Neutrino Pair Emission by a Stellar Plasma*

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Various current models and generalizations of a universal weak Fermi interaction predict a first-order weak coupling between electrons and ν-$\bar{\nu}$ pairs. The radiation of such pairs by a hot, partially degenerate relativistic plasma is calculated for temperatures and densities that appear to be relevant for stellar evolution. Neutrino-pair emission by collective electron modes, especially transverse plasma excitations, is found to be the main mechanism for neutrino radiation by a dense stellar plasma when electron-positron production is small either because the temperature is too low ($T \lesssim 10^8$ °K) or degeneracy supresses it. The neutrino luminosity of a star can greatly exceed its photon luminosity for a central core temperature greater than 10^8 °K.

I. INTRODUCTION

THE Fermi coupling of electron and neutrino pairs is a consequence of many postulated forms for a universal Fermi interaction. An intermediate heavy boson, the representation of a universal four-fermion interaction in the form[1] $J_\lambda J^\lambda$, or the equivalence of $(\mu\nu_\mu)$ and $(e\nu_e)$ in all weak interactions, would lead to an interaction between the pairs[2] $(e\nu)(e\nu)$ to lowest order in g. A simple rearrangement then gives an interaction in the form $g(ee)(\nu\nu)$. An electron could then radiate a neutrino pair as well as electromagnetic radiation, although with an enormously decreased probability. The detection of such radiation in any terrestrial experiment seems at best remote, but Pontecorvo,[3] Chiu and Morrison,[4] and others have emphasized that in stars such radiation may be of great significance in certain stages of stellar evolution. The mean free path of low-energy neutrinos is sufficiently large that they will always escape from a star without interaction, whereas the electromagnetic radiation diffuses out very slowly from the hot stellar core. In a star that has evolved off the main sequence, such neutrino pair emission may become the chief means of energy loss at certain times. Specific calculations have been performed for the neutrino pair emission from electron-positron pair annihilation,[4,5] by a photon in a Coulomb field,[6,7] by photon-electron collisions,[8,9] and by neutrino bremsstrahlung.[4,10] Here we investigate the special effects of the very hot ($T>10^7$°K), very dense ($\rho>10^5$ g/cc) plasma from which the emission is presumed to take place.

The collective modes of the plasma (plasmons) can play a significant and even dominant role in the neutrino pair radiation. As long as the plasma frequency ω_0 is not negligible in comparison with $\kappa T/\hbar$ the collective behavior of the plasma is often more significant than effects from single photons or electrons. In Fig. 1 the region where such collective effects are expected to be significant is exhibited. It coincides with the region of temperatures and densities that have been conjectured for the cores of white dwarfs, red giants, and prenova or supernova stars.

The neutrino-pair decay of a free photon is forbidden by gauge invariance or energy-momentum conservation only if $\omega^2=k^2c^2$ or $\omega^2<k^2c^2$. But in a plasma the dielectric constant is approximately $1-\omega_0^2/\omega^2$, so that the transverse electromagnetic waves (transverse plasmons) have a spectrum of the form $\omega^2=\omega_0^2+k^2c^2$. For ω less than ω_0 there is no propagation; for $\omega>\omega_0$ the wave has the

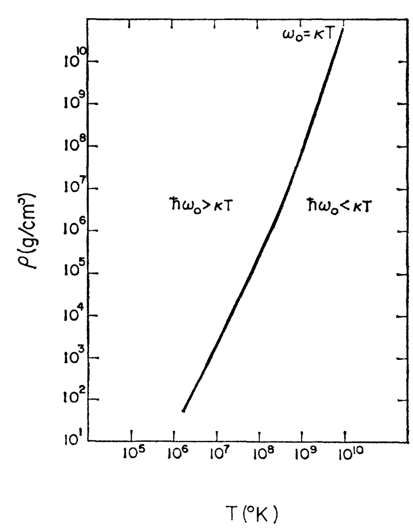

FIG. 1. The density ρ at which for a given temperature T the plasma frequency $\omega_0=\kappa T/\hbar$.

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relation between frequency and wave number of a particle of rest mass $\hbar\omega_0/c^2$, and such a plasmon can decay into a ν-$\bar{\nu}$ pair. A similar situation exists for the longitudinal plasmon. Moreover, the plasmon decay rate into neutrino pairs can be simply calculated once ω_0 is known.

In Sec. II the formalism for the calculation of the dispersion law of plasmons in a relativistic plasma is reviewed. The quantization of these collective modes is discussed in Sec. III, and related to ν-$\bar{\nu}$ emission in Sec. IV. Section V presents explicit approximate formulas for the relation between ω and k of the plasmon, together with numerical values for the neutrino-pair emission. In Sec. VI we discuss the validity of the approximation that the plasmons are adequate normal modes, i.e., that they have negligible damping. Application to stellar models and comparison with other mechanisms of neutrino-pair emission is given in Secs. VII and VIII.

II. COLLECTIVE MODES AND ELECTROMAGNETIC WAVES IN A STELLAR PLASMA

Conventional quantum electrodynamics assumes, in the absence of electromagnetic interactions, a vacuum in which all negative energy states are filled and all positive energy states are empty. The same formalism describes the QED of a relativistic electron gas when the vacuum is replaced by a new state in which, in addition, certain positive energy states are filled in a manner given by the distribution function of the Fermi gas. At sufficiently high temperatures, $\kappa T \gg m_e c^2$, electron-positron pairs must be included in this non-interacting ground state. The hole formalism of Feynman diagrams is applicable for the description of interactions, with the "hole" now standing for the absence of either a positive or a negative electron from the ground state.

The Green's function for the propagation of an electron is modified to take account of the sea of electrons through which it moves. This Green's function is[11]

$$G(x)=\frac{i}{(2\pi)^4}\int_{(\infty)} d^4p \exp[-ip_\lambda x_\lambda+i\mu t]$$
$$\times\frac{1}{2E_{\mathbf{p}}}(m+E_{\mathbf{p}}\gamma_4-\boldsymbol{\gamma}\cdot\mathbf{p})\frac{1-n^-(\mathbf{p})}{p_4+\mu-E_{\mathbf{p}}+i\delta}$$
$$+\frac{n^-(\mathbf{p})}{p_4+\mu-E_{\mathbf{p}}-i\delta}(m-E_{\mathbf{p}}\gamma_4-\boldsymbol{\gamma}\cdot\mathbf{p})$$
$$\times\frac{1-n^+(\mathbf{p})}{p_4+\mu+E_{\mathbf{p}}-i\delta}+\frac{n^+(\mathbf{p})}{p_4+\mu+E_{\mathbf{p}}+i\delta}. \quad (1)$$

[11] We use the following conventions: Greek indices range from 1 to 4, Latin from 1 to 3; metric $(---+)$; $\gamma_5=i\gamma^1\gamma^2\gamma^3\gamma^4$. The momentum k will occur in the following ways: $k^\mu=(\mathbf{k},\omega)$, $\hat{k}=\mathbf{k}/|\mathbf{k}|$, $k=|\mathbf{k}|$. Boltzmann's constant is κ. Units $\hbar=c=1$ with $m_e=1$, $e^2=1/137$.

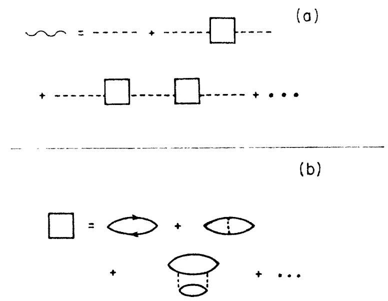

FIG. 2. Feynman diagrams that contribute to the dielectric constant in a plasma. The dotted lines are photons; the wavy line is a transverse or a longitudinal plasmon; the solid lines represent electrons or positrons and the holes in the unperturbed electron gas.

Here $E_{\mathbf{p}}=(\mathbf{p}^2+m_e^2)^{1/2}$, δ is a positive infinitesimal, μ is the chemical potential, $n^+(\mathbf{p})$ is the distribution in momentum space of positrons, and $n^-(\mathbf{p})$ is the distribution of electrons.

We now consider the propagation of an electromagnetic disturbance in such a medium, with which it continuously interacts. This interaction may be described as the sum of the Feynman diagrams in Fig. 2(a), in which each box represents in turn the sum of all irreducible Feynman graphs between two photon lines such as are pictured in Fig. 2(b). The operator corresponding to one box is a tensor that we call $\Pi_{\mu\nu}(\mathbf{k},\omega)$, which is a function of both the energy and the wave number of the electromagnetic wave. The corresponding field equation satisfied by the plane electromagnetic wave is, in momentum space,

$$(k_\lambda k^\lambda g_{\mu\nu}-k_\mu k_\nu-4\pi\Pi_{\mu\nu})A_\nu=j_\mu, \quad (2)$$

where A_ν is the electromagnetic four-potential and j_μ is the external four-current. Because gauge invariance implies that $\Pi_{\mu\nu}$ is normal to k,

$$k^\mu\Pi_{\mu\nu}=0,$$

and because the medium is considered isotropic and $\Pi_{\mu\nu}=\Pi_{\nu\mu}$, one may put the polarization operator in the form[12]

$$\Pi_{\mu\nu}(\mathbf{k},\omega)=(4\pi)^{-1}\omega^2(\epsilon^t-1)(P_T)_{\mu\nu}+(4\pi)^{-1}k_\lambda k^\lambda(\epsilon^l-1)(P_L)_{\mu\nu}, \quad (3)$$

where the projection operators are given by

$$(P_L)_{\mu\nu}=e_\mu e_\nu, \quad e_\mu=[k_\lambda k^\lambda]^{-1/2}(\omega\hat{k},k)$$
and
$$(P_T)_{ij}=\delta_{ij}-\hat{k}_i\hat{k}_j, \quad (P_T)_{i4}=(P_T)_{44}=0, \quad (4)$$

and ϵ^t and ϵ^l, the dielectric constants, are functions of $|\mathbf{k}|$ and ω, and of the momentum distribution in the electron background plasma. Substituting expression (3) into the field equation (2), and setting the external

[12] We follow here V. N. Tsytovich, Zh. Experim. i Teoret. Fiz. **40**, 1775 (1961) [translation: Soviet Phys.—JETP **13**, 1249 (1961)].

current equal to zero, we find that the equation has two independent solutions: One is a longitudinal plasmon which has the dispersion relation

$$\epsilon^l(\omega,\mathbf{k})=0, \tag{5}$$

and the other is a transverse plasmon which in the case of large k is the usual transverse electromagnetic wave with the dispersion relation

$$\omega^2\epsilon^t(\omega,\mathbf{k})=k^2. \tag{6}$$

III. QUANTIZATION

For neutrino emission by a stellar plasma the most effective collective modes have energies that are not large compared with $\kappa T/\hbar$, and the quantization of the mode amplitude is a dominant feature. Equations (5) and (6) give the spectral dispersion relation for the plasmons, i.e., $\omega(k)$, but not its amplitudes. A plasmon of frequency ω consists of oscillating electromagnetic fields coupled with electrons moving with the same frequency ω. Quantized electromagnetic waves in a dispersive medium have been described by Watson and Jauch.[13] The plasmon vector potential operator is the sum of transverse and longitudinal parts, $A=A_l+A_t$. The longitudinal part can be written

$$A_l{}^\mu(\mathbf{x},t)=(2\pi)^{-3/2}\int d^3k\,[\omega^2\partial\epsilon^l/\partial\omega]^{-1/2}\times\{\eta_l{}^\mu(\mathbf{k})a(\mathbf{k})\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x})]+\eta_l{}^\mu(\mathbf{k})a^\dagger(\mathbf{k})\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x})]\}. \tag{7}$$

In this expression $a^\dagger$, a are creation and destruction operators, respectively, for longitudinal plasmons,

$$[a(\mathbf{k}),a^\dagger(\mathbf{k}')]=\delta^3(\mathbf{k}-\mathbf{k}'), \tag{8}$$

and $\eta_l{}^\mu$ is a polarization vector: $\eta_l{}^\mu=e^\mu$ defined in (4). The normalization factor in the integrals is such that the plasmon's energy (electromagnetic+interaction +electron energy) equals $(n+1/2)\hbar\omega$. The derivation of its form is given in the Appendix.

The transverse electromagnetic potential operator is

$$A_t{}^\mu(\mathbf{x},t)=(2\pi)^{-3/2}\sum_s\int d^3\mathbf{k}\,[\omega(2\epsilon^t+\omega\partial\epsilon^t/\partial\omega)]^{-1/2}\times\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x})]\{\eta_{ts}{}^\mu(\mathbf{k})b_s(\mathbf{k})\times\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x})]+\eta_{ts}{}^\mu(\mathbf{k})b_s{}^\dagger(\mathbf{k})\times\exp[i(\omega t-\mathbf{k}\cdot\mathbf{x})]\}, \tag{9}$$

where b, $b^\dagger$ are destruction and creation operators, respectively, and s is a polarization index with values 1 and 2; $\eta_{t1}{}^\mu$ and $\eta_{t2}{}^\mu$ are two polarization unit vectors

[13] K. M. Watson and J. M. Jauch, Phys. Rev. 75, 1249 (1949).

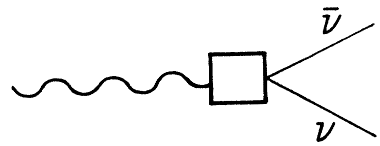

FIG. 3. Feynman diagram for the neutrino-pair decay of a plasmon.

that for $\eta^\mu=(\boldsymbol{\eta},0)$, satisfy

$$\boldsymbol{\eta}_{t1}\cdot\boldsymbol{\eta}_{t2}=0,$$
$$\boldsymbol{\eta}_{ts}\cdot\mathbf{k}=0,$$

and

$$\boldsymbol{\eta}_{ts}{}^2=1\quad(s=1,2). \tag{10}$$

IV. NEUTRINO PAIR EMISSION FROM THE COLLECTIVE MODES

The coupling of the electron-positron field to the plasma electromagnetic field A is, of course, still given by the interaction Hamiltonian,

$$\mathcal{H}_I=(4\pi)^{1/2}e\bar{\psi}_e\gamma_\mu\psi_eA^\mu. \tag{11}$$

By using the above formalism it is possible to compute processes involving plasmons by the usual rules of QED, among them plasmon neutrino-pair decay. The proposed $(e\nu)(e\nu)^\dagger$ coupling can be written

$$\begin{aligned}\mathcal{H}_W&=g(2)^{-1/2}\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_\nu\bar{\psi}_\nu\gamma^\mu(1+\gamma_5)\psi_e+\text{H.c.}\\&=-g(2)^{-1/2}\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_e\bar{\psi}_\nu\gamma^\mu(1+\gamma_5)\psi_\nu+\text{H.c.},\end{aligned} \tag{12}$$

where $g=3.08\times10^{-12}m_e{}^{-2}$ is the weak coupling constant.

The decay rate of the transverse plasmon and that of the longitudinal plasmon are computed separately. Each is conveniently subdivided further into the rate from vector and pseudovector couplings. We write the weak interaction Hamiltonian as the sum of two parts,

$$\begin{aligned}\mathcal{H}_W=&-g(2)^{-1/2}\bar{\psi}_e\gamma_\mu\psi_e\bar{\psi}_\nu\gamma^\mu(1+\gamma_5)\psi_\nu\\&-g(2)^{-1/2}\bar{\psi}_e\gamma_\mu\gamma_5\psi_e\bar{\psi}_\nu\gamma^\mu(1+\gamma_5)\psi_\nu.\end{aligned} \tag{13}$$

The matrix element for the process pictured in Fig. 3 has two parts: one from the electron vector current[14] part of the $\mathcal{H}_W$ and the other from the electron pseudovector part of $\mathcal{H}_W$. When the matrix element is squared, the cross term of these two parts is a pseudoscalar and contributes nothing when integrations to compute decay rates are carried out. The axial vector current vanishes in nonrelativistic limit (the collective modes have charge but no net spin variation), and because the momenta of interest are not highly relativistic we expect that the contribution of the electron axial vector current to the decay rates is much smaller than that from the electron vector current in stellar plasmas.

[14] Electromagnetic corrections to the vector current, which to lowest order in $\mathcal{H}_W$, but all orders in e (Z_3 is the charge renormalization of electrodynamics) effectively replace g by $gZ_3{}^2$ are explicitly ignored. We also ignore the possibility of direct $(ee)(\nu\nu)$ coupling, which might cancel the $(e\nu)(e\nu)^+$ coupling. [B. Pontecorvo, Phys. Letters 1, 287 (1962), and S. Bludman, Nuovo Cimento 9, 433 (1958).]

We consider the vector current contribution first and deal with axial vector contribution separately.

The plasmon can decay via the mechanism pictured in Fig. 3, which is just Landau damping into neutrino pairs, where—if we are considering only electron vector contributions— the box represents exactly the sum of irreducible Feynman graphs pictured in Fig. 2(b) and represents the operator $\Pi_{\mu\nu}(\mathbf{k},\omega)$ defined above. The decay rate of longitudinal plasmons, τ_l^{-1} (from the vector current), is conveniently expressed in terms of ϵ^l. The dispersion relation (5) allows one to set $\epsilon^l=0$, and the result is found by the usual rules of QED to be

$$\tau_l^{-1}=(8/3)g^2(4\pi e)^{-2}(\omega^2\partial\epsilon^l/\partial\omega)^{-1}(\omega^2-\mathbf{k}^2)^3. \tag{14}$$

Similarly, the decay rate of a transverse plasmon τ_t^{-1} can be calculated as

$$\tau_t^{-1}=(8/3)g^2(4\pi e)^{-2}[\omega(2\epsilon^t+\omega\partial\epsilon^t/\partial\omega)]^{-1}\times(\omega^2-\mathbf{k}^2)[\omega^2(\epsilon^t-1)]^2. \tag{15}$$

To obtain the energy loss rate per unit volume from neutrino-pair radiation we integrate over the density of plasmons. Neglecting the damping of the plasmon states, one obtains the plasmon density from a boson thermal distribution. The neutrino-pair emission rates per unit volume from transverse and longitudinal plasmons are then

$$Q_t=\int\tau_t^{-1}\omega[e^{\omega\beta}-1]^{-1}(2\pi)^{-3}(8\pi)k^2dk, \tag{16}$$

and

$$Q_l=\int\tau_l^{-1}\omega[e^{\omega\beta}-1]^{-1}(2\pi)^{-3}4\pi k^2dk, \tag{17}$$

where $\beta=(\kappa T)^{-1}$. The dielectric constants ϵ^t, ϵ^l are functions both of k and of ω and depend also on the momentum distribution of electrons in the plasma. The energy ω is given as an implicit function of k in the dispersion relations Eqs. (5) and (6).

V. THE DISPERSION RELATIONS OF THE LONGITUDINAL AND TRANSVERSE PLASMA MODES

To calculate the emissivities Q_l and Q_t we must now estimate $\epsilon^{l,t}(\mathbf{k},\omega)$. In the sum for $\Pi_{\mu\nu}$ represented in Fig. 1(b) all terms but the first, the single-loop integral, are dropped. In the limit of infinite density this should be an excellent approximation. For our applications to stellar plasmas, the interelectron spacing is always very small compared with the Bohr radius. With this approximation an integral form for the dielectric constants has been computed by Tsytovitch.[12] We quote the relevant results here:

$$\epsilon^{l,t}=1-\frac{4\pi e^2}{\omega^2}\int f(E_\mathbf{p})d^3\mathbf{p}\left\{\frac{E_\mathbf{p}-E_{\mathbf{p-k}}}{(E_\mathbf{p}-E_{\mathbf{p-k}})^2-\omega^2}\Lambda^{l,t}+\frac{E_\mathbf{p}+E_{\mathbf{p-k}}}{(E_\mathbf{p}+E_{\mathbf{p-k}})^2-\omega^2}\Lambda_+^{l,t}\right\}+\delta\epsilon_B^{l,t}, \tag{18}$$

$$\Lambda_\pm^l=1\pm\frac{E^2_\mathbf{p}+(\mathbf{p}\cdot\mathbf{k})-2(\mathbf{p}\cdot\mathbf{k})^2/\mathbf{k}^2}{E_\mathbf{p}E_{\mathbf{p-k}}}, \tag{19}$$

$$\Lambda_\pm^t=1\pm\frac{m^2-(\mathbf{p}\cdot\mathbf{k})+(\mathbf{p}\cdot\mathbf{k})^2/\mathbf{k}^2}{E_\mathbf{p}E_{\mathbf{p-k}}},$$

and

$$f(E_\mathbf{p})=2(2\pi)^{-3}[n^-(\mathbf{p})+n^+(\mathbf{p})], \tag{20}$$

where symbols are defined as for Eq. (1). The terms $\delta\epsilon_B^{l,t}$ represent vacuum polarization effects and are divergent. This divergence is removed in the well-known way by charge renormalization. The remaining finite vacuum polarization effect is not important compared with the effect of the polarization of the plasma for a stellar plasma.

For stellar plasma we content ourselves with an approximate result to avoid numerical integration. In the applications of interest the conditions $k\ll m$ and $\omega\ll 2m$ obtain, so we can neglect terms in $\mathbf{k}$ and ω in the integrands in the expressions for ϵ^t, ϵ^l. In this case both dielectric constants reduce to the well-known form

$$\epsilon=1-\omega_0^2/\omega^2, \tag{21}$$

where

$$\omega_0^2=4\pi e^2\int d^3\mathbf{p}\,\frac{f(E_\mathbf{p})}{E_\mathbf{p}}\left(1-\frac{1}{3}\frac{\mathbf{p}^2}{E_\mathbf{p}}\right). \tag{22}$$

For a degenerate Fermi sea this becomes

$$\omega_0^2=4e^2p_F^3/(3\pi E_F), \tag{23}$$

where by E_F we mean the relativistic energy, including rest energy, of an electron with the Fermi momentum p_F $[E_F=(p_F^2+m^2)^{1/2}]$. This plasma frequency is used in Fig. 1.

To get an idea of the validity of neglecting the higher-order terms in k^2 and ω^2, we can carry out the integrals of Eq. (18), retaining terms of order k^2 and ω^2. Under this approximation

$$\epsilon^t=1-\frac{4\pi e^2}{\omega^2}\int d^3\mathbf{p}\,f(E_\mathbf{p})\times\left\{\frac{1}{E_\mathbf{p}}\left[1-\frac{1}{3}\frac{\mathbf{p}^2}{E_\mathbf{p}^2}\right]+\frac{\omega^2}{4E_\mathbf{p}^3}\left[1-\frac{1}{3}\frac{\mathbf{p}^2}{E_\mathbf{p}^2}\right]+\frac{k^2}{2E_\mathbf{p}^3}\left[\frac{\mathbf{p}^2}{E_\mathbf{p}^2}-1-\frac{1}{3}\frac{\mathbf{p}^4}{E_\mathbf{p}^4}\right]+\frac{k^2\mathbf{p}^2}{\omega^2E_\mathbf{p}^3}\left[\frac{1}{3}-\frac{1}{5}\frac{\mathbf{p}^2}{E_\mathbf{p}^2}\right]\right\}, \tag{24}$$

$$\epsilon^l=1-\frac{4\pi e^2}{\omega^2}\int d^3\mathbf{p}\,f(E_\mathbf{p})\times\left\{\frac{1}{E_\mathbf{p}}\left[1-\frac{1}{3}\frac{\mathbf{p}^2}{E_\mathbf{p}^2}\right]+\frac{\omega^2}{4E_\mathbf{p}^3}\left[1-\frac{1}{3}\frac{\mathbf{p}^2}{E_\mathbf{p}^2}\right]+\frac{k^2}{2E_\mathbf{p}^3}\left[\frac{2\mathbf{p}^2}{E_\mathbf{p}^2}-1-\frac{\mathbf{p}^4}{E_\mathbf{p}^4}\right]+\frac{k^2\mathbf{p}^2}{\omega^2E_\mathbf{p}^3}\left[1-\frac{3}{5}\frac{\mathbf{p}^2}{E_\mathbf{p}^2}\right]\right\}. \tag{25}$$

These expressions can easily be integrated for a zero-temperature Fermi distribution. If we consider a density of 4×10^5 g/cc of pure helium, p_F is 0.59 m_e and integration of these equations shows that as long as ω and $\mathbf{k}$ are small compared with m_e the approximation of Eq. (21) is good to a few percent. If we now use Eq. (23) for $\epsilon^{t,l}$ in the dispersion relations, we find that for the longitudinal plasmons $\omega^2=\omega_0^2+\alpha k^2$, where α is always much less than 1, and for the transverse ones $\omega^2=\omega_0^2+k^2$. With these expressions for the dielectric constant and the dispersion relations, the longitudinal neutrino-pair emission energy loss rate can be written

$$Q_l=\frac{g^2}{3\pi e^2}\frac{1}{(2\pi)^3}\frac{1}{e^{\omega_0\beta}-1}\int_0^{\omega_0}dk\,k^2(\omega_0^2-k^2)^3$$

$$=\frac{g^2}{3\pi e^2}\frac{1}{(2\pi)^3}\frac{1}{e^{\omega_0\beta}-1}\omega_0^9\left(\frac{16}{315}\right)$$

$$Q_l/(\text{ergs/cc sec})=3.15\times10^{20}(\omega_0/m_ec^2)^9(e^{\omega_0\beta}-1)^{-1}. \quad (26)$$

The upper limit $k=\omega_0$ is set by energy-momentum conservation. For $k<\omega_0$ the term αk^2 can be neglected, in comparison with ω_0^2, with small error.

We use the above dielectric constant (21) and dispersion relation (6) and the expression $[\exp(\omega\beta)-1]^{-1}=\sum_{n=1}^{\infty}\exp(-n\omega\beta)$ in Eq. (16) to obtain the following form for the transverse emissivity:

$$Q_t=2g^2(3\pi e^2)^{-1}(2\pi)^{-3}\omega_0^6\sum_{n=1}^{\infty}\int_0^{\infty}\exp(-n\beta\omega)k^2dk. \quad (27)$$

The integral in this expression can be expressed in terms of modified Hankel functions of the zeroth and first order, or by the following approximation. We distinguish two cases: $\omega_0\beta\ll1$ and $\omega_0\beta\gg1$. If $\omega_0\beta\ll1$, then the exponential factor is negligible for all values of k until $k\gg\omega_0$, when $\omega\approx k$. Then

$$Q_t=\frac{4g^2}{3\pi e^2}\frac{\omega_0^6}{\beta^2}\frac{1}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{1}{n^3}$$

or

$$Q_t/(\text{ergs/cc sec})=2.96\times10^{22}(\omega_0/m_ec^2)^6(\beta m_ec^2)^{-3}. \quad (28)$$

If $\omega_0\beta\gg1$, then as k increases the exponential factor will suppress the integrand when k is still very much smaller than ω_0, so that we can approximate $\omega=\omega_0+k^2/2\omega_0$. The emissivity is then

$$Q_t=\frac{2g^2}{3\pi^2e^2}\frac{\omega_0^6}{(2\pi)^3}\sum_{n=1}^{\infty}\exp(-n\omega_0\beta)\int_0^{\alpha}\exp\left(-\frac{k^2\beta n}{2\omega_0}\right)k^2dk$$

$$\approx(2\pi)^{1/2}\frac{g^2}{3\pi e^2}\frac{\omega_0^{7.5}}{(2\pi)^3}\beta^{-1.5}e^{-\omega_0\beta}$$

or

$$Q_t/(\text{ergs/cc sec})=1.54\times10^{22}(\omega_0/m_ec^2)^{7.5}(\beta m_ec^2)^{-1.5}\times\exp(-\omega_0\beta). \quad (29)$$

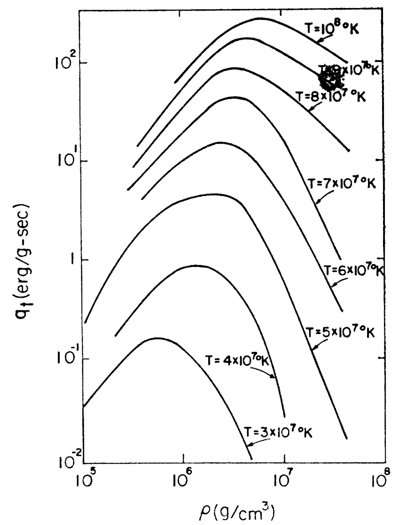

FIG. 4. Emissivity from transverse plasmons q_t as a function of density ρ and temperature.

Neither of these approximations is accurate in the region $\omega_0\beta\approx1$; we make a smooth interpolation of two approximations through this region on a log-log plot in Fig. 4.

The energy rates per unit mass q_t $(=Q_t/\rho)$ and q_l $(=Q_l/\rho)$ have been plotted in Figs. (4) and (5) for a medium of $(Z/A)=(1/2)$. It should be noted that temperature-density regions are included where the gas is nondegenerate. However, as long as $\kappa T\lesssim m_ec^2$ $(T\approx10^{10}\ ^\circ\text{K})$, Eq. (24) for the plasma frequency is quite insensitive to the temperature. The exact $f(E_p)$ in Eq. (23) would give only a slightly smaller value for ω_0.

We return now to the effect of the axial vector current at the weak-interaction vertex. For the longi-

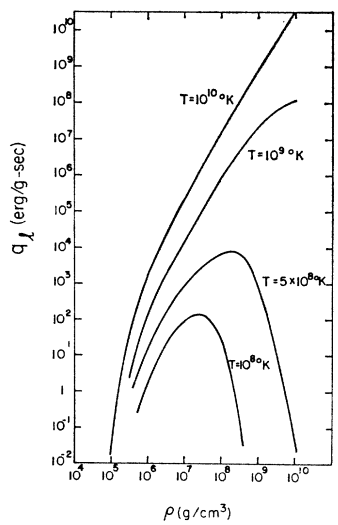

FIG. 5. Emissivity from longitudinal plasmons q_l as a function of plasma density and the temperature.

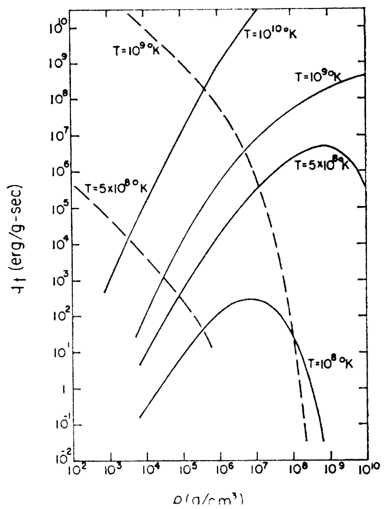

FIG. 6. Comparison of emissivity from transverse plasmons q_t with that from electron-positron pair annihilation (dotted lines), as a function of plasma density and temperature.

tudinal plasmon the decay rate due to the axial vector current can be seen to vanish from momentum conservation and parity considerations. For transverse plasmons the axial vector current does contribute to the decay rate, denoted τ_5^{-1}. If in computing the loop integral

$$\Pi_{\mu\nu}'(\mathbf{k},\omega)=e^2(2\pi)^{-4}\int d^4p\,\mathrm{Tr}[\gamma_\mu G(p+k)\gamma_\nu\gamma_5 G(p)],$$

$G(p)$ being the Fourier transform of $G(x)$, we retain terms to order k, the rate τ_5^{-1} is

$$\tau_5^{-1}=\frac{8}{27}(2\pi)^4g^2e^2\frac{k^2\omega_0^2}{\omega}\left(\frac{p_F^3}{p_F^2+m^2-(\omega/2)^2}\right)^2. \quad (30)$$

Assuming $\omega\ll m$, the luminosity can again be integrated, and it is seen to be considerably smaller than Q_t and Q_l computed earlier for the range of temperature and pressure of interest:

$$Q_5/(\text{ergs/cc sec})=2\times10^{16}\left(\frac{p_F^3c}{E_F^2m_e}\right)\left(\frac{\omega_0}{m_ec^2}\right)^{4.5}\times(\beta m_ec^2)^{-2.5}e^{-\omega_0\beta}. \quad (31)$$

We drop this term.

VI. VALIDITY OF THE PLASMON EXCITATIONS AS NORMAL MODES

Finally, we consider the validity of treating the plasmons as normal modes in our computation. We have assumed that a plasmon is a reasonable approximation to a normal mode of the system and have ignored the possibility that plasmons can decay by other channels than neutrino-pair creation. Even with the loop integral approximation (second order in e) for $\epsilon^{t,l}$, we have ignored an imaginary part for $\epsilon^{t,l}$. This imaginary part for $\epsilon^{t,l}$ represents plasmon decay into real electron positron pairs and Landau damping.[15] Energy momentum conservation implies for both transverse and longitudinal plasmons that real electron-pair creations can occur only if $\omega_0>2m$, which is never the case in our study. Landau damping occurs only if $k>\omega$; this never obtains for transverse plasmons. In longitudinal plasmons this mechanism is relevant when $k>\omega_0\approx\omega$, but only for $k<\omega_0\approx\omega$ is neutrino-pair creation possible, and so this damping mechanism will not affect the momentum distribution in the region of interest. It has been recently pointed out[16] that in nondegenerate nonrelativistic plasmas damping due to third-order (in e) effects (collision damping), in which two of the plasma electrons are excited, may be more important than Landau damping. The nondegenerate nonrelativistic case has been calculated. It would be expected that degeneracy would reduce the number of final accessible states and that the available formulas will overestimate damping in that case. If the degeneracy is neglected, $\mathrm{Im}\omega/\mathrm{Re}\omega_0$ is small (e.g., $\rho=10^5$ g/cc, $T=10^8\,^\circ$K, $\mathrm{Im}\omega/\mathrm{Re}\omega_0\approx10^{-4}$).

VII. COMPARISON WITH OTHER MECHANISMS FOR NEUTRINO-PAIR EMISSION

The neutrino-pair energy loss rate can be compared to that due to four other mechanisms[17] that have been proposed. Neutrino bremsstrahlung was proposed by Pontecorvo, and rates were calculated by Gandel'man and Pineau.[10] Their results give an energy-loss rate

$$q/(\text{ergs/g sec})=5.9\times10^{-42}(\rho/\nu\mu_e\ \text{g/cc})(T/^\circ\text{K})^{4.5}, \quad (32)$$

where $\nu^{-1}=\sum_i c_iZ_i^2/A_i\mu_e^{-1}=\sum_i c_iZ_i/A_i$, and c_i is the fractional concentration of the elements by weight. Chiu and Stabler,[9] and independently Ritus,[8] have computed neutrino Compton radiation $\gamma+e\rightarrow e+\bar{\nu}+\nu$. The results given by Ritus are

$$q/(\text{ergs/g sec})=1.67\times10^{-64}\mu_e^{-1}(T/^\circ\text{K})^8, \quad (33)$$

for nondegenerate electrons;

$$q(\text{ergs/g sec})=6.9\times10^{-71}(\rho)^{-1/3}(T/^\circ\text{K})^9\mu_e^{-2/3}, \quad (34)$$

for highly degenerate nonrelativistic electrons.

Emissivity due to neutrino-pair creation by γ rays in a Coulomb field has been calculated by Matinyan and Tsilosani.[7] Their result is[17a]

$$q/(\text{ergs/g sec})=2\times10^{-50}\nu^{-1}(T/^\circ\text{K})^6. \quad (35)$$

[15] See, e.g., reference 12.

[16] D. F. Dubois, V. Gilinsky, and M. G. Kivelson, Phys. Rev. Letters 8, 419 (1962).

[17] These are in addition to the URCA process of Gamow and Schönberg, which gives a single neutrino from electron capture by individual nuclei. The energy-loss rate is essentially zero in He, and otherwise very dependent on the assumed presence of specific nuclei with low threshold for electron capture.

[17a] *Note added in proof.* Dr. Leonard Rosenberg (private conversation) has pointed out the lack of gauge invariance in this result. He finds a smaller result at $T=10^9\,^\circ$K and a T^{10} temperature dependence. G. Marx and J. Nem'rath estimate $q=10^3$ ergs/g sec

These rates are smaller than the energy-loss rate due to plasmon ν-pair decay for $\log_{10}\rho \gtrsim 4[\log_{10}T - 7]$. None of these earlier calculations has taken account of the effect of the dense plasma on the electron propagator or of the effect of the plasma on the electromagnetic waves, especially the plasma cutoff, which greatly decreases the energy-loss rate due to neutrinos for densities and temperature that lie above the curve of Fig. 1.

Chiu[6] has investigated the energy-loss rate due to electron-positron pair annihilation suggested by Chiu and Morrison.[5] His results have been plotted in Fig. 6, together with the emissivities of Eqs. (28) and (29). For low densities and for high temperatures the electron-positron effect is larger than the plasmon effect. Below 10^9°K the collective radiation always dominates for $\rho > 5\times10^6$ g/cc and generally falls off very much less rapidly with decreasing temperature at smaller densities.

VIII. APPLICATIONS TO STELLAR EVOLUTION

In the theory of stellar structure the density and temperature of stellar interiors is such that the neutrino emissivity is negligible for stars on the main sequence. Only at later stages of evolution during periods of core temperature much greater than 10^7°K and high density may the energy loss due to neutrino-pair emission play a significant role. Some effects of neutrino-pair emission have been discussed by Chiu[18] for stars in which core temperatures exceed 10^9°K. Two types of stars are recognized as possessing dense hot cores: white dwarfs and red giants. White dwarfs are presumed to be cooling by surface radiation with no interior energy generation. Estimated central densities in a white dwarf are about 10^5 to 10^7 g/cc. The observed bright white dwarf, Sirius B, which is estimated to have a core temperature of 1.8×10^7°K and a density of 10^5 g/cc, has a photon luminosity about 10^3 times as large as the calculated neutrino luminosity.[19] Most other white dwarfs are estimated to have lower interior temperatures, and consequently an even smaller ratio of neutrino-to-photon luminosity. Only if the internal temperature approached 5×10^7°K would neutrino-pair emission play a significant role in the cooling process.

A red giant burns hydrogen outside of a dense 10^5- to 10^6-g/cc core of inert helium. As more and more helium is added to the core it contracts, and heats by conversion of gravitational to thermal energy. The heating continues until the core reaches a temperature of about 8×10^7°K, when helium burning commences and the star enters the next stage of evolution. Two types of red giants may be distinguished by their masses: those with mass greater than twice the mass of the sun, and those with mass less than 1.1 solar mass. The core temperature in the heavy red giants is such that the electrons are not degenerate, nor do they ever become degenerate as core contraction proceeds. If a thermal energy sink such as neutrino emission is placed in the core, the core contracts somewhat faster, and heats rather than cools (according to the virial theorem). Under nondegenerate conditions the neutrino-pair emission simply accelerates the evolution of the star to the point at which helium burning starts generating energy at a rate very much greater than the rate of energy loss by neutrino emission. In light red giants (mass$\lesssim$mass of the sun) the core is largely degenerate. In a degenerate core a neutrino energy sink results in cooling, since very little gravitational contraction takes place. According to a preliminary calculation by Hoyle and Schwarzschild,[20] the core density varies between 10^5 and 10^6 g/cc as the temperature grows in the temperature range 10^7 to 10^8°K, because of gravitational contraction before He burning begins. The time scale for this contraction is estimated to be between 10^6 and 10^7 yr. The rate of increase in thermal energy of the core from a gravitational contraction would then be of the same order of magnitude as the loss of thermal energy by neutrino-pair emission, which can approach 100 ergs/g sec in this interval. Whether or not it would substantially delay or even prevent the ignition of He in the degenerate core of a light red giant depends upon a reasonably detailed model of the density temperature and time scale for the development of the red-giant core.

The relevance of neutrino-pair emission from the electron-positron annihilation to the evolution of a heavier star into a nova or supernova has been emphasized by Chiu.[18] Here the temperature of the star of interest is above 10^9°K. For a core density of $\rho \approx 2\times10^6$ g/cc and $T = 10^9$°K, the radiation of neutrino-antineutrino pairs by plasmons is greater than 3×10^6 ergs/g sec, which is about the same as that from $e^- - e^+$ annihilation. The neutrino luminosity of the star would be about 10^6 times the luminosity of the sun. For $\rho \gtrsim 2\times10^6$ g/cc, $T = 10^9$°K, the plasmon $\nu - \bar{\nu}$ radiation exceeds that from $e^- - e^+$ annihilation, in part because electron degeneracy surpresses the formation of $e^- - e^+$ pairs, and in part because the plasmon-pair emission rises as the density and plasma frequency increase. For slightly lower temperature the plasmon emission begins to dominate at much lower densities.

The effect of neutrino pair emission on the abundance of elements in stars has been considered by Chiu and Stothers.[21] They show that addition of $e^- - e^+$ annihilation to a stellar model can considerably reduce the time scale of certain phases of evolution and hence modify the relative abundance of the elements. Time scales of evolution associated with pair annihilation and with

at $T = 1.0\times10^9$ °K and $q = 10$ ergs/g sec at $T = 0.6\times10^9$ °K [Proceedings of 1962 Annual International Conference on High-Energy Physics at CERN (CERN, Geneva, 1962)].

[18] H. Y. Chiu, Ann. Phys. (N. Y.) **15**, 1 (1961); **16**, 321 (1961).

[19] E. Schatzman, *White Dwarfs* (North-Holland Publishing Company, Amsterdam, 1958).

[20] M. Schwarzschild, *Structure and Evolution of the Stars* (Princeton University Press, Princeton, New Jersey, 1958), p. 217.

[21] R. Stothers and H. Y. Chiu, Astrophys. J. **135**, 963 (1962).

TABLE I. Time scales of evolution for a stellar density of $\rho=2\times10^6$ g/cc.

T(°K)	Pair annihilation only		Plasmon decay only	
	$q_p/q_\odot$	τ(yr)	$q_t/q_\odot$	τ(yr)
6×10^8	5×10^2	2×10^7	1×10^5	7×10^4
8×10^8	4×10^5	3×10^4	3×10^5	3×10^4
10^9	3×10^7	400	7×10^5	2×10^4
1.2×10^9	5×10^7	20	1×10^6	9×10^3

plasmon decay are displayed in Table I for a density of 2×10^6 g/cc and a variety of temperatures. At $T=6\times10^8$°K, plasmon decay reduces the time scale of evolution to 10^5 yr. Density rises with the cube of the temperature in a contracting nondegenerate stellar core. At slightly higher densities pair-annihilation emissivity is suppressed and plasmon decay increases (Fig. 6), so that neutrino pair emission by plasmon decay would be the dominant factor in controlling the evolutionary time scale in this temperature-density regime.

We are happy to thank Professor L. Henyey, H. Reeves and G. Wallerstein for an informative discussion.

APPENDIX

The normalization factor in the plasmon operators, Eqs. (7) and (9), is the square root of the classical energy density, including the effect of the electric field, the kinetic energy of the charges, and their potential energy. This has been given for transverse waves by Landau and Lifshitz,[22] and by Watson and Jauch.[13] We present here a slightly different derivation which is also applicable to the longitudinal waves. We suppose the D field and the E field to be related by

$$D(t,\mathbf{x})=E(t,\mathbf{x})+\int_0^\infty d\tau\int_{(\infty)} d^3\mathbf{y}\, f(\tau,\mathbf{y})E(t-\tau,\,\mathbf{x}-\mathbf{y}). \quad \text{(A1)}$$

The dielectric constant is defined by

$$\epsilon(\mathbf{k},\omega)=1+\int_0^\infty d\tau\int_{(\infty)} d^3\mathbf{y}\, f(\tau,\mathbf{y})\exp(i\omega\tau-i\mathbf{k}\cdot\mathbf{y}). \quad \text{(A2)}$$

We consider an electric wave of gradually increasing amplitude

$$E=E_0\exp(-i\omega_0 t+i\mathbf{k}\cdot\mathbf{x}+\lambda t),$$

with λ infinitesimal and positive, and E_0 constant in time. The energy density at x at a time T, $U(x,T)$ will then be

$$U(\mathbf{x},T)=\int_{-\infty}^{T}\mathrm{Re}E(t,\mathbf{x})[d\,\mathrm{Re}D(t,\mathbf{x})/dt]dt. \quad \text{(A3)}$$

Now we have

$$\mathrm{Re}E(d\,\mathrm{Re}D/dt)=\tfrac{1}{2}\,\mathrm{Re}(EdD/dt+\tfrac{1}{2}\,\mathrm{Re}(E^*dD/dt). \quad \text{(A4)}$$

The time average of the first term on the right side of this equation vanishes. After a contour integration, the second term gives

$$U(\mathbf{x},T)=\tfrac{1}{4}E_0^2e^{2\lambda T}[\mathrm{Re}\epsilon(\omega+i\lambda,\,\mathbf{k})+\omega_0\,\mathrm{Im}\epsilon(\omega_0+i\lambda,\,\mathbf{k})/\lambda]. \quad \text{(A5)}$$

If we now neglect absorption, $\mathrm{Im}\epsilon=0$, and letting $\lambda\rightarrow0$, we have

$$U(\mathbf{x},T)=\tfrac{1}{4}E_0^2\left[\mathrm{Re}\epsilon(\omega_0,\mathbf{k})+\omega_0\frac{\partial}{\partial\omega}\epsilon(\omega,\mathbf{k})\Big|_{\omega=\omega_0}\right]. \quad \text{(A6)}$$

[22] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, New York, 1960), p. 255.