Tunneling through Thin Films with Traps

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Tunneling currents through barriers containing one-dimensional potential wells with effective cross-sectional areas are calculated by using the WKB connection formulas to extend the wave functions of the incident free electrons to the other side of the barrier. The lifetime of electrons in the potential wells is assumed to limit the rate at which electrons may tunnel through the barrier. It is proposed that the qualitative features of this calculation may be applicable to tunneling currents through metal-insulator-metal thin-film sandwiches when the insulating film contains defects capable of trapping electrons. It is found that if the quasi-stable levels of the potential well lie near the Fermi level of the metal, for small voltages applied across the insulator, then the current may be greatly increased by the presence of these potential wells and most of the current will flow at energies near the quasi-level. A sample calculation was done to demonstrate these features for a square well in a rectangular barrier. A possible extension of this model allows the current to be proportional to the product of the densities of states in the metals on both sides of the barrier.

I. INTRODUCTION

RECENTLY Fisher and Giaever[1] compared their experimental results of tunneling currents through metal-aluminum oxide-metal thin-film sandwiches with Holm's[2] theoretical calculations of tunneling currents through single potential barriers. The current-voltage characteristics, the dependence of the current on the barrier thickness, and the weak temperature dependence were concluded to be in agreement with the theory. However, the experimental current values were found to be much larger than the theory predicted. By using an effective electron mass in the oxide equal to 1/9 the free-electron mass, the experimental current values were brought into agreement with the theory. Giaever and Megerle[3] also found that the tunneling current was proportional to the product of the densities of states of the metals on both sides of the insulating oxide film. This last result was obtained on samples which had either one or both metals in the superconducting state.

In this paper the tunneling current through a one-dimensional barrier containing a potential well is computed and compared with the tunneling current through a single barrier, which contains no potential well. A number of investigators[2,4–6] have made theoretical studies of tunneling currents through thin films by representing the thin film by a single one-dimensional potential barrier. Sommerfeld-Bethe and Holm calculated the current through a thin film by assuming that the tunneling current from metal i was proportional to the transmission coefficient of the barrier multiplied by the number of electrons which were available at the barrier in metal i. The total current was then the difference between the currents in both directions through the film. Bardeen found the current by calculating the transition rate from an eigenstate describing an electron on one side of the barrier to an eigenstate describing this electron on the other side of the barrier. It was assumed that the perturbation of the eigenstates caused by the barrier was such that the wave functions were confined mostly to one side of the barrier and the barrier region. In the case to be discussed the presence of potential wells in the barrier gives rise to large transmission coefficients, which allow the wave functions to spread out appreciably to both sides of the barrier. Consequently, the approach used by Sommerfeld-Bethe and Holm is more suitable and will be used in this paper to compute the tunneling current.

In Sec. II of this paper, the general formulation of the tunneling current is shown and the current through a single barrier is computed. In Sec. III, a model for the potential well is introduced and the tunneling current through a barrier containing this potential well is calculated. A comparison of the results of Secs. II and III is made in Sec. IV showing that traps may increase the tunneling current. Possible extensions of this model are considered in Sec. V. The effect of energy changes suffered by the electron has not been considered in this investigation.

II. TUNNELING CURRENT THROUGH BARRIERS WITHOUT TRAPS

A. General Formulation of Tunneling Current

The general form of the Sommerfeld and Bethe approach is now presented. The current flow from metal i to metal j is assumed to be

$$J_{ij}=e\int T(E_x)N_i(E_x)dE_x, \tag{1}$$

where $T(E_x)$ is the transmission coefficient for the energy $E_x=p_x^2/2m$ associated with the electron momentum p_x in the x direction; m and e are the electron

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[1] J. C. Fisher and I. Giaever, J. Appl. Phys. **32**, 172 (1961).
[2] R. Holm, J. Appl. Phys. **22**, 569 (1951).
[3] I. Giaever and K. Megerle, Phys. Rev. **122**, 1101 (1961).
[4] J. Bardeen, Phys. Rev. Letters **6**, 57 (1961).
[5] W. A. Harrison, Phys. Rev. **123**, 85 (1961).
[6] A. Sommerfeld and H. Bethe, in *Handbuch der Physik*, edited by von Geiger and Schied (Verlag Julius Springer, Berlin, 1933), 2nd ed., Vol. 24, Part 2, p. 450.

mass and charge, and $N_i(E_x)dE_x$, the supply function, represents the number of electrons in the energy range (E_x, E_x+dE_x) which are available for tunneling through the barrier. The supply function is set equal to the number of electrons which strike unit area of the barrier per second from the free-electron gas in the metal. This is found to be

$$N_i(E_x)dE_x = v_x dp_x \int f_i g_i dp_y dp_z, \tag{2}$$

where f_i is the occupation probability of a state in metal i, g_i is the density of states in momentum space per unit volume in metal i, and v_x is the electron velocity in the x direction. Using $v_x = dE_x/dp_x$, we find the difference between the currents in either direction to be

$$J = e\int T(E_x)dE_x \int g(f_1 - f_2)dp_y dp_z. \tag{3}$$

If both metals are assumed to be identical free electron metals and if we assume that the applied voltage is small, then the current becomes

$$J \approx (4\pi me^2V/h^3) \times \int T(E_x)\{1+\exp[(E_x-\eta)/kT]\}^{-1}dE_x, \tag{4}$$

where η is the Fermi level and V is the applied voltage. We may also assume that the transmission coefficient does not change appreciably with voltage and that the only effect of a small applied voltage is to shift the Fermi levels of the two metals. The current will then be proportional to the voltage for small voltages.

B. Tunneling Current through Rectangular Barriers

In this section Eq. (4) is used to compute the tunneling current through a rectangular barrier. Two different methods, the WKB method and the matching of wave functions at the classical turning points, were used to find the transmission coefficient of this barrier. The technique of matching wave functions is more appropriate for computing free-electron transmission through rectangular barriers than the WKB method, but a comparison of the resulting currents indicates that the WKB method is a reasonably good approximation. The transmission coefficient found by the WKB method is

$$T(E_x) = \exp[-2aW(V_0 - E_x)^{1/2}], \tag{5}$$

where a is the barrier thickness, V_0 is the barrier height, $W = (2m)^{1/2}/\hbar$, and m is the electron mass. Using Eq. (4) and setting the temperature to 0°K, we find the following approximate value of the current:

$$J \approx [e^2V(2m\phi)^{1/2}/ah^2]\exp(-2aW\phi^{1/2}), \tag{6}$$

where $\phi = V_0 - \eta$. Equation (6) is the result obtained by Sommerfeld and Bethe for small voltages and rectangular barriers. It is also found to first order in $(3kT/\phi)$ that the current through a rectangular barrier is independent of temperature.

III. TUNNELING CURRENT THROUGH BARRIERS WITH TRAPS

A. General Formulation of Tunneling Current

The current from metal i to metal j through a barrier with traps is set equal to Eq. (1). However, the presence of traps changes not only the transmission coefficient but also the supply function. New expressions for the transmission coefficient and the supply function are found in this section. As before, the total current is the difference between the currents in either direction.

Let us consider what happens to the electronic wave functions when there is a trap in the barrier. The trapped electron's wave function is large in the well and decays in an exponential manner away from the well in the classically inaccessible regions of the barrier. The amplitude of this wave function in the region outside the barrier depends on the "tunneling path length" to the potential well; that is, those classically accessible regions nearer the trap will have the larger wave function amplitudes. Using this general picture, we find the wave function of an electron with energy equal to the trapping level, the quasi-level of the potential well, to behave somewhat like this: The incoming wave strikes the barrier, grows exponentially as it approaches the trap, resonates with a large amplitude inside the well, decays exponentially toward the other side of the barrier, and then travels away from the barrier. By moving the potential well to the middle of the barrier the amplitude of the incoming wave can be made equal to that of the outgoing wave. In such a manner traps can introduce large transmission coefficients. The resonance of the wave corresponds to the electron being trapped for some time in the potential well. For electron energies different than the trapping level, the wave function will decay exponentially in the usual way in passing through the barrier. This general behavior is the same for a three- or a one-dimensional trap.

For simplicity, the three-dimensional potential well will be replaced by a one-dimensional well. The resulting treatment is then very similar to Bohm's discussion of the metastable states of nuclei which uses a one-dimensional potential well for the nucleus.[7] It should be emphasized that this approximation is an oversimplification of the actual situation. A more realistic calculation would use not only a three-dimensional model, but also a more appropriate method of finding the wave function in the barrier region. The WKB approximation

[7] D. Bohm, *Quantum Theory* (Prentice-Hall, Inc., New York, 1951), pp. 283–295.

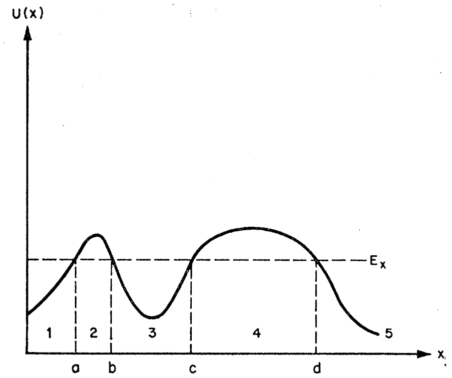

FIG. 1. Potential energy barrier with trap. The classical turning points for energy E_x are at a, b, c, and d.

is not really valid since the wavelength of the electron is the same order of magnitude as the dimensions of the trap. The possibility of additional reactions taking place, such as thermal ionization of the trapped electron, has been completely neglected in this simple treatment. However, it is hoped that the qualitative behavior of this model will provide some insight into the actual case.

For purposes of computation the total barrier will be assumed to be composed of sections with and without traps. The total current through this composite barrier is then the sum of the current densities through the average tunneling barrier of each region multiplied by that region's effective cross-sectional area.

In the regions without traps the transmission coefficient is

$$T(E_x)=\exp\left(-2\int_a^d |k'|dx\right), \tag{7}$$

where $k'\hbar=\{2m[E_x-U'(x)]\}^{1/2}$ and $U'(x)$ is the average potential barrier. For a free-electron metal, the supply function given in Sec. II becomes

$$N_i(E_x)dE_x=(4\pi mkT/h^3)dE_x \ln \times\{1+\exp[(\eta_i-E_x)/kT]\}. \tag{8}$$

For small voltages, we may use Eq. (4) to find the current using the transmission coefficient given by Eq. (7).

In the regions with traps the average barrier is assumed to be that shown in Fig. 1. To replace the actual potential barrier by the one shown in Fig. 1 is equivalent to assuming that the "effective mass" approximation is valid. If the width of the well is much larger than the crystal lattice spacing such that the quasi-stable state allows the wave function to spread over many lattice sites, then the use of the "effective mass" approximation is probably justified. The transmission coefficient of this barrier is found in Appendix A;

$$T^t(E_x)=4\{[S(A^2+B^2)/AB]^2+(4ABC)^2\}^{-1}. \tag{9}$$

In order to find the supply function, we must consider the lifetime of the trap. Since the supply function represents the number of electrons which are available for tunneling through the barrier per unit energy range per second, it is easy to see that if the electron is delayed in crossing the barrier then the supply function will decrease. The potential well not only delays the electron in crossing the barrier but also limits the number of electrons which can pass through the well at any given time. If the well is occupied, then no other electrons can enter it until the well becomes empty again. For example, a negative-ion vacancy in an alkali halide crystal can trap either one electron (F center) or two electrons (F' center). However, if an F' center is formed, the negative-ion vacancy can not trap another electron until it loses one of its trapped electrons. In this example, the effective number of electrons which could pass through the trap is somewhere between one and two.

There are then two mechanisms which influence the supply function, the supply of electrons from the metal up to the barrier and the delay in crossing the barrier caused by the lifetime of the trap. Only the former mechanism has been considered in the previous derivations. If the supply of electrons from the metal were infinite, the current would be limited only by the trap lifetime. In such a case the rate at which electrons are able to tunnel through a trap is assumed to be the effective number of electrons which can cross the barrier through one trap divided by the delay time in crossing the barrier and by the cross sectional area of the trap,

$$N_i(E_x,\tau)dE_x=(\alpha P_i/\tau\sigma)dE_x. \tag{10}$$

The probability for an electron which is able to tunnel to have an energy in the range (E_x, E_x+dE_x) is P_idE_x, the effective cross sectional area of the trap is σ, the effective number of electrons which can pass through the same trap at the same time is α, and the delay time of these electrons as they pass through the trap is τ. This delay time is computed in Appendix B. For a free-electron metal the supply function limited only by the delay time in the trap is found to be

$$N_i(E_x,\tau)dE_x=(1/\tau\beta_i)N_i(E_x)dE_x, \tag{11}$$

where $\beta_i=[32\pi m\sigma(E_xE_{Fi})^{1/2}]/3\alpha h^3$ and E_{Fi} is the height of the Fermi level above the bottom of the conduction band in metal i. Now the conductivity is proportional to the supply function multiplied by the transmission coefficient. Since the conductivity limited by traps in the barrier can be thought of as being in series with the conductivity limited by the supply of electrons from the metal to the barrier, we may write the effective supply function as

$$N_i(\text{eff})=N_i(E_x)N_i(E_x,\tau)/[N_i(E_x)+N_i(E_x,\tau)]. \tag{12}$$

If the metals on both sides of the barrier are identical, then the difference between the currents in either direction through one trap of cross section σ is

$$J^t(1)=(\sigma/\alpha)\int KT^t(E_x)dE_x/(1+\beta\tau), \quad (13)$$

here $K=(4\pi m\sigma e^2V/h^3)\{1+\exp[(E_x-\eta)/kT]\}^{-1}$. When $N_i(E_x)=N_i(E_x,\tau)$ the delay time will be denoted by $\tau^*=1/\beta_i$. For $\tau>\tau^*$ the supply function is mainly limited by the trap lifetime. For $\tau<\tau^*$ the supply function is mainly limited by the supply of electrons from the metal to the barrier.

The total current density through a barrier with traps will depend upon the distribution as well as the concentration of traps. If the concentration of traps is not too large, then we may write

$$J_{\text{tot}}\approx J+\int C(x)J^t(1)dx, \quad (14)$$

where $C(x)$ is the number of traps per unit area of the barrier in the range $(x, x+dx)$ in the barrier and J is the current density through the barrier in the regions without traps.

In contrast to the temperature independence of tunneling currents through rectangular barriers, there are two possible ways in which tunneling currents through traps may be temperature dependent: (1) Since the concentration of potential wells or ionized traps may be temperature dependent, the tunneling current in Eq. (14) may also be temperature dependent. (2) In a later section it is shown that if the quasi-levels of the traps lie near the Fermi level then the majority of the current may be due to electrons with energies around these quasi-levels. Since the distribution of electrons around the Fermi level is temperature dependent, the tunneling current resulting from these electrons may also be temperature dependent.

B. Total Tunneling Current through a Trap

An approximate expression for the total tunneling current through one trap is found. As derived in Sec. IV and demonstrated in Fig. 2, the current per unit energy interval is much larger for energies near the quasi-levels than for energies away from the quasi-levels. Consequently, the majority of the current will flow at the nth quasi-level energy if the nth quasi-level is near the Fermi level. The total amount of current which flows through such a trap will then be approximately the maximum current per unit energy interval multiplied by the half maximum width of the current peak ΔE_n,

$$J^t(1,n)\approx 4KA^2B^2\Delta E_n\left[(A^2+B^2)^2+4A^2B^2\beta(A^2+B^2)\int_b^c dx/v\right]^{-1}, \quad (15)$$

where $v=\{2[E_x-U(x)]/m\}^{1/2}$. If $\beta\int_a^b dx/|v|$, $\beta\int_b^c dx/v$, and $(A^2+B^2)/AB$ are all small compared to $(AB)^{1/2}$, then an approximate expression for the half maximum width is

$$\Delta E_n\approx(\hbar/AB)[\beta(A^2+B^2)]^{1/2}\left[\int_b^c dx/v\right]^{-1/2}. \quad (16)$$

The right-hand sides of Eqs. (15) and (16) are to be evaluated at the nth quasi-level energy. The resulting total current through one trap is then

$$J^t(1,n)\approx(\hbar\beta K/AB)\left[(A^2+B^2)\left(\beta\int_b^c dx/v\right)^3\right]^{-1/2}. \quad (17)$$

IV. COMPARISON OF TUNNELING CURRENTS THROUGH BARRIERS WITH AND WITHOUT TRAPS

A. Current per Unit Energy Interval

To evaluate the effect of traps on the current, we shall compare the integrands of Eqs. (13) and (4). The transmission coefficient in Eq. (4) is set equal to the value given in Eq. (7) for barriers with no traps. One finds

$$dJ^t(1)/dJ_\sigma=T^t(E_x)(1+\beta\tau)^{-1}\exp\left[2\int_a^d|k'|dx\right], \quad (18)$$

where dJ_σ is the current per unit energy interval through a barrier of cross-sectional area σ which contains no traps. If

$$\exp\left[2\int_a^d|k'|dx-\int_b^c|k'|dx\right]\geq AB, \quad (19)$$

then we find for $\tau<\tau^*$ that

$$dJ^t(1)/dJ_\sigma\geq(1/4)\exp\left[2\int_b^c|k'|dx\right]. \quad (20)$$

The current through traps is then larger than the current through barriers without traps for $\tau<\tau^*$. The factor (1/4) results from the different methods used to compute the transmission coefficients. If the transmission coefficient used in Eq. (4) is set equal to the transmission coefficient given in Appendix A for zero size traps, then the factor becomes unity. Equation (19) assumes that the barrier height does not increase when a trap is added. This need not be the case. The positive and negative changes in the work function of metals when impurities are added to the surface indicate that the barrier can be increased as well as decreased.[8]

When $\tau>\tau^*$, we find using Eq. (19) that

$$dJ^t(1)/dJ_\sigma\geq(T^t(E_x)/\beta\tau)AB\exp\left[\int_b^c|k'|dx\right]. \quad (21)$$

[8] L. P. Smith, *Handbook of Physics*, edited by E. U. Condon and H. Odishaw (McGraw-Hill Book Company, New York, 1958), pp. 8–80.

In order to estimate the value of this expression, we shall assume that the trap is a square well such that

$$\int_b^c dx/v = (c-b)(m/2E_x)^{1/2} = t_0, \qquad (22)$$

and that $\Omega = \sigma(c-b)$, $k_F = WE_F^{1/2}$. Equation (21) then becomes

$$dJ^t(1)/dJ_\sigma \geq \frac{(3\alpha\pi^2/\Omega k_F^3)A^2B^2 \exp\left[2\int_b^c |k'|dx\right]}{A^2+B^2+(1/t_0)\left[A^2\int_c^d dx/|v| + B^2\int_a^b dx/|v|\right]}. \qquad (23)$$

The term t_0 is the classical time for the electron to travel across the trap. The other two integrals in the denominator of Eq. (23) are the classical times for an electron to travel across the potential wells which would result if the potential barrier were turned upside down. These transit times are probably all of the same order of magnitude so that the denominator of Eq. (23) is approximately of magnitude (A^2+B^2). If we let $\Omega = 20$ Å^3 and $E_F = 5$ eV, then the front coefficient of Eq. (23) containing these terms is the order of 0.1. However, since the term $A^2B^2/(A^2+B^2)$ is sufficiently large to dominate, we see that the current is enhanced by the presence of traps for $\tau > \tau^*$. As we may have anticipated, the current through a barrier with a trap is enhanced at least by the factor resulting from a reduction in the tunneling path length. When the energy of the electron is near the quasi-levels of the trap, the current is enhanced much more than for energies away from the quasi-levels.

A sample calculation was done for a rectangular barrier with a square well in its center. The height of the barrier, as measured from the bottom of the conduction band of the metal, was 10 eV, the Fermi level was 9 eV, the total tunneling path was 40 Å, the trap width was 3 Å, the bottom of the trap well was 0.4 eV below the bottom of the conduction band, the trap cross section was 9 Å^2, and the effective number α was 1. There were two quasi-levels in the trap, at 0.64 and at 8.94 eV. The current through one trap per unit energy interval was plotted vs the electron energy as curve A in Fig. 2. The width of the current peak at the lower quasi-level is 3.2×10^{-14} eV and the width of the current peak at the upper quasi-level is 1.3×10^{-4} eV. The current per unit energy interval through a rectangular barrier of height 10 eV, cross-sectional area 9 Å^2, and tunneling path 43 Å was plotted as curve B in Fig. 2 for comparison. For this example the current through a barrier with traps is increased by about $S\times10^5$, where S is the effective area of the traps per unit area of the barrier, $S = C\sigma/$(area of the barrier). The number of traps per unit area of the barrier is C.

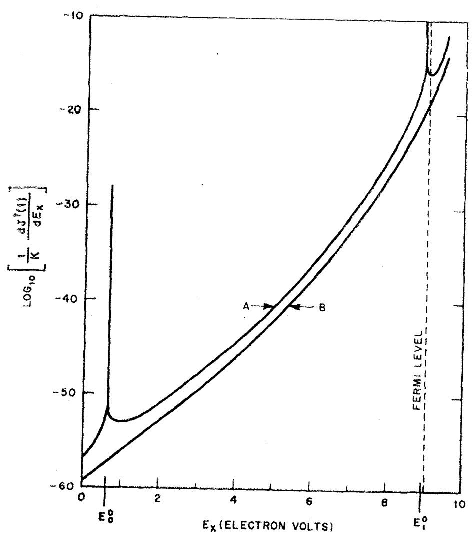

FIG. 2. Tunneling current per unit energy interval of tunneling electrons. Curve A—rectangular barrier with square well trap, total barrier thickness = 43 Å. Curve B—rectangular barrier without trap, total barrier thickness = 43 Å.

There are usually high densities of defects in thin films so that a value of $S = 10^{-3}$ is not unreasonable. This value for S produces a trap increased current, which is one hundred times larger than the current through a rectangular barrier of the same total width.

B. Total Current

A comparison between the total current through one trap and the total current through a rectangular barrier without a trap but of the same size indicates that the current is greatly increased if the quasi-level of the trap is near the Fermi level of the metal. The experimental data of Fisher and Giaever is considered with respect to this model and it is shown that their estimated effective mass needed for theoretical agreement is probably too small.

A comparison of the total current through a rectangular barrier containing a square well trap in the center, from Eq. (17), with the total current through a rectangular barrier of the same tunneling path length and cross section, from Eq. (6), gives

$$J^t(1,n)/J_\sigma = G\exp\{2aW[(V_0-E_F)^{1/2} - (3/4)(V_0-E_n)^{1/2}]\}, \qquad (24)$$

where E_n is the nth quasi-level energy and

$$G = (2E_F/\phi)^{\frac{1}{2}}(\sigma a/\alpha\pi^2)W^3E_FE_n^{1/2}\left[\beta\int_b^c dx/v\right]^{-3/2}.$$

When the quasi-level is near the Fermi level of the metal, $E_n \approx E_F$, the trap will increase the total current through the barrier by a substantial amount. In such a case the current density through a barrier with traps becomes approximately

$$J^t(n) \approx (SGVe^2/ah^2)(2m\phi)^{1/2} \exp[-(3a/2)W\phi^{1/2}], \quad (25)$$

where S is the effective area of the traps per unit area of the barrier. The term SG is usually not too small so that by comparison with Eq. (6) we see that the current will be approximately the same as a current through a rectangular barrier of width $(3a/4)$.

Fisher and Giaever could bring their data into agreement with the theoretical tunneling currents through rectangular barriers by using an effective mass equal to $(1/9)m$. These same data can therefore be brought into agreement with Eq. (25) by using an effective mass equal to $\frac{1}{5}m$. By considering the capacitance measurements used by Fisher and Giaever to determine the barrier thickness, the value of the effective mass needed for agreement with theory may be further increased. The capacitance measurements will give some average of the barrier thickness which is larger than the minimum thickness. But, since the current is exponentially dependent on thickness, the minimum thickness will be the barrier thickness for most of the current. Consequently, the barrier thickness used by Fisher and Giaever probably was too large and their estimate of the effective mass was probably too small.

V. POSSIBLE EXTENSIONS

A. Current Proportional to the Density of States

If the quasi-levels of the traps are near the Fermi levels of the metals and if the majority of the current does result from the current peaks around the quasi-levels, that is the supply function is limited by the trap life time, then one may expect the current to be proportional to the product of the density of states in the two metals. This result can be contructed by using the supply function

$$N_i(p_x,\tau)dp_x = Q(dp_x/\tau)\int f_i(1-f_j)\rho_i\rho_j dp_y dp_z, \quad (26)$$

where Q is a constant and ρ_i and ρ_j are the densities of states in the two metals on either side of the barrier. The current resulting from a current peak through one trap is then

$$J^t(1,n) \approx Q[T^t(p_x)\Delta p_x/\tau]_{E_n}\int \rho_1\rho_2(f_1-f_2)dp_y dp_z. \quad (27)$$

In terms of the transition rate viewpoint of Bardeen,[4] the transition rate is limited by the lifetime of the trap and not the electron velocity. As Harrison[5] showed, when the transition rate is limited by the electron velocity, the reciprocal relationship between the density of states and the velocity removes the current dependence on the density of states. In the case of traps, the transition rate may be set equal to a constant over the width of the sharp current peak around the quasi-level. This current is then proportional to the product of the densities of states. Since the quasi-levels near the Fermi level may be broadened in an actual sample, either by variations in the barrier shape or by phonon interactions with the trap, this proportionality between current and the density of states may hold over a range of applied voltages corresponding to the broadening of the quasi-levels in the sample. If the density of states does not change very much within this short range, then one may not see any structure in the current-voltage characteristic. This result is in agreement with the experiments of Giaever and Megerle.[3] The traps in the aluminum oxide films used by Giaever may have been either impurity ions from the metal or oxygen-ion vacancies.

B. Photoconductivity

The presence of traps in the barrier region could permit the tunneling process to be sensitive to light. By irradiating the barrier with light capable of ionizing the traps the lifetime of electrons in the traps could be reduced and the number of traps effective in the tunneling process could be increased. An increase in current would then result from irradiating the tunneling barrier.

VI. CONCLUSIONS

The model examined in this paper suggests that traps in thin insulating films may increase the tunneling current through these films and that this tunneling current may be proportional to the product of the density of states in the metals over a limited electron energy range. The current increase is greatest when the trapping level is near the Fermi level and when the traps are near the center of the barrier if the applied voltage is small. In this case the majority of the electrons will tunnel through the barrier at the trapping level energy.

It is felt, due to the oversimplified nature of this model and the experimental difficulties involved in determining the necessary parameters, that any correlation between the results of this model and experimental data may be fortuitous. However, in view of the large order of magnitude of the "resonance" effect in the model, it is possible that an actual trap in a thin film may display this type of behavior. It is hoped that this paper may provide some insight into the influence of traps in the thin insulating films used for tunneling experiments.

APPENDIX A. TRANSMISSION COEFFICIENT OF BARRIER WITH POTENTIAL WELL

The WKB method is applied to the barrier shown in Fig. 1. The classical turning points for an electron of energy $E_x = p_x^2/2m$ are a, b, c, and d. Let the wave function in region 1 be that for a running wave traveling to $-\infty$,

$$\psi_1 = p^{-1/2} \exp\left[i \int_x^a k dx - i\pi/4 \right]. \quad \text{(A1)}$$

Using the WKB connection formulas this wave function is extended into region 5,

$$\psi_5 = (1/2p^{1/2}) \left\{ F \exp\left[i \int_d^x k dx - i\pi/4 \right] + H \exp\left[-i \int_d^x k dx + i\pi/4 \right] \right\}, \quad \text{(A2)}$$

where

$$F = S(B^2 - A^2)/AB - iC(4AB + 1/4AB),$$

$$H = S(A^2 + B^2)/AB - iC(4AB + 1/4AB),$$

$$A = \exp\left[\int_a^b |k| dx \right], \quad B = \exp\left[\int_c^d |k| dx \right],$$

$$C = \cos\left[\int_b^c k dx \right], \quad S = \sin\left[\int_b^c k dx \right],$$

$$\hbar k = \{2m[E_x - U(x)]\}^{1/2}.$$

The resulting transmission coefficient is

$$T^t(E_x) = 4/|H|^2. \quad \text{(A3)}$$

APPENDIX B. TRAP LIFETIME

To find the lifetime of the trap, wave packets are constructed from the wave functions in regions 1 and 5. The time lag of this packet in traversing the barrier is then equal to the lifetime of the trap.[7] The incident wave packet is

$$\psi_i = \int f(E - E_0) p^{-1/2} dE \times \exp\left[-i \int_d^x k dx + i\pi/4 - iEt/\hbar \right], \quad \text{(B1)}$$

and the transmitted wave packet is

$$\psi_t = \int f(E - E_0) p^{-1/2} dER \times \exp\left[i \int_x^a k dx - i\pi/4 + i\phi - iEt/\hbar \right], \quad \text{(B2)}$$

where $Re^{i\phi} = 2/H$,

$$\tan\phi = [4A^2B^2/(A^2 + B^2)] \tan\left(\pi/2 - \int_b^c k dx \right).$$

The center of the wave packet is found where the phase has an extremum. The time behavior of the center of the incident wave packet is

$$t = \int_x^d dx/v, \quad \text{(B3)}$$

and the transmitted wave packet gives

$$t = \int_x^a dx/v + \hbar \partial\phi/\partial E. \quad \text{(B4)}$$

We see that the delay time in traveling from d to a is $\hbar \partial\phi/\partial E = \tau$. This delay time τ is found to be

$$\tau = [T^t(E_x)/4] \left\{ 4(A^2 + B^2) \times \int_b^c dx/v - 4\left[A^2 \int_c^d dx/|v| + B^2 \times \int_a^b dx/|v| \right] \sin\left[2 \int_b^c k dx \right] \right\}. \quad \text{(B5)}$$

The delay time is equal to the lifetime of electrons trapped in the potential well in the barrier. We see that at certain energies quasi-stable states are formed. This is similar to the metastable states of certain nuclei. We also see that τ in Eq. (B5) can be negative. This does not affect the results of this paper since τ is always positive around the quasi-levels where it is used.