Inelastic Electron Scattering from V^{51}†

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Shell-model predictions of the relative inelastic electron scattering cross section for excitation of the first four excited states of V^{51} are made using effective single-particle operators and the known configurations of these states. The predicted cross sections for excitation of three of these levels are compared with measured scattering cross sections determined at scattering angles less than 60° using electrons of primary energy 183, 300, and 600 MeV.

I. INTRODUCTION

RECENT studies of the inelastic scattering of high-energy electrons from heavy nuclei[1] have shown that the predominant transitions to discrete nuclear states excited in the scattering process are electric and, for the most part, the gamma rays de-exciting these states directly to the ground state have greatly enhanced transition rates. Multipole assignments and transition rates may be determined that are in agreement with measurements by other techniques in those cases where comparison is possible. In I, it was shown that all of the states strongly excited in five different isotopes near $A=59$ and $A=208$ could be interpreted as collective states. Indeed, no transitions were observed which could be shown to be single-particle transitions. The strong transitions in odd-even nuclei could be interpreted as leading to a group of closely lying levels obtained by weakly coupling the odd particle to the collective 2+ state. In this case there is a simple sum rule saying that the sum of the intensities of the transitions leading to all levels of such a group should be equal to the intensity of the $0 \rightarrow 2$ transition in neighboring even-even nuclei. This sum rule was actually found to hold experimentally in several cases.

In view of this behavior, it was of much interest to investigate the inelastic scattering on a nucleus whose levels can be assigned simple shell-model configurations. Nuclei with proton (or neutron) number between 20 and 28, and neutrons (or protons) in closed shells can be very well described by $f_{7/2}^n$ configurations. The energy levels of such nuclei can be calculated on the basis of the shell model by assuming effective two-body interactions between nucleons.[2] In particular, vanadium-51 has three $f_{7/2}$ protons moving in the potential formed by the (magic number) 28 neutrons and the closed proton subshells. The structure of the low-lying excited states of V^{51} have been calculated in detail on the basis of the shell model and the energies and spin assignments are in agreement with experiment.[3] The levels of V^{51} up to 3 MeV are given in Fig. 1, taken from reference 3. The spins of levels which belong to the $f_{7/2}^3$ configurations are indicated. It is seen that, along with the ground state, the first four excited levels belong to the $f_{7/2}^3$ configuration. These states cannot be obtained by the coupling of an $f_{7/2}$ proton to the 2+ state of ${}_{22}Ti_{28}{}^{50}$ (which belongs to the $f_{7/2}$ configuration). Due to the antisymmetry of the states, enforced by the Pauli principle, the $f_{7/2}^3$ states contain pairs of $f_{7/2}$ protons in states with $J=0$, 2, 4, 6 according to the coefficients of fractional parentage. The calculation of the transition intensities is thus more involved than in the case of a single particle weakly coupled to a collective state.

The enhanced $E2$ transition probabilities occur even in light nuclei and even in cases where the energies agree well with the individual-particle model (shell model). From the point of view of the shell model, there are two possible ways to consider those enhanced transitions. It is possible that although we can calculate energies using shell-model wave functions by replacing the free nucleon interaction by effective interactions,

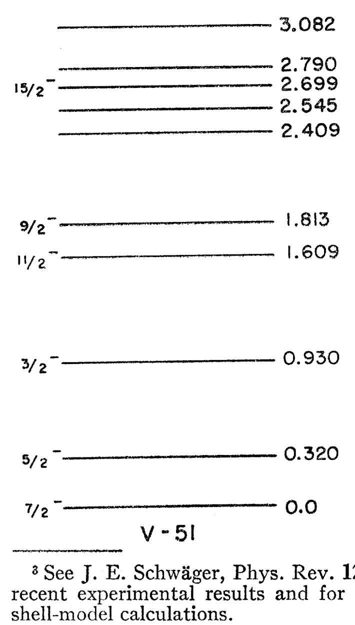

FIG. 1. The energy levels of V^{51}. (See reference 3.) The present experiment is concerned with the levels at 0.930, 1.609, and 1.813 MeV, all members of the $f_{7/2}^3$ proton configurations. The spins of other members of the configuration are indicated.

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[1] H. Crannell, R. Helm, H. Kendall, J. Oeser, and M. Yearian, Phys. Rev. **123**, 923 (1961). Referred to in the text as I.
[2] The situation is summarized and references given by I. Talmi, in *Proceedings of the Rehovoth Conference on Nuclear Structure* (North-Holland Publishing Company, Amsterdam, New York, 1958), pp. 31–45.
[3] See J. E. Schwäger, Phys. Rev. **121**, 569 (1961) for the most recent experimental results and for a number of references on shell-model calculations.

TABLE I. Predicted values of the squares of the reduced matrix elements and transition strengths for transitions within the $(f_{7/2})^3$ configuration in V^{51}. Values of $B(E2)$ are given both for the upward and downward transitions. All quantities are measured in units of the single nucleon value $(e^2\times 0.02\times 10^{-48})$ cm^4.

J	7/2	3/2	5/2	9/2	11/2
Reduced matrix element squared	1/9=0.11	12/35=0.34	22/15=1.47	26/63=0.41	4/3=1.33
$B(E2, J\to 7/2)$		0.086	0.244	0.041	0.111
$B(E2, 7/2\to J)$		0.0425	0.184	0.051	0.166

there is no such simplification for transition probabilities. The configuration interaction due to the short range correlations may give rise to energy shifts which can be approximated by effective two-body interactions. However, the enhancement of the transition probabilities arising from this configuration interaction may be completely independent for different states of a shell-model configuration. It may be strong for some special states (e.g., the 2+ states of even-even nuclei) and still be weak for others.

On the other hand, there may be some order in these enhancements so that they could be described in terms of the shell model. It may be that shell-model wave functions can be used for calculating transition probabilities if instead of the free nucleon operators, we use equally enhanced effective single-particle operators.[4] The enhancements are sometimes described as due to core polarizations. If the core polarizations due to the individual nucleons outside closed shells are actually independent of each other, we can replace their effect by considering effective single-particle operators. We try to check, by looking at experimental data, whether we can calculate transition probabilities by using effective single-particle operators with shell-model wave functions. We thus calculate the relative scattering cross sections for exciting the four lowest excited states of V^{51} (which can be obtained by $E2$ excitation from the ground state), and compare them with the experimental data. Furthermore, we can see whether the results of Coulomb excitation of the V^{51} levels[5] agree with this approach of using effective single nuclear operators. In the present paper, we report the theory and results of the electron-scattering measurements.

II. THEORY

We calculate the $E2$ matrix elements between the ground states and excited states of V^{51}. We take these states to be the antisymmetric states of the $f_{7/2}{}^3$ proton configuration. The operator whose matrix elements squared give the $E2$ transition rates is taken to be a sum of single nucleon operators. The $E2$ operator which should be used with the *real* wave function is

$$e\sum_{i=1} r_i^2 Y_{2m}(\theta_i\phi_i), \tag{1}$$

where r_i, θ_i, ϕ_i are the coordinates of the ith proton. We consider instead an effective operator to be used with the shell-model wave functions,

$$\sum_{i=1}^{3} T_m^{(2)}(i). \tag{2}$$

Here the summation is extended only over the three $f_{7/2}$ protons since the other protons are now in closed shells and their contribution vanishes. The operator $T_m^{(2)}(i)$ is the m component of an irreducible tensor of deg 2 operating on the coordinates of the ith proton. Within a given configuration (the $f_{7/2}{}^3$ configuration in the present case) the matrix elements of $T_m^{(2)}$ are proportional to those of $er^2Y_{2m}(\theta\phi)$. The proportional factor may be called *effective charge*, but it should be remembered that this factor can be different for different multipole radiations and even different for different configurations.

In order to calculate the transition rate, we have to calculate the reduced matrix element of (2). Using fractional-parentage coefficients for the initial and final states, we obtain for the j^3 configuration

$$\begin{aligned}(j^3J'||\sum_{i=1}^{3} T_m^{(2)}||j^3J) &= 3(j||T_m^{(2)}||j)\sum_{J_1}(j^3J'[|j^2(J_1)jJ')(j^2(J_1)jJ|]j^3J)\\ &\times(-1)^{J_1+j+J'}[(2J'+1)(2J+1)]^{1/2}\\ &\times\begin{Bmatrix} j & J' & J_1\\ J & j & 2\end{Bmatrix}.\end{aligned} \tag{3}$$

In the present case $J'=j=7/2$ and J can assume the values 3/2, 5/2, 9/2, and 11/2. Using the coefficients of fractional parentage for $j=7/2$,[6] we obtain the reduced matrix elements for the possible values of J. The squares of these matrix elements are all proportional to $|(f_{7/2}||T_m^{(2)}||f_{7/2})|^2$ which is the square of the single particle reduced matrix element. The proportionality coefficients are given in Table I. The value for $J=7/2$ is also included.

The rate per unit time of the $E2$ transition $J_i\to J_f$ is directly proportional to

$$B(E2J_i\to J_f)=\frac{1}{2J_i+1}|(J_i||\sum_i T_m^{(2)}(i)||J_f)|^2. \tag{4}$$

[4] I. Talmi and I. Unna, Ann. Rev. Nuclear Sci. **10**, 353–408 (1960).

[5] *Proceedings of the Second Conference on Reactions between Complex Nuclei, Gatlinburg, Tennessee, 1960* (John Wiley & Sons, New York, 1960).

[6] A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. (London) **A214**, 515 (1952).

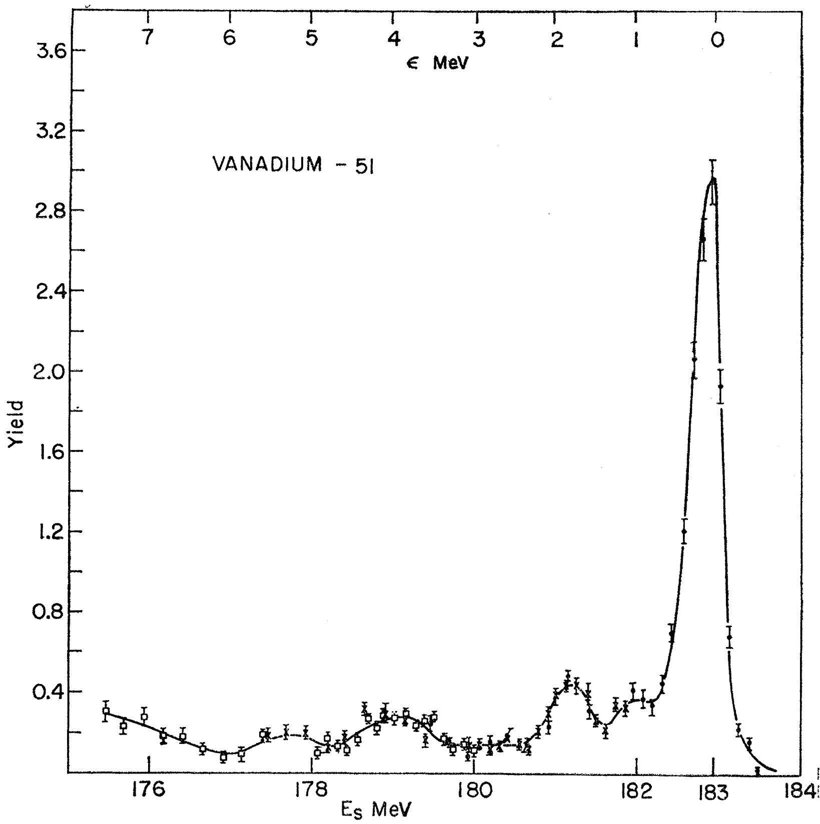

FIG. 2. In this and succeeding figures through Fig. 7 we show raw and corrected spectra of electrons scattered elastically and inelastically from V^{51} and from carbon and polyethylene. E_s is the scattered electron energy, and ϵ is the nuclear excitation energy after the scattering. Three overlapping runs with the 27-channel detector are indicated by different symbols. The smooth curve is a visual fit to the data. The present figure refers to scattering of 183-MeV electrons from V^{51} at a laboratory angle of 60°.

Thus, for the transitions $7/2\to J$ considered here, the rates are proportional to the squares of the reduced matrix elements given in Table I. We see that the predictions in the present case are, indeed, very different from those for weak coupling to the collective state. In that case we have

$$(0jJ_i=j||T_m^{(J_0)}||J_0jJ)=(-1)^{j+J+J_0}\times[(2j+1)(2J+1)]^{1/2}\begin{Bmatrix}0 & J_0 & J_0\\ J & j & j\end{Bmatrix},$$

$$(0||T_m^{(J_0)}||J_0)=\left(\frac{2J+1}{2J_0+1}\right)^{1/2}(0||T_m^{(J_0)}||J_0),\quad(5)$$

where J_0, the spin of the collective state, is equal to the multipole order (2 in our case). Therefore, the rates of the $j\to J$ transitions are proportional to $2J+1$. Dividing (5) by $2j+1$ and summing over J, we obtain for the case of weak coupling

$$\sum_J B(j\to J)=\sum_{J=|j-J_0|}^{j+J_0}\frac{2J+1}{(2j+1)(2J_0+1)}(0||T_m^{(J_0)}||J_0)^2=(0||T_m^{(J_0)}||J_0)^2=B(0\to J_0).\quad(6)$$

Equation (6) is the mathematical expression of the sum rule mentioned above. The $B(E2)$ for the gamma transitions from the excited states of V^{51} to the ground state are obtained from (4) for $J_i=J$. In the case of the weak coupling to the collective state it follows from (5) that they are all equal. In the model used here, the results are quite different as can be seen from Table I where the relevant $B(E2)$'s are listed in terms of $|(f_{7/2}||T_m^{(2)}||f_{7/2})|^2$.

The numbers appearing in Table I can give only the ratios of transition rates. We cannot predict the absolute rates because we do not know the effective operator $T_m^{(2)}$. We can only check and see whether the ratios agree with the values predicted on the assumption that the effective operators are sums of single particle operators. In addition, we can check whether the effective $E2$ operators obtained from several nuclei are the same. In particular, we know that $E2$ transitions in even-even nuclei are considerably enhanced and we can check whether the effective $E2$ operator determined from V^{51} is equally enhanced. A simple way to define enhancement is by comparing it to the "single-particle estimate." What is actually referred to is the value obtained by using the free nucleon operator (1) with shell-model wave functions (usually those of the harmonic oscillator).

We shall thus calculate the value of

$$|(f_{7/2}||r^2Y^{(2)}||f_{7/2})|^2.$$

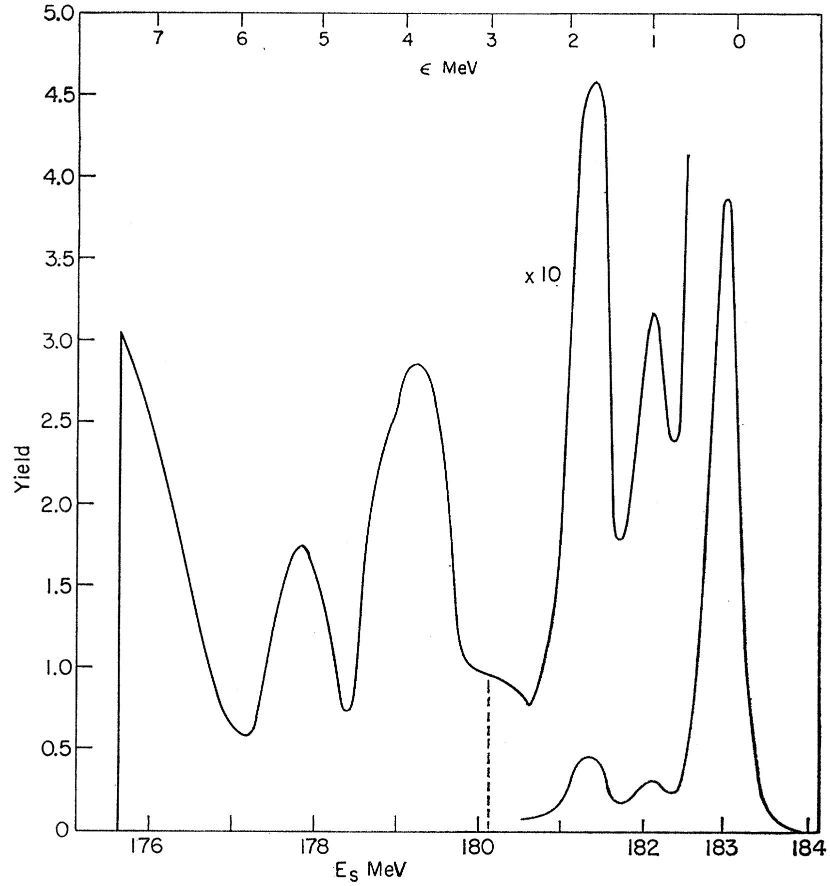

FIG. 3. The smooth curve of Fig. 2 with bremsstrahlung and Schwinger corrections applied.

As is well known, this is given by[7]

$$|(j||r^2Y^{(2)}||j)|^2 = \frac{5(2j+1)^2(2l+1)^2}{4\pi}\begin{pmatrix} l & l & 2 \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{Bmatrix} l & j & \frac{1}{2} \\ j & l & 2 \end{Bmatrix}^2 \times\left[\int_0^\infty r^2R(r)^2dr\right]^2 \quad (7)$$

for $j=7/2$ and $l=3$.

Using harmonic oscillator wave functions, we obtain

$$\int_0^\infty r^2R(r)^2dr=\frac{4n+2l+3}{4\nu}=\frac{9}{4\nu},$$

where ν is the parameter of the oscillator wave functions.[8] Using the value $e^2(\nu^{1/2}/\pi)=0.3$ MeV obtained in reference 8 for the $f_{7/2}$ shell, we obtain

$$\int_0^\infty r^2R(r)^2dr=16.4\times10^{-26}\text{ cm}^2.$$

Putting this value, as well as the appropriate values of the Wigner and Racah coefficients in (7), we obtain

$$|(f_{7/2}||r^2\mathbf{Y}^{(2)}||f_{7/2})|^2=0.02\times10^{-48}\text{ cm}^4. \quad (8)$$

This is the unit by means of which we shall measure the enhancement of $|(f_{7/2}||T_m^{(2)}||f_{7/2})|^2$.

III. APPARATUS

The electron-scattering techniques used in the present experiment were very similar to those described in I. The modifications of the experimental equipment and data reduction from those described in I will be discussed in detail in a later paper.[9] We will only outline them here. The electrons scattered from the primary beam of the Stanford MKIII accelerator by the various targets used in the present experiment were momentum-analyzed by a 72-in. 180° double focusing spectrometer and were detected by a counter-telescope array. Each element of the scintillator array defined a momentum interval in the spectrometer focal plane. $\Delta p/p$ was equal to 0.067%. Eighteen of the counters were closely spaced, the remaining nine were spaced apart by approximately one channel width. The exact positions were determined optically and were checked by a series of calibration

[7] See, e.g., J. P. Elliott and A. M. Lane, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 241.

[8] B. C. Carlson and I. Talmi, Phys. Rev. **96**, 436 (1954).

[9] H. W. Kendall (to be published).

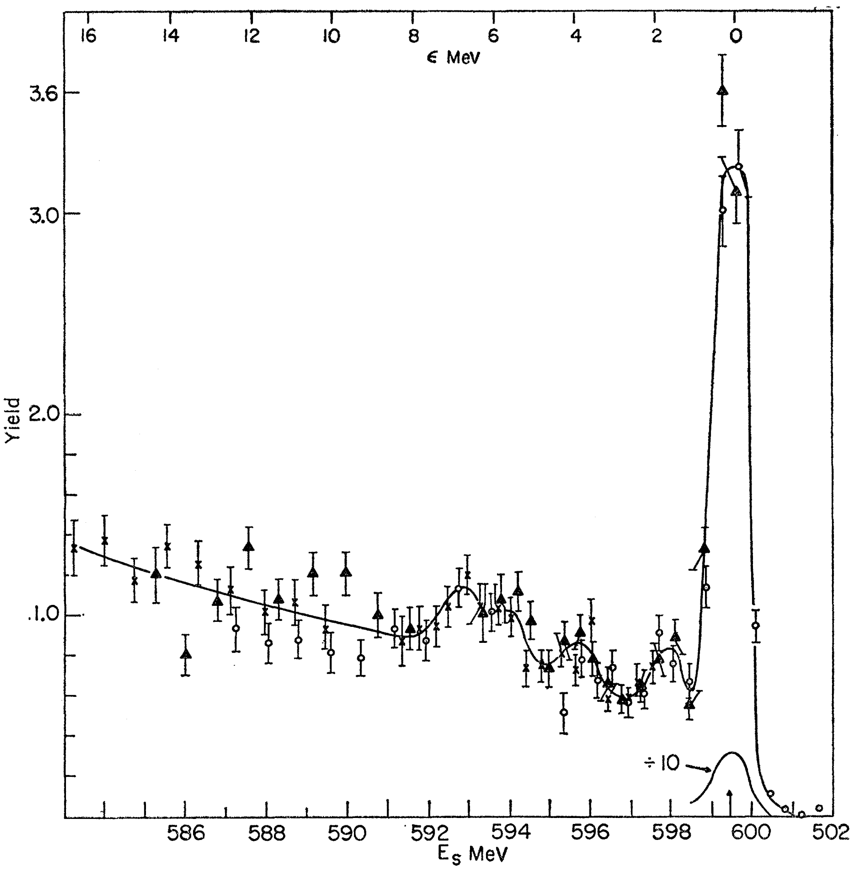

FIG. 4. Scattering of 600-MeV electrons from V^{51} at a laboratory angle of 31°. Cf. caption to Fig. 2.

runs. Two Čerenkov counters were filled with FC-75, a low index-of-refraction liquid fluorocarbon having a lower response to background gamma rays than Lucite. One counter backed the closely spaced scintillators; the other the remaining ones.

The remainder of the electronic equipment was similar to that described in I.

The vanadium target was quite thin (0.0102 radiation lengths) and the carbon and polyethylene targets used to determine the efficiencies of the counter array and to furnish the protons for the absolute cross-section determinations were matched to approximately the same thickness: 0.0106 and 0.0100 radiation length, respectively.

The spread in energy of the primary beam was set in the range from 0.1 to 0.2% and the uncertainty in momentum of the scattered electrons, from all effects combined, was from 0.16 to 0.25%.

Data reduction was done by the Stanford Computation Center's Burroughs 220 digital computer. The various channel efficiencies were computed and corrections applied to the raw data. The momentum spectrum of scattered electrons for each run was plotted and a visual fit to the data corrected for radiative and Schwinger effects using a second computer program. From these corrected spectrum the absolute cross sections for elastic and inelastic scattering could be determined by comparison with the scattering from the free protons in polyethylene, as in I. The raw and corrected spectra of two of the four runs on vanadium are shown in Figs. 2 through 5. These spectra show peaks corresponding to excitation of a number of higher energy states in vanadium. These data will be analyzed in a later paper.[2] Figure 6 shows the data from the scattering of protons in polyethylene and from the matched carbon target used to normalize the two vanadium runs for $E_0=183$ MeV, and Fig. 7 shows the corrected spectrum of electrons scattered from the protons after the carbon background subtraction.

The level at 0.93 MeV was observed only during the $E_0=183$-MeV runs and was masked by the elastic peak during the other runs. The two levels at 1.609 and 1.813 MeV were not completely resolved in any of the runs. The peaks at the two $E_0=183$-MeV runs and the $E_0=300$-MeV run each fell at 1.7 MeV and were noticeably flat and (for the lower energy runs) slightly wider than the associated elastic peaks. As there are no other levels beside those at 1.609 and 1.183 MeV which could contribute to the observed peak at 1.7 MeV, we concluded that we were exciting both levels with approximately equal intensity. While the cross sections and inelastic F^2 (as defined in I) may be determined with uncertainties of about 15% for the combined yield, the branching ratio cannot be found as precisely. Figure 8

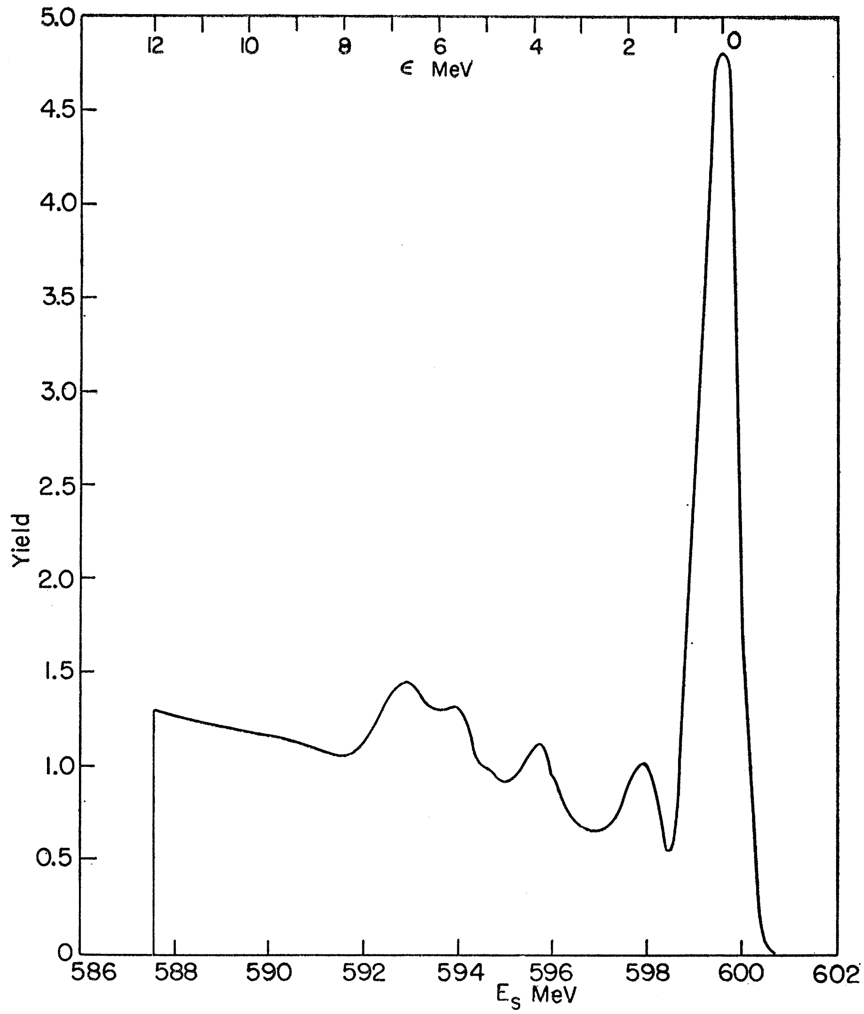

FIG. 5. The smooth curve of Fig. 4 with bremsstrahlung and Schwinger corrections applied. Cf. caption to Fig. 5.

shows the measured F^2 for the elastic, the 0.93 MeV, and the (combined) 1.609- and 1.813-MeV transitions. Theoretical inelastic F^2 are shown also. The F^2 are given as a function of $qA^{1/3}$, where q is the momentum transfer in the scattering process. In order to make meaningful the comparison of the 300- and 600-MeV data with the 183-MeV data we have corrected the values of q appropriate to these higher incident energies using the "local wavelength correction" described in I. The data may, after the corrections, be treated as if they were all taken at $E_0=183$ MeV. The corrections are small and are indicated on Fig. 8 by horizontal arrows which show the directions and magnitudes of the corrections.

The errors in Fig. 8 are standard deviations found from the number of events detected in the peaks and from estimates of the background subtractions made during the radiative correction programs. The theoretical predictions were made in a manner similar to those in I: The predictions for $E2$ transitions are shown in Fig. 7 as solid and dashed lines. As in I we regard the radius and edge thickness as parameters to be determined by the average values of these quantities found from Born approximation analysis of elastic scattering. The strengths of the transitions determine the normalization of the $|F|^2$ and these strengths are here regarded as quantities to be determined by experiment. The strength parameters are the β_2 and were used to compute the gamma-ray transition rates for the decay from the excited states to the ground state.

The data for the 1.609- and 1.813-MeV (combined) transitions are fitted somewhat better by the choice of a slightly smaller radius parameter in the theoretical F^2. We have not done this because there is no theory that supports such a change and because the present fit is not far outside a reasonable estimate of the errors of the measurements. The best fit alters the values of β_2 less than 10% from those reported here. We have included a contribution from this uncertainty in radius to our estimation of the errors in the β_2's.

From the values of β_2, we have derived the gamma-ray transition rates Γ_m and the values of $B(E2)$ for the corresponding upward transitions.

IV. RESULTS AND CONCLUSIONS

The results of the present measurements are shown in Table II which gives values of β_2, Γ_m, (the measured transition rates for the corresponding gamma-ray decay branches to the ground state) and the predicted and measured values of $B(E2)$ in terms of the single particle value. There are several sources of uncertainties in the measured values. The most important contributions to the uncertainties in the β_2 are from statistical fluctuations in the measured values which are enhanced in the β_2 by the necessary background subtractions, and, for the 1.609- and 1.813-MeV transitions, from our inability to resolve the two clearly. The uncertainties in the values of β_2 are standard deviations from the above effects and from the uncertainty in our knowledge of the correct value of the radius parameters. We have listed the β_2's for the 1.609- and 1.813-MeV transitions separately and for the combined yield.

The uncertainties in Γ_m and $B(E2)$ have contributions not only from those associated with the β_2, but also from deficiencies in the theory used in the analysis. On the basis of comparisons of measurements by electron scattering and by other techniques (discussed in detail in I) we estimate the uncertainties from these sources to be about 30%. These uncertainties are listed separately in Table I. An additional comparison is available from the present results: the value of $B(E2)$ for the 0.930 transition may be compared with three measurements by Coulomb excitation techniques (cf. Table IV). The agreement is excellent.

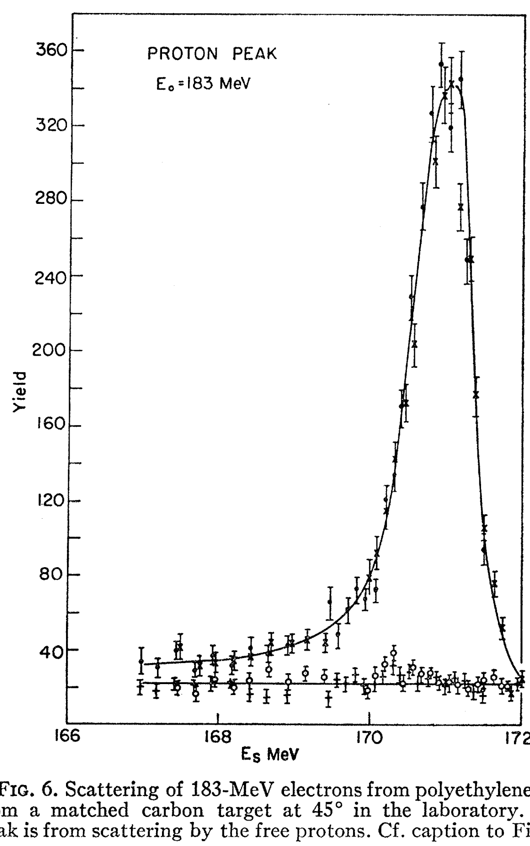

FIG. 6. Scattering of 183-MeV electrons from polyethylene and from a matched carbon target at 45° in the laboratory. The peak is from scattering by the free protons. Cf. caption to Fig. 2.

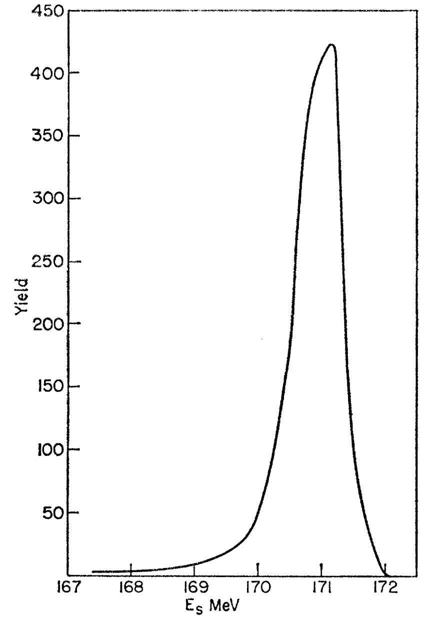

FIG. 7. The smooth curve of Fig. 6 with carbon background subtracted and bremsstrahlung and Schwinger corrections applied. The elastic and inelastic cross sections for the scattering from V^{51} were determined for the 183-MeV data from the yield from the e-p scattering and its known cross section. Cf. caption to Fig. 2.

Table III gives the comparisons between theory and experiment using the measurements of the present experiment except in the case of the 0.320-MeV transition. No data for this transition is available from electron scattering measurements. Comparisons are made both from the detailed study of the $(f_{7/2})^3$ configuration and the weak-coupling theory. The comparisons are made in such a way that the table entries may be compared with unity. There are a number of objections to the weak-coupling theory, discussed below. However, the data displayed in Table III do not distinguish between the two theories. It is seen that the measurements show deviations from the predictions of the one-particle-operator theory by as much as 70%. The maximum discrepancy is observed for the 0.930-MeV transition; the one whose value of $B(E2)$ is established with much smaller uncertainties than any other.

The V^{51} level spectrum cannot be explained by assuming a weak coupling of the odd proton to a $J_0=2$ collective state. The spread of the observed levels, from 0.3 to 1.8 MeV, is as big as the 0–2 separation in neighboring even-even nuclei. This can be well understood if the 2+ state in Ti^{50} is a state of the $f_{7/2}{}^2$ configuration rather than a collective vibration involving mainly excitations of the nucleons in closed shells. The odd proton in V^{51} is in the same orbit as the other two $f_{7/2}$ protons and its interaction with them is as strong as their mutual interactions. Another qualitative indica-

TABLE II. Experimental Values of β_2, Γ_m, $B(E2)$, and G. The table gives the electron-scattering results for transitions in V^{51}. The ϵ are the transition energies. The uncertainties in the β_2 are standard deviations from counting statistics and from uncertainties in the radius parameter required in the predicted $|F|^2$. These uncertainties are reflected in Γ_m, the gamma-ray decay rates, in the reduced transition rates $B_m(E2)$ for the upward transitions, and in the values of G, the enhancements of the $B_m(E2)$. Additional uncertainties in these latter quantities are written last and are discussed in the text. The $B_m(E2)$ are given in terms of the single-particle values $e^2\times0.02\times10^{-48}$ cm^4. The G are found using the single-particle predictions from Table I.

ϵ (MeV)	j	β_2	Γ_m (sec^{-1})	$B_m(E2)$	G
0.930	3/2	$(7.2\pm1.1)\times10^{-3}$	$(1.6\pm0.24\pm0.48)\times10^{11}$	$0.467\pm0.08\pm0.14$	$11.0\pm1.7\pm3.3$
1.609, 1.813 (combined)		$(1.6\pm0.25)\times10^{-2}$		$1.03\pm0.16\pm0.3$	$4.84\pm0.76\pm1.4$
1.609	11/2	$(8.0\pm3.0)\times10^{-3}$	$(9.23\pm3.5\pm2.8)\times10^{11}$	$0.516\pm0.19\pm0.15$	$3.12\pm1.2\pm0.9$
1.813	9/2	$(8.0\pm3.0)\times10^{-3}$	$(2.00\pm0.84\pm0.67)\times10^{12}$	$0.516\pm0.19\pm0.15$	$10.1\pm3.7\pm3.3$

tion is the fact that no low-lying 7/2− excited state, which could be obtained by coupling the odd $f_{7/2}$ proton to the 2+ collective state, is observed in V^{51}. In the $f_{7/2}{}^3$ configuration another 7/2− state, besides the ground state, is forbidden by the Pauli principle.

Quantitative evidence on the nature of the V^{51} spectrum comes from a detailed analysis of the energy levels. As mentioned in Part I, the levels calculated by assuming pure shell-model $f_{7/2}{}^3$ configuration and effective two-body forces taken from $f_{7/2}{}^2$ configurations (as in Ti^{50} or Ca^{42}) agree very well with the measured energies of the low-lying levels of V^{51}. The main conclusion that can thus be drawn from the results is that $E2$ transition rates between rather good shell-model $f_{7/2}{}^3$ states are strongly enhanced. These rates are much bigger than those obtained by the "single-particle" estimates. This means that the effective operators for $E2$ transitions which should be used with shell-model wave functions are very different from the single-particle operator (1).

The next question is whether the effective $E2$ operators can be well approximated by (effective) single-particle operators. Looking at Table III and Table IV, we see that this approximation is not very good in the present case. Adopting the experimental values, in spite of the large possible errors, we obtain that the "effective charge" for the various transitions is constant only within ±30%. Putting it in another way, only part of the $B(E2)$ values can be accounted for by effective single-particle operators (with effective charge of about 2.5e). The enhancement due to this part can be ascribed to independent core polarizations by the individual $f_{7/2}$ protons. Such enhancement is then expected, on the average, for all $E2$ transitions and static quadrupole moments of nuclei with the (unfilled) proton $f_{7/2}$ shell.

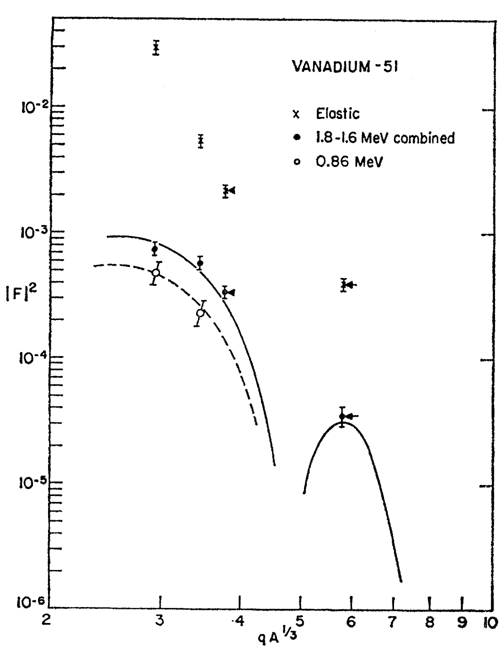

FIG. 8. The elastic and inelastic $|F|^2$ electron scattering from V^{51}. The abscissa, $qA^{1/2}$, is given for the momentum transfer q in reciprocal fermis. The data taken at primary energies of 300 and 600 MeV were corrected as described in the text to allow comparison with the 183-MeV data. The magnitude and direction of the corrections are indicated by horizontal arrows. The smooth and dotted curves are predictions for $E2$ transitions using a radius parameter $r_0=1.20$ F. See the text for a more complete discussion of this figure.

TABLE III. Comparison of Theory and Experiment. The table gives the values of G, the enhancement of the reduced transition rates, using predictions discussed in the text for the $(f_{7/2})^3$ configurations and also for the weak-coupling model. The prediction from the study of the $(f_{7/2})^3$ configurations is that the values of G should be the same. A less precise prediction, discussed in the text, indicates that the values should be close to the G observed for a neighboring even-even nucleus. For the 1.43-MeV $E2$ transition in Cr^{52} this value is approximately 6.5 (see Table IV). We have rather arbitrarily displayed $G/6.5$ so the results may be compared with unity. The values of G using the weak-coupling theory have been found from the value of $B(E2, 0\rightarrow2)$ from the 1.43-MeV transition in Cr^{52}, taken to be 6.5, in units of the single-particle value, $e^2\times0.02\times10^{-48}$ cm^4. (Cf. Table IV.) The predicted values of $B(E2, J\rightarrow j)$ were found from this value using Eq. (6) in the text, and compared with the measured $B(E2, J\rightarrow j)$ to yield G (weak coupling). These values also may be compared with unity.

ϵ (MeV)	j	$G/6.5$	G, weak coupling
0.320[a]	5/2	0.50 ± 0.12	1.6 ± 0.4
0.930[b]	3/2	1.69 ± 0.26	1.87 ± 0.32
1.609	11/2	0.48 ± 0.18	0.69 ± 0.25
1.813	9/2	1.55 ± 0.57	0.83 ± 0.30
1.609, 1.813 (combined)		0.745 ± 0.12	0.748 ± 0.12

[a] Data from Adams *et al.*, see footnote b of Table IV.
[b] Measured values in good agreement with others; see Table IV.

TABLE IV. The table gives the values of $B(E2)$ and G for transitions in V^{51} and Cr^{52} measured by other workers. Values of G are given in brackets. The values of G for the 0.930-MeV transition in V^{51} compare favorably, for the most part, with the value 11.0 measured in the present experiment.

			$B(E2)$ and G[a]				
ϵ MeV	Nuclide	$B_T(E2)$	Gove and Broude[b]	Adams *et al.*[b]	Lemberg[b]	Stelson[c]	McGowan *et al.*[d]
0.320	V^{51}	0.184	0.65 (3.53)	0.60±0.15 (3.26±0.8)		0.65 3.53	
0.930	V^{51}	0.043	0.19[e] (4.4)	0.40 (9.3)	0.55 (12.1)		
1.43	Cr^{52}	0.50		3.0 ±0.75 (6.0 ±1.5)	3.2±0.6 (6.4±1.2)		3.62±0.39 (7.25±0.8)

[a] $B(E2)$ measured in units of the single-particle value $e^2\times0.02\times10^{-48}$ cm^4.
[b] H. E. Gove and C. Broude (Chalk River); B. M. Adams, D. Eccleshall, and M. J. L. Yates (Aldermaston); I. Kh. Lemberg (Leningrad); *Reactions between Complex Nuclei: Proceedings of the Second Conference on Reactions between Complex Nuclei, May 2–4, 1960, Gatlinburg, Tennessee* (John Wiley & Sons, Inc., New York, 1960).
[c] P. H. Stelson (Oak Ridge) (private communication).
[d] C. McGowan *et al.* (Oak Ridge), reported at the Conference on Nuclear Lifetimes, Gatlinburg ,Tennessee, October 1961 (to be published).
[e] Value may be in error. H. E. Gove (private communication).

The other part of the $B(E2)$ values may either increase or decrease the values obtained by using the single-particle operators. This other part is due to other effects, like configuration interaction, which depend on the particular states involved. These effects cannot be described by single-particle operators and must be given by two- and three-particle terms in the effective operator for the pure $f_{7/2}^n$ configurations.

In order to have a more complete and systematic picture, it would be interesting to have also the $B(E2)$ values of Ti^{50} and Fe^{54} as well as an accurate measurement of the V^{51} quadrupole moment. These nuclei have closed neutron shells and $f_{7/2}^n$ proton configurations. The $f_{7/2}^n$ neutron configurations are found in the Ca isotopes. It would be instructive to have $B(E2)$ values and quadrupole moments measured in the Ca isotopes.

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Polarization of Protons from Deuteron Stripping Reactions with a Zero Orbital Angular Momentum Transfer*

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Angular dependence of the polarization of protons from the $l=0$ stripping reactions, $Al^{27}(d,p)Al^{28}{}_{g.s.+1st}$ and $Si^{28}(d,p)Si^{29}{}_{g.s.}$ has been measured at a deuteron energy of 15 MeV. Polarization changes the sign at angles close to each minimum of the angular distribution, remaining the same in the angular region corresponding to each stripping peak. The magnitude of polarization is large (20 to 30%) near the angles at which the sign change takes place.

I. INTRODUCTION

IN recent years, considerable attention has been given to polarization of the outgoing particles from the deuteron stripping reaction.[1] Study of the polarization is considered very useful for a better understanding of the stripping reaction. Two different effects are respon-

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[1] For example, a fine introduction to this phenomena is found in the following article: N. Austern, "Direct Reactions; Fast Neutron Physics" (to be published).