Theory of Breakdown Wave Propagation*

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(Received June 18, 1962)

The propagation of electrical breakdown waves in a gas is analyzed by assuming that the wave front of the breakdown wave is an electron shock wave. A simple three-fluid hydrodynamical model is used in which the partial pressure of the electron gas behind the shock zone is the primary source of the motion. A theoretical wave velocity of 2×10^7 m/sec is predicted for an applied field of 10^4 V/m in H_2 at a pressure of 0.2 mm Hg. Since the propagation mechanism of the breakdown wave is mechanical, the model explains propagation equally well into either a positive or negative electric field. Qualitative agreement between theoretical and experimental breakdown wave velocities is obtained.

INTRODUCTION

FOR the past century, researchers have observed the propagation of luminosity fronts associated with the electrical breakdown of a gas. Thompson[1] observed that the luminosity front may travel at a speed as high as one half the speed of light. Beams[2] studied luminosity fronts in air and hydrogen and reported that the breakdown wave always moves from the electrode to which the potential is applied toward the electrode at ground potential, regardless of the polarity of the applied voltage. Von Zahn[3] showed that there is no Doppler shift in the spectrum lines which implies that the luminosity front does not result from motion of the particles emitting the observed radiation. Schonland[4] has made extensive studies of the lightning discharge, measuring the velocity of propagation of the luminosity front, sometimes called a pilot streamer (a pilot streamer is a front traveling into undisturbed gas) and the propagation velocity of the secondary front which travels down the partially ionized channel left by an earlier pilot streamer. Schonland[4] has made some predictions of the minimum propagation velocity which he obtains from qualitative energy considerations, but he has not given a theory for the propagation of these waves.

Luminosity studies of spark breakdown by Loeb[5] have indicated that potential waves are present during this phenomenon. The pertinent articles published by Loeb's group are too numerous to list separately; hence, only a few specific references are given. Loeb[5] terms the dendritic luminosity fronts observed in spark breakdown "streamers." Loeb and Meek,[5] and independently Raether,[5] first proposed a qualitative model for streamer propagation as a mechanism for electrical breakdown in a gas. The mechanism of the streamer process proposed by Loeb for a point-anode plane-cathode geometry is as follows: Photons emitted by the excited gas molecules diffuse outward from the anode, ionizing and exciting new molecules. The photons emitted by the newly excited molecules diffuse further into the gas, and the cycle of diffusion, excitation, and ionization is repeated. The net result of this process is the propagation of a photoionization wave from the anode to the cathode.

Westberg[6] recently published a rather complete study of potential waves present in the transition of a glow discharge to an arc. Fowler and Hood[7] have observed luminosity fronts propagating into a field free region.

This note introduces a new quantitative theory for potential wave propagation with particular emphasis on waves propagating into unionized regions. The ideas presented here are outgrowths of earlier work by Fowler, Paxton, and Hughes[8] and Fowler and Fried.[9] The model presented treats the potential wave front as an electron shock wave. The essence of this theory is the application of hydrodynamics to the analysis of breakdown waves. The two important ionization processes active in this model are photoionization and electron impact ionization. In the case of molecular gases, both ionization processes are active to various degrees depending on the gas and ambient conditions. In the case of atomic gases where photoionization of the gas by its own photons is not possible, either electron impact ionization or a combination of electron impact and photoionization is the active process. The potential waves predicted by this model will be compared with the experimental luminosity fronts, potential waves, and streamers observed by Thompson,[1] Beams,[2] White,[10] and Loeb.[11] This theory is a first-order approximation to an electron shock wave applicable to very restrictive conditions given below. However, this first-order approximation should certainly apply to the

* This study has been supported in part by the Office of Naval Research.

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[11] L. B. Loeb, R. G. Westberg, and H. C. Huang, Phys. Rev. **123**, 43 (1961).

low-pressure breakdowns between widely spaced electrodes observed by Thompson[1] and Beams.[2] This mechanism is also expected to be present in Loeb's[5,11] streamer propagation, to some extent. The theory of breakdown waves given here may furnish one new element, but is certainly not the whole explanation of the phenomena of streamer production.

MODEL

The breakdown wave front will be treated as an electron shock wave regardless of the nature of the ionization process. The case of electron impact ionization in an electric field will be treated here with apparent differences for the photoionization case noted.

We presume that near the electrode where the potential gradient in the gas is greatest ionization of a small quantity of gas occurs and the electrons produced are given kinetic energy by the electric field. This localized high-temperature electron gas expands producing an electron shock wave which propagates into the undisturbed gas, partially ionizing the overrun neutral gas molecules. The energy necessary for driving the shock wave is given directly to the electrons in the shock zone by the external electric field. The electron shock is followed by a rarefaction wave, but the analysis of the rarefaction wave will not be included here.

We have used a three-fluid, hydrodynamical model which is applied to a quasi-steady state three-component system. The state of the system is designated quasi-steady because, even though the electron gas has achieved a steady state, the heavy particles, due to their large inertia, are in a transient state. The time constants associated with the motion of the heavy particles are so large, however, that there is only a slight change in the kinetic energy of these particles during their brief interaction with the electron shock wave.

EQUATIONS

The steady-state equations of mass, momentum, and energy transfer for continuous media are easily derived.[12] For the case where the electron pressure is much greater than the partial pressures of the other species, where there is no electrical current, where there is negligible heat flow, and where inelastic energy losses can be neglected, the transfer equations are given by

$$(d/dx)(MNV+M_iN_iV_i+mnv)=0, \quad (1)$$

$$(d/dx)(MNV^2+M_iN_iV_i^2+mnv^2+nkT_e-\tfrac{1}{2}\epsilon_0E^2)=0, \quad (2)$$

$$(d/dx)(MNV^3+M_iN_iV_i^3+mnv^3+5nvkT_e)=0, \quad (3)$$

where M is the mass of a neutral atom, M_i is the mass of a positive ion, m is the mass of an electron, N is the neutral atom density, N_i is the positive ion density, n is the electron density, V is the flow velocity of the neutral atoms, V_i is the flow velocity of the ions, v is the flow velocity of the electrons, T_e is the electron temperature, and E is the electric field strength.

In the rest frame of the shock wave, cold neutral atoms enter the shock zone from the front side and a partially ionized gas leaves the rear side of the shock zone. If we integrate the above steady-state transfer equations across the shock zone, the following equations of mass, momentum, and energy balance are obtained:

$$MN_0V_0=MNV+M_iN_iV_i+mnv, \quad (4)$$

$$MN_0V_0^2=MNV^2+M_iN_iV_i^2+mnv^2+nkT_e+\tfrac{1}{2}\epsilon_0(E_0^2-E^2), \quad (5)$$

$$MN_0V_0^3=MNV^3+M_iN_iV_i^3+mnv^3+5nvkT_e, \quad (6)$$

where T_e is the electron temperature in the plasma behind the electron shock wave. Quantities in front of the shock zone are designated with a zero subscript, while quantities behind the shock zone have no subscript. All of the flow velocities are given relative to the shock front.

Under the quasi-steady-state conditions described above, the flow velocity of the massive particles is not altered by the passing of the shock wave, and to a very good approximation, the following assumption is valid:

$$V_0=V=V_i. \quad (7)$$

The condition of zero electrical current implies

$$nv-N_iV_i=0. \quad (8)$$

Substituting Eqs. (7) and (8) into Eq. (4), the mass transfer equation, gives

$$N_i=N_0-N=fN_0, \quad (9)$$

where f is the degree of ionization in the gas.

Substituting Eqs. (7), (8), and (9) into the momentum transfer equation (5) gives

$$mfN_0V_0^2=mnv^2+nkT_e+\tfrac{1}{2}\epsilon_0(E_0^2-E^2), \quad (10)$$

and substituting these same restrictions into Eq. (6), the energy transfer equation, gives

$$mfN_0V_0^3=mnv^3+5nvkT_e. \quad (11)$$

Solving Eqs. (10) and (11) for the velocity of the electron shock wave as a function of the remaining variables yields

$$V_0^2=[2m(3kT_e-2W)]^{-1}\{B+[B^2+4W^2(15k^2T_e^2-10kT_eW)]^{1/2}\}, \quad (12)$$

where

$$B=16k^2T_e^2-10kT_eW-W^2,$$

and

$$W=\epsilon_0(E_0^2-E^2)/2fN_0.$$

The quantity W in Eq. (12) is unknown because the electric field behind the shock zone and the degree of ionization of the gas behind the shock wave have not been determined. To include the explicit determination

[12] See Appendix.

of these quantities would require additional equations related to the collision processes and the rarefaction wave as well as an integration of Poisson's equation. However, experimental evidence[13] has shown that the potential-wave propagation is of the same general character regardless of the polarity of the applied potential which indicates a secondary dependence of the propagation velocity on the direct effect of the electric field. The cited experiments give a slightly greater velocity for a negative applied voltage than for a positive voltage. Considering these experimental results, Eq. (12) is here tentatively simplified by assuming W negligible; however, in the next section qualitative dependence of propagation velocity on W is noted.

With the discussed approximations, Eq. (12) reduces to

$$V_0^2=16kT_e/3m. \tag{13}$$

With Eq. (13) and the modified transfer equations, the electron density in the region behind the shock wave can be determined to be

$$n=4N_i=4fN_0. \tag{14}$$

The electrons in the shock zone are given kinetic energy by the action of the electric field. The electron temperature for electrons on which a strong electric field acts briefly has been derived by Fowler, Paxton, and Hughes,[8] and is given by an expression essentially the same as that of Compton[14]:

$$kT_e=\tfrac{1}{3}(M/3m)^{1/2}\lambda_{\text{eff}}eE. \tag{15}$$

Here λ_{eff} is the effective electron mean free path and is given approximately by

$$1/\lambda_{\text{eff}}=\sigma N+1/a, \tag{16}$$

where σ is the total electron collision cross section and a is the tube diameter.

COMPARISON WITH EXPERIMENT

There is very little available data giving both the propagation velocity and the applied field for breakdown waves moving into an unionized gas. Some low-pressure data are given by Beams[2] and are listed in the first three lines of Table I. The atmospheric pressure data in Table I have been taken from White.[10] The streamer velocity of 3.5×10^5 m/sec listed in the table for atmospheric hydrogen is the velocity of a secondary streamer which was propagating down the partially ionized channel left by an earlier, faster pilot streamer. The data for streamers in argon were taken from Loeb, Westberg, and Huang.[11] There have been considerable experimental data taken on lightning discharges,[4] but the electric fields are unknown in these cases.

TABLE I. Propagation velocity of breakdown waves in H_2 and Ar gases with different pressures and field strengths.

Applied field (V/m)	Gas	Pressure (mm Hg)	Experimental velocity (m/sec)	Theoretical velocity (m/sec)	Source
1×10^4	H_2	0.2	4.9×10^7	2×10^7	Beams[a]
1×10^4	H_2	0.5	4.5×10^7	1.7×10^7	Beams[a]
1×10^4	H_2	1.5	4.0×10^7	1.4×10^7	Beams[a]
1×10^6	H_2	760	3.5×10^5	1.5×10^6	White[b]
2×10^5	Ar	300	1.0×10^6	2.7×10^6	Loeb *et al.*[c]

[a] See reference 2.
[b] See reference 10.
[c] See reference 11.

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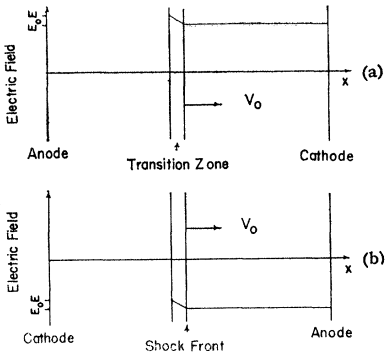

FIG. 1. Spatial distribution of electric field in the region in front of the breakdown wave and in the transition zone of the wave front for a point-plane geometry: (a) Point-anode, plane-cathode; (b) point-cathode, plane-anode.

The streamer velocity of 3.5×10^5 m/sec listed in the table for atmospheric hydrogen is the velocity of a secondary streamer which was propagating down the partially ionized channel left by an earlier, faster pilot streamer. The data for streamers in argon were taken from Loeb, Westberg, and Huang.[11] There have been considerable experimental data taken on lightning discharges,[4] but the electric fields are unknown in these cases.

Considering present uncertainties in regard to the electron temperatures and fields, only rough qualitative agreement between the theory and experimental results can be expected. From Table I we see that such agreement exists for the available data.

As mentioned above, experimental evidence[13] indicates that potential wave velocities are slightly greater for negative impulses than for positive ones. Equation (14) along with Poisson's equation predicts a dependence of the electric field shown pictorially in Fig. 1. In the case of a negative impulse, $|E|$ is less than $|E_0|$ which makes W positive. If a positive pulse is applied, $|E|$ is greater than $|E_0|$ and W becomes negative.

Taking $|W|=kT_e$ gives, for positive W,

$$V_0^2=5.8kT_e/m,$$

and for negative W,

$$V_0^2=5.2kT_e/m.$$

Therefore, theory confirms that the breakdown waves propagating from the negative electrode should, in fact, be the faster.

CONCLUSIONS

Treating the potential wave which precedes electrical breakdown in a gas as an electron shock wave and using a hydrodynamical model for analysis seems extremely promising in that it is possible to predict the propagation velocities at extreme ends of a vast

range of pressures and electric field strengths, within the large experimental error. The theory elucidates the near light-speed propagation velocities reported first by Thompson[1] and later by Beams[2] which were at one time supposed to imply electromagnetic waves of some unknown kind. It is also in harmony with the findings of Von Zahn[3] that the radiating particles are not in motion. It adequately explains the propagation of luminosity fronts from both positive and negative electrodes which were observed by Beams[2] and which have been a major obstacle for people attempting to explain this phenomenon.

Since inelastic energy losses are negligible to the first order, the propagation velocity of the breakdown wave exhibits no dependence on the ionization process in this order and is independent of the degree of ionization. Both photoionization and electron impact ionization are compatible with the first order model.

The assumption of zero total current restricts the validity of the theory to cases where the electric field in the region in front of the breakdown wave is not changing in time. This condition is achieved in a tube with widely separated electrodes if the velocity is measured midway between the electrodes.

In the case of potential waves propagating into field-free regions, the theory given is applicable, but the electron temperature is determined by energy balance of the inelastic collisions instead of by the electric field. In applied electric fields the field energy is swept up by the moving wave, whereas in the field-free case photons enter the wave zone from the back side. In both cases the energy entering the wave zone is utilized in the ionization and heating of the electrons.

The theory can be extended to cover potential waves propagating into a partially ionized gas by including the terms involving electrical current and magnetic fields in the momentum and energy transfer equations. In the case of the secondary streamer, the wave is propagating into a region which is partially ionized and there is electrical current flowing through the wave front.

APPENDIX

The steady-state equations for mass, momentum, and energy transfer in one dimensional flow of a continuous media are derived by requiring conservation of mass and balancing the change of momentum and energy carried by the fluid with the local sources of these quantities.

For the steady state, conservation of mass is given by

$$(d/dx)(MNV+M_iN_iV_i+mnv)=0. \tag{17}$$

The momentum equation is given by

$$(d/dx)(MNV^2+M_iN_iV_i^2+mnv^2)dydz$$
$$=\left(-\frac{dp}{dx}-\frac{dp_i}{dx}-\frac{dp_a}{dx}-neE+N_ieE\right)dydz, \tag{18}$$

where p_i is the ion partial pressure and p_a is the neutral atom partial pressure.

In the one-dimensional geometry, Poisson's equation is given by

$$dE/dx=(e/\epsilon_0)(N_i-n). \tag{19}$$

With Poisson's equation the last two terms in Eq. (18) reduce to

$$(N_i-n)eE=\epsilon_0E\frac{dE}{dx}=\frac{d}{dx}\frac{\epsilon_0E^2}{2}. \tag{20}$$

With Eq. (20), Eq. (18) reduces to

$$(d/dx)(MNV^2+M_iN_iV_i^2+mnv^2+p+p_i+p_a-\tfrac{1}{2}\epsilon_0E^2)=0. \tag{21}$$

For the case of electron impact ionization, the energy balance equation is given by

$$(d/dx)(\tfrac{1}{2}MNV^3+\tfrac{1}{2}M_iN_iV_i^3+\tfrac{1}{2}mnv^3+\tfrac{3}{2}nvkT_e+\tfrac{3}{2}N_iV_ikT_i+\tfrac{3}{2}NVkT_a)dydz=\left(-\frac{dpv}{dx}-\frac{dp_iV_i}{dx}-\frac{dp_aV}{dx}-nveE+N_iV_ieE-\kappa g\frac{dnv}{dx}\right)dydz, \tag{22}$$

where T_i is the temperature of the ion gas, T_a is the temperature of the neutral gas, κ is the net energy loss from the fluid per collision for electron neutral collisions, and g is the ratio of total collisions to ionizing collisions. The function dnv/dx is the number of ionizing collisions per unit time per unit volume of fluid.

Noting that the electrical current density is given by

$$J=e(nv-N_iV_i), \tag{23}$$

Eq. (22) simplifies to

$$(d/dx)(MNV^3+M_iN_iV_i^3+mnv^3+5nvkT_e+5N_iV_ikT_i+5NVkT_a)=-2EJ-2\kappa gdnv/dx. \tag{24}$$

For large values of $T_e(kT_e\gg e\phi_i$, where ϕ_i is the ionization potential of the neutral atoms), all collisions become ionizing collisions with g approaching unity and κ approaching $e\phi_i$. In this case the last term on the right in Eq. (24) reduces to

$$-2\kappa gdnv/dx\approx-2de\phi_inv/dx,$$

which when combined on the left side of the equation appears in a term given by

$$(d/dx)[nv(5kT_e+2e\phi_i)],$$

but the assumption leading to this result was

$$kT_e\gg e\phi_i,$$

so that the last term on the right of Eq. (24) can be neglected for the case of high electron temperatures.

For small values of $T_e(kT_e \ll e\phi_i)$, all collisions become elastic and the term $\kappa g dnv/dx$ approaches zero and can be neglected.

For the ranges of T_e discussed the energy equation reduces to

$$(d/dx)(MNV^3+N_iM_iV_i^3+nmv^3+5nvkT_e +5N_iV_ikT_i+5NVkT_a)=-2EJ. \quad (25)$$

For the case of photoionization, the inelastic energy loss will be smaller than in the electron impact case. Since the inelastic energy loss was shown to be negligible for the regions of interest for electron impact ionization, it will also be negligible for photoionization. Thus the first-order energy equation will be given by Eq. (25) when photoionization dominates as well as when electron impact ionization dominates.

PHYSICAL REVIEW VOLUME 128, NUMBER 3 NOVEMBER 1, 1962

High-Density Corrections in Plasma Spectroscopy*

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(Received May 25, 1962)

The various relationships between spectroscopically measured quantities and the temperature and density of a collision-dominated plasma in local thermal equilibrium are discussed. An internally consistent system of corrections is derived for Saha equations, partition functions, and equations of state on one hand, and line intensities, line profiles, continuum intensities, and optical refractivities. The errors due to remaining uncertainties in these corrections are shown to be usually below 1%. They are therefore negligible compared to those stemming from uncertainties in atomic theory (except for hydrogen or hydrogenic ions) and an order of magnitude smaller than was suggested by discrepancies between previously used corrections to ionization energies of atoms or ions in dense plasmas.

INTRODUCTION

THE concept of local thermal (or thermodynamic) equilibrium plays a vital role in plasma spectroscopy. If it is applicable, all particle distribution functions can be calculated from total densities and temperature, which may both be local functions of time. In other words, the state of the plasma is then fully described by mass density, chemical composition, and temperature. This does not only make it relatively easy to determine the state of the plasma by a few spectroscopic measurements,[1] but also often facilitates the use of such plasmas as spectroscopic light sources for the measurement of atomic parameters[2] like oscillator strengths and line profiles or for the establishment of absolute intensity standards.[3]

Local thermal equilibrium may be expected if collision induced transitions are more frequent than radiative ones, which will in laboratory plasmas usually not lead to thermal equilibrium populations since there one practically never deals with an equilibrium, i.e., blackbody, radiation field. However, often radiative transitions will be negligible, and the level populations will be governed by collision induced processes. If the velocity distribution of the most important reaction partners is Maxwellian (in the nondegenerate case), the principle of detailed balance applies, and the steady-state solution of the rate equations yields the same populations that pertain to a system in complete thermodynamic equilibrium at a temperature equal to the kinetic temperature of the reaction partners.

Since most of these processes are dominated by collisions with electrons, it is their kinetic temperature that is of primary interest. If the relevant velocity distributions at any point and instant are sufficiently close to being Maxwellian and if spatial and time variations are also sufficiently weak as to enable instantaneous and local steady-state populations to be reached, then the assumption of local thermal equilibrium will always be valid as long as radiative rate processes are not important.

THERMODYNAMIC POTENTIALS

Before the distributions over the various possible states are computed for the equivalent thermodynamic equilibrium system, it should be realized that the only quantities which can, at least in principle, be stated without any ambiguity are total number densities N_i of the various chemical species i (hydrogen, helium, etc.) and the temperature. They should, therefore, be chosen as independent variables. Already in the calculation of pressure and internal energy some uncertainty has to be expected, not to mention the distributions over bound

* Jointly sponsored by the Office of Naval Research and the National Science Foundation.

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