Phase-Parameter Representation of Proton-Proton Scattering from 9.7 to 345 MeV. II*

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(Received June 11, 1962)

Phase parameters for fits YLAM and YRB1 to p-p scattering data are presented in the form of graphs suitable for quantitative application. (Additional tabulated material is available from the American Documentation Institute.) Modifications in YLAM arising from the use of a potential are recorded with special attention to the lower energies. Modifications in fit YLAM suggested by newer polarization data are described. Evidence from p-p data regarding the spin-orbit component of the interaction is compared with that from n-p data in the context of the question of validity of charge independence for this component.

I. INTRODUCTION

SINCE the publication of phase-parameter representations of proton-proton scattering by the Yale group,[1] a number of requests for more detailed numerical information have been and continue to be received. In view of this and the appearance of new experimental data,[2,3] which suggest a modification of the earlier fits, graphs and tables of the phase parameters for some of the fits have been prepared and are in part reproduced below. Supplementary numerical material is also being made available.[4]

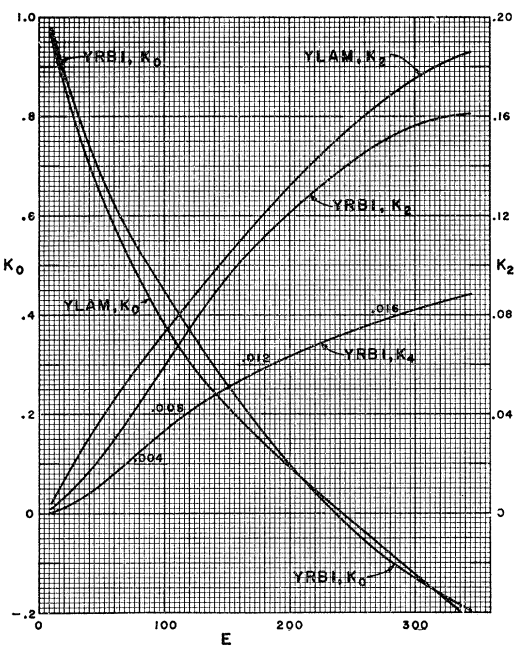

Fig. 1. Phase shifts K_0, K_2, K_4 in radians plotted against incident energy in the laboratory system in MeV. For YLAM phase shift K_4 is not plotted. Its values are obtainable from Table I. Scale of K_4 is shown next to the graph.

The notation used is essentially the same as in reference 1. Triplet phase shifts for orbital angular momentum $L\hbar$ and total angular momentum $J\hbar$ are denoted by ${}^3\delta^L{}_J$; for coupled states the matrix U is parametrized in terms of the quantities ${}^3\theta^L{}_J$ and ρ_J; for the superscripts the spectroscopic equivalent of L, such as P for $L=1$, is often used; singlet phase shifts for angular momentum $J\hbar$ are designated by K_J. The phase

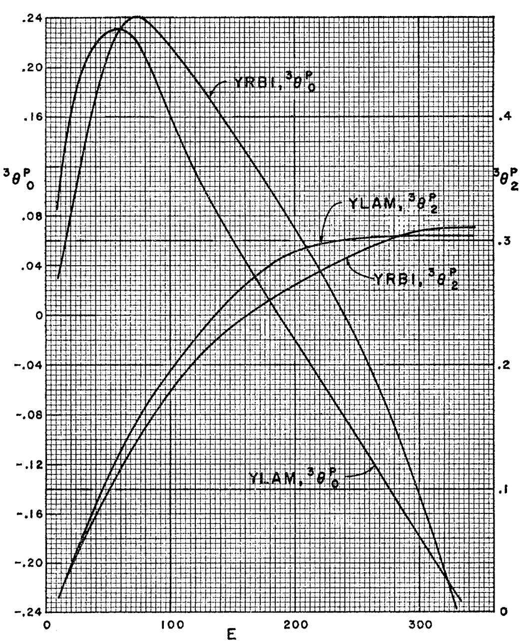

Fig. 2. Phase parameters ${}^3\delta^P{}_0$ and ${}^3\theta^P{}_2$ in radians plotted against incident energy in the laboratory system in MeV. In the figure ${}^3\delta^P{}_0$ is indicated as ${}^3\theta^P{}_0$.

* This research was supported by the U. S. Atomic Energy Commission and by the U. S. Army Research Office (Durham).

1 G. Breit, M. H. Hull, Jr., K. E. Lassila, and K. D. Pyatt, Jr., Phys. Rev. **120**, 2227 (1960).

2 W. Benenson, R. H. Walters and T. H. May, Phys. Rev. Letters **8**, 66 (1962).

3 P. Christmas and A. E. Taylor (private communication) and Atomic Energy Research Establishment Research Group Progress Report, Nuclear Physics Division AERE-PR/NP1 (unpublished). The values used in the analysis reported on are as of November, 1961. They include corrections for energy spread at 37.0 and 52.5 MeV.

4 This material has been deposited as Document No. 7282 with the ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington 25, D. C. A copy may be secured by citing the Document number and by remitting $2.50 for photoprints, or $1.75 for 35 mm microfilm. Advance payment is required. Checks or money orders are to be made payable to: Chief, Photoduplication Service, Library of Congress.

TABLE I. OPEP values of K_4.

E(MeV)	K_4
25	0.00073
50	0.00270
100	0.00674
150	0.00989
200	0.01223
250	0.01400
300	0.01535
330	0.01601

shifts are always meant to be added to the Coulomb phase to obtain the actual phase.

II. GRAPHICAL REPRESENTATION OF YLAM AND YRB1

Figures 1, 2, 3, 4, and 5 show the phase parameters for the results of searches YLAM and YRB1 of reference 1, these appearing as the more likely among the five previously[1] presented. The statistical uncertainties are not shown so as to facilitate reading of the graphs. The description of the searches may be found in the footnotes to Table III of reference 1 and other parts of Sec. IV of that paper. Angles are shown in radians and the incident energy in the laboratory coordinate system in MeV.

In Fig. 1 are shown K_0 and K_2 for YLAM and YRB1 and K_4 for YRB1. For YLAM the OPEP (one-pion exchange potential) value of K_4 was used. The graph for it would be very close to that for YRB1 and runs slightly above that shown through most of the energy range. Enough values of K_4 for YLAM are listed in Table I to make graphical interpolation practical.

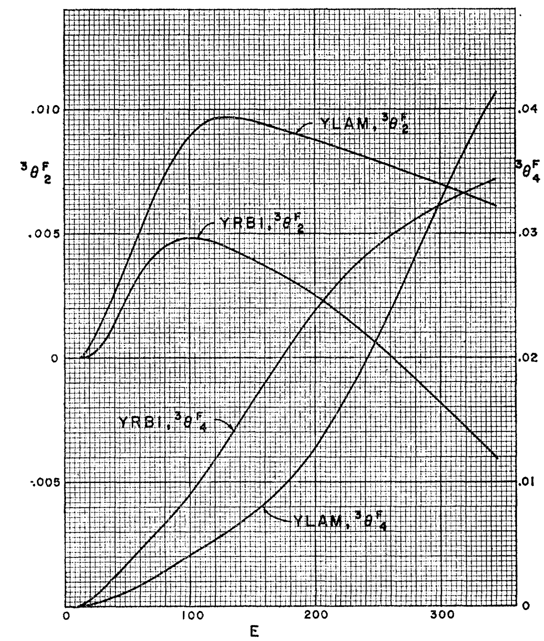

FIG. 4. Phase parameters ${}^3\theta^F{}_2$ and ${}^3\theta^F{}_4$ in radians plotted against incident energy in the laboratory system in MeV.

Figure 2 contains plots of ${}^3\delta^P{}_0$ and ${}^3\theta^P{}_2$. In Fig. 3, the phase shifts ${}^3\delta^P{}_1$ and ${}^3\delta^F{}_3$ are shown. Figure 4 gives a graphical representation of ${}^3\theta^F{}_2$ and ${}^3\theta^F{}_4$ while Fig. 5 represents the coupling parameters ρ_2 and ρ_4. Phase

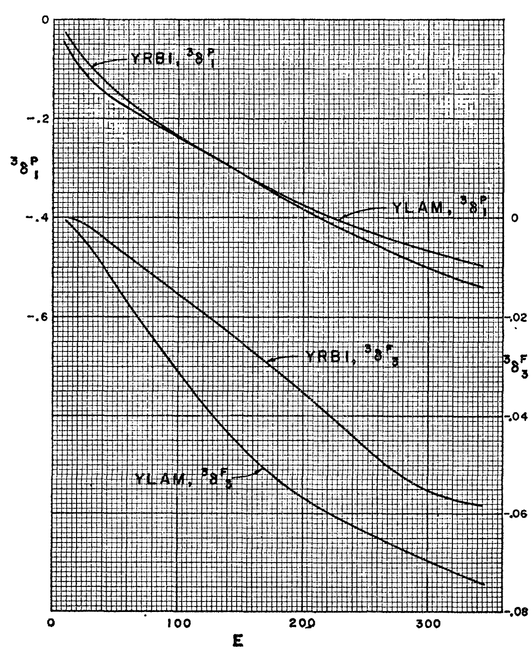

FIG. 3. Phase shifts ${}^3\delta^P{}_1$ and ${}^3\delta^F{}_3$ in radians plotted against incident energy in the laboratory system in MeV.

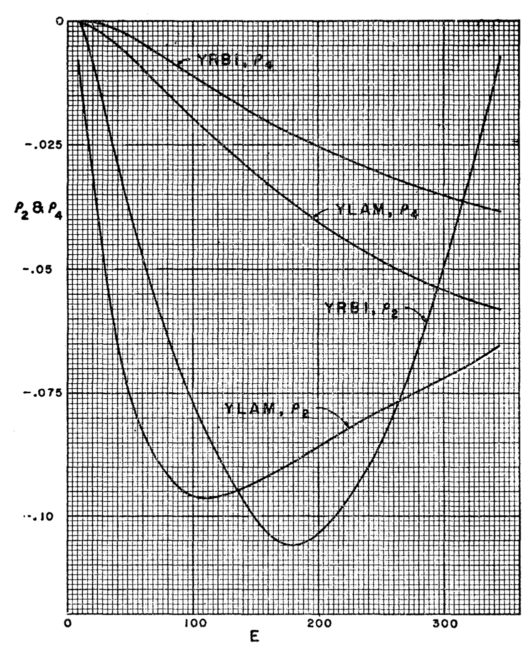

FIG. 5. Phase parameters ρ_2 and ρ_4 plotted against incident energy in the laboratory system in MeV.

TABLE II. Phase parameters for $T=1$ states calculated from a potential adjusted to fit YLAM. The spin-orbit potential was omitted in the calculations for $J \geq 3$. Values are in radians.

E (MeV)	K_0	K_2	K_4	${}^3\delta^P{}_0$	${}^3\delta^P{}_1$	${}^3\theta^P{}_2$	ρ_2	${}^3\theta^F{}_2$	${}^3\delta^F{}_3$	${}^3\theta^F{}_4$	ρ_4	${}^3\theta^H{}_4$
2	0.7983											
4	0.9322											
5	0.9525	0.0007		0.0316	−0.0181							
6	0.9614											
8	0.9621											
10	0.9523	0.0029		0.0743	−0.0418							
15	0.9128	0.0058		0.1120	−0.0637							
25	0.8258	0.0126	0.0007	0.1653	−0.1009	0.0441	−0.0339	0.0019	−0.0048	0.0005	−0.0020	0.0001
50	0.6440	0.0326	0.0028	0.2082	−0.1681	0.1032	−0.0716	0.0068	−0.0144	0.0022	−0.0080	0.0006
100	0.3870	0.0750	0.0082	0.1577	−0.2533	0.2004	−0.1037	0.0143	−0.0333	0.0078	−0.0235	0.0025
150	0.1988	0.1132	0.0142	0.0718	−0.3149	0.2603	−0.1017	0.0172	−0.0479	0.0137	−0.0377	0.0050
200	0.0465	0.1440	0.0202	−0.0146	−0.3674	0.2897	−0.0874	0.0157	−0.0586	0.0191	−0.0489	0.0077
250	−0.0836	0.1675	0.0259	−0.0958	−0.4154	0.2974	−0.0707	0.0104	−0.0662	0.0239	−0.0574	0.0103
300	−0.1984	0.1844	0.0311	−0.1712	−0.4606	0.2902	−0.0551	0.0020	−0.0715	0.0218	−0.0635	0.0127
330	−0.2615	0.1917	0.0340	−0.2138	−0.4868	0.2808	−0.0467	−0.0043	−0.0738	0.0305	−0.0662	0.0140

parameters not shown in the graphs have been given their OPEP values.

Arrangements referred to previously[4] make available tables of values of phase parameters for fits YLAM, YRB1, YLA, and YRB3 at 20 experimental energies covering the range from 9.68 to 337.5 MeV. It should be emphasized that the work reported on in reference 1 was primarily intended to provide an over-all representation of the data above 9.68 MeV and that the joining to data at lower energies was not as smooth as is justifiable by the accuracy of the precision measurements at the lower energies. This defect has been largely removed in the work of Lassila, Hull, Ruppel, McDonald, and Breit[5] in which a potential is made to fit the data below 9.68 MeV and YLAM above this energy. A few values obtained by means of this potential showing characteristic differences from YLAM are shown in Table II.

As was reported in reference 5, the best fit to YLAM with a potential was obtained by omitting the spin-orbit potential in states for $J \geq 3$. In order to allow an estimate of the importance of the spin-orbit potential in these states and for possible applications where the omission might be inadvisable, values of the phase-parameters calculated with inclusion of the linear spin-orbit potential are given in Table III. Values of the same quantities computed without using the spin-orbit potential are listed in Table II.

TABLE III. Phase parameters for $T=1$ states with $J \geq 3$ calculated from a potential[a] adjusted to fit YLAM. The spin-orbit potential adjusted to YLAM for states with $J<3$ was included in the calculation.

E (MeV)	${}^3\delta^F{}_3$	${}^3\theta^F{}_4$	ρ_4	${}^3\theta^H{}_4$
25	−0.0048	0.0008	−0.0005	
50	−0.0145	0.0026	−0.0078	0.0005
100	−0.0340	0.0108	−0.0233	0.0023
150	−0.0500	0.0226	−0.0376	0.0047
200	−0.0628	0.0378	−0.0487	0.0068
250	−0.0733	0.0567	−0.0566	0.0084
300	−0.0822	0.0795	−0.0618	0.0094
330	−0.0869	0.0951	−0.0636	0.0097

[a] See reference 5.

[5] K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).

III. INCLUSION OF SOME NEW DATA

The data under discussion are those of Benenson, Walters, and May[2] and of Christmas.[3] They are both of special value in supplying information regarding either the polarization parameter P_{p-p} or nearly its equivalent in the energy region below 66 MeV. As has been mentioned in reference 2 the almost complete lack of such measurements has made the searches published in reference 1 especially uncertain regarding the amplitude of the first order effect of the spin-orbit potential which is primarily responsible for P. The inclusion of the newer data on P may be supposed, therefore, to make the fit more suitable for drawing conclusions regarding the effective spin-orbit potential.

The data of Christmas[3] deal directly with p-p scattering and have been used as follows. The $L=1$ phase parameters for YLAM were expressed in terms of an L and J independent part on which were superposed first Born approximation effects of spin-orbit and tensor interactions. In this sense the phase parameters have been resolved into central, tensor, and spin-orbit parts. The last of these will be referred to as δ_{LS}. A number of gradient searches were tried first. At 52.5 MeV δ_{LS} decreased from 0.050 for YLAM to 0.049 without an indication of rapid improvement. It was then attempted to accelerate the search by comparison of the experimental $P(\theta)$ with the calculated. On this basis a decrease by about 30% in δ_{LS} would be expected. When this was tried the agreement with $\sigma(\theta)$ was spoiled too much. Several trials indicated that a 10% reduction in δ_{LS} should give a better compromise between the polarization and cross section requirements. The resultant P waves were then improved by successive variations of one of them at a time, with special emphasis on ${}^3\delta^P{}_0$ and ${}^3\theta^P{}_2$. Only polarization data at 52.5 and 37.0 MeV were used at this stage. After graphical smoothing this "grid" type of search was followed by complete gradient searches employing all available p-p scattering data from 9.69 to 133 MeV inclusive. Since the fit to the new polarization data was still not as good as desired, δ_{LS} was decreased from its new value of 0.047 to 0.040. This was followed by a set of grid searches, graphical smoothing and gradient searches on K_0, ${}^3\delta^{P}{}_0$, ${}^3\delta^{P}{}_1$, ${}^3\theta^{P}{}_2$, ρ_2 and K_2. At this point the weighted sum of squares of deviations for $\sigma(\theta)$ at 9.69, 39.4 and 68.3 MeV increased by about 30% as compared with its YLAM value giving a local D of nearly 1. Further refitting was stopped since it would have given an improbable disagreement with these very good data. The final δ_{LS} at 52.5 MeV was 0.0443. Since it was found in the Yale potential searches[5] that ${}^3\theta^{F}{}_2$ corresponding to the potential differed appreciably from its YLAM value and that this difference did not markedly affect the mean square error, the Yale potential values were used for this quantity. The phase parameters obtained as described were smoothed graphically to join YLAM just above 172 MeV. The new fit is referred to below as YLAM (P), the "P" being intended to indicate emphasis on polarization. The maximum deviation of K_0 from YLAM occurs at about 95 MeV and is approximately equal to the uncertainty of the YLAM value listed in Table IV of reference 1. The changes in K_0 at 39.4 and 68.3 MeV are towards an f-function extrapolation[6] of lower energy data. The maximum change in K_2 is smaller than the YLAM uncertainty. For ${}^3\delta^{P}{}_0$ the position of the maximum has been shifted towards lower energies by $\sim$10 MeV. For ${}^3\delta^{P}{}_0$ and ${}^3\delta^{P}{}_1$ the deviations from YLAM are essentially within the uncertainties listed in Table IV of reference 1. For ${}^3\theta^{P}{}_2$ the larger deviations exceed those of Table IV of reference 1 at times by as much as a factor 2. This may perhaps be due partly to the coupling of this parameter to ${}^3\theta^{F}{}_2$ which has been deliberately changed from its YLAM value by amounts close to the nominal uncertainty. In Table IV phase parameter values for YLAM(P) are listed for comparison with YLAM. Below each YLAM(P) entry there appears the corresponding value for YLAM. The values for this table have been obtained by means of graphs and are not as accurate as machine values would be. It is believed, however, that they are sufficiently accurate for physical applications. For $E>200$ MeV the values are the same for the two fits.

TABLE IV. Values of phase parameters for YLAM(P). Below each value there appears the corresponding value for YLAM.

E (MeV)	K_0	${}^3\delta^{P}{}_0$	${}^3\delta^{P}{}_1$	${}^3\theta^{P}{}_2$	ρ_2	K_2
25	0.823	0.205	−0.102	0.043	−0.0389	0.0168
	0.822	0.186	−0.109	0.050	−0.0401	0.0178
50	0.657	0.227	−0.152	0.101	−0.0752	0 0367
	0.638	0.230	−0.163	0.110	−0.0755	0.0384
100	0.404	0.159	−0.236	0.199	−0.0964	0.0706
	0.380	0.160	−0.238	0.195	−0.0959	0.0724
150	0.214	0.060	−0.310	0.260	−0.0929	0.1027
	0.214	0.060	−0.310	0.254	−0.0929	0.1034

TABLE V. Phase parameters at 16, 24 MeV for YLAM, YLAM′, and YLAM(P).

	16 MeV			24 MeV		
	YLAM(P)	YLAM	YLAM′	YLAM(P)	YLAM	YLAM′
$\delta^{P}{}_0$	0.146	0.142	0.1267	0.202	0.183	0.2115
$\delta^{P}{}_1$	−0.076	−0.078	−0.0776	−0.099	−0.106	−0.1060
$\theta^{P}{}_2$	0.0254	0.026	0.0241	0.041	0.047	0.0414
ρ_2	−0.0225	−0.0230	−0.0221	−0.0371	−0.0382	−0.0368
$\theta^{F}{}_2$	0.0008	0.0002	0.0009	0.0017	0.0009	0.0015

Employing the data[2] on n-p polarization at 16 and 24 MeV, a modification of YLAM became necessary.[7] The fit obtained was called YLAM′. Table V shows a comparison of phase parameter values for YLAM′ with those for YLAM and YLAM(P). At 24 MeV there is considerable similarity between YLAM(P) and YLAM′, both regarding the values of parameters and regarding the value of P. At 16 MeV the similarity is lost and the values of P at the n-p experimental values of $\theta=100°$, 120°, and 140° are somewhat below YLAM for YLAM(P) and somewhat above for YLAM′. However, these differences are all appreciably smaller than the standard deviation of the experimental values. On the other hand, at 24 MeV the difference is barely outside the experimental uncertainty while both YLAM(P) and YLAM′ are much closer than YLAM

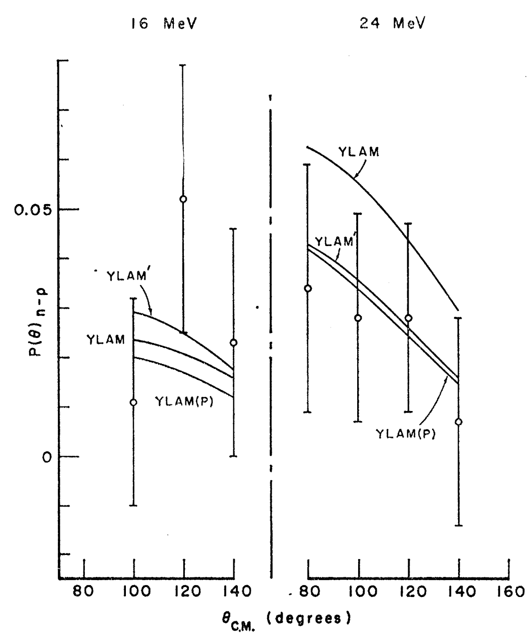

FIG. 6. YLAN3M values of $P_{n-p}(\theta)$ employing $\tau=1$ parameters according to YLAM, YLAM′ and YLAM(P) compared with experiment.[2]

[6] G. Breit, E. U. Condon and R. D. Present, Phys. Rev. **50**, 825 (1936). The f function is defined in Eq. (7.6) of this reference.

[7] M. H. Hull, Jr., F. A. McDonald, H. M. Ruppel, and G. Breit, Phys. Rev. Letters **8**, 68 (1962).

to the most probable values of P. The disagreements at 16 MeV thus do not appear to be as statistically significant as the agreement at 24 MeV. There appears to be here, therefore, a partial, though still rather inaccurate, confirmation of charge independence of nuclear forces as applied essentially to the spin-orbit part of the nucleon-nucleon interaction. The situation is illustrated in Fig. 6. If experiments of higher accuracy could be performed, more accurate information regarding the applicability of charge independence to this component of the nucleon-nucleon interaction would be obtained.

It should be remarked that plots of P against angle at 37 and 52.5 MeV show a marked trend for $P(45°)$ to be low as compared with theoretical expectation. This lack of internal consistency in the comparison of theory and experiment is an additional reason for regarding the attempt at the test of charge independence for spin-orbit component of the interaction as preliminary.

ACKNOWLEDGMENTS

The writers are indebted to Dr. Taylor and Dr. Christmas for communicating to them their data before publication and for permission to refer to them, to Miss J. Gibson for expert mathematical assistance, and to F. A. McDonald and R. G. Brandt for their help at various stages of the work.

PHYSICAL REVIEW VOLUME 128, NUMBER 2 OCTOBER 15, 1962

Phase-Parameter Representation of Neutron-Proton Scattering from 13.7 to 350 MeV. II*

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Phase parameters for fits YLAN3M and YLAN1 to n-p scattering data are presented in the form of graphs suitable for quantitative application. (Additional tabulated material is available from the American Documentation Institute.) Modifications in YLAN3M arising from the use of a potential are recorded with special attention to lower energies.

I. INTRODUCTION

SINCE the publication of phase-parameter representations of neutron-proton scattering by the Yale group,[1] a number of requests for more detailed numerical information have been and continue to be received. In order to satisfy these demands more efficiently, graphs and tables of the phase parameters for some of the fits have been prepared and are in part reproduced below. Supplementary material is also being made available.[2]

The notation used is essentially the same as in reference 1. Triplet phase shifts for orbital angular momentum $L\hbar$ and total angular momentum $J\hbar$ are denoted by ${}^3\delta^L{}_J$; for coupled states, the matrix U is parametrized in terms of the quantities ${}^3\theta^L{}_J$ and ρ_J; for the superscripts, the spectroscopic equivalent of L, such as P for $L=1$, is often used; singlet phase shifts for angular momentum $J\hbar$ are designated by K_J.

II. GRAPHICAL REPRESENTATION OF YLAN1 AND YLAN3M

Figures 1, 2, 3, 4 and 5 show the phase parameters for the results of searches YLAN1 and YLAN3M of reference 1, the latter appearing as the more likely among the six previously presented. From a purely statistical viewpoint of the value of the mean-square deviation YLAN1 is among the poorer fits, but the purely statistical criterion may not be very reliable in this case. While it belongs in the same class with YLAN3M and YLAN3 regarding the classification of fits according to values of ρ_3 and K_3, the sign for ρ_1 for it is like that of YLAN3M rather than of YLAN3 and is more likely to be the right one if one may judge from predictions of potential models.

In Fig. 1 are shown K_1 and K_3. Figure 2 contains plots of ${}^3\theta^S{}_1$ and ${}^3\theta^D{}_1$. Figure 3 displays ${}^3\delta^D{}_2$ and ${}^3\theta^D{}_3$. Figure 4 gives a graphical representation of the coupling parameters ρ_1, ρ_3 and ρ_5, while Fig. 5 represents ${}^3\theta^G{}_3$, ${}^3\delta^G{}_4$, and ${}^3\theta^G{}_5$. In all cases, angles are in radians and the incident energy in the laboratory system is in MeV. Phase parameters not shown in the graphs have been given their OPEP (one-pion exchange potential) values.

* This research was supported by the U. S. Atomic Energy Commission and by the U. S. Army Research Office (Durham).

[1] M. H. Hull, Jr., K. E. Lassila, H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **122**, 1606 (1961).

[2] This material has been deposited as Document No. 7282 with the ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington 25, D. C. A copy may be secured by citing the Document number and by remitting $2.50 for photoprints, or $1.75 for 35 mm microfilm. Advance payment is required. Checks or money orders are to be made payable to: Chief, Photoduplication Service, Library of Congress.

[3] K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).