Polarization and Differential Cross Sections in *n-d* Scattering*

A. J. ELWYN, R. O. LANE, AND A. LANGSDORF, JR.
Argonne National Laboratory, Argonne, Illinois

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The polarization and differential cross section for neutrons scattered from deuterium are measured at five laboratory angles (22°, 48°, 80°, 100°, and 150°) for incident neutron energies of 0.5, 1.0, and 1.95 MeV. The neutrons were produced in the $Li^7(p,n)Be^7$ reaction. The partially polarized beam emitted at 51° was used first directly and then with the spins precessed through 180° in the transverse field of an electromagnet in order to determine the asymmetry in the intensities after scattering from deuterium. The results at 0.5 and 1.0 MeV give zero polarization within 2–3%. At 1.95 MeV small positive polarizations up to 4% are observed. The differential cross sections measured are more peaked at large angles than in earlier work. These data are compared with earlier calculations and a new phase-shift analysis of these measurements is made.

I. INTRODUCTION

MEASUREMENTS of the polarization of neutrons elastically scattered from deuterons have yielded conflicting results in the neutron energy region below 5 MeV. At 3 and 4 MeV Bucher *et al.*[1] have obtained polarizations of a few percent at most. This is consistent with results of Brüllman *et al.*[2] at 3.27 MeV. At 2 MeV the results obtained by Cranberg[3] at five angles are not inconsistent with polarizations of zero, while Darden *et al.*[4] have measured a small negative polarization at this energy. White *et al.*,[5] however, find a very large polarization (about 50%) near 2 MeV which is definitely in disagreement with the other measurements. At 1 MeV Darden *et al.* find a polarization of about10%. Recent measurements of Ferguson and White[6] are in agreement with those of Darden at 1 MeV, and give values of about 16% at 0.6 MeV. Beghian *et al.*[7] obtain a small positive polarization of the order of 5% or so at 1.175 MeV. In most of these earlier measurements the errors were rather large.

It has been pointed out by Massey[8] that the measurement of the polarization in nucleon-deuteron elastic scattering should lead to information on the size of the tensor-force components in the nucleon-nucleon interaction. While the general theory of *n-d* scattering including tensor forces has been formulated,[9] numerical calculations leading to definite predictions from this formalism have not yet been published. An approximate calculation, which takes into account deuteron distortion as well as tensor forces, has been made by Delves and Brown.[10] The predictions are in general inconsistent with the above mentioned experiments, except perhaps at 2 MeV.

The present experiments were undertaken with the purpose of improving the accuracy of the polarization measurements while at the same time checking the calculation of Delves and Brown. The selected energies of the incident neutrons were 0.5, 1.0, and 1.95 MeV. At each energy, the polarization was determined at five scattering angles.

Since the unpolarized differential cross section could be obtained at the same time as the polarization, it was also measured at each of the three incident energies above. In this energy region there have been few previous measurements of angular distributions in *n-d* scattering. Adair *et al.*,[11] using a gas-recoil counter, obtained results from 0.2 to 2.5 MeV, but the precision of the data was not too high. Other recoil measurements were performed with greater precision by Tunnicliffe[12] and by Allen *et al.*[13] at energies from 0.1 to 1.0 MeV. Seagrave and Cranberg[14] have made accurate measurements at 2.45 and 3.27 MeV by a time-of-flight technique. There have been some other measurements between 2.5 and 14 MeV.[15]

The theories of *n-d* and *p-d* scattering that have been most successful from the point of view of comparison with experimental angular distributions have been formulated in terms of central forces alone for the internucleon interaction. (One consequence of this is that

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[2] M. Brüllmann, H.-J. Gerber, D. Meier, and P. Scherrer, Helv. Phys. Acta **32**, 511 (1959).

[3] L. Cranberg, Phys. Rev. **114**, 174 (1959).

[4] S. E. Darden, C. A. Kelsey, and T. R. Donoghue, Nuclear Phys. **16**, 351 (1960).

[5] R. E. White, A. Chisholm, and D. Brown, Nuclear Phys. **7**, 233 (1958).

[6] A. T. G. Ferguson and R. E. White, Nuclear Phys. **33**, 477 (1962).

[7] L. E. Beghian, K. Sugimato, M. Wachter, and J. Weber, Bull. Am. Phys. Soc. **7**, 333 (1962). In a private communication these authors have revised their deuterium measurement from the value given in this reference to a value of $P_1(30°_{\text{lab}})P_2(60°_{\text{lab}}) = 0.015 \pm 0.01$.

[8] H. S. W. Massey, in *Proceedings of the International Conference on Nuclear Forces and the Few Nucleon Problem* (Pergamon Press, London, 1960), Vol. II, p. 345.

[9] B. H. Bransden, K. Smith, and C. Tate, Proc. Roy. Soc. (London) **A247**, 73 (1958); K. Smith and M. Peshkin, Argonne National Laboratory Report ANL-5910 (unpublished).

[10] L. M. Delves and D. Brown, Nuclear Phys. **11**, 432 (1959).

[11] R. K. Adair, A. Okazaki, and M. Walt, Phys. Rev. **89**, 1165 (1953).

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[13] W. D. Allen, A. T. G. Ferguson, and J. Roberts, Proc. Phys. Soc. (London) **A68**, 650 (1955).

[14] J. D. Seagrave and L. Cranberg, Phys. Rev. **105**, 1816 (1957).

[15] J. C. Allred, A. H. Armstrong, and L. Rosen, Phys. Rev. **91**, 90 (1953); E. Wantuch, *ibid.* **84**, 169 (1951); M. M. Gordan and W. D. Barfield, *ibid.* **86**, 679 (1952); S. L. Martin, E. H. S. Burhop, C. B. Alcock, and R. L. F. Boyd, Proc. Phys. Soc. (London) **A63**, 884 (1950); J. H. Coon and H. H. Barshall, Phys. Rev. **70**, 592 (1946); I. Hamouda, J. Halter, and P. Scherrer, *ibid.* **79**, 539 (1950); I. Hamouda and G. de Montmollin, *ibid.* **83**, 1277 (1951).

they can lead only to predictions of zero polarization in the scattering.) These theories, discussed originally by Buckingham and Massey,[16] have been extended mainly by Buckingham *et al.*[17] and others[18] in Great Britain, and by Christian and Gammel.[19] Measurements have been in reasonable agreement with these calculations above 2 MeV. At lower energies the experiments are less accurate, and the theory more approximate.

The present experiments performed in this low-energy region are reasonably accurate. The results are compared with extrapolations of the presently available theories, and with *p-d* scattering measurements. A phase-shift analysis of the present measurements is also made.

II. THE EXPERIMENT

The experimental arrangement has been described previously,[20,21] and will be mentioned only briefly. Figure 1 is a schematic diagram of the setup. The protons accelerated in the Argonne 4.5-MeV Van de Graaff machine are incident on normal Li, in the form of lithium nitride, placed at the center of a shielded source tank. Partially polarized neutrons are produced by the $Li^7(p,n)Be^7$ reaction. Those emitted at an angle of 51° to the incident proton beam pass between the poles of an electromagnet, which forms part of a collimator system, and impinge on scatterers in the form of thin slabs of the material of interest. These slabs, rotated so that their normals make an angle of 30° with the incident beam, are placed at the center of a circular track around which five neutron detectors can be positioned. The detectors are arrays of BF_3 proportional counters in oil moderator, located at the center of neutron shield tanks filled with borated water. The signals from the five detectors are amplified and fed into scalers. The forward neutron flux is monitored by two oil-moderated BF_3 counters, arranged to be insensitive to back-scattered neutrons from the scatterers or from the large detector tanks. A long counter (not shown in Fig. 1), positioned to sample the backward neutron flux from the Li target, served as a second monitor. The apparatus is arranged so that the neutron-producing reactions and the neutron scatterings are coplanar.

The procedure for any given scatterer consisted in simultaneously measuring the intensity of scattered neutrons (i.e., the counting rates) at five angles, first with the electromagnet off and then with the magnet

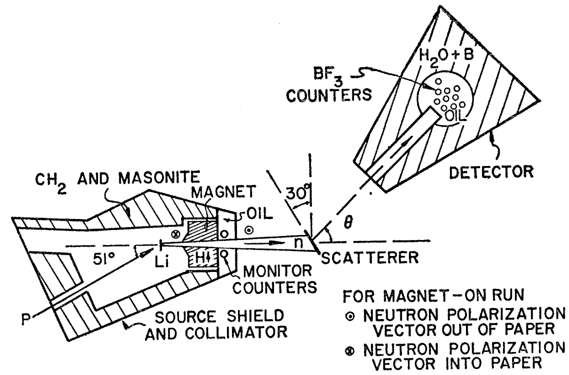

FIG. 1. Schematic top view of the experimental arrangement, showing one of the large detector tanks and the placement of the electromagnet between the neutron source and the scatterer. The direction of the polarization vector of the neutrons produced in the $Li^7(p,n)Be^7$ reaction is initially into the paper, but after passing through the magnetic field it will point out of the paper.

turned on at a value of magnetic field sufficient to precess the neutron spins through 180°. Therefore, at any given scattering angle θ, the ratio of the counting rate with the magnet off to that with the magnet on is equivalent to a left-right asymmetry ratio. This ratio $L(\theta)/R(\theta)$ is related to the product $P_1(51°)P_2(\theta)$ by

$$\frac{L(\theta)}{R(\theta)}=\frac{1-P_1(51°)P_2(\theta)}{1+P_1(51°)P_2(\theta)}, \tag{1}$$

where $P_1(51°)$ is the polarization of the neutrons emitted at 51° to the incident protons in the $Li^7(p,n)Be^7$ reaction, and $P_2(\theta)$ is the polarization that would result if an unpolarized beam of neutrons incident on the scatterer were scattered through an angle θ. The choice of algebraic sign for $P_1(51°)$ and $P_2(\theta)$ is consistent with the Basel convention[22] for the positive direction of the polarization vector. The unpolarized differential cross section at any angle θ is proportional to the sum of the magnet-off and magnet-on runs. It was possible therefore to obtain the differential cross section from the same data as the polarization.

The scatterers were in the form of thin sheets of deuterated polyethylene (CD_2). To get the net counts for *n-d* scattering, the properly normalized counting rates of neutrons scattered from carbon were subtracted from the CD_2 data. During most of the experiment the CD_2 and C samples used were 1/16 in. thick, except at the highest energy (1.95 MeV) where carbon scatterers 1/8 in. thick were utilized. (The experiment was repeated with 1/32-in. CD_2 samples and 1/16-in. C scatterers, as will be discussed below.) The transmission of both the CD_2 and the C samples at the three neutron energies studied was always greater than 93% so that the effects of multiple scattering were minimized.

[16] R. A. Buckingham and H. S. W. Massey, Proc. Roy. Soc. (London) **A179**, 123 (1941).
[17] R. A. Buckingham, S. J. Hubbard, and H. S. W. Massey, Proc. Roy. Soc. (London) **A211**, 183 (1952).
[18] A. H. deBorde and H. S. W. Massey, Proc. Phys. Soc. (London) **A68**, 769 (1955); P. G. Burke and H. H. Robertson, *ibid.* **A70**, 777 (1957); F. A. Haas and H. H. Robertson, *ibid.* **A73**, 160 (1959); J. W. Humberston, *ibid.* **78**, 1157 (1961).
[19] R. S. Christian and J. L. Gammel, Phys. Rev. **91**, 100 (1953).
[20] R. O. Lane, A. J. Elwyn, and A. Langsdorf, Jr., Phys. Rev. **126**, 1105 (1962); A. Langsdorf, Jr., A. J. Elwyn, and R. O. Lane (to be published).
[21] A. J. Elwyn and R. O. Lane, Nuclear Phys. **31**, 78 (1962).

[22] *Proceedings of the International Symposium on Polarization Phenomena of Nucleons, Basel, 1960* [Helv. Phys. Acta Suppl. **6**, 436 (1961)].

Throughout the experiment the scattering of neutrons by protons (n-p scattering) was studied as a check on possible systematic effects. CH_2 scatterers, nominally 1/16 in. thick, were used. (Again the experiment was repeated with CH_2 samples 1/32 in. in thickness.) The transmission of these samples was greater than about 90% at all energies.

The rise-curve method at threshold was used periodically throughout the experiment to measure the thickness of the neutron-producing Li targets. This thickness was 80–100 keV for 1.9-MeV protons. The laboratory scattering angles were 22°, 48°, 80°, 100°, and 150°. The experiment was performed at three incident neutron energies: 0.5, 1.0, and 1.95 MeV.

The following experimental procedure was used to check on possible time-dependent systematic effects. The scattered intensities first with the magnet on and then with the magnet off were determined: (1) with the CD_2 samples in place, (2) with the carbon scatterer in place, (3) with no sample, and (4) with the CH_2 scatterer in position. Each of these runs (i.e., magnet on *or* magnet off) took from 4 to 10 min and was for a prescribed accumulation of integrated proton beam. It was further required that the ratio of the counting rates in the two monitors remain constant from one run to the next. The above sequence of runs was repeated 6–10 times at each incident energy. In this fashion a maxis mum of 500 000 and a minimum of 120 000 total count-were accumulated for the sum of the magnet-on and magnet-off runs at any angle. The counting rate with no sample (air background rate) was quite low in most cases. It varied from 10 to 20% of the counting rate with the 1/16-in. CD_2 sample in place in most cases but rose to 35% at 22° for incident energies of 0.5 and 1.0 MeV and at 150° for 1.95 Mev.

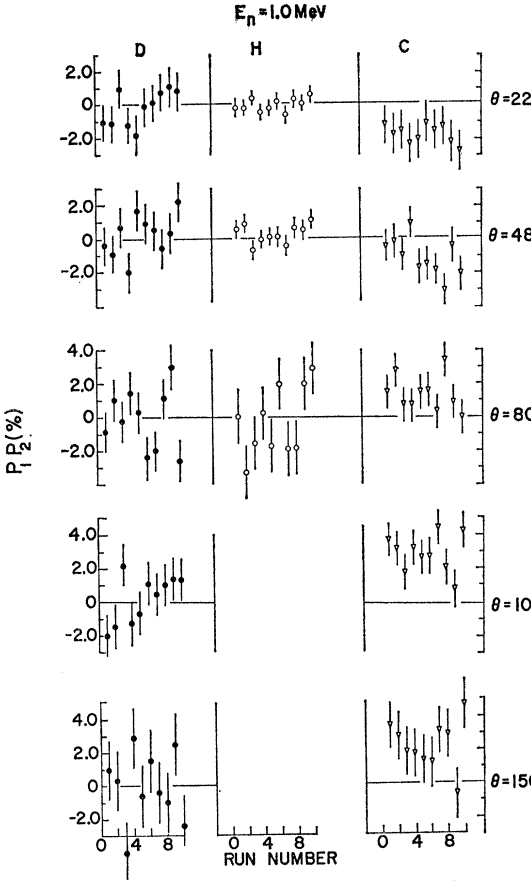

FIG. 3. The values of $P_1(51)P_2(\theta)$ as a function of run number at each of the laboratory scattering angles at 1.0 MeV, for n-d scattering (D), n-p scattering (H), and n-carbon scattering (C).

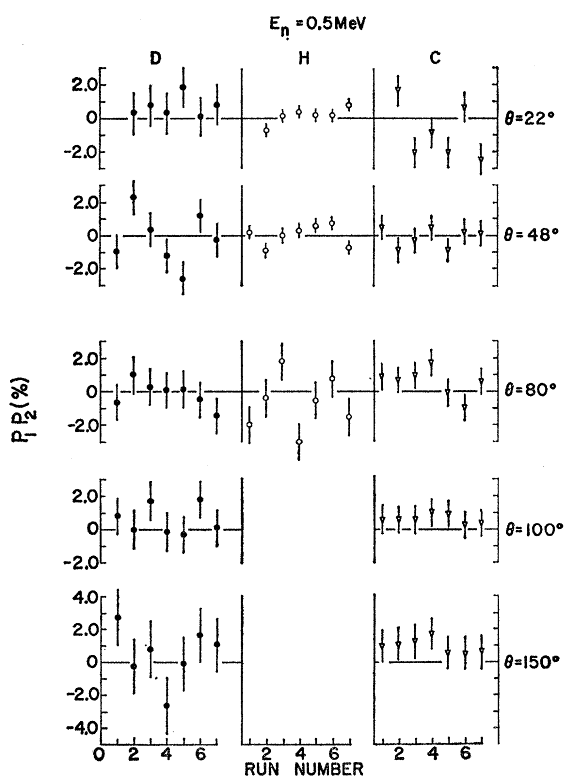

FIG. 2. The values of $P_1(51)P_2(\theta)$ as a function of run number at each of the laboratory scattering angles at 0.5 MeV, for n-d scattering (D), n-p scattering (H), and n-carbon scattering (C).

After each sequence of runs, calculations leading to left-right ratios and $P_1(51°)P_2(\theta)$ products for n-d, n-p, and n-carbon scattering were performed immediately with an IBM-1620 computer. It was possible therefore to keep a constant check on the behavior of the apparatus and, if necessary, to repeat runs because of inconsistencies that otherwise would not have become apparent until some later time. Figures 2, 3, and 4 show the results for $P_1(51°)P_2(\theta)$ as obtained from the raw data for n-d (shown as column D on the figures), n-p(H), and n-carbon (C) scattering at 0.5, 1.0, and 1.95 MeV for each of the laboratory scattering angles. These results indicate that there are no time-dependent systematic effects, at least none larger than the statisti-

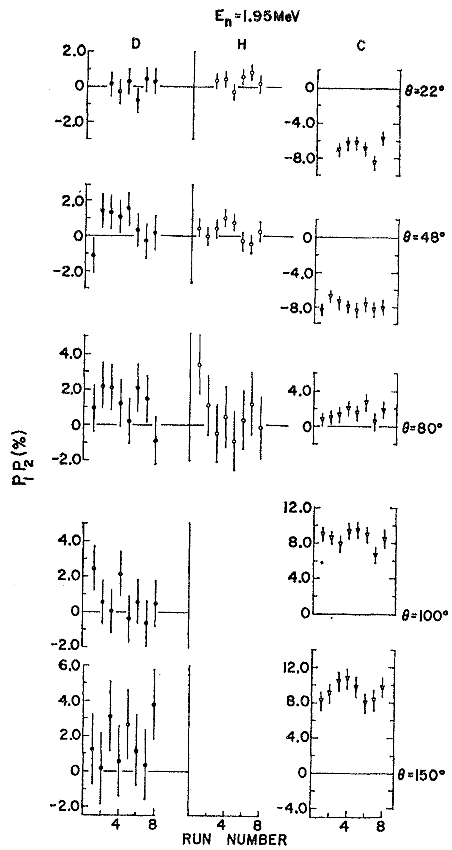

FIG. 4. The values of $P_1(51)P_2(\theta)$ as a function of run number at each of the laboratory scattering angles at 1.95 MeV, for *n-d* scattering (D), *n-p* scattering (H), and *n*-carbon scattering (C).

cal accuracy of each point. The $P_1(51°)P_2(\theta_2)$ values for *n-p* scattering are shown only at the first three scattering angles. No neutrons can be singly scattered from protons to angles greater than 90° in the laboratory system.[23]

III. DATA ANALYSIS AND RESULTS

A. Polarization

Column 4 of Table I presents the values of $P_1(51°)P_2(\theta)$ obtained as properly weighted averages of the data shown in Figs. 2–4 for *n-d* scattering. The errors are based on counting statistics only.

In order to check that the small values of $P_1(51)P_2(\theta)$ shown in the table are not due to a malfunctioning of the magnet or its magnetic field control circuit (which might result in nonprecession of the neutron spins), or to depolarization within the magnet because of the spread in neutron energies of the beam, the values of $P_1(51)P_2(\theta)$ for *n*-carbon scattering (obtained by properly averaging the results in Figs. 2–4) were compared with previous measurements. The present results for carbon are in excellent agreement with the older five-angle spin-precession measurements,[24] and also with asymmetry measurements performed without the use of a magnet to precess the spins.[21] As a further check the polarization in the scattering of neutrons from lithium was determined from time to time throughout the present experiment and is in excellent agreement with previous results.[25] Moreover, the depolarization expected when a beam of neutrons with a mean energy of, for example, 500 keV and an energy spread of 100 keV passes through the magnet was calculated to be only 0.6%. Such a value is in agreement with previous tests on the magnet system,[20] and falls well within the experimental errors in the present work.

Other experimental sources of systematic error were monitored by studying the polarization in the scattering of neutrons from hydrogen. At the energies employed in the present experiment, the polarization in *n-p* scattering should be zero at all angles. Again from Figs. 2–4 the average values of the products $P_1(51°)P_2(\theta)$ were determined at three angles: 22°, 48°, and 80°. To check these products at other angles, a supplementary experiment was carried out at incident energies of 0.47, 0.97, and 1.63 MeV. The results of all measurements are shown in Fig. 5, where $P_1(51)P_2(\theta)$ is plotted against the cosine of the scattering angle in the c.m. system. The results for both 1/16- and 1/32-in. thicknesses of CH_2 scattering samples are shown at 1.95 MeV. The differences between the two samples can probably be attributed to a systematic effect due to more multiple scattering in the 1/16-in. scatterer. Except for one point at 0.97 MeV and one (with poor statistical accuracy) at 1.95 MeV, the results are consistent with a polarization in *n-p* scattering of $(0.0\pm2.0)\%$ at 0.5 and 1.0 MeV, and $(0.0\pm3.0)\%$ near 1.95 MeV. From these considerations it is felt that no large sources of experimental systematic difficulties are affecting the measurements.

Because of the large transmissions (greater than 93%), i.e., small scattering, of the CD_2 and carbon scattering samples, and because the polarization in *n*-carbon scattering is very small at 0.5 and 1.0 MeV, the uncertainties due to multiple scattering should be well within the statistical uncertainties of the $P_1(51)P_2(\theta)$ products at these energies. At 1.95 MeV, however, even though the transmissions of the scatterers are greater than 95%, the polarization in *n*-carbon scattering reaches a maximum of +40%,[24] and this could lead to systematic errors due to multiple scattering. To check this, the experiment was rerun at 1.95 MeV with good statistical accuracy with CD_2 and CH_2 samples 1/32 in. thick, and carbon scatterers 1/16 in.

[23] Actually, the ratio of the measured counting rate at 150°, to that at 22°, was 0.025 for the CH_2 scatterer $\frac{1}{16}$ in. thick. This small amount is due in all probability to the effects of multiple scattering.

[24] R. O. Lane, A. J. Elwyn, and A. Langsdorf, Jr. (to be published).

[25] R. O. Lane, A. J. Elwyn, and A. Langsdorf, Jr., Bull. Am. Phys. Soc. 6, 430 (1961).

TABLE I. Values of $P_1(51)P_2(\theta)$ and $P_2(\theta)$ for *n-d* scattering.

(1) E_n (MeV)	(2) θ_{lab} (deg)	(3) $\theta_{c.m.}$ (deg)	(4) P_1P_2 (1/16-in. sample, uncorrected) (%)	(5) P_1P_2 (1/32-in. sample, uncorrected) (%)	(6) P_1P_2 (1/16-in. sample, corrected) (%)	(7) P_1P_2 (1/32-in. sample, corrected) (%)	(8) $P_1(51)$ (%)	(9) P_2 (1/16-in. sample) (%)	(10) P_2 (1/32-in. sample) (%)
0.5	22	33	0.7 ± 0.5		0.8 ± 0.6		24.	3.2 ± 2.4	
	48	70	0.2 ± 0.4		0.2 ± 0.4			0.8 ± 1.8	
	80	110	-0.2 ± 0.4		-0.2 ± 0.4			-0.7 ± 1.8	
	100	130	0.6 ± 0.4		0.6 ± 0.4			2.4 ± 1.8	
	150	165	0.1 ± 0.6		0.1 ± 0.6			0.6 ± 2.6	
1.0	22	33	-0.1 ± 0.4		-0.2 ± 0.5		33.	-0.5 ± 1.4	
	48	70	0.2 ± 0.4		0.3 ± 0.5			0.9 ± 1.4	
	80	110	-0.1 ± 0.4		0.2 ± 0.5			-0.5 ± 1.5	
	100	130	0.2 ± 0.4		0.3 ± 0.5			0.8 ± 1.4	
	150	165	-0.3 ± 0.6		-0.4 ± 0.7			-1.1 ± 2.1	
1.95	22	33	0.0 ± 0.3	0.5 ± 0.4	0.0 ± 0.3	0.6 ± 0.4	37.	0.0 ± 0.9	1.6 ± 1.1
	48	70	0.6 ± 0.3	1.3 ± 0.5	0.7 ± 0.4	1.5 ± 0.6		1.8 ± 1.1	4.1 ± 1.7
	80	110	1.2 ± 0.5	1.5 ± 0.7	1.4 ± 0.6	1.8 ± 0.9		3.7 ± 1.5	4.8 ± 2.3
	100	130	0.7 ± 0.5	0.7 ± 0.7	0.8 ± 0.5	0.8 ± 0.8		2.1 ± 1.4	2.1 ± 2.2
	150	165	1.7 ± 0.7	-0.3 ± 1.2	1.9 ± 0.8	-0.3 ± 1.4		5.1 ± 2.2	-0.9 ± 3.8

thick.[26] The results are shown in column 5, Table I. The values are slightly larger than those obtinead with the 1/16-in. scatterer, but within the statistical accuracy the values follow the same trend as a function of angle as do the previous measurements. The results at 1.95 MeV with both the 1/16- and 1/32-in. samples indicate that a real, albeit small, polarization exists in *n-d* scattering at this energy, and that effects of multiple scattering fall well within the experimental accuracy of the points.

Other effects that could lead to systematic errors were considered. Since the detectors could not distinguish between elastically scattered neutrons and any neutron group at lower energy, it was necessary to correct the measured values of $P_1(51)P_2(\theta)$ for the lower energy group of neutrons corresponding to the $Li^7(p,n)Be^{7*}$ reaction leaving Be^7 in a state at 0.43 MeV.[27] The correction, made on the assumption that this second group of neutrons is unpolarized,[3] varied from a minimum (1% of the value of the polarization) at 0.5 MeV to a maximum (18% of the value) at 1.0 MeV.

Also, according to the specifications, the CD_2 samples[28] used contained a 2% contamination of H. The measured $P_1(51)P_2(\theta)$ products were corrected for this effect by use of the known *n-p* cross sections[29] under the assumption that the neutrons scattered from hydrogen are unpolarized. This correction reached a maximum (11% of the value of the polarization) at 22° for $E_n=0.5$ MeV, and was much smaller at other angles and energies.

Considerations were given to the possibility that the chemical formula for the deuterated polyethylene used was not exactly CD_2, although our normalization procedure was based on this assumption. Even in the unlikely event that the ratio of D to C should differ from 2.0 by as much as 5%, the measurements of $P_1(51)P_2(\theta)$ should not be changed by more than 6% of their values. This effect lies well within the statistical accuracy of the results.

The general conclusion from the above discussion is

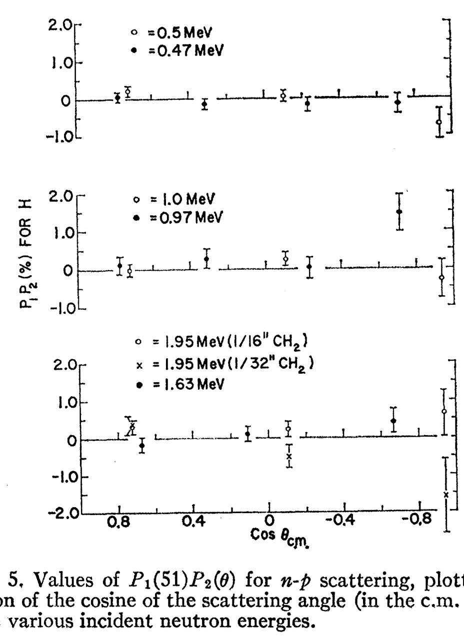

FIG. 5. Values of $P_1(51)P_2(\theta)$ for *n-p* scattering, plotted as a function of the cosine of the scattering angle (in the c.m. system) for the various incident neutron energies.

[26] The experiment was also rerun at 0.5 and 1.0 MeV with these thinner samples. Although the statistical accuracy of the P_1P_2 values was not very good, the results are in agreement with the measurements with the thicker sample. At 1.95 MeV, under the new conditions, the counting rates for the background runs were a much larger percentage of the sample-in runs, and great care was exercised to monitor instabilities and to keep the Van de Graaff operating under stable conditions.

[27] A. Smith (private communication); P. R. Bevington, W. W. Rolland, and H. W. Lewis, Phys. Rev. **121**, 871 (1961).

[28] The CD_2 samples were obtained from U. S. Nuclear Corporation, Burbank, California.

[29] *Neutron Cross Sections*, compiled by D. J. Hughes and J. A. Harvey, Brookhaven National Laboratory BNL-325 and Supplement (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1955 and 1957).

that the largest source of error in the measured polarization products for n-d scattering are the counting statistics. Systematic errors fall well within the statistical accuracy associated with each value of $P_1(51)P_2(\theta)$. It might be mentioned that other spurious asymmetries (such as those arising from the finite size of the scatterer and detector) which contribute in the usual measurements of left-right asymmetry,[30] are avoided completely by the use of a magnet.

The corrected values of $P_1(51)P_2(\theta)$ are presented in columns 6 and 7 in Table I. Values of $P_1(51°)$ from previous measurements, both at Argonne[21] and elsewhere,[31] are shown in column 8.[32] Dividing the corrected $P_1(51)P_2(\theta)$ products by these values of $P_1(51°)$ leads to final values for $P_2(\theta)$, the polarization of the neutrons in n-d scattering. These are shown in columns 9 and 10. The errors are statistical only.

B. Differential Cross Section

The net numbers of counts as a function of angle at 0.5, 1.0, and 1.95 MeV for n-d scattering were obtained from the sums of the magnet-on and magnet-off gross counts for the CD_2 and C scattering samples. In all cases the total number of net counts was sufficiently large that the statistical error was always less than 0.8%. Similarly, the net counts for n-carbon scattering have very small statistical errors.

For both n-d and n-carbon scattering, the total number of counts at each angle was corrected for the relative nonuniformity of the five neutron detectors at each of the three incident energies, and for the energy dependence of the relative efficiency of any given detector. The degree of nonuniformity of the five detectors was measured by an extension of a procedure that has been previously described.[21] The energy dependence of the relative efficiency of any given detector has been determined previously,[33] and is shown as Fig. 2 of reference 33. In addition, the n-d counts were also corrected for the 2% hydrogen content in the CD_2 sample; and both the n-d and n-carbon data were corrected for the second group of neutrons in the $Li^7(p,n)Be^7$ reaction, which leads to the Be^7 level at 0.43 MeV. For these latter effects, the maximum correction to the n-d results was a 10% change in the measured counting rate for the point at 22° and 0.5 MeV. For most of the other data points the correction was less than 2%.

[30] J. E. Monahan and A. J. Elwyn, Argonne National Laboratory Report, ANL-6420, 1961 (unpublished); J. E. Monahan and A. J. Elwyn, Nuclear Instr. and Methods **14**, 348 (1961); J. E. Evans, Atomic Energy Research Establishment Report AERE-R3347, 1960 (unpublished).

[31] H. R. Striebel, S. E. Darden, and W. Haeberli, Nuclear Phys. **6**, 188 (1958); J. A. Baicker and K. W. Jones, *ibid.* **17**, 424 (1960); S. M. Austin, S. E. Darden, A. Okazaki, and Z. Wilhelmi, *ibid.* **22**, 451 (1960).

[32] These values of $P_1(51°)$ are the values from references 21 and 31 averaged over a neutron energy spread of 100 keV. The final values of $P_2(\theta)$ are also to be interpreted as averages.

[33] R. O. Lane, A. Langsdorf, Jr., J. E. Monahan, and A. J. Elwyn, Ann. Phys. **12**, 135 (1961).

TABLE II. Measured values of $\sigma(\theta)$ for n-d scattering.

E_n (MeV)	θ_{lab} (deg)	$\theta_{c.m.}$ (deg)	$\sigma(\theta)$ (1/16-in. sample) (b/sr)
0.5	22	33	0.110±0.006
	48	70	0.157±0.008
	80	110	0.268±0.013
	100	130	0.380±0.019
	150	165	0.617±0.031
1.0	22	33	0.144±0.007
	48	70	0.135±0.007
	80	110	0.224±0.011
	100	130	0.344±0.017
	150	165	0.625±0.031
2.0	22	33	0.201±0.010
	48	70	0.140±0.007
	80	110	0.149±0.008
	100	130	0.245±0.012
	150	165	0.558±0.028

Absolute differential cross sections for n-d scattering were obtained by comparison with the known total cross section of a standard scatterer, in this case carbon. This method has been described by Lane *et al.*[33] The scattering cross section integrated over all angles was normalized to the independently measured integrated carbon cross sections, which in turn were set equal to the known total cross section for carbon.[29] The procedure was checked by comparing the resulting total cross section for deuterium with the known total cross section.[29] The agreement was excellent; the differences at each of the three energies were less than 2%.

The final n-d results were not corrected for the effects of multiple scattering or beam attenuation. Such effects are expected to be small because of the high transmission (greater than 93%) of the 1/16-in. CD_2 and C samples that were used. As an experimental check of these effects, the final results with the 1/16-in. samples were compared with the differential cross sections obtained when the 1/32-in. CD_2 sample (96% transmission) was used as a scatterer.[34] The results of the two measurements agreed to within 4% at all energies and angles. It was assumed, therefore, that neglect of multiple scattering introduces a systematic error of about this magnitude into the results.

Since the errors due to counting statistics are less than 0.8%, systematic effects constitute the major source of experimental error in these n-d measurements. Effects of this sort can be associated with the accuracy of the corrections for the energy dependence of efficiency and the nonuniformity of the detectors, as discussed above, as well as with the effects of multiple scattering. When the properly corrected n-p cross section[35] measured with CH_2 scatterers 1/32 in. thick was compared with the known total cross section for hydrogen[29] agreement was better than 2%. Thus, an

[34] Again, enough counts were taken with the 1/32-in. scatterer to yield differential cross sections of high statistical accuracy.

[35] These cross sections were measured by the method previously discussed in connection with the P_1P_2 measurements.

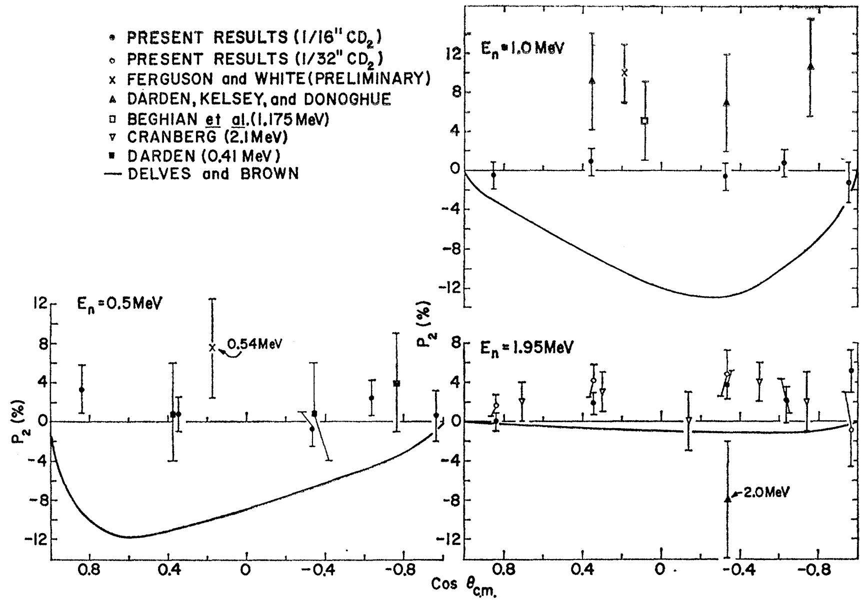

FIG. 6. The polarization $P_2(\theta)$, in percent, plotted as a function of $\cos\theta_{c.m.}$ for *n-d* scattering near 0.5, 1.0, and 1.95 MeV. The points summarize measurements to date near these energies, and the solid curves are the results of the approximate calculation of Delves and Brown (reference 10). Beghian *et al.* (reference 7) give $P_1(30°_{lab})P_2(86°_{c.m.}) = 0.015 \pm 0.010$. Taking $P_1(30°) \approx \frac{1}{4}$ gives $P_2(86°_{c.m.})$ as shown in the figure. The error bars on the present results are based only on counting statistics, which contributed most of the total errors on these points.

error of this size can be associated with our knowledge of the efficiency correction.[36] The uniformity of the detectors is probably known to an accuracy of 0.8% at all three incident energies. From the above discussion, an estimate of systematic effects gives an rms probable error of about 5% at each angle for all energies.

Table II gives the final *n-d* differential cross sections. The errors shown are based on the estimate of a 5% systematic error. The angles shown in the table are the nominal mean angles as determined by the experimental setup. Since the angular resolution is 3–5°, the true mean angles are not expected to differ from these by more than 1.5° at 22° or by more than 0.5° at the other scattering angles.[33]

III. DISCUSSION

A. Polarization

Figure 6 shows the results for $P_2(\theta)$ for *n-d* scattering, taken from columns 9 and 10 in Table I. Shown also are the other experimental determinations of this quantity for similar incident neutron energies. Recently, Darden and Donoghue[37] have measured $P_2(\theta_2)$ for *n-d* scattering for three angles at 0.41 MeV. These results are shown in Fig. 6. It is clear that their results at this energy agree with ours at 0.5 MeV. The measurement by Ferguson and White[6] at 0.54 MeV is not definitely in disagreement with our measurements since the errors on the points overlap. However, those authors have also obtained values of polarizations of about 16% at 0.6 MeV, 13% at 0.75 and 0.87 MeV, and 10% at 1.0 MeV (all at $\theta_{c.m.} = 80°$). These results indicate a trend not apparent from our measurements.[37a] Moreover, their result at 1.0 MeV is not consistent with our values, although it is in agreement with the measurements of Darden *et al.*[4] The recent measurement by Beghian *et al.*[7] at 1.12 MeV is slightly lower than that of Darden *et al.* The errors on the point of Beghian *et al.* overlap those of Darden *et al.* and those of the present results. Our results have smaller errors and are consistent with a polarization of $(0.0 \pm 2.5)\%$ at 0.5 MeV and $(0.0 \pm 2.0)\%$ at 1.0 MeV.

At 1.95 MeV, the values of $P_2(\theta)$ obtained from both the 1/16- and 1/32-in. CD_2 samples are consistent with a percentage of polarization of approximately $+(4.0 \pm 2.0) \sin\theta$. This is in agreement with the results of Cranberg,[3] but in disagreement with a single point by Darden *et al.*[4]

The solid curves in Fig. 6 for all three energies are the theoretical predictions of Delves and Brown.[10,37b] This calculation is approximate and its predictions should be considered as qualitative only. Even so, the data at 0.5 and 1.0 MeV are in disagreement with the curves. The calculation at 1.95 MeV is not inconsistent with

[36] Actually this error is due to the effects of multiple scattering as well as the efficiency correction since the former have not been explicitly taken into account in the *n-p* results.

[37] S. E. Darden and T. R. Donoghue (private communication). The authors wish to thank Dr. Darden and Dr. Donoghue for permission to include in this paper their recent unpublished data.

[37a] *Note added in proof.* Recently we have measured the polarization for neutrons of 0.750 MeV scattered from deuterium at 5 angles. The results are $P_2(36°) = +2.4 \pm 1.8$, $P_2(80°) = +2.1 \pm 2.0$, $P_2(116°) = +2.0 \pm 2.0$, $P_2(143°) = -3.5 \pm 2.4$, and $P_2(165°) = -0.4 \pm 3.1$ where polarizations are in percent and angles in the center-of-mass system.

[37b] *Note added in proof.* Since the writing of this article, some confusion has developed concerning the sign of the polarization in the calculation of Delves and Brown. See L. M. Delves, Nuclear Phys. **33**, 482 (1962). It should be further noted that as a consequence of one of the arguments in this latest development there would also be confusion as to the sign of both P_1 and the measured values $P_2(\theta)$ as shown in Fig. 6.

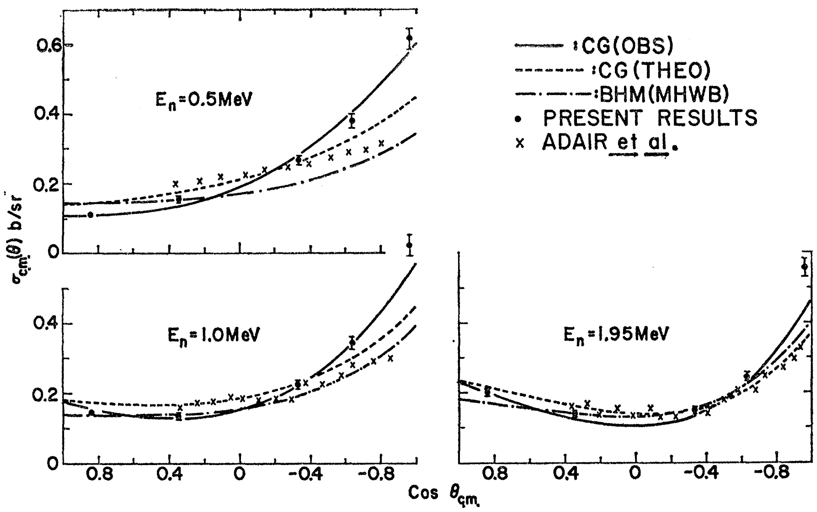

FIG. 7. The differential cross sections in the center-of-mass system as a function of $\cos\theta_{c.m.}$ at $E_n=0.5$, 1.0, and 1.95 MeV. The solid circles are the present measurements. Where no error bars appear, the errors are smaller than the size of the circles. The curves are calculated from phase shifts taken from the results of Buckingham *et al.* (reference 17) and Christian and Gammel (reference 19) and are explained in the text.

the observed polarization in magnitude but appears to have the opposite sign.

As mentioned in the introduction, a reasonable degree of success has been attained in comparing *n-d* differential cross sections with theories involving only central forces for the internucleon interaction. This situation is consistent with the present conclusions of zero or at least very small polarizations, although it has been pointed out by Bose[38] and others that the effect of a spin-orbit coupling (which may produce polarization) should indeed be small in its influence on the unpolarized differential cross section.

Because of the symmetry between *p-d* and *n-d* scattering, it is expected that the polarization of the protons in the *p-d* case should be similar to that of the neutrons in *n-d* scattering. The only measurements are by Shafroth *et al.*[39] near 3.5 MeV. They conclude that the proton polarization should be less than 10% in absolute magnitude. This is in agreement with the *n-d* results of Brüllman *et al.*[2] at 3.27 MeV as well as with the results reported here.

B. Differential Cross Section

The unpolarized differential cross sections from Table II are plotted as a function of $\cos\theta$ as the solid points in Fig. 7. Also shown are the experimental results which Adair *et al.*[11] obtained with a gas recoil counter. These authors estimate that their measurements have an uncertainty of about 15% except near 180° ($\cos\theta=-1$) where the poor resolution of their detector could account for cross sections too low by a somewhat larger value. Near 90° their results are in agreement with the present ones, but our measurements show considerably more peaking at larger angles. Other measurements by Tunnicliffe[12] and by Allen *et al.*[13] (see Fig. 9) from 0.1 to 1.0 MeV also show considerably more anisotropy than the results of Adair *et al.* and are therefore more in agreement with the present experiment.

The curves shown in Fig. 7 were calculated from phase shifts obtained from the work of Buckingham, Hubbard, and Massey[17] (the curves marked BHM in Fig. 7) and of Christian and Gammel[19] [the curves marked CG(obs) and CG(theo) in Fig. 7], the relationship used for the differential cross section in *n-d* scattering being

$$\sigma(\theta)=\frac{1}{k^2}\left\{\frac{2}{3}\left|\frac{1}{2i}\sum_l (2l+1)[\exp(2i\,{}^4\delta_l)-1]\mathcal{P}_l(\cos\theta)\right|^2 +\frac{1}{3}\left|\frac{1}{2i}\sum_l (2l+1)[\exp(2i\,{}^2\delta_l)-1]\mathcal{P}_l(\cos\theta)\right|^2\right\}. \quad (2)$$

In Eq. (2), k is the wave number of the incident neutron, ${}^4\delta_l$ and ${}^2\delta_l$ are the phase shifts for the quartet and doublet spin states, respectively, the $\mathcal{P}_l(\cos\theta)$ are the Legendre polynomials, and l is the orbital angular momentum of the neutron relative to the center of mass of the deuteron. At each of the energies involved in the present experiment the sums in (2) are performed through $l=2$, i.e., they include D-wave neutrons. There are thus six phase shifts—${}^4\delta_0$, ${}^2\delta_0$, ${}^4\delta_1$, ${}^2\delta_1$, ${}^4\delta_2$, and ${}^2\delta_2$. It should be noted that writing Eq. (2) in this way neglects spin-orbit coupling in the *n-d* interaction. The inclusion of such a term would split the above phases into components, each labeled by a value of the total angular momentum J of the system. However, as has been mentioned above, the theories of *n-d* scattering (as described, for example, by Buckingham *et al.* and by Christian and Gammel) include only central forces in the discussion, and therefore only the above six phases will be involved.

The curve marked BHM(MHWB) in Fig. 7 should be considered only as an approximate representation of

[38] A. K. Bose, Z. Naturforsch. **16a**, 95 (1961).
[39] S. M. Shafroth, R. A. Chalmers, E. N. Strait, and R. E. Segel, Phys. Rev. **118**, 1054 (1960).

the theory, especially at energies below 2.0 MeV. The phase shifts used were taken from Table III of reference 17 for the case of a symmetric exchange force (MHWB). The P- and D-wave phases were tabulated only at energies above 2.0 MeV. Values at the lower energies of interest were obtained from these by a smooth extrapolation to zero energy. Those authors, moreover, do not claim high reliability for their theory below 2.0 MeV.[17] Even though the agreement with the measurements is not very good (at energies below 2.0 MeV) it is considerably better than a calculation (not shown) using values of the phase shifts (from Table III, reference 17) for the case of an ordinary type of exchange force. This conclusion is in general agreement with previous results which seemed to indicate that exchange forces of the Serber or symmetric type must be used.[40]

The curve marked CG(obs) in Fig. 7 is based on an analysis of previous p-d and n-d scattering measurements by Christian and Gammel.[19] The S-wave phase shifts used in the calculation were obtained from the curves marked "N-D Experimental" in Fig. 6 of reference 19.[41] The P- and D-wave phases used for the present calculation were derived from the results of a phase-shift analysis of proton-deuteron scattering (Tables II and III, reference 19) by a procedure described by Christian and Gammel and by de Borde and Massey.[18] The values used for the phases ${}^2\delta_1$, ${}^2\delta_2$, and ${}^4\delta_2$ are those calculated in Born approximation with a Yukawa interaction and Serber exchange mixture; the phase ${}^4\delta_1$ differs only slightly from such a value in a similar calculation. The curve marked CG(theo) in Fig. 7 differs from CG(obs) in that the value of the phase shift ${}^4\delta_1$ is also that obtained in a Born-approximation calculation, and the S-wave phases are based on a theoretical calculation.[42]

As Fig. 7 shows, the present measurements are in very good agreement (except perhaps at the largest angle) with the calculated CG(obs) curves at all energies. The S-wave phases used in the calculation are consistent with one of two sets of experimental zero-energy n-d scattering lengths,[43] namely, the quartet length ${}^4a=(6.4\pm0.3)\times10^{-13}$ cm and the doublet length ${}^2a=(0.7\pm0.3)\times10^{-13}$ cm. These experimental values (at least the value of ${}^4a$) are in good agreement, furthermore, with n-d zero-energy scattering lengths derived from the p-d scattering lengths; and the latter in turn were consistent with the p-d phase-shift analysis described above.[19] The generally good agreement between the calculation and the present n-d measurements, therefore, indicates consistency with the results of p-d scattering, as is to be expected on the basis of the symmetry in the two reactions.

It should perhaps be pointed out that the S phases used in the calculation BHM, shown in Fig. 7, lead to zero-energy scattering lengths in poor agreement with either of the two possible sets of experimental values.[8,40] The scattering lengths corresponding to the curve CG(theo), on the other hand, are consistent (at least for ${}^4a$) with the experimental set discussed above.[19] However, the calculation by Christian and Gammel leads to too high a value of the triton binding energy,[8] while the calculations of Buckingham *et al.* are more in agreement.

A phase-shift analysis of the present measurements was carried out with the IBM-704 computer. The two S phases—the phases which can be most easily related to other experimental data (i.e., zero-energy scattering lengths, triton binding energy, etc.)—were held at fixed values while the P- and D-wave phases were varied simultaneously to get a good fit to the experimental points. A least-squares fitting procedure[44] utilizing a gradient-search method was employed with the IBM-704 computer for the phase-shift analysis.

Excellent fits to the experimental angular distributions were obtained for two sets of fixed S phases. The phase shifts giving the best fit are shown as Fit A and Fit B in Table III. The fixed S phases for Fit A correspond to those used in the CG(obs) calculation, while for Fit B the S phases correspond to those used in the BHM(MHWB) calculation. It was not possible to get a fit to the data at 0.5 and 1.0 MeV with fixed S phases corresponding to the CG(theo) calculation, although a good fit was found at 1.95 MeV.

Figure 8(a) compares the measured points with the results obtained by use of the phases from Fit A and Fit B. At 0.5 MeV the two solutions are indistinguishable. At 1.0 and 1.95 MeV, the two sets predict slightly different angular dependences; but an experimental attempt to determine which of the solutions is preferable would be quite difficult. Figure 8(b) shows the energy dependence of the two sets of phase shifts. The solid and dot-dashed curves correspond to the phases CG(obs) and BHM, respectively, which led to the corresponding curves in Fig. 7. The differences between these phase shifts and the final ones can be seen by comparing the curves with the open circles (Fit A) and the triangles (Fit B). The final quartet P- and D-wave phase shifts are practically unchanged from one fit to the other and agree with those for the CG(obs) case. The final doublet P-wave phases are approximately equal in the two cases, and do not vary much from the CG(obs) and BHM curves although the energy dependences are somewhat different. The major difference in the two solutions is reflected in the behavior of the

[40] K. B. Mather and P. Swan, *Nuclear Scattering* (Cambridge University Press, New York, 1958), Chap. 8.

[41] This curve is based on one set of experimental zero-energy scattering lengths, and a phase-shift analysis of 14-MeV n-d scattering.

[42] They are taken from the curve marked "N-D Theoretical" in Fig. 6 of reference 19.

[43] D. G. Hurst and N. Z. Alcock, Can. J. Phys. **29**, 36 (1951); E. O. Wollan, C. G. Shull, and W. C. Koehler, Phys. Rev. **83**, 700 (1951).

[44] E. A. Crosbie and J. E. Monahan, Argonne National Laboratory Report ANL-208, 1959 (unpublished).

TABLE III. Phase shifts (radians) from analysis of n-d angular distributions.

E_n (MeV)	Type	CG(obs) (rad)	CG(theo) (rad)	BHM(MHWB) (rad)	Fit A (rad)	Fit B (rad)
0.5	${}^4\delta_0$	−0.604	−0.620	−0.485	−0.604	−0.485
	${}^2\delta_0$	−0.086	−0.177	−0.415	−0.086	−0.415
	${}^4\delta_1$	0.15	0.09	0.065	0.1311±0.0007	0.172 ±0.001
	${}^2\delta_1$	−0.04	−0.04	−0.02	−0.093 ±0.002	−0.064 ±0.001
	${}^4\delta_2$	−0.008	−0.008	−0.014	−0.0130±0.0005	−0.0177±0.0009
	${}^2\delta_2$	0.004	0.004	0.0045	0.062 ±0.001	0.052 ±0.001
1.0	${}^4\delta_0$	−0.808	−0.884	−0.685	−0.808	−0.685
	${}^2\delta_0$	−0.127	−0.285	−0.58	−0.127	−0.58
	${}^4\delta_1$	0.25	0.17	0.135	0.245 ±0.003	0.248 ±0.006
	${}^2\delta_1$	−0.07	−0.07	−0.04	−0.026 ±0.008	−0.034 ±0.009
	${}^4\delta_2$	−0.0185	−0.0185	−0.027	−0.034 ±0.001	−0.042 ±0.004
	${}^2\delta_2$	0.009	0.009	0.0095	0.114 ±0.007	0.032 ±0.009
1.95	${}^4\delta_0$	−1.027	−1.181	−0.955	−1.027	−0.955
	${}^2\delta_0$	−0.197	−0.431	−0.81	−0.197	−0.81
	${}^4\delta_1$	0.37	0.295	0.27	0.371 ±0.016	0.334 ±0.017
	${}^2\delta_1$	−0.11	−0.11	−0.09	−0.009 ±0.038	−0.079 ±0.033
	${}^4\delta_2$	−0.046	−0.046	−0.0515	−0.073 ±0.010	−0.095 ±0.011
	${}^2\delta_2$	0.025	0.025	0.02	0.235 ±0.031	0.021 ±0.029

doublet D-wave phases (and the associated fixed S-wave phases). For set A, a good fit to the data implies values of ${}^2\delta_2$ which are large and not at all in agreement with those given by CG(obs) or BHM(MHWB). For set B, the values of ${}^2\delta_2$ are approximately equal to the values predicted by the two curves CG(obs) and BHM, although the energy dependence is somewhat different.

Nothing is said here of the uniqueness of the solutions A and B. There are perhaps other sets of phases that will result in equally good fits. We should point out, however, that both solutions (A and B) had good local minima. That is, the P- and D-wave phases always converged to the same final solution even when a number of different starting values for these phases were chosen.

Attempts to fit the data by a phase-shift analysis with only S- and P-wave phases were unsuccessful. It is necessary to include D waves to get a fit. This is obvious from Fig. 9, in which the coefficients B_l in the expansion of the differential cross section into a series of Legendre polynomials are plotted as a function of incident energy. As can be seen, the experimental angular distributions yield nonzero values of B_3 and B_4 at each energy. This can only occur if D-wave neutrons contribute to the interaction. The coefficient B_0 in Fig. 9 is equal to the total cross section divided by 4π. The solid curve shown corresponds to the known neutron total cross section for deuterium.[29] As mentioned previously, the agreement with the present results is excellent. Shown also in Fig. 9 are the B_l for the work of

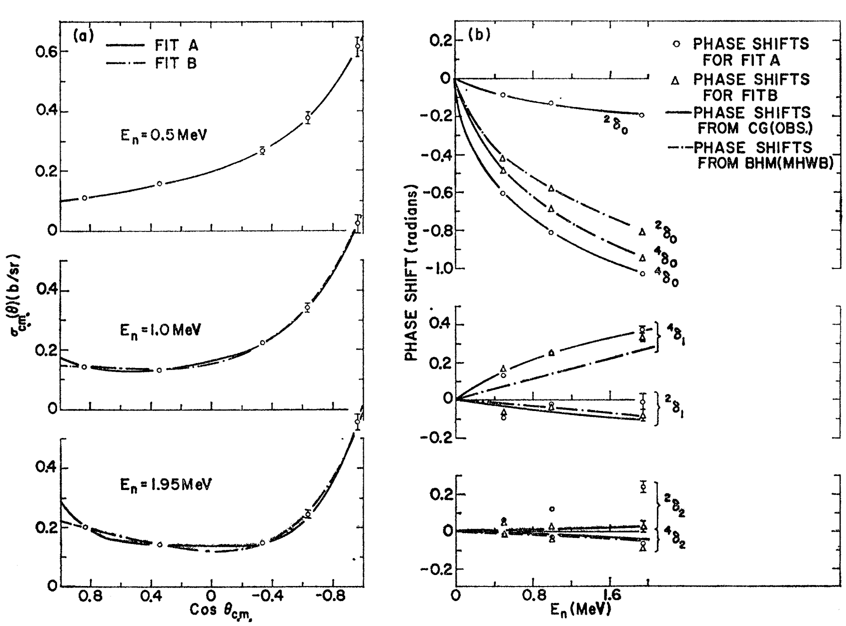

FIG. 8. (a) The measured differential cross sections compared with the results of the phase-shift analysis. The open circles correspond to the measured differential cross sections. The curves were calculated according to Eq. (2) from the phase shifts corresponding to Fit A and Fit B in Table III. At 0.5 MeV the curves corresponding to the two sets of phases are indistinguishable. (b) The values of the phase shifts of various order as a function of energy. The open circles correspond to the phases from Fit A and the triangles to those from Fit B. The curves show the energy dependence of the phase shifts corresponding to the curves labeled similarly in Fig. 7. The dot-dashed and the solid curves are indistinguishable for the doublet D phase ${}^2\delta_2$.

Allen *et al.*[13] and for that of Seagrave and Cranberg.[14] For the most part these results fit rather well with the expected extrapolations of our data.

SUMMARY

The present experimental results show zero polarizations at 0.5 and 1.0 MeV in *n-d* elastic scattering, and a small positive value of 4% (maximum) at 1.95 MeV. Except for the result at 1.95 MeV, the present measurements do not agree with the calculation of Delves and Brown, and even at 1.95 MeV the predictions are opposite in sign to the measured polarizations. As pointed out, the numerical evaluation of a more accurate formulation of the *n-d* scattering problem including tensor forces has not as yet been carried out.

The angular distribution measurements, to which we have assigned probable errors of 5% (at most), are consistent with at least two sets of *S*-wave phase shifts. One set corresponds to the theoretical phases of Buckingham *et al.* (marked BHM on Figs. 7 and 8). The other set, leading to the curves marked CG(obs) in Figs. 7 and 8, are consistent with the experimentally observed zero-energy scattering lengths, ${}^4a=(6.4\pm0.3)\times10^{-13}$ cm and ${}^2a=(0.7\pm0.3)\times10^{-13}$ cm. The higher-order phases (*P*- and *D*-wave phases), corresponding to this latter set of *S*-wave phases, are consistent with the results of a phase-shift analysis of proton-deuteron angular distributions except for the doublet *D*-wave phase shift. At each of the three incident energies the present results predict values of ${}^2\delta_2$ larger than those in the *p-d* case, and therefore larger than predicted in a Born-approximation calculation. Theoretical calculations leading to these large values, as well as predictions of the doublet *S*-wave phase shifts consistent with the observed doublet zero-energy scattering length, are not available.

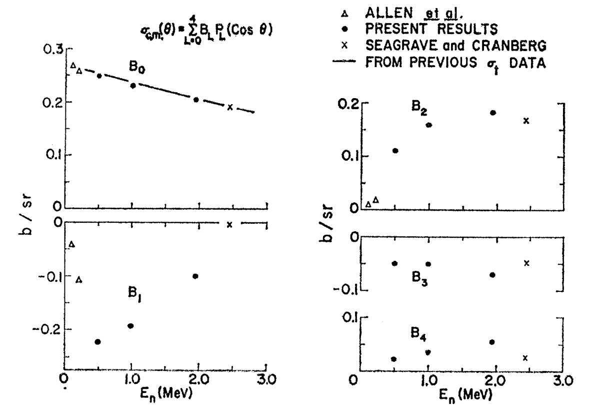

FIG. 9. The values B_L in the center-of-mass system from the expansion of the differential cross section into a series of Legendre polynomials according to $\sigma_{c.m.}(\theta)=\sum_{L=0}^{4} B_L\mathcal{P}_l(\cos\theta)$. The solid curve in the B_0 plot is from reference 29. The **×**'s correspond to an expansion of the cross sections of Seagrave and Cranberg (reference 14), and the △'s correspond to an expansion of those of Allen *et al.* (reference 13).

ACKNOWLEDGMENTS

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