[21]

5. *The 1384.48-keV Level*

This level decays by gamma emission to the 2^+, 4^+, and 6^+ members of the ground-state rotational band. This is consistent only with a spin assignment of 4. The ratio[24] of the intensity of the 1077.8-keV transition ($4 \rightarrow 4^+$) to that of the 1291.9-keV transition ($4 \rightarrow 2^+$)

---

removed unless explicitly stated to the contrary. For $M1$ radiation, this energy factor is the cube of the ratio of the gamma-ray energies; for $E2$ radiation the factor is the fifth power of the ratio of the γ-ray energies.

is 12±2 for $E2$ radiation. This is compatible with either a $K=0$ or $K=2$ assignment if some dipole radiation is assumed to be present in the $4 \rightarrow 4^+$ transition. If $E2$ or $M2$ radiation is assumed, the ratio of the intensity of the 752.3-keV transition ($4 \rightarrow 6^+$) to the intensity of the 1291.9-keV transition ($4 \rightarrow 2^+$) is 10±4. Although this ratio is larger than any of the appropriate ratios in Table V, it is more compatible with a $K=0$ assignment than with a $K=2$ assignment. This assignment of $K=0$, spin$=4$ suggests that two additional states with spins of 0 and 2, may exist below the 1384-keV state. These levels were not generated in the previously mentioned procedure. For this reason the value $K=0$ for this level is held in question. An alternative explanation of the observed ratios is possible if one assumes $K>2$ for this level. Quadrupole radiation would then be fobidden and higher order multipoles could enter into the discussion.

6. The 1402.25-keV Level

The two gamma transitions from the 1402.25-keV level to the 0^+ and 2^+ members of the ground-state band indicate a spin assignment of 1 or 2 for this level. After the removal of the energy factor,[24] the observed ratio of the intensity of the (1 or 2) $\rightarrow 2^+$ (1310 keV) transition to the intensity of the (1 or 2) $\rightarrow 0^+$ (1402 keV) transition is 1.9±0.2 if $M1$ radiation is assumed and 2.2±0.2 if $E2$ radiation is assumed. Comparison of these ratios with Table V shows that only a spin assignment of 2, with $K=0$ or 2, is appropriate. The decay of an upper 4^+ state (1473.32 keV) to the 1402.25-keV level is consistent only with a 2^+ assignment. The positive-parity assignment for this level is further substantiated by a relatively strong transition (147.78 keV) from another upper level and will be discussed in more detail later. The $2^+ \rightarrow 4^+$ gamma transition (≈ 1095 keV) to the ground-state band was not observed with the crystal spectrometer. Careful analysis of the data sets an upper limit of $\leqslant 0.05$ photons per 100 neutron captures for the intensity of this γ ray. The ratio of the intensity of the $2^+ \rightarrow 4^+$ transition to the $2^+ \rightarrow 0^+$ transition is, therefore, $\leqslant 0.21$. This is consistent with $K=2$ but not with $K=0$. The assignment is thus $K=2$ spin$=2^+$.

The ratio (2.2) of $2^+ \rightarrow 2^+$ to $2^+ \rightarrow 0^+$ is much closer to the predicted ratio (1.4) for pure $E2$ radiation than are the similar ratios observed for gamma decays from $K=0$ bands. This demonstrates the effectiveness of the $\Delta K \leqslant \lambda$ selection rule in retarding the $M1$ radiation between $K=0$ and $K=2$ bands and indicates that band mixing is considerably smaller here than for the two $K=0$ bands just below it (as discussed in Secs. IVD-*3* and IVD-*4*).

7. The 1420.64-keV Level

The 1420.64-keV level is similar to the 1402.25-keV level in that only two gamma transitions are observed to decay to the ground-state band. The 0^+ and 2^+ spins of these lower levels indicate a spin assignment of 1 or 2 for the 1420.64-keV level. The ratio[24] of the intensities of the (1 or 2) $\rightarrow 2^+$ transition (1327 keV) to that of the (1 or 2) $\rightarrow 0^+$ transition (1420 keV) is 1.2±0.2 if $M1$ radiation is assumed and 1.4±0.2 if $E2$ radiation is assumed. This is consistent only with an assignment of spin$=2$, $K=0$ or 2. The $(2 \rightarrow 4^+)/(2 \rightarrow 0^+)$ ratio is $\leqslant 0.15$. These two ratios suggest a $K=2$, spin$=2$ assignment for this level. The observed intensity ratio of the two transitions to the ground-state band (1327.5 and 1420.9 keV) is within 10% of the value listed in Table V for $E2$ transitions.

Two low-energy gamma rays contribute to the decay of the 1420.64-keV level. They proceed to the 2^+ level at 1269.34 keV and to the 0^+ level at 1196.27 keV. If these two levels are members of the same $K=0$ band, then the ratio of the relative intensities of the two low-energy gamma rays mentioned above can be compared with Table V. The ratio of the intensity of the 151.29-keV transition ($2 \rightarrow 2^+$) to that of the 224.37-keV transition ($2 \rightarrow 0^+$), after the removal of the energy factor,[24] is 34±6 if $M1$ radiation is assumed and 75±20 if $E2$ radiation is assumed.

The amount of band mixing needed to produce the above mentioned ratio of 75 for gamma-ray energies of about 200 keV will produce a much smaller intensity ratio (≈ 3) for gamma-ray energies of ≈ 1 MeV. This effect results from the different energy dependence of the two types of competing radiation $[T(E2) \propto E^5;\ T(M1) \propto E^3]$.[24] The relatively small energy spacing between the two upper bands will also tend to increase the band mixing and thus the amount of $M1$ radiation present in the $2^+ \rightarrow 2^+$ transition.

The 1420.64-keV level reported here is associated with the (1430±14)-keV level observed in the conversion-electron experiments.[6] It is possible to use the gamma-ray intensities obtained with the bent-crystal spectrometer and the multipole assignments inferred from Fig. 5 to predict the ratio of the conversion electron intensities of the 1327- and 1420-keV transitions. Here, one associates the 1327.5- and 1420.9-eV gamma transitions with the (1335±13)- and (1430±14)-keV transitions observed in the conversion-electron work.[6] The good agreement between the ratio of 1.0±0.2 predicted by the bent-crystal data and the experimentally observed ratio[6] of 0.9±0.2 forms a check on the consistency of the two experiments. It also allows one to use the conversion coefficients measured in the conversion-electron work[6] to assign a positive parity to the 1420.64-keV level.

8. The 1431.25-keV Level

Two high-energy gamma rays (1338 and 1125 keV) appear to decay from this level to the ground-state band. The intensity ratios of 0.07±0.01 (if $M1$ radiation is assumed) and 0.10±0.02 (if $E2$ radiation is assumed) for the (?) → 4^+ to (?) → 2^+ transitions do not agree very well with any of the ratios that appear in Table V. The upper limit on the intensity of a transition from the 1431-keV level to the 0^+ ground state is 0.18%. The ratio of the intensity of the 1125.4-keV transition $[(2)\rightarrow 4^+]$ to that of the unobserved 1431-keV transition $[(2)\rightarrow 0^+]$ is $\geqslant 3.6\pm 0.6$. This ratio is also in poor agreement with any of the corresponding ratios in Table V (see column $I_f/I_f'=4/0$). An assignment of $K=0$, spin$=2$ is the most consistent with the observed ratios. The 1338.2-keV gamma transition observed with the bent-crystal spectrometer is associated with the (1345±13)-keV transition observed in the conversion-electron work[6] (Table III). In the level scheme of Hf^{178} proposed by Gallagher, Nielsen, and Nielsen,[6] the 1345-keV transition proceeds from a 0^+ level at 1440±14 keV to the 2^+ member of the ground-state band (93 keV). The $0^+\rightarrow 0^+$ transition (conversion electron) was also observed by the above authors. They set a lower limit of $\geqslant 0.011$ on the K-conversion coefficient of the 1440 keV transition. This restricts the choice of multipoles to either $E0$ or to high multipoles of the order of $M4$ and $E6$. The bent-crystal data can be used in conjunction with the conversion-electron work[6] to raise the lower limit on this conversion coefficient to $\geqslant 0.23$. This calculation assumes that the 1338-keV gamma ray $(0^+\rightarrow 2^+)$ is $E2$. An $E1$ assignment for the 1338-keV gamma ray would give a conversion coefficient of $\geqslant 0.09$ for the 1440-keV transition. This is still very much larger than any conceivable conversion coefficient except that of an $E0$ transition. In view of the previously mentioned similarities between the conversion-electron results and the decay scheme deduced from the capture gamma rays, an assignment of $K=0$, spin$=0^+$ is made for the 1431.25-keV level. The 1125-keV gamma is then assumed to fit here only by accident (note dashed line in Fig. 5).

9. *The 1473.13- and 1473.32-keV Levels*

The level (or group of levels) around 1473 keV appeared to have four transitions to the ground-state-band. This is one too many for any one level. The possible low-energy γ transitions from this region to the nearby levels also suggested the existence of two levels. The proposed lower (1473.13 keV) level decays to 0^+ and 2^+ states while the upper (1473.32 keV) level decays to a 2^+ state and a spin$=4$ state. The only strong γ transition (1167 keV) to the ground-state band decays to the 4^+ state at 306.59 keV. This suggests that one of the levels is a spin$=4$ state. A spin of 4 was assigned to the 1473.32-keV level. The relatively high intensity of the low-energy gamma that proceeds to the spin$=4$ level at 1384 keV supports this view. The ratio[24] of the intensity of the $(4)\rightarrow 6^+$ transition to that of the $4\rightarrow 4^+$ transition (to the ground-state band) is 0.19 ± 0.3 if $E2$ radiation is assumed and 0.10 ± 0.2 if $M1$ radiation is assumed. This is most consistent with an assignment of $K=0$, spin$=4$. The 204.05-keV transition to the 2^+ level at 1269.34 keV makes the 1473.32-keV level a good candidate for the 4^+ member of the $K=0$ band with its base state at 1196.27 keV. The level spacing between this 4^+ level (1473.3 keV) and the 2^+ level at 1269 keV is about 20% larger than one would predict from the energy separation of the 2^+ level (1269 keV) and the 0^+ level (1196 keV). Part of this discrepancy may be accounted for by the interaction of this state with two near-by states (1384 and 1513 keV) with similar spins. An alternative explanation would be to assume that another 4^+ level is present in the level scheme at the appropriate energy (≈1440 keV). With this 4^+ assignment for the 1473.3-keV level one would expect the 1381-keV $(4^+\rightarrow 2^+)$ transition to be more intense. The ratio of the intensity of the $4^+\rightarrow 6^+$ (841.1 keV) transition to that of the $4^+\rightarrow 2^+$ (1381.8 keV) transition is 7 ± 2 ($E2$ radiation). This is considerably larger than the predicted ratio of 1.8 for this assignment but is even less consistent with any other assignment. Some of this apparent discrepancy may be due to the difficulty encountered in resolving the weak lines in the energy region near 1381 keV.

If much of the intensity of the 1381-keV gamma is associated with the 1473.32-keV level, then only a small amount will be left for the decay from the proposed level at 1473.13 keV. The $[(1\text{ or }2)\rightarrow 2^+]/[(1\text{ or }2)\rightarrow 0^+]$ intensity ratio for gamma decays from the 1473.13-keV level is estimated in this manner to be $\leqslant 0.55$. This suggests a 1^+ level with $K=1$. The intensity ratio[24] of the two relatively strong low-energy transitions to the 2^+ level at 1269.34 keV and 0^+ level at 1196.27 keV is 2.6 ± 0.4 if $M1$ radiation is assumed and 4.8 ± 0.8 if $E2$ radiation is assumed. This is most nearly consistent with a spin assignment of 2^+ with $K=0$ or 2. The assignment of $K=1$ and spin$=2$ is also possible. The conflict in spin assignments suggested above depends heavily on the intensity of a poorly resolved gamma ray (1381 keV). The spin assignment of the 1473.13-keV level is therefore only tentative and no further arguments should be based on it until the conflict is resolved.

10. *The 1512.92-keV Level*

The 1512.92-keV level decays to the 2^+, 4^+, and 6^+ states and is therefore given a 4^+ assignment. The $(4^+\rightarrow 6^+)/(4^+\rightarrow 4^+)$ ratio[24] of gamma-ray intensities is 0.14 ± 0.02 while the $(4^+\rightarrow 6^+)/(4^+\rightarrow 2^+)$ ratio[24] is 2.1 ± 0.4 (both ratios being calculated for $E2$ radiation). This indicates a $K=0$ assignment for this level. The 190.07-keV transition to the 2^+ state at 1322.85 keV suggests that the 1512.92-keV level is the 4^+ member of the $K=0$ band with its 0^+ state at 1237.26 keV. The level spacing between this 4^+ level (1512.9 keV) and the 2^+ level at 1322.8 keV is in good agreement with the value predicted for this separation by the level spacing of the 2^+ level (1322.8 keV) and the 0^+ level at 1237.2 keV.

E. Tentative Levels

Four additional levels appear in Fig. 5. They were suggested by a detailed comparison of energy differences between the low-energy gamma rays.[19] Many more levels are similarly suggested above these levels. As more states are added to the level scheme, the number of possible ways of combining gamma rays to generate new levels increases and the chance of making an error increases with it. The arbitrary cutoff at 1575 keV is an attempt to keep the level scheme within bounds. The main point in extending the level scheme this far is to look for the missing members of the excited-state bands.

The gamma-ray spectrum between 1.5 and 2.0 MeV appears to consist of many weak (<1.0%) transitions which were not resolved by the bent-crystal spectrometer. This lack of resolved transitions to the ground-state band reduces the general reliability of the level scheme above 1512 keV. The major loss is that one no longer has a straightforward method of determining K values for the levels. In theory one could use the relative transition probabilities to the two $K=0$ excited-state bands with base states (0^+ levels) at 1197 and 1237 keV to suggest K values. This method is severely restricted by the relatively small energy spacing between the $K=0$, 4^+ states and the states under examination. In practice one has only the ratio of intensities to the 2^+ and 0^+ members of the $K=0$ bands. As mentioned above, these ratios are often ambiguous and, therefore, of only limited use. Nevertheless, these ratios do suggest assignments for two of the upper levels.

1. *The 1514.56-keV Level*

Three gamma-ray transitions proceed from this level to lower states with spin assignments of 2^+ and 0^+. This limits the spin assignment for the 1514.56-keV level to 1 or 2. If the 0^+ state at 1196.27 keV and the 2^+ state at 1269.34 keV are assumed to be members of the same rotational band, then the relative intensities of the two gamma rays (245.23 and 318.29 keV) which proceed to these levels can be used to suggest a spin assignment for the 1514.56-keV level. The ratio of the gamma intensity of the 245.23-keV transition $[(?)\rightarrow 2^+]$ to the 318.29-keV transition $[(?)\rightarrow 0^+]$ is 66 ± 10 if $M1$ radiation is assumed and 111 ± 20 if $E2$ radiation is assumed. These ratios are quite similar to those observed for the low-energy gamma decay of the 1420.64-keV state ($K=2$, spin$=2^+$) discussed in Sec. IVD-7. Again, the assignment of $K=1$, spin$=1$ can be ruled out on the basis of the above-mentioned intensity ratio, so that only the spin$=2$ assignment is compatible with the data. No straightforward choice between $K=0$, 1, or 2 can be made because the possible $E2$ enhancement of the two transitions (318.29 and 245.23 keV) cannot be predicted. If one assumes that there is no $E2$ enhancement of these gamma rays, then the best assignment for the 1514.56-keV level is $K=1$ or 2. A $K=1$ assignment would make the 1514.56-keV level a candidate for the 2^+ member of a $K=1$ band with a 1^+ state at 1473.13 keV. Such an association is speculative at best.

2. *The 1550.02-keV Level*

All of the low-energy transitions that depopulate this level proceed to states with a spin$=2$ assignment. The relatively strong 147.78-keV transition (0.16%) to the 1402.25-keV level (with $K=2$, spin$=2^+$) suggests that the 1550.02-keV level may be the 3^+ member of this $K=2$ band. Such an assignment would explain the absence of transitions to the two 0^+ states at 1237.26 and 1196.27 keV.

3. *The 1569.78-keV Level*

The three low-energy transitions that depopulate the 1569.78-keV level proceed to states with spin$=2^+$ and 4. This is consistent with a spin assignment of 2, 3, or 4^+ for the 1569.78-keV level. The relatively strong 149.14-keV transition (0.10%) to the 1420.64-keV 2^+ level with $K=2$ suggests an assignment of $K=2$, spin $=3^+$ for the 1569.78-keV level. Again, this assignment must be considered as speculative, as indicated by the double parentheses on the assignment in Fig. 5.

4. *The 1575.73-keV Level*

The 1575.73-keV level decays to four lower states at 1196.27 keV ($K=0$, spin$=0^+$), 1269.34 keV ($0,2^+$), 1431.25 keV ($0,0^+$), and 1473.13 keV ($1,1^+$ or $2,2^+$). This limits the spin of the 1575.73-keV level to 1 or 2. The relatively strong transition to the 0^+ level at 1431.25 keV suggests that the 1573.73-keV level is the 2^+ member of a $K=0$ band with its 0^+ state at 1431.25 keV. The 306.37- and 379.44-keV transitions proceed to the 2^+ and 0^+ states of a $K=0$ band and therefore the ratio of their intensities can be used to suggest an assignment for the 1575.73-keV level. The ratio of the intensity of the 306.37-keV gamma to the intensity of the 379.44-keV gamma is 1.7 ± 0.2 for $M1$ radiation and 2.7 ± 0.3 for $E2$ radiation. The $M1$ ratio is three times as large as the ratio ($R=0.5$) predicted for a $K=1$, 1^+ assignment but quite compatible with an assignment of $K=(0,1,2)$, spin$=2^+$. The tentative assignment of $K=0$, 2^+ suggested by the relatively strong 144.49-keV transition (0.27%) is, therefore, compatible with all the information available.

V. GAMMA-GAMMA COINCIDENCE STUDIES

A number of coincidence experiments have been performed on the $\mathrm{Hf}^{177}(n,\gamma)\mathrm{Hf}^{178}$ reaction with thermal and epi-thermal neutrons.[25] The 3-dimensional recording and analyzing system of the Argonne chopper group was

---

[25] R. K. Smither (unpublished).

used for this work.[26] Only preliminary results are available to date but a number of the important features of this level scheme of Hf^{178} (Fig. 5) have been confirmed. In particular, the strong coincidence of the 93- and 213-keV gammas with the group of gamma rays between 1.0 and 1.3 MeV is quite evident. Further, there is some evidence for coincidences between the high-energy gamma rays (6–7 MeV) and the low-energy gamma rays which connect the upper levels in Fig. 5. The coincidence rate of the high-energy gammas with a 140–150 keV gamma or gammas is particularly striking. This gamma gamma coincidence can be associated with the level at 1575.73 keV ($K=0$, spin$=2^+$) and/or the level at 1569.78 keV ($K=2$, spin$=3^+$) and/or the level at 1550.02 keV ($K=2$, spin$=3^+$).

VI. GENERAL DISCUSSION AND CONCLUSIONS

Three requirements were imposed on the levels in Fig. 5 during the construction of the level scheme. First, each level was required to have at least three connections (observed transitions) with the rest of the level scheme. The average number of such connections for the levels in Fig. 5 is five. The one exception to this rule is the level at 1058 keV. This level is believed to have an 8^+ spin assignment and is therefore expected to make only one connection ($8^+ \rightarrow 6^+$) to the rest of the level scheme. The second requirement on each level was that all gamma-ray transition energies should agree with the level spacings to within 1.5 probable errors. The average energy difference between the level spacings and the gamma-ray energies is 0.4 times the probable error. This better-than-expected agreement is believed to be due to the cancellation of systematic errors. The third requirement was that the sum of the total intensities (γ+conversion electron) of the transitions feeding a level must be equal to or less than the sum of the intensities of the transitions depleting the level, to within the errors of the measurements.

Although the level scheme below 1.6 MeV (Fig. 5) is believed to be incomplete (see high-spin levels, Fig. 4), considerable progress has been made in its development. Two low-lying excited-state $K=0$ bands have been proposed with base states at 1196.3 and 1237.3 keV. The moments of inertia associated with these bands are, respectively, 5% and 10% larger than that associated with the ground-state rotational band. This suggests slightly larger nuclear deformations for these bands. Above 1400 keV, many new levels appear. Another $K=0$ band with a base (0^+) state at 1431.3 keV and two $K=2$ bands with base (2^+) states at 1402.3 and 1420.6 keV are suggested by this group of levels. The nuclear deformations of these upper bands (as derived from their level spacings) are very similar (within 4%) to each other and 40% smaller than the deformation observed for the ground-state band. The collective nature of these states is evident in the strong enhancement of the low-energy transitions between the lower members of these bands. A good example of this collective enhancement is the 73.07-keV transition from the 1269.34-keV level (with $K=0$, spin$=2^+$) to the 1196.27-keV level (with $K=0$, spin$=0^+$). This $E2$ transition competes with an $E2$ transition (1269.5 keV) to the ground state ($K=0$, spin$=0^+$). The observed gamma intensity of the 73.07-keV transition is only 1/20 that of the 1269.5-keV transition. After the removal of the E^5 energy factor, the ratio of the reduced transition probabilities (73.07 keV/1269.5 keV) is 6.7×10^4.

TABLE VII. A summary of the reduced gamma transition ratios observed for the decay of spin 2 states to the ground-state rotational band of Hf^{178}. The word "reduced" refers to the removal of the energy dependence of the ratio (see text). Part (A) contains only ratios in which both transitions correspond to a spin change of two units. Part (B) contains those ratios in which some dipole radiation may be present. All theoretical ratios are for $E2$ radiation.

	(A) Reduced gamma transition ratios $(2\rightarrow 4)/(2\rightarrow 0)$		
	$K=0$	$K=1$	$K=2$
Theory $E2$	2.6	1.1	0.07
Exp. level (keV)			
1268	2.4 ± 0.3		
1325	2.6 ± 0.3		
1402			$\leqslant 0.21$
1420			$\leqslant 0.15$

	(B) Reduced gamma transition ratios $(2\rightarrow 2)/(2\rightarrow 0)$		
	$K=0$	$K=1$	$K=2$
Theory $E2$	1.4	0.28	1.4
Exp. level (keV)			
1268	7.4 ± 0.6		
1323	2.6 ± 0.3		
1402			2.2 ± 0.2
1420			1.4 ± 0.2

The 10 levels at 1237.3, 1322.8, 1384.5, 1402.2, 1473.3, 1512.9, 1514.6, 1550.0, 1569.8, and 1575.7 keV have not been observed previously. This fact is noted in the level scheme (Fig. 5) by an asterisk following the energy of the level. All the K-value assignments for the levels above 1200 keV are new because the experimental data available previously were not sufficient for their determination. The spin assignments obtained from the bent-crystal data for the levels at 1196.3, 1269.3, 1431.2, and 1473.1 keV agree with the spin assignments obtained for these levels from the conversion-electron work.[6] In only one case, the level at 1420.6 keV, is there a discrepancy between the previous spin assignment and the Argonne bent-crystal work (this paper). The previous assignment[6] was 1^+, while the bent-crystal data are much more consistent with a 2^+ assignment (see Sec. IV). The observation of the low-energy gamma transitions between the upper levels of the scheme is also new, although the conversion-electron data[6] did indicate some unresolved structure in the low-energy region.

---

[26] L. M. Bollinger and R. E. Cote, Argonne National Laboratory Report ANL-6275 (unpublished).

It is the observation of these low-energy transitions that allows one to determine which levels belong to the same rotational band.

The success experienced in determining K values for levels through the investigation of relative gamma-ray intensities is very encouraging. The results for states with initial spin$=2$ are summarized in Table VII. In all cases the $(I\rightarrow I+2)/(I\rightarrow I-2)$ ratios of $E2$ gamma intensities lead to clear-cut assignments. The further fact that the observed $(2^+\rightarrow 2^+)/(2^+\rightarrow 0^+)$ ratios for transitions from the $K=2$ bands to the $K=0$ ground-state band were quite close to the ratio predicted for $E2$ radiation alone (experimental ratios$=2.2$ and 1.4, as compared with the theoretical ratio$=1.4$) demonstrates that $\lambda\geqslant\Delta K$ selection rule is effective for these transitions. This result can be interpreted to mean that these $K=2$ bands do not mix appreciably with the ground-state $K=0$ band or with a possible nearby $K=1$ band.

In the level scheme of Hf^{178}, as presented in Fig. 5, there are 121 possible transitions that correspond to a spin change of 2 units or less. Of these, 93 are within the energy range of the bent-crystal spectrometer. Of these 93 transitions, 54 (or 60%) are present in the final level scheme (Fig. 5). Some additional selection rules[27] (besides spin, parity, and K value) are expected to be present in the level scheme of Hf^{178} and may account for the absence of some of the missing gamma transitions. As the levels in Fig. 5 are either confirmed or rejected by future experiments, it may be possible to employ the unused members of the gamma spectrum to extend the level scheme of Hf^{178} to higher energies.

ACKNOWLEDGMENTS

The authors wishes to thank Dr. R. D. Lawson for many valuable discussions concerning the interpretation of the work. The author would also like to thank Dr. C. A. Mallmann and Dr. D. Kurath for many helpful suggestions and comments concerning this experiment. The author wishes to acknowledge the considerable assistance rendered by A. P. Magruder in the conversion of the raw spectrometer data into useful numbers. The author also wishes to thank the Argonne chopper group for the use of their 3-dimensional analyzer and to thank Dr. L. M. Bollinger and Dr. R. T. Carpenter for their assistance in setting up the coincidence runs.

---

[27] G. Alaga, Nucl. Phys. 4, 625 (1957).