# Magnetoresistance of *p*-Type Si in the Hopping Region

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The magnetoresistance in the hopping region of B-doped Si was found to be negative. This contrasts with the behavior in *n*-type Ge, where a positive magnetoresistance is characteristic of the hopping conduction region. No anisotropy is observed in fields up to 17 kG in the *p*-type Si. The resistance tends to saturate at about 15 kG. The change in dc resistance at that field is about 10% from the zero-field value.

MAGNETORESISTANCE was measured in *p*-type Si in the region of hopping conduction. In contrast to the magnetoresistance of hopping in *n*-Ge, which is rather large and positive,[1] the magnetoresistance in *p*-Si is small and negative. While negative magnetoresistances had been found also at low temperatures in *n*-type germanium,[2] such behavior is characteristic in that material only for concentrations for which banding occurs.[3] In *p*-type silicon, on the other hand, it occurs at concentrations where the hopping mechanism is operative.

The experiments were performed under the following conditions: Partially compensated samples with $\sim 2\times 10^{15}$ and $\sim 10^{17}$ boron atoms per cc were used. Experiments were performed extensively on the $10^{17}$ material, while the $2\times 10^{15}$ sample was used mainly to determine whether the negative sign of the magnetoresistance should be attributed to some banding that might occur in the more heavily doped sample. The magnetoresistance in the $2\times 10^{15}$ material was, however, also found to be negative and similar in magnitude.

Most measurements were performed at the boiling point of He, with magnetic fields up to 18 kG. The directions of the magnetic and electric fields with respect to each other and to the crystallographic axis are indicated in the figures. Except for one measurement with a dc electric field, all measurements were performed with an ac electric field, at frequencies between 100 cps and 100 kc/sec. The apparatus used was similar to the one described in a previous paper.[4]

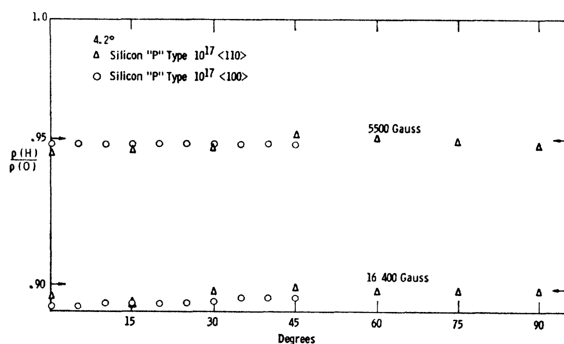


FIG. 1. The magnetoresistance at 4.2° of silicon with $\sim 10^{17}$ borons/cc, at 100 cps. The upper curves are for 5.5 kG, the lower for 17 kG. The points represent the transverse magnetoresistance with $E\|\langle 100\rangle$ (circles) and $E\|\langle 110\rangle$ (triangles). 0° on the ordinate is in a [100] direction. The arrows represent longitudinal magnetoresistance ($\rightarrow$ for $E\|\langle 100\rangle$; $\leftarrow$ for $E\|\langle 110\rangle$).

The magnetoresistance is almost independent of the direction of the magnetic field ($C=D=0$) as is illustrated in Fig. 1. The figure represents measurements at 4.2°, 100 cps on sample $10^{17}$. The change of resistance with magnetic field is presented in Fig. 2. It is quadratic only in very low fields and comes to a saturation around 13 kG. The figure also indicates the frequency dependence of the magnetoresistance in the heavily doped sample. A corresponding set of measurements was performed with dc electric fields. The resulting curve is rather similar to the one that represents the 100 cps measurements, but is not plotted here because of the relatively poor accuracy of our dc measurements.

The rather pronounced frequency dependence in the heavily doped sample (none was observed in the $2\times 10^{15}$ sample, at least within the reduced sensitivity for this sample) does not come as a surprise. The sample has a concentration such that the 100 cps is close to the dc situation, whereas the 100-kc/sec behavior is in the range treated in reference 4. In the dc case, the carriers have to perform many consecutive hops in order to contribute to the conductivity. The magnetoresistance in this case measures how much the hopping rates are impeded (or enhanced) by the magnetic field. At the

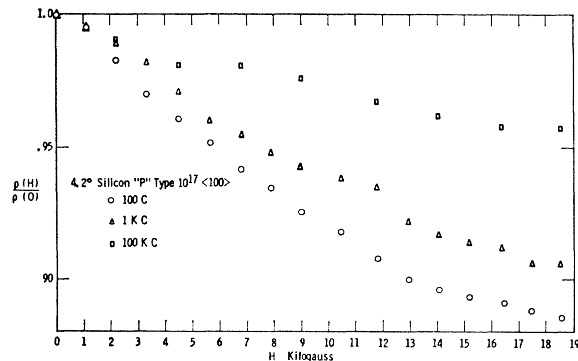


FIG. 2. The variation of resistance with magnetic field at various frequencies. $N_a \cong 10^{17}$ cm$^{-3}$.

[1] R. J. Sladek and R. W. Keyes, Phys. Rev. **122**, 437 (1961).
[2] W. Sasaki, in *Proceedings of the International Conference on Semiconductors, Prague, 1960* (Czechoslovakian Academy of Physics, Prague, 1961), p. 159.
[3] W. Sasaki (to be published).
[4] M. Pollak and T. H. Geballe, Phys. Rev. **122**, 1742 (1961).

higher frequencies, on the other hand, it has been shown in reference 4, that the major contribution to the conductivity comes from hops between such pairs of impurities, that the hopping rate back and forth between them corresponds to the applied frequency. If the hopping rates are changed by the magnetic field (or by some other cause) the resulting change in conductivity will just be a measure of the change of population of carriers, whose hopping rate corresponds to the applied frequency.

The mechanism of the negative magnetoresistance is not understood. In terms of Mikoshiba's theory,[5] there are two major causes for a magnetoresistance of hopping conduction: the phase effect and the size effect. The phase effect can be immediately discarded, as it must cause a positive magnetoresistance and furthermore must be vanishing for $H\|E$. The size effect could be responsible for the negative magnetoresistance, if one assumes that the magnetic field splits the heavy and light holes raising the latter, so that the relative contribution of the light hole to the localized wave function increases with magnetic field, thus increasing the overlap integrals. It is hard to see, however, how such an explanation could be made consistent with the magnetic field dependence. It also remains to be seen, whether the negative magnetoresistance is characteristic of $p$-type Si, or whether it is a freak of the boron impurity.

Helpful discussions with T. Holstein are gladly acknowledged. The authors are also indebted to J. Albrecht for his able and extensive help with the apparatus.

[5] N. Mikoshiba and S. Gonda, Phys. Rev. **127**, 1954 (1962).

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# Magnetoabsorption and the Band Gap of Bi

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Data are presented to show that the magnetoabsorption of Bi in the infrared is due to direct interband transitions between Landau levels rather than from de Haas-van Alphen type oscillations of the Fermi level as previously hypothesized. These new data permit a more accurate determination of the energy gap and yield a value of 0.024 eV for this energy.

A REINVESTIGATION of the magneto-oscillations in the infrared transmission of Bi observed by Boyle and Rodgers[1] shows that they result from resonant interband transitions between Landau levels of the valence and conduction bands rather than from de Haas–van Alphen type oscillations in the Fermi level as previously hypothesized. These new data extend to lower energies the measurements of interband transitions made by Brown *et al.*[2] using reflection techniques. Because of the nonparabolic shape of the Bi energy bands, this extension permits a more accurate determination of the energy gap. A value of 0.024 eV is obtained in contrast to 0.046 eV obtained by Brown *et al.*

At low temperatures Bi becomes transparent to infrared radiation between approximately 20 and 50$\mu$. Close to the short-wavelength cutoff of this pass band, oscillations in the transmission at fixed wavelength are observed when the magnetic field strength is varied as shown in Fig. 1. These oscillations are periodic in $1/H$. The hypothesis that they arise from a de Haas–van Alphen type variation of the Fermi level requires that their period be independent of photon energy. The data of Fig. 2, which show the position of the transmission minima (in terms of $H$) vs photon energy, do not satisfy this criterion. Instead, these data are

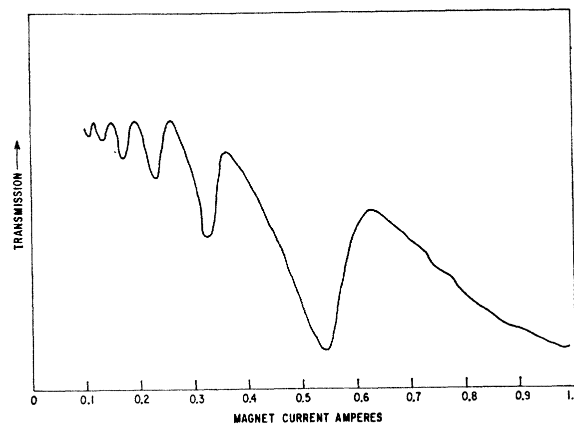


FIG. 1. Magnetic field variation of the transmission of Bi, $H\|$binary axis$\perp$trigonal axis; Poynting vector of the 20.7 $\mu$ radiation$\|$trigonal axis; bath temperature 4.2°K. 1 A$\approx$12 kG. Sample thickness$\approx$10$\mu$.

[1] W. S. Boyle and K. F. Rodgers, Phys. Rev. Letters **2**, 338 (1959).
[2] R. N. Brown, J. G. Mavroides, M. S. Dresselhaus, and B. Lax, Phys. Rev. Letters **5**, 243 (1960).